

Renormalization Group Running of the Neutrino Mass Operator in Extra Dimensions

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Abstract

We study the renormalization group (RG) running of the neutrino masses and the leptonic mixing parameters in two different extra-dimensional models, namely, the Universal Extra Dimensions (UED) model and a model, where the Standard Model (SM) bosons probe an extra dimension and the SM fermions are confined to a four-dimensional brane. In particular, we derive the beta function for the neutrino mass operator in the UED model. We also rederive the beta function for the charged-lepton Yukawa coupling, and confirm some of the existing results in the literature. The generic features of the RG running of the neutrino parameters within the two models are analyzed and, in particular, we observe a power-law behavior for the running. We note that the running of the leptonic mixing angle θ_{12} can be sizable, while the running of θ_{23} and θ_{13} is always negligible. In addition, we show that the tri-bimaximal and the bimaximal mixing patterns at a high-energy scale are compatible with low-energy experimental data, while a tri-small mixing pattern is not. Finally, we perform a numerical scan over the low-energy parameter space to infer the high-energy distribution of the parameters. Using this scan, we also demonstrate how the high-energy θ_{12} is correlated with the smallest neutrino mass and the Majorana phases.

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I. INTRODUCTION

With the recent start-up of the Large Hadron Collider (LHC), experimental physics has started the search for the domain beyond the current Standard Model (SM) of particle physics. In addition to searching for the Higgs boson, the LHC will supply us with important information about the nature of physics above the TeV energy scale. Among the most popular high-energy extensions of the SM are extra-dimensional models. The idea that spacetime could have more than four dimensions was first proposed by Theodore Kaluza [1] and Oskar Klein [2] at the beginning of the twentieth century. In the 1980's, this idea gained popularity through the emergence of string theory, and at the end of the 1990's, several extra-dimensional models, which could potentially be detected at the next generation of high-energy experiments, were proposed [3–8]. These models are mainly motivated by the fact that they could provide solutions to different problems in the SM, such as the hierarchy problem and the lack of a good particle dark matter candidate.

An interesting aspect of extra dimensions is their impact on the RG running of physical parameters. It has been shown that the Kaluza–Klein (KK) towers give rise to an effective power-law running of the parameters for energies above the first KK level [9]. Hence, extra dimensions may increase the RG running dramatically, resulting in large effects at relatively low energy scales.

One of the rare examples of experimental evidence for physics beyond the SM comes from neutrino physics. The observation of neutrino oscillations strongly indicates that neutrinos are massive and that lepton flavors are mixed. Since it is not possible to describe neutrino masses within a renormalizable framework using the SM particle content only, neutrino oscillations imply new physics beyond the SM. The fact that the neutrino masses are bounded to be unnaturally small in comparison to the other SM fermion masses, together with the possibility that neutrinos could be their own antiparticles, has given rise to extensive model building within the neutrino sector. For example, small neutrino masses could naturally be generated through the so-called seesaw mechanisms [10–13], where neutrinos couple to new degrees of freedom. Small neutrino masses could also be generated in certain extra-dimensional models [14–17], which have the advantage of potentially being observable at the LHC.

Since the neutrino parameters are measured in low-energy scale experiments, the RG

running effects should be taken into account properly in studying neutrino mass models at high-energy scales [18–25]. Therefore, in this paper, we study the RG running of the neutrino parameters in extra-dimensional models. We employ an effective description for neutrino masses in terms of the dimension-five Weinberg operator, and investigate the RG running of the neutrino parameters in two different extra-dimensional models. In particular, we derive the beta function for the Weinberg operator in the Universal Extra Dimensions (UED) model [8], which has previously not been performed in the literature. In order to determine which high-energy values that are consistent with current experimental data within the models, we use a Markov Chain Monte Carlo to infer the favored high-energy parameter values from the low-energy parameter bounds using our analytical results.

The rest of the work is organized as follows: In Sec. II, we describe the models that we have studied, and present the beta functions for the neutrino mass operator in both models. In Sec. III, we give the RGEs for the neutrino masses and the leptonic mixing parameters, which are obtained from the beta function for the neutrino mass operator. Then, in Sec. IV, we show our numerical results for the RG running. Finally, in Sec. V, we summarize our work and state our conclusions. In addition, in Appendix A, we give the full set of RGEs that we have used.

II. RGEs IN EXTRA-DIMENSIONAL THEORIES

In order to illustrate the features of RGEs in extra-dimensional theories, we consider two representative models. First, we investigate the UED model, in which all the SM fields probe the extra spatial dimensions. Then, we study a model, in which the SM bosons propagate in the bulk, while the SM fermions are localized to a four-dimensional brane [9].

In general, the KK towers in an extra-dimensional theory are expected to be cut off at some energy scale Λ in order to keep the theory renormalizable, with the nature of the cutoff scale depending on the specific ultraviolet (UV) completion of the model. For the purpose of illustration, we take $\Lambda = 50$ TeV for both models in this work.

A. Model I: Universal Extra Dimensions

The UED model is constructed by promoting all the SM fields to a higher-dimensional flat spacetime, and hence, all the SM particles acquire towers of KK modes. Here, we consider the simplest case with only a single extra spatial dimension, which is assumed to be compactified on an S^1/\mathbb{Z}_2 orbifold with radius R . In this framework, KK parity, which is defined as $(-1)^n$ for the n th KK level, is conserved after compactification. The mass scale of the first excited KK level, given by R^{-1} , is bounded to be larger than approximately 300 GeV [26]. In this work, we use the value $R^{-1} = 1$ TeV.

In addition to the operators that are renormalizable at the level of the SM, we introduce the dimension-five Weinberg operator responsible for neutrino masses

$$-\mathcal{L}_\nu = \frac{1}{2} (\bar{L}\phi) \hat{\kappa} (\phi^T L^c) + \text{h.c.}, \quad (1)$$

where L denotes the lepton doublet fields, ϕ denotes the Higgs doublet, and $\hat{\kappa}$ is a matrix in flavor space. After electroweak symmetry breaking, the neutrino mass matrix is obtained as

$$m_\nu \equiv \kappa v^2, \quad (2)$$

where $v \simeq 174$ GeV is the vacuum expectation value of the Higgs field and $\kappa \equiv \hat{\kappa}/\pi R$. The neutrino mass operator defined in Eq. (1) can be realized through certain extra-dimensional seesaw mechanisms at the cutoff scale Λ [16], as well as through the standard seesaw mechanisms. In the language of an effective theory, Eq. (1) is essentially the same for the different seesaw models.

In a five-dimensional spacetime, there are no chiral fermions. In order to reproduce the phenomenology of the SM for energies below R^{-1} , a five-dimensional Dirac fermion has to be introduced for each chiral fermion in the SM, and the KK expansions of these Dirac fermions are chosen in such a way that chiral fermions are obtained at the zero-mode level. Hence, the number of degrees of freedom in the fermion sector is doubled at the excited KK levels in comparison to the SM. In addition, each SM gauge field has a fifth component, which appears as a real scalar from the four-dimensional point of view. Again, the zero-modes of these scalars can be removed by suitable choices of KK expansions, but they appear at the excited KK levels. In particular, compared to the SM-like couplings, additional vertices involving SM fermions and the fifth components of gauge fields appear.

In the UED model, the beta function for the neutrino mass operator can be written as

$$16\pi^2 \frac{d\kappa}{d \ln \mu} = \beta_\kappa^{\text{SM}} + \beta_\kappa^{\text{UED}}, \quad (3)$$

where β_κ^{SM} denotes the SM beta function [18, 19, 21]

$$\beta_\kappa^{\text{SM}} = -\frac{3}{2}\kappa(Y_\ell^\dagger Y_\ell) - \frac{3}{2}(Y_\ell^\dagger Y_\ell)^T \kappa - (3g_2^2 - 2T - \lambda) \kappa, \quad (4)$$

where

$$T = \text{tr} \left(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_\ell^\dagger Y_\ell \right), \quad (5)$$

with Y_f (for $f = u, d, \ell$) denoting the Yukawa coupling matrices of the up-type quarks, down-type quarks, and charged leptons, respectively. Here, g_i are the gauge couplings and λ denotes the Higgs self-coupling constant. The second term on the right-hand side of Eq. (3), $\beta_\kappa^{\text{UED}}$, comes from the contributions of the excited KK modes. As mentioned above, the KK spectrum at the excited levels differs from the SM, and this is reflected in the contributions of the KK modes to the beta function. We have calculated $\beta_\kappa^{\text{UED}}$ with the following result

$$\beta_\kappa^{\text{UED}} = s \left[-\frac{3}{2}\kappa \left(Y_\ell^\dagger Y_\ell \right) - \frac{3}{2} \left(Y_\ell^\dagger Y_\ell \right)^T \kappa - \left(\frac{1}{4}g_1^2 + \frac{11}{4}g_2^2 - 4T - \lambda \right) \kappa \right], \quad (6)$$

where $s = \lfloor \mu/\mu_0 \rfloor$ counts the number of KK levels contributing to the beta function for a given energy μ . Here, $\mu_0 = R^{-1} = 1 \text{ TeV}$. For large μ/μ_0 , s is well-approximated by the continuous expression μ/μ_0 . In evaluating physical parameters from lower to higher energy scales, new KK excitations enter the theory at each KK threshold, giving additional quantum corrections, and hence, the coefficients of the beta functions are modified depending on how many KK modes that are excited. The result is that the RG running of κ follows a power-law behavior, controlled by s . This is a distinctive feature of RG running in extra-dimensional theories. Compared to the RG running behavior in the SM, where the coefficients of the beta functions are nearly constant, the power-law behavior results in a significant boost in the RG running, which could possibly be tested at near-future experiments.

The differences in the coefficients for the gauge couplings between Eqs. (4) and (6) are due to the additional Feynman diagrams involving the fifth components of the electroweak gauge bosons shown in Fig. 1. The additional factor of 2 in the coefficient for T is due to the fact that the chiral fermions are replaced by Dirac fermions at each excited KK level.

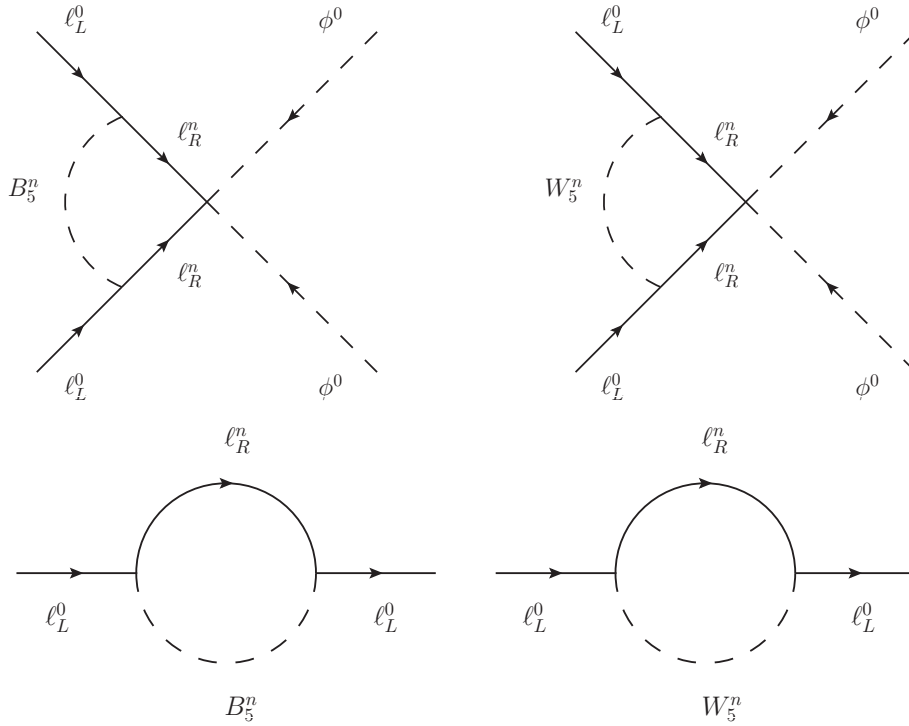


FIG. 1. The Feynman diagrams including the fifth components of the electroweak gauge bosons that contribute to the beta function for κ .

The beta functions for the Yukawa couplings Y_f as well as the gauge couplings are listed in Appendix A. Note that the beta functions for the Yukawa couplings differ between Refs. [27] and [28]. We have confirmed the computations in Ref. [28], namely,

$$\beta_{Y_\ell}^{\text{UED}} = sY_\ell \left[\frac{3}{2} Y_\ell^\dagger Y_\ell + 2T - \frac{33}{8} g_1^2 - \frac{15}{8} g_2^2 \right]. \quad (7)$$

B. Model II: Fermions on the brane

In this model, the SM fermions are confined to a four-dimensional brane, while all the SM bosons probe the bulk [9]. Again, we assume the extra dimension to be compactified on an S^1/\mathbb{Z}_2 orbifold with radius R . Present collider bounds allow the masses of the lowest KK excitations to be as low as 500 GeV [9]. As for model I, we take $\mu_0 = R^{-1} = 1$ TeV in this work. Thus, the heavy KK modes of the SM particles could be accessible at forthcoming collider experiments, and the unification of gauge couplings could also be achieved at a low-energy scale [29, 30].

Again, we write the beta function for κ as

$$16\pi^2 \frac{d\kappa}{d \ln \mu} = \beta_\kappa^{\text{SM}} + \beta_\kappa^{\text{II}}, \quad (8)$$

where the SM beta function β_κ^{SM} is given in Eq. (4). The contribution from the excited KK modes reads [9, 31, 32]

$$\beta_\kappa^{\text{II}} = 2s \left[-\frac{3}{2}\kappa(Y_\ell^\dagger Y_\ell) - \frac{3}{2}(Y_\ell^\dagger Y_\ell)^T \kappa - (3g_2^2 - \lambda) \kappa \right]. \quad (9)$$

In comparison to the SM beta function, the term proportional to T is missing here. This term comes from the fermion loop contributions to the self-energy of the Higgs boson, and since the fermions have no KK excitations in this model, there is no such contribution for $n > 0$. Also, the remaining terms in Eq. (9) are larger by a factor of 2, which is due to a rescaling of all interactions involving KK excitations by a factor $\sqrt{2}$, coming from the canonical normalization of all KK modes [33]. Note that this factor does not exist in model I.

III. RUNNING NEUTRINO PARAMETERS

We proceed our discussion to the RG running of the neutrino parameters. In general, one can choose to work in a basis where the charged lepton Yukawa coupling matrix Y_ℓ is diagonal, i.e., $Y_\ell = \text{diag}(y_e, y_\mu, y_\tau)$. In this basis, the leptonic mixing matrix [34, 35] stems from the diagonalization of the neutrino mass matrix, i.e.,

$$U^\dagger m_\nu U^* = D_\nu \equiv \text{diag}(m_1, m_2, m_3), \quad (10)$$

with m_i being the neutrino masses. Inserting Eq. (10) into the beta function for κ , one obtains the evolution for the leptonic mixing matrix due to the KK modes as

$$\frac{dU}{dt} \equiv \dot{U} = 2sUX, \quad (11)$$

with $t = \ln \mu / 16\pi^2$, and

$$\text{Re}X_{ij} = -\frac{3}{2}\eta\zeta_{ij}\text{Re}\left(U^\dagger Y_\ell^\dagger Y_\ell U\right)_{ij}, \quad (12)$$

$$\text{Im}X_{ij} = -\frac{3}{2}\eta\zeta_{ij}^{-1}\text{Im}\left(U^\dagger Y_\ell^\dagger Y_\ell U\right)_{ij}, \quad (13)$$

where $\eta = 1$ in model I, and $\eta = 2$ in model II, and the indices i and j run over 1, 2, and 3. The factors ζ_{ij} are defined as $\zeta_{ij} = (m_j + m_i)/(m_j - m_i)$. Note that the charged lepton

mass spectrum is strongly hierarchical, i.e., $m_e \ll m_\mu \ll m_\tau$, which allows us to make a reasonable approximation by ignoring the electron and muon Yukawa couplings in Eqs. (12) and (13). In what follows, we will assume $Y_\ell = \text{diag}(0, 0, y_\tau)$ for simplicity. Furthermore, the ζ factors play a key role in the RG running of the neutrino mixing angles, since in the case of a nearly degenerate neutrino mass spectrum, i.e., $m_1 \simeq m_2 \simeq m_3$, $\zeta_{ij} \gg 1$, and therefore, the RG running effects will be enhanced dramatically. If the neutrino mass spectrum is hierarchical, e.g., $m_1 \ll m_2 \ll m_3$, $\zeta_{ij} \simeq 1$ holds and the RG running effects on the leptonic mixing matrix are not observable. In the following analytical analysis, we will assume a nearly degenerate neutrino mass spectrum.

In order to figure out the RG running behaviors of the leptonic mixing parameters, we employ the standard parametrization, in which U is parametrized by three mixing angles and three CP-violating phases as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} \\ e^{i\sigma} \\ 1 \end{pmatrix}, \quad (14)$$

with $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ ($ij = 12, 13, 23$). Combining Eqs. (11) and (14), we arrive at the running for the leptonic mixing angles

$$\dot{\theta}_{12} \simeq \frac{3}{2}\eta s \zeta_{12} s_{12} c_{12} s_{23}^2 c_{\rho-\sigma}^2 y_\tau^2, \quad (15)$$

$$\dot{\theta}_{23} \simeq \frac{3}{2}\eta s \zeta_{13} s_{23} c_{23} (s_{12}^2 c_\rho^2 + c_{12}^2 c_\sigma^2) y_\tau^2, \quad (16)$$

$$\dot{\theta}_{13} \simeq \frac{3}{2}\eta s \zeta_{13} s_{12} c_{12} s_{23} c_{23} (c_\sigma c_{\delta+\sigma} - c_\rho c_{\delta+\rho}) y_\tau^2, \quad (17)$$

where $c_x \equiv \cos x$. Here, we have taken $\zeta_{13} \simeq \zeta_{23}$ and neglected the smallest mixing angle θ_{13} to a reasonably good approximation. In the limit of four dimensions, i.e., $s = \eta = 1$, the RGEs for θ_{ij} in the SM are reproduced [36, 37]. One observes that, compared to the RG running of the mixing angles in four-dimensional theories, an additional enhancement factor s enters the RGEs, which may lead to significant RG running effects. However, as the beta functions for the mixing angles are proportional to the four-dimensional ones, some qualitative features (e.g., the dependences of the running mixing angles on the neutrino mass hierarchy and the Majorana CP-violating phases) remain the same, and were already known in four-dimensional theories [18–25].

For the RG running of the leptonic mixing angles, one of the key features is that θ_{12} increases with increasing energy scale, independently of the neutrino mass hierarchy and the CP-violating phases. Consequently, a small θ_{12} at the high-energy scale is disfavored. On the other hand, the tri-bimaximal ($s_{12} = 1/\sqrt{3}$, $s_{23} = 1/\sqrt{2}$, $s_{13} = 0$ [38–40]) and the bimaximal ($s_{12} = s_{23} = 1/\sqrt{2}$, $s_{13} = 0$) mixing patterns could be natural candidates for flavor symmetries at some high-energy scale. As for θ_{23} and θ_{13} , the RG running effects are milder, since their RG running is boosted by ζ_{13} , which is much smaller than ζ_{12} .

For the sake of completeness, we also give the analytical RGEs for the neutrino masses

$$\dot{m}_1 \simeq -\frac{3}{2}\eta s m_1 s_{12}^2 s_{23}^2 y_\tau^2 + m_1 \alpha_\kappa, \quad (18)$$

$$\dot{m}_2 \simeq -\frac{3}{2}\eta s m_2 s_{23}^2 c_{12}^2 y_\tau^2 + m_2 \alpha_\kappa, \quad (19)$$

$$\dot{m}_3 \simeq -\frac{3}{2}\eta s m_3 c_{23}^2 y_\tau^2 + m_3 \alpha_\kappa, \quad (20)$$

where α_κ is flavor universal, and can be found in Appendix A. Similarly to the RG running of the mixing angles, the power-law factor s enhances the RG corrections. However, compared to the RGEs for the mixing angles, there is no enhancement factor ζ . Furthermore, the flavor non-trivial parts in Eqs. (18)-(20) are suppressed by the charged-lepton Yukawa coupling y_τ in comparison to α_κ . In particular, if one ignores the terms proportional to y_τ^2 , and obtain a very simple form for Eqs. (18)-(20) as $\dot{m}_i \simeq m_i \alpha_\kappa$. The solution to this equation is roughly estimated by

$$\frac{m_i(\Lambda)}{m_i(M_Z)} \simeq \left(\frac{\Lambda}{M_Z} \right)^{\alpha_\kappa}. \quad (21)$$

Therefore, the RG running of the neutrino masses is only sensitive to α_κ , independently of the neutrino mass spectrum and the mixing parameters.

In the following numerical analysis, we will mainly concentrate on the RG corrections to the flavor structure of the leptonic mixing matrix in the two above models.

IV. NUMERICAL ANALYSIS

In our numerical computations, we make use of the full sets of RGEs without any approximations. The input values for the neutrino parameters and SM observables at the $\mu = M_Z$ scale are taken from Refs. [41, 42]. In addition, for the Higgs mass, we use the representative value $m_H = 140$ GeV. Direct information on the absolute neutrino mass scale

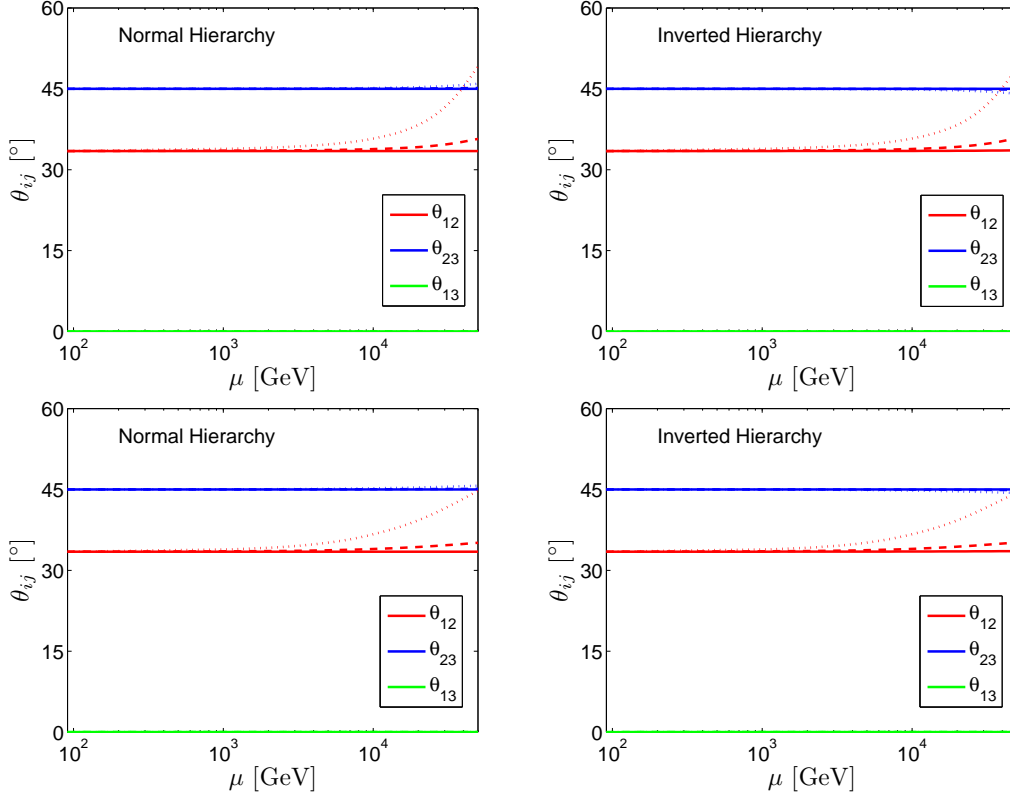


FIG. 2. The RG evolution of the three leptonic mixing angles as functions of the energy scale from M_Z to Λ for the normal mass hierarchy (left column) and the inverted mass hierarchy (right column) in model I (upper row) and model II (lower row). The solid, dashed, and dotted curves correspond to the mass of the lightest neutrino to be 0, 0.2 eV, and 0.5 eV, respectively. Since there is no strong experimental evidence for a non-vanishing θ_{13} , we take $\theta_{13} = 0$ as the input value at the M_Z scale. In addition, all the CP-violating phases are taken to be zero.

can be derived from tritium beta decay experiments [43, 44], i.e., $m_\nu < 2.3$ eV (at 95 % C.L.). Indirect constraints from the CMB data of the WMAP experiment and the large scale structure surveys also lead to an upper limit on the sum of neutrino masses. In our numerical studies, we conservatively take $m_i < 0.5$ eV.

In Fig. 2, we show the RG evolution of the neutrino mixing angles. In good agreement with our analytical results, θ_{12} is the mixing angle, which is the most sensitive to the RG corrections, and it increases with increasing energy in both the normal hierarchy ($m_1 < m_2 < m_3$) and inverted hierarchy ($m_3 < m_1 < m_2$) cases. In contrast, θ_{23} and θ_{13} are rather stable under the RG running, which reflects the fact that there is no strong enhancement

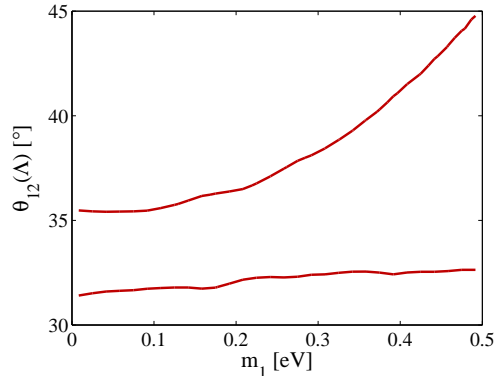


FIG. 3. The dependence of θ_{12} on m_1 at $\Lambda = 50$ TeV in model I. The band between the two curves corresponds to the allowed parameter space at 90 % posterior probability. Here, a normal mass hierarchy is assumed.

factor ζ_{12} in their RGEs. Furthermore, the running direction of θ_{23} depends on the mass hierarchy, namely, an increasing (decreasing) θ_{23} is obtained in the case of normal mass hierarchy (inverted mass hierarchy). This feature can be understood from the sign of ζ_{13} , which is positive in the normal hierarchy case, and negative in the inverted hierarchy case.

Note that the RG running effects on the mixing angles in model I are generally larger than those in model II, although the coefficients of the beta functions in model II are twice as large due to η . This can be understood from the different RG running behavior of y_τ in the two models. Concretely, in model I, the flavor independent parts of β_{y_τ} contain the trace of charged-fermion Yukawa couplings (c.f., the T term in Eq. (A13)), whereas such a contribution does not exist in model II, due to the absence of fermion KK excitations. Therefore, y_τ receives larger RG corrections in the UED model and a more sizable y_τ could naturally be expected at higher scales, which eventually leads to more significant RG corrections to θ_{ij} in the UED model.

In Fig. 3, we present the dependence of the high-energy θ_{12} on m_1 in the normal mass hierarchy case. In the numerical calculations, we use the MonteCUBES software [45] as a basis for a Markov Chain Monte Carlo (MCMC), and generate $5 \cdot 10^4$ samples per value of the mass m_1 , while the neutrino oscillation parameters have priors corresponding to present bounds. The band between the two curves corresponds to the allowed parameter space at

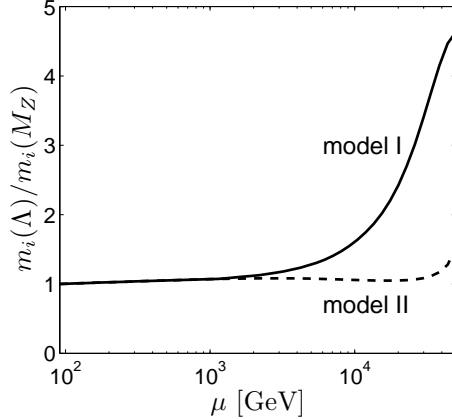


FIG. 4. The RG evolution of the ratio $m_i(\Lambda)/m_i(M_Z)$ on m_1 for model I (solid curve) and model II (dashed curve). Here, we use $m_1 = 0.2$ eV and a normal mass hierarchy.

90 % posterior probability¹. One can observe that remarkable changes in θ_{12} can be achieved at the higher energy scales if m_1 is sufficiently large. For a proper choice of m_1 , the tri-bimaximal or the bi-maximal mixing pattern can be easily achieved at higher-energy scales. However, in contrast to the conclusion obtained in Ref. [32], large leptonic mixing angles at low-energy scales cannot be generated from very small mixing angles near the cut-off scale. This is due to the naive but unrealistic two-flavor picture employed in Ref. [32]. Explicitly, in the two-flavor framework, a small mixing angle can be obtained at the cut-off scale for $m^2/\Delta m^2 \sim 10^4$. Thus, when we are working in the standard three-flavor framework, the RG corrections to θ_{13} and θ_{23} are related to $(m_3 + m_2)^2/\Delta m_{32}^2$, which can be at most at order 10^2 for $m_i < 0.5$ eV, and hence not sufficiently large to result in visible effects. As for θ_{12} , the RG running is related to the ratio $(m_2 + m_1)^2/\Delta m_{21}^2$, which could be large enough to give significant corrections for a nearly degenerate neutrino mass spectrum. However, as we have mentioned, the RG running always leads to a larger value of θ_{12} at the cut-off scale, independently of the choices of neutrino mass hierarchy. Therefore, the tri-small mixing pattern is not compatible with the models under consideration. We further remark that this feature is generic and independent of the choices of other physical parameters.

For the sake of completeness, we illustrate the running of the neutrino masses in Fig. 4. As expected, the RG running behavior is universal for the three neutrino masses within a specific

¹ The 90 % posterior probability region is the smallest region containing 90 % of the posterior probability distribution, i.e., given the priors, there is a 90 % probability that the parameters are within this region.

It is to Bayesian statistics what confidence regions is to frequentist statistics.

model. For model I, the RG corrections could increase the neutrino masses dramatically, whereas in model II, the RG corrections are not significant for the neutrino masses. This is consistent with our analytical result that α_{κ} is larger in model I, due to the contributions to T from the top quark Yukawa coupling. We have also checked that this conclusion does not change for different hierarchies of the neutrino masses.

Finally, since the RG running of θ_{12} is very sensitive to the Majorana CP-violating phases, it is of interest to observe the correlations between the CP-violating phases and the RG running of the leptonic mixing angles. To this end, we show in Fig. 5 the correlations between θ_{12} at the cutoff scale and the phase difference $\rho - \sigma$, with $5 \cdot 10^4$ samples per value of $\rho - \sigma$. The bands in the plot indicate the favored parameter spaces (at 90 % posterior

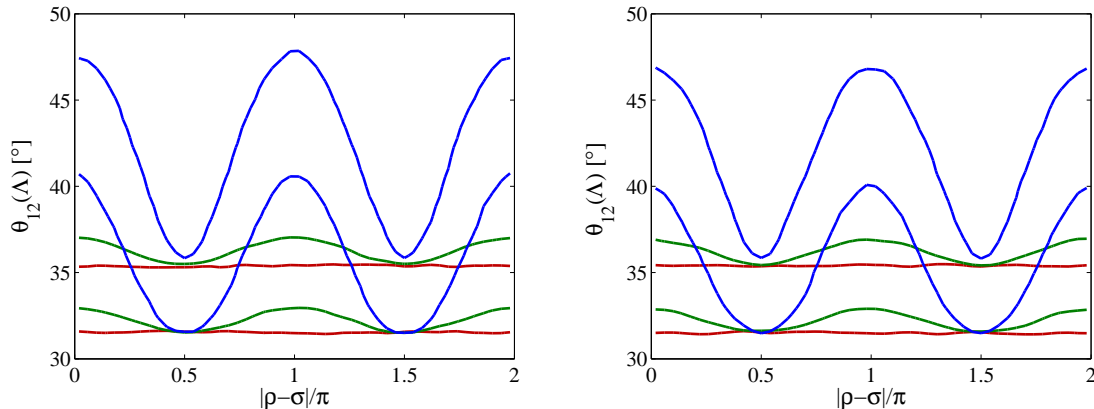


FIG. 5. The 90 % posterior probability regions for $\theta_{12}(\Lambda)$ as depending on the Majorana CP-violating phase difference $\rho - \sigma$ in model I (left panel) and model II (right panel). The bands between red, green, and blue curves correspond to the allowed parameter space for $m_1 = 0$, 0.2 eV, and 0.5 eV, respectively.

probability) for $m_1 = 0$ (red), 0.2 eV (green), and 0.5 eV (blue), respectively. As expected, no visible RG running effects are observed for a hierarchical neutrino spectrum. As for the degenerate neutrino spectrum, sizable RG corrections may exist depending on the difference between two Majorana CP-violating phases. A peak in $\theta_{12}(\Lambda)$ appears around $\rho - \sigma = n\pi$, while the RG corrections are damped if $\rho - \sigma = (n + 1)\pi/2$, with n being an integer.

V. SUMMARY AND CONCLUSION

In this work, we have studied the RG running of neutrino parameters in two representative extra-dimensional models extended by an effective neutrino mass operator. In particular, we have derived the full set of RGEs for the Yukawa coupling matrices and the neutrino mass operator. Both analytical and numerical analyzes of RG corrections to the neutrino mixing angles and masses have been performed according to our RGEs. We have found that, due to the power-law behavior and a sizable enhancement factor, θ_{12} is the most sensitive mixing angle to the RG running, especially in the nearly degenerate limit, e.g., $\theta_{23} = \pi/4$ can be accommodated at high energy scales if $m_1 \simeq 0.5$ eV. In contrast, the RG corrections to θ_{23} and θ_{13} are negligible in both of the models. In addition, the RG running effects on θ_{12} in model I are generally larger than those in model II, due to the charged-lepton Yukawa couplings. Most interestingly, θ_{12} does not decrease with increasing energy scale, regardless of the choice of the neutrino mass hierarchy and the CP-violating phases. Therefore, mixing patterns with small θ_{12} at the cutoff scale are not compatible with low-energy experiments, whereas the tri-bimaximal and bi-maximal patterns turn out to be favorable, depending on the specific choice of model parameters. We have also presented the connection between the running θ_{12} and the Majorana CP-violating phases, which indicates that the RG correction is damped if the difference between the Majorana phases is close to $(2n+1)\pi/2$. We conclude that quantum corrections should not be neglected in studies of extra-dimensional neutrino mass models. Our results allow to carry out an integrated investigation of fermion masses and flavor mixing in the framework of extra dimensions.

In the current work, we have only considered theories with one spatial extra dimension. In the cases of models with more than one extra dimension, the RG corrections are generally more substantial. The conclusions of our study depend on the compactification scheme, and other choices of boundary conditions would lead to different phenomena, which are however beyond the scope of the current work.

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Appendix A: Full set of one-loop RGEs for the extra-dimensional models

The one-loop RGEs for the Yukawa coupling matrices Y_f ($f = u, d, \ell$) and the neutrino mass operator κ can be expressed in a general form as

$$16\pi^2 \frac{dY_f}{d \ln \mu} = \beta_f^{\text{SM}} + \tilde{\beta}_f = \beta_f^{\text{SM}} + \alpha_f Y_f + Y_f N_f, \quad (\text{A1})$$

$$16\pi^2 \frac{d\kappa}{d \ln \mu} = \beta_\kappa^{\text{SM}} + \tilde{\beta}_\kappa = \beta_\kappa^{\text{SM}} + \alpha_\kappa \kappa + \kappa N_\kappa + N_\kappa^T \kappa, \quad (\text{A2})$$

where the SM beta functions are [18, 19, 21]

$$\beta_u^{\text{SM}} = Y_u \left(\frac{3}{2} Y_u^\dagger Y_u - \frac{3}{2} Y_d^\dagger Y_d - \frac{17}{12} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 + T \right), \quad (\text{A3})$$

$$\beta_d^{\text{SM}} = Y_d \left(-\frac{3}{2} Y_u^\dagger Y_u + \frac{3}{2} Y_d^\dagger Y_d - \frac{5}{12} g_1^2 - \frac{9}{4} g_2^2 - 8g_3^2 + T \right), \quad (\text{A4})$$

$$\beta_\ell^{\text{SM}} = Y_\ell \left(\frac{3}{2} Y_\ell^\dagger Y_\ell - \frac{15}{4} g_1^2 - \frac{9}{4} g_2^2 + T \right), \quad (\text{A5})$$

$$\beta_\kappa^{\text{SM}} = -\frac{3}{2} \kappa \left(Y_\ell^\dagger Y_\ell \right) - \frac{3}{2} \left(Y_\ell^\dagger Y_\ell \right)^T \kappa + (\lambda - 3g_2^2 + 2T) \kappa, \quad (\text{A6})$$

with $T = \text{tr} \left(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_\ell^\dagger Y_\ell \right)$. The contributions from KK excitations are given by

$$N_u = \frac{3}{2} \eta s Y_u^\dagger Y_u - \frac{3}{2} \eta s Y_d^\dagger Y_d, \quad (\text{A7})$$

$$N_d = -\frac{3}{2} \eta s Y_u^\dagger Y_u + \frac{3}{2} \eta s Y_d^\dagger Y_d, \quad (\text{A8})$$

$$N_\ell = \frac{3}{2} \eta s Y_\ell^\dagger Y_\ell, \quad (\text{A9})$$

$$N_\kappa = -\frac{3}{2} \eta s Y_\ell^\dagger Y_\ell, \quad (\text{A10})$$

where $\eta = 1$ for model I and $\eta = 2$ for model II. Here, we have defined the scale parameter $s = [\mu/\mu_0]$. The flavor diagonal coefficients α 's for model I read

$$\alpha_u = \eta s \left(-\frac{101}{72} g_1^2 - \frac{15}{8} g_2^2 - \frac{28}{3} g_3^2 + 2T \right), \quad (\text{A11})$$

$$\alpha_d = \eta s \left(-\frac{17}{72} g_1^2 - \frac{15}{8} g_2^2 - \frac{28}{3} g_3^2 + 2T \right), \quad (\text{A12})$$

$$\alpha_\ell = \eta s \left(-\frac{33}{8} g_1^2 - \frac{15}{8} g_2^2 + 2T \right), \quad (\text{A13})$$

$$\alpha_\kappa = \eta s \left(-\frac{1}{4} g_1^2 - \frac{11}{4} g_2^2 + 4T + \lambda \right), \quad (\text{A14})$$

whereas for model II, we have

$$\alpha_u = \eta s \left(-\frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right), \quad (\text{A15})$$

$$\alpha_d = \eta s \left(-\frac{5}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right), \quad (\text{A16})$$

$$\alpha_\ell = \eta s \left(-\frac{15}{4}g_1^2 - \frac{9}{4}g_2^2 \right), \quad (\text{A17})$$

$$\alpha_\kappa = \eta s (\lambda - 3g_2^2). \quad (\text{A18})$$

For the sake of completeness, we also present the RGEs for the gauge couplings

$$16\pi^2 \frac{dg_i}{d \ln \mu} = \left(b_i^{\text{SM}} + \eta s \tilde{b}_i \right) g_i^3, \quad (\text{A19})$$

where $(b_1^{\text{SM}}, b_2^{\text{SM}}, b_3^{\text{SM}}) = (41/6, -19/6, -7)$, while $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (27/2, 7/6, -5/2)$ in model I and $(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) = (1/6, -41/6, -21/2)$ in model II. Furthermore, we also need the running of the Higgs self-coupling,

$$16\pi^2 \frac{d\lambda}{d \ln \mu} = \beta_\lambda^{\text{SM}} + \tilde{\beta}_\lambda, \quad (\text{A20})$$

where the SM contribution reads [46]

$$\begin{aligned} \beta_\lambda^{\text{SM}} = & 6\lambda^2 - \lambda (3g_1^2 + 9g_2^2) + \left(\frac{3}{2}g_1^4 + 3g_1^2g_2^2 + \frac{9}{2}g_2^4 \right) \\ & + 4\lambda T - 8\text{tr} \left[3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2 + (Y_\ell^\dagger Y_\ell)^2 \right]. \end{aligned} \quad (\text{A21})$$

In addition, the extra-dimensional contributions are

$$\begin{aligned} \tilde{\beta}_\lambda = & 6\eta s \lambda^2 - \eta s \lambda (3g_1^2 + 9g_2^2) + \eta s (2g_1^4 + 4g_1^2g_2^2 + 6g_2^4) \\ & + 8\eta s \lambda T - 16\eta s \text{tr} \left[3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2 + (Y_\ell^\dagger Y_\ell)^2 \right], \end{aligned} \quad (\text{A22})$$

for model I, and

$$\tilde{\beta}_\lambda = 6\eta s \lambda^2 - \eta s \lambda (3g_1^2 + 9g_2^2) + \eta s \left(\frac{3}{2}g_1^4 + 3g_1^2g_2^2 + \frac{9}{2}g_2^4 \right), \quad (\text{A23})$$

for model II.

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