Beamforming utilizing channel norm feedback in multiuser MIMO systems

Master of Science Thesis
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January 2007

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LUND
UNIVERSITY

Abstract

Cellular wireless communication like GSM and WLAN has become an important part of the infrastructure. The next generation of wireless systems is believed to be based on multiple-input multiple-output (MIMO), where all units are equipped with multiple antennas. In contrast to the single antenna case, MIMO systems may exploit beamforming to concentrate the transmission in the direction of the receiver. The receiver may in turn use beamforming to maximize the received signal power and to suppress the interference from other transmissions. The capacity of a MIMO system has the potential of increasing linearly with the number of antennas, but the performance gain is limited in practice by the lack of channel information at the transmitter side.

This thesis considers downlink strategies where the transmitter utilizes channel norm feedback to perform beamforming that maximizes the signal-to-noise ratio (SNR) for a single beam. Two optimal strategies with feedback of, either the channel squared norm to each receive antenna, or the maximum of them are introduced and analyzed in terms of conditional covariance, eigenbeamforming, minimum mean-square error (MMSE) estimation of the SNR and the corresponding estimation variance. These strategies are compared under fair conditions to the upper bound and strategies without feedback or with pure SNR feedback. Simulations show that both strategies perform well, even if spatial division multiple access (SDMA) is required to exploit the full potential.

The beamforming strategies are generalized to the multiuser case where a scheduler schedule users in time slots in which their channel realization seems to be strong and thereby support high data rates. The gain of exploiting multiuser diversity is shown in simulations.

The thesis is concluded by a generalization to a multi-cell environment with intercell interference. Optimal and suboptimal receive beamforming is analyzed and used to propose approximate beamforming strategies based on channel norm feedback.

Acknowledgments

This thesis was carried out at the School of Electrical Engineering at the Royal Institute of Technology (KTH). It is the final part of my education at the Faculty of Engineering (LTH) at Lund University and will lead to a Master degree in Engineering Mathematics (Civilingenjör i Teknisk Matematik). I would like to express my gratitude to my supervisor David Hammarwall for introducing me to his work and for all valuable help, advice and discussions. I would like to thank Professor Björn Ottersten for suggesting the topic and for giving me the opportunity to write the thesis at KTH. I would also like to thank my examiner Professor Ove Edfors and my supervisor PhD Fredrik Tufvesson at LTH for their valuable comments on my rather extensive thesis.

Finally and most importantly, I would like to thank my girlfriend for support and encouragement under these intensive months and for suggesting that the thesis may be carried out at KTH.

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Chapter 1

Introduction

This chapter will provide an introduction to wireless communication and multiple antenna systems. Some of the terminology will be introduced, fundamental properties will be stated and some examples of their usage will be given. The chapter will be concluded by an overview of the problem considered in the thesis, major contributions and the outline of the remaining chapters.

1.1 Wireless communication

The thesis will consider wireless communication, which may be described as transmission of data without the use of cables. Many different wireless communication systems have been proposed and used throughout the years, starting with the wireless telegraph in the end of the 19th century. The main property of wireless systems is that the information is transmitted as electromagnetic waves from an antenna and received by another antenna at a place not necessarily known to the transmitter. The path between these antennas is known as the radio *channel*. The received waves will be different from the transmitted due to reflections, interference and noise on the channel and that the transmission power decays with distance.

The type of wireless communication considered in this thesis is based on *cellular networks*, which is probably the most common type of system. Mobile phone networks (GSM, UMTS) as well as wireless computer networks (WLAN) are examples of this kind of communication. Many other communication links may be described as special cases or generalization of cellular networks.

A cellular network consists of subscribers (users) with communication devices (mobile phones) who are allowed to move around freely in the region covered by the network. The subscribers do not communicate directly with each other. Instead there are a number of fixed base stations that receive data from subscribers and direct it to the correct receivers. These stations are spread in the environment to provide reliable link quality and good communication coverage. In its simplest form the network of base stations and area covered by them may be described as a hexagon lattice as in Figure 1.1a. In reality, the stations are more irregularly

Figure 1.1. Base stations in a cellular network. If the each subscriber is served by nearest station, the area is divided into cells. These cells may form a hexagon lattice (a), but in reality the structure is more irregular (b).

positioned, as in Figure 1.1b.

Each subscriber communicates with the base station that, for the moment, can provide the strongest communication link. This will in many cases correspond to the nearest station, but the environment will create fuzzy cell boundaries that in general are much more irregular than those marked in Figure 1.1b. If the receiver is wired (like a standard telephone), then the station may redirect the data directly to it through the wired network. Otherwise the station will forward the data (wireless or wired) to the station with the strongest link to the wireless receiver. Hence, when analyzing a wireless communication system there are only two categories of communication: *uplink* transmission from subscribers to a base station and *downlink* transmission from a base station to subscribers.

An alternative to cellular communication would be to demand transmission to occur directly from the transmitter to the receiver. There are, however, many advantages with cellular networks. The power usage is dramatically reduced when the transmitter and the receiver are separated by large distances. The data throughput may be increased since users in different cells may transmit simultaneously using the same frequency band without creating any large disturbances. Several subscribers within the same cell may also communicate simultaneously with the base station by sharing, for example, time and frequency in a standardized way. See Section 5.1 for a survey of multiuser access techniques.

1.2 Physical model for a wireless channel

In this section, the physical properties of a wireless communication channel will be discussed. If the transmission from subscriber to base station, or vice versa,

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Figure 1.2. Two different wireless communication scenarios. In (a) the signal can only reach the receiver through the direct path, while in (b) the received signal will also include a sum of several reflected versions of the transmitted signal.

occurs in free space, i.e., with no walls or other objects that can reflect the antenna radiation, then the receive antenna will receive a single wave that has traveled directly from the transmitter to the receiver. The received signal will be exactly the same as the transmitted - apart from the influence of noise and that the energy will decay with the distance between the transmitter and the receiver. This situation is shown in Figure 1.2a.

This model may be accurate in transmission between satellites, but would be highly unreasonable in land based communication. Such an environment contains a lot of objects and since the antenna radiation is spread over an area these objects will reflect a portion of the signal. The transmission will be exposed to *scattering*. The received signal will be the sum of a large number of waves that have traveled different paths and are arriving at different angles and with different phases. This is known as a *multipath* signal. In many situation there is no line-of-sight path between the transmitter and the receiver, so everything that is received has in fact been reflected one or several times. It is possible to make models of different scattering situation, like the one shown in Figure 1.2b. Chapter 2 will introduce a model based on rich scattering that will be used throughout the thesis.

1.3 Near- and far-field

The electromagnetic field emitted by an antenna during transmission may be approximated in different ways depending on the distance to the antenna and the size

of it [1]. The field is typically divided into two parts and their properties will be reviewed in this section.

Let the carrier wavelength of the transmitted signal be λ_c and let the length of the largest antenna dimension be D . The zone nearest the antenna is called the *near-field region* and includes all space up to a radial distance of a small number of λ_c and D (the largest of them will decide) [1]. In this region the energy decays very rapidly with distance and there may be local energy fluctuations. The field distribution is, in other words, highly dependent on the distance and direction. The dependence will however reduce with distance and in the *far-field region* the field distribution is essentially independent of the distance to the antenna. This approximation is assumed to be reasonable when the radial distance is much larger than λ_c and D .

An important property of the far-field region is that the incoming signal to the receiver may be approximated as a *plane wave*, i.e., an infinite set of parallel rays that propagate in a certain direction and have a phase that only depends on position in that direction. This property will be used later on, when deriving the relation between the received signals in an array of antennas.

There is no exact boundary between the near-field and far-field regions. As stated above, the boundary depends on the wavelength and antenna size, but it will also be a matter of approximation precision. Since the frequencies used in wireless communication are given in terms of GHz (0.9 or 1.8 for GSM, 2.4 or 5 for WLAN) the wavelength is a fraction of a meter, so it should be considered a safe assumption that the receiver will be in the far-field region of the transmitter [2] (there may however be scatterers in the near-field of the receiver).

The existence of the two regions can be easily motivated in terms of electromagnetic field theory [1]. The field may be modeled using several terms that decay with different exponents of the radial distance r_d . When the distance is large, the term with the smallest exponent will dominate. This leads to the conclusion that the power decay in the far-field will be inversely proportional to r_d^2 , which also has been supported empirically in some situations. In an environment with much scattering the power may decay (due to power absorption and destructive interference) with a larger exponent in the far-field or even exponentially with distance [2].

1.4 Multiple-Input Multiple-Output communication

In traditional cellular networks both base stations and subscribers are communicating using a single antenna each, which is known as *Single-Input Single-Output* (SISO) communication. There is however no theoretical reason why several antennas cannot be used on each device, which is the type of communication considered throughout the thesis. If both the receiver and the transmitter in a wireless communication link use an array with multiple antennas, then the communication system is known as *Multiple-Input Multiple-Output* (MIMO). There are also intermediate situations, MISO and SIMO, where only the transmitter or the receiver uses multiple

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antennas.

Jack Winters at Bell Labs is often mentioned as the pioneer of multi antenna communication with his work from 1984, [3], and with a patent filed in 1987. Since then it has been considered how the additional antennas can be used to improve the range and data throughput (on a given power usage and bandwidth). According to [4] the capacity of MIMO systems has the potential of increasing linearly with the number of spatial subchannels created by the multiple antennas. In principle the additional antennas may be used to transmit several simultaneous signals, but these will in practice interfere with each other. This thesis will consider methods based on *beamforming*, where all transmit antennas transmit the same signal but phase shifted to create constructive interference at the receiver. Such methods are efficient when the channel to each receive antenna is similarly distributed and may also be used to transmit several simultaneous beams in different directions.

The research on MIMO communication is rapidly increasing at the same time as the cost of producing receivers and antenna arrays has decreased. Many wireless communication systems are expected to be based on MIMO in the future. Beamforming has already been incorporated into existing systems, like GSM and WLAN, since it allows multiple antennas to be used at the base station without creating any incompatibility problems.

1.5 Models for antenna arrays

The structure of a multiple antenna array will affect its capability of, for example, receiving transmission in different directions. In this thesis, two kinds of antenna arrays are considered: Uniform Linear Array (ULA) and Uniform Circular Array (UCA). ULA is probably the most commonly used type of antenna array in MIMO theory, but it has also become common to compare it with UCA when evaluating the performance of MIMO systems.

In a ULA the antennas are placed uniformly on a straight line with the distance d , while the antennas in a UCA are placed uniformly over a circle with radius α . These array models are shown in Figure 1.3a and 1.4a, respectively, together with the radial coordinates system used throughout the thesis.

The structure of an antenna array can be specified by the array response $\mathbf{a}(\theta)$, where θ is the angle of arrival. This vector has an element for each antenna in the array and the element describes the relative difference in phase when receiving a plane wave arriving at angle θ . The array response also depends on the elevation angle φ between the transmitter and the receiver, but here a two dimensional environment is assumed with $\varphi = \pi/2$. The situation for ULA and UCA is shown in Figure 1.3b and 1.4b, respectively. Since the array response describes relative differences it is only defined up to a phase shift of all elements.

Next, the array response for ULA and UCA will be derived, with some inspiration from [5] and [6]. Let the carrier wavelength be λ_c and the number of antennas be n . In the case with ULA, the signal received at the second antenna (from left)

Figure 1.3. Uniform Linear Array. (a) ULA with four antennas at uniform distance d , with a total array length of $3d$. (b) The antenna array receives a plane wave generated by a narrowband signal. The distance to each antenna is different, so there will be phase difference in the received signals. The angular coordinate system starts at the vertical axis.

Figure 1.4. Uniform Circular Array. (a) UCA with eight antennas placed uniformly on a circle with radius α . (b) The antenna array receives a plane wave by a narrowband signal. The distance to each antenna is different, so there will be phase difference in the received signals. These differences are measured relative the origin.

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must travel a distance of $d \sin(\theta)$ longer than the one received at the first antenna. Expressed in wavelengths the difference will therefore be $d \sin(\theta)/\lambda_c$, as shown in Figure 1.3. Since the difference in distance the incoming signal has to travel to two adjacent antennas is constant, the array response may be expressed as

$$\mathbf{a}_{\text{ULA}}(\theta) = \left[e^{j \frac{2\pi d}{\lambda_c} \sin(\theta) \cdot 0} \dots e^{j \frac{2\pi d}{\lambda_c} \sin(\theta) \cdot (n-1)} \right]^T. \quad (1.1)$$

The array response of the UCA can be derived in a similar way. Let the centre of the circle be the reference point when calculating the distance differences the plane wave has traveled to the different antennas. The n antennas are spread uniformly over the circle, i.e., the i th antenna will be at angle $2\pi(i-1)/n$. In Figure 1.4 it can be seen that the difference in distance compared to the origin will be $-\alpha \cos(\theta - 2\pi(i-1)/n)$, so the array response may be expressed as

$$\mathbf{a}_{\text{UCA}}(\theta) = \left[e^{-j \frac{2\pi \alpha}{\lambda_c} \cos(\theta-0)} \dots e^{-j \frac{2\pi \alpha}{\lambda_c} \cos(\theta-2\pi \frac{n-1}{n})} \right]^T. \quad (1.2)$$

1.6 Degrees of freedom and diversity

This section will consider two fundamental concepts in the design of communication systems: degrees of freedom and diversity. The degrees of freedom may be described as the number of dimensions that are exploited, while diversity is a way of enhancing the probability of transmitting/receiving data over channel realizations with good properties.

Time-frequency dimensions

The two basic signaling dimensions are time and frequency. These may be exploited for orthogonal transmission by dividing all available time and frequency into slots. The orthogonality makes it possible to exploit all such time-frequency slots without creating any disturbances between them. The orthogonality of time-frequency stands in contrary to the spatial dimensions described below that often are correlated in practice.

Diversity may be created in the time-frequency dimensions by introducing redundancy. Due to the random nature of the channel its performance will change over time. It is probable that the channel gain will become weak in comparison to the noise for some amount of time or frequency. By transmitting the same data in several alternative and mutual independent ways it is however much more unlikely that all of them will have low performance. This will improve the probability of correct detection of the transmitted data and thereby allow higher data rates.

Time diversity may be achieved by repeating (or coding) each symbol in time with a sufficient interleaving, i.e., with a time distance such that the channel realizations are fairly independent. Frequency diversity may be achieved in a similar way by transmitting coded symbols over independent frequency bands.

Spatial dimensions

As mentioned in Section 1.4, an important purpose of introducing multiple antennas is to exploit spatial dimensions. Theoretically, each pair of additional transmit and receive antennas will give an additional spatial dimension. Unfortunately, the structure of these spatial dimensions depends on the channel and is in practice only partially known to the transmitter. In the ideal case, with perfect channel information at the transmitter, the communication may be spatially orthogonal and based on water-filling [2], but as will be discussed later it is often unreasonable to approximate this with channel estimations. The spatial dimension will therefore be correlated in practice.

This thesis will use the additional antennas to maximize the channel performance for just one signal, i.e., to exploit spatial diversity by transmitting the same signal over all antennas. This may especially be useful when the antenna separation is large or when the receiver is surrounded by a rich amount of scatterers. In either case, the channel to each receive antenna may be assumed to be independent. According to [7] this is a valid assumption for antenna separations larger than several wavelengths.

Multiuser diversity

In a system with several users the available dimensions needs to be divided among them in order to avoid interference. Since the users in general are independently located in the environment, their channels may be considered as approximately independent. Hence, multiuser diversity may be exploited by giving each user the opportunity to transmit when they are experiencing particular strong channels realizations.

1.7 Problem formulation and contributions

The thesis considers beamforming and rate estimation in the downlink of a wireless narrowband MIMO system. The channel to each receive antenna is assumed to be independent and identical complex Gaussian distributed (see Appendix A.1). The channel statistics may be estimated at both the transmitter and the receiver, but only the receiver is capable of estimating the instantaneous channel realization. The receiver may however feed back channel information that allows the transmitter to adapt its behavior to properties of the current channel realization.

The thesis will analyze and compare feedback strategies that maximize the overall throughput in the communication system, under the condition that each base station only transmits a single simultaneous beam. Several beamforming strategies based on limited feedback of some kind of channel norm will be considered. The general purpose of such feedback is to give the transmitter some understanding of the current channel strength and its spatial properties. This information may be exploited by the transmitter to perform efficient beamforming that maximizes the data throughput. The strategies will be analyzed to provide closed-form ex-

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pressions of the minimum mean-square error (MMSE) estimate of the SNR/SINR at the transmitter and the estimation variance. The strategies will be compared under fair conditions (equal probability of overestimation the maximum supported rate) in simulations based on local scattering and ULA/UCA, but these simulation assumptions have no effect on the analysis.

The proposed feedback strategies will be derived in a single-cell and single-user environment and then generalized to the multi-user case where scheduling is used to exploit multiuser diversity. The strategies will also be generalized to the multi-cell and multi-user case with intercell interference.

The thesis contributes analytically by deriving expressions of the conditional channel covariance matrix based on feedback of either a set of squared channel norms to each receive antenna or just the maximum of them. The conditional fourth order moments are also analyzed and exploited to control the estimation error.

1.8 Outline

This chapter has considered fundamental and historical facts of wireless communication and systems with multiple antennas. The channel has been described using physical and electromagnetic properties. The array response for two types of antenna arrays, ULA and UCA, has been derived and will later be used in simulations. An overview of two communication concepts has been given: degrees of freedom and diversity. Finally, the problem formulation and the contributions by the thesis have been reviewed. The rest of the thesis will be structured as follows.

In Chapter 2, a mathematical communication model for a narrowband MIMO system will be derived and some assumptions regarding the channel statistics will be made. Quality measures such as signal-to-noise ratio (SNR), channel capacity and outage probability will be introduced. The chapter will be concluded by a discussion of what information is immediately available at the transmitter and receiver, and what information may be sent as feedback between them. Chapter 3 will then introduce a somewhat simplified model that will be used in simulations throughout the thesis. Channel covariance matrices will be derived for ULA and UCA.

The main part and contribution of the thesis is included in Chapter 4. The downlink of a MIMO system with beamforming, limited channel norm feedback and a single user will be analyzed. Optimal communication strategies (in sense of SNR maximization) will be derived based on different feedback variables and will be compared to the upper bound. Especially two of these strategies are introduced by the thesis and will be analyzed as generalizations of the work in [8]. The minimum mean-square error (MMSE) of the SNR estimate at the transmitter will be analyzed for the different strategies and variance expressions will be derived. Two different approaches of avoiding over-estimation will be considered and used to compare the considered strategies under fair conditions in terms of equal outage probability.

In Chapter 5, the strategies analyzed in the previous chapter will be used in

a system with multiple users. First, the most common multi-user communication schemes will be reviewed. Then, two methods of dividing time, frequency, etc., between multiple users will be described: Maximum throughput scheduling and Proportional fair scheduling. The chapter will be concluded by a simulation that compares the CDF of the user mean throughput and the cell throughput of the previously analyzed strategies. The gain of multiuser diversity will be clear.

Finally, the work of the thesis will be generalized in Chapter 6 by considering the downlink of a MIMO system with beamforming, limited feedback, multiple users and interference from adjacent cells. The chapter begins with a short introduction that shows how previous concepts still may be used with small modifications. By assuming that the base stations cannot cooperate, the transmit beamforming will be similar to the single-cell case. The receive beamforming needs however to be analyzed further. First, the optimal receive strategy will be derived and compared in simulations to two simplified suboptimal strategies. Then, the available channel information will be discussed and two partially new feedback strategies will be introduced, analyzed and compared.

The thesis is summarized by Chapter 7 that consists of a retrospective survey, conclusions and suggestions of future work. The definition of Complex Gaussian distribution and some derivations have been gathered in the Appendix.

Chapter 2

Narrowband MIMO channel model

This chapter will introduce a communication system based on narrowband MIMO communication and beamforming. This model will be used in the analysis throughout the thesis. The chapter begins with a stepwise derivation of the communication model with assumptions regarding the channel statistics. Then some quality measures, which will be used in the following chapters, will be introduced: SNR, channel capacity and outage probability. The last part of the chapter will discuss to what extent the transmitter and the receiver may estimate the channel, and what amount of information may be sent as feedback between them.

The chapter ends with a summary, in Section 2.7, that will give a brief review of all concepts that will be of importance in the rest of the thesis. Hence, the eager reader may omit the rigorous and well-known derivations in this chapter and accept the model given in the summary as reasonable.

2.1 Basic principles and modeling

In this section, the basic principles of the digital communication model considered in this thesis will be discussed and derived by the approach used in [2]. Here a model based on a single transmitted signal $s(t)$ traveling over a channel with *additive white Gaussian noise* (AWGN) and resulting in a single received signal $r(t)$ will be considered. The model will then be extended to include multiple antennas.

Consider a real valued band-limited continuous signal $s_p(t)$ that is to be transmitted over a passband channel $[f_c - W/2, f_c + W/2]$. The bandwidth is W (Hz) and the center frequency f_c satisfies $f_c > W/2$ and is known as the *carrier frequency*. Let the Fourier transform of $s_p(t)$ be $S_p(f)$. Another signal $s_b(t)$ may be defined by having the Fourier Transform

$$S_b(f) = \begin{cases} \sqrt{2}S_p(f + f_c) & f + f_c > 0, \\ 0 & f + f_c \leq 0. \end{cases} \quad (2.1)$$

These two signals contain exactly the same information since the spectrum of a real valued signal satisfies $S_p(f) = S_p^*(-f)$. The complex valued signal $s_b(t)$ is band-

limited in $[-W/2, W/2]$ and is known as the *complex baseband equivalent*. Observe that $s_p(t)$ may be reconstructed from $s_b(t)$:

$$\begin{aligned} S_p(f) &= \frac{1}{\sqrt{2}} \left(S_b(f - f_c) + S_b^*(-f - f_c) \right), \\ s_p(t) &= \frac{1}{\sqrt{2}} \left(s_b(t) e^{j2\pi f_c t} + s_b^*(t) e^{-j2\pi f_c t} \right) = \sqrt{2} \Re \left\{ s_b(t) e^{j2\pi f_c t} \right\}. \end{aligned} \quad (2.2)$$

Now consider an AWGN channel from $s_p(t)$ to the received signal $r_p(t)$:

$$r_p(t) = \int_0^\infty h_p(\tau, t) s_p(t - \tau) d\tau + n_p(t), \quad t > 0,$$

where $h_p(\tau, t) \in \mathbb{R}$ and $n_p(t) \in \mathcal{N}(0, N_0)$ is a white process. The equivalent channel in $s_b(t)$ and $r_b(t)$, with $r_b(t)$ being the complex baseband equivalent to $r_p(t)$, may be expressed as

$$r_b(t) = \int_0^\infty h_p(\tau, t) e^{-j2\pi f_c \tau} s_b(t - \tau) d\tau + n_p(t) e^{-j2\pi f_c t}, \quad t > 0, \quad (2.3)$$

by using (2.2) and the corresponding expression for $r_b(t)$. Although the signal transmitted over the channel in a digital communication system is continuous it will only be an analog representation of a discrete sequence of symbols. Therefore it would make more sense to analyze a sampled communication model with a sample frequency $f_s > W$. Then, according to the Nyquist-Shannon sampling theorem [9], there exists an equivalent discrete channel model to (2.3), at least if the received signal is assumed to be ideally low-pass filtered to remove high frequency noise. The signal may be reconstructed as

$$s_b(t) = \sum_n s_b\left(\frac{n}{f_s}\right) \text{sinc}(f_s t - n), \quad t > 0.$$

By low-pass filtering and sampling of the received signal in (2.3), the following model is derived

$$r_b\left(\frac{m}{f_s}\right) = \sum_n s_b\left(\frac{n}{f_s}\right) \int_0^\infty h_p\left(\tau, \frac{m}{f_s}\right) e^{-j2\pi f_c \tau} \text{sinc}(m - n f_s \tau) d\tau + \tilde{n}\left(\frac{m}{f_s}\right), \quad m \in \mathbb{Z}^+,$$

where the noise $\tilde{n}(t)$ is the ideally low-pass filtered version of $n(t) e^{-j2\pi f_c t}$. The real and imaginary part of this random process will be the projection of a white process on an orthogonal basis. Hence the real and imaginary parts will be independent and the noise will become white complex Gaussian distributed (see Appendix A.1).

Now, by defining the discrete channel as

$$h(n, m) = \int_0^\infty h_p\left(\tau, \frac{m}{f_s}\right) e^{-j2\pi f_c \tau} \text{sinc}(m - n f_s \tau) d\tau,$$

2.1. BASIC PRINCIPLES AND MODELING

Figure 2.1. Example of the symbol structure and the symbol-to-bits correspondence with Quadrature Amplitude Modulation (QAM): (a) 4-QAM, (b) 16-QAM.

by introducing the new notation $s(n) = s_b(n/f_s)$, $r(m) = r_b(m/f_s)$ and $n(m) = \tilde{n}(m/f_s)$ and by some abuse of notation ($n \rightarrow \tau$, $m \rightarrow t$) the discrete channel from above may be expressed as

$$r(t) = \sum_{\tau} s(\tau)h(\tau, t) + n(t), \quad t \in \mathbb{Z}^+, \quad (2.4)$$

where $r(t), s(t), h(\tau, t) \in \mathbb{C}$ and $n(t) \in \mathcal{CN}(0, N_0)$. The information is carried in the real and imaginary part of $s(t)$. The receiver uses the received discrete signal $r(t)$ to estimate the transmitted signal. To make the estimation of the transmitted signal easier it may in practice not take any value. Instead $s(t)$ consists of *symbols* chosen from a discrete symbol space like for example in Figure 2.1, where each symbol corresponds to a sequence of bits.

Finally, assume that the bandwidth W is narrow, which leads to the conclusion that the transmission time of a symbol will be much larger than time dispersion from signals traveling multiple paths (see Section 1.2). Then the inter-symbol interference is small and it is reasonable to assume that there is no time dispersion at all¹.

Using the assumption of no time dispersion, $h(\tau, t) = 0$ for $\tau \neq 0$ and the communication model in (2.4) is transformed into

$$r(t) = h(0, t)s(t) + n(t) = h(t)s(t) + n(t), \quad t \in \mathbb{Z}^+, \quad (2.5)$$

which will provide much easier calculations later on.

¹This feature is especially useful in systems based on Orthogonal frequency-division multiplexing (OFDM), where the available bandwidth is divided into many orthogonal narrowband subchannels. OFDM is widely used in wireless systems, for example in WLAN, DVB and DAB.

2.1.1 Extended communication model

The communication model in (2.5) will now be extended into a MIMO system. Assume that the transmitter and the receiver have n_T and n_R antennas, respectively. Since the average transmit power may be considered as a design parameter it will be denoted P and hence the transmitted signal may be described as zero-mean and satisfying $E\{|s(t)|^2\} = 1$. Using these assumptions, (2.5) may be expressed as

$$\mathbf{r}(t) = \sqrt{P}\mathbf{H}(t)\mathbf{s}(t) + \mathbf{n}(t), \quad (2.6)$$

where the transmit signal is $\mathbf{s}(t) \in \mathbb{C}^{n_T}$, the channel gain matrix is $\mathbf{H}(t) \in \mathbb{C}^{n_R \times n_T}$, the received signal is $\mathbf{r}(t) \in \mathbb{C}^{n_R}$ and the AWGN is $\mathbf{n}(t) \in \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$.

2.2 Channel statistics

This section will discuss and make assumptions regarding the statistical properties of the channel matrix $\mathbf{H}$. When choosing an appropriate model for the channel statistics it is adjustment between the ease of performing analysis and the amount of details the model will cover. Throughout the thesis, the rather simple *Rayleigh fading* model will be used. Communication schemes that are optimal in this model will probably be suboptimal in more general or specific situations. The analysis performed in the thesis may however provide important insight in which strategies that are advantageous and which are definitely not.

When considering the statistics, it is important to note that there are three different time scales. The additive noise is white and its realization will be independent for each symbol. The channel matrix will be much more slowly changing. It is often assumed that there is a coherence time [2], which is defined as the time interval it takes for the channel realization to change significantly. This time is typically long enough to make it worthwhile to estimate the channel. The channel statistics changes even more slowly and will therefore be quite easy to estimate it adaptively over time.

The thesis will not make any exact assumptions regarding how the channel and its statistics changes over time. The time index of the channel will however be left out from now on to indicate that the channel is semi-constant during a sufficient amount of time to make it interesting to analyze. The model in (2.6) may then be expressed as

$$\mathbf{r}(t) = \sqrt{P}\mathbf{H}\mathbf{s}(t) + \mathbf{n}(t). \quad (2.7)$$

The channel matrix $\mathbf{H}$ is a combination of n_R channels, one to each of the receive antennas. By denoting the channel vector to the i th receive antenna $\mathbf{h}_i^H$, the matrix may be expressed as

$$\mathbf{H} = \begin{pmatrix} \mathbf{h}_1^H \\ \vdots \\ \mathbf{h}_{n_R}^H \end{pmatrix}.$$

2.2. CHANNEL STATISTICS

2.2.1 Rayleigh fading

This section will first present the Rayleigh fading model that will be used throughout the thesis and then justify its relevance in communication. In the model, the channel vectors are assumed to be zero-mean, independent and identically complex Gaussian distributed, i.e., $\mathbf{h}_i \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\mathbf{h}})$. The name of the model comes from the fact that the magnitude of each channel vector element $x \in \mathcal{CN}(0, \sigma^2)$ is a Rayleigh random variable $|x| \in R(\sigma/\sqrt{2})$:

$$F_{|X|}(x) = P(|X| \leq x) = P\left(\underbrace{\frac{|X|^2}{\sigma^2/2}}_{\in \chi^2(2)} \leq \frac{x^2}{\sigma^2/2}\right) = 1 - e^{-x^2/\sigma^2}, \quad x \geq 0, \quad (2.8)$$

$$f_{|X|}(x) = \frac{d}{dx}F_{|X|}(x) = \frac{2x}{\sigma^2}e^{-x^2/\sigma^2}, \quad x \geq 0.$$

In [4] the Rayleigh fading model is claimed to be well accepted for most environments without a line-of-sight path to the receiver. According to [2] the model is quite reasonable in environments with rich scattering, which may be justified quite intuitively as follows.

Assume that the multipath signal received at time t consists of the sum of components that has followed $L(t)$ different paths, where $L(t)$ is very large for rich scattering. The impact of some of these components will be of time-varying nature since there are objects that are in motion. The scatterers are typically independent and by assuming identical distributions the channel may be approximated as a complex Gaussian distribution, according to the Central Limit Theorem. This is perhaps not a completely realistic assumption, but there are other theorems (like the Lindeberg-Feller Central Limit Theorem [10], [11]) that suggest approximations with Gaussian distributions for sums of non-identically distributed variables.

The assumption of rich scattering may be reasonable near a mobile user in an urban environment. The mobile user will typically be surrounded by objects that may reflect the signal. The multipath signal received at each antenna will consist of components that arrive from all possible directions. If these directions are spread uniformly, then the received signal will not contain any spatial information. If the scattering is rich enough it is also sufficient to assume that the channel to each of the mobile antennas is approximately independent, although identically distributed. This assumption will be increasingly reasonable with increasing antenna separation. Base stations, on the other hand, are often elevated from the surroundings with the purpose of minimizing the scattering near the station. Hence, the signal to each of its antennas may be quite correlated so a corresponding assumption of independence would demand an antenna separation of many wavelengths [2]. Hence the channel independence is only assumed at the mobile and may therefore mainly be exploited when considering downlink beamforming.

2.2.2 Rician fading

If there is a line-of-sight path between the transmitter and the receiver the Rayleigh fading model is often inaccurate due to the strong direct path. In such situations the model may be extended to

$$\mathbf{h} = \sqrt{\frac{K}{K+1}} \bar{\mathbf{h}} + \sqrt{\frac{1}{K+1}} \tilde{\mathbf{h}}, \quad \text{with } \bar{\mathbf{h}} = \mathbf{a}(\theta), \tilde{\mathbf{h}} \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\mathbf{h}}),$$

where $\mathbf{a}(\theta)$ is the array response of the transmitter and K determines the amount of the received power that came through the direct path, in relation to the overall received power. Observe that a large K -value will make the channel almost deterministic, while $K = 0$ corresponds to Rayleigh fading. This generalized model is known as the *Rician fading* model [2].

2.2.3 Power decay

As mentioned in Section 1.3 the expected received signal power decays with the distance. The receiver is assumed to be in the far-field region of the transmitter, so the distance is approximately the same in all scattering paths. Therefore the power decay may be seen as a function $f(r_d)$ of the radial distance r_d . This property will not be used in any sense in the analysis, but will be considered in the simulations.

2.3 Beamforming

This section will extend the communication model to include beamforming [12]. Recall the communication model in (2.7) and observe that it transmits a vector with symbols at each time step. Assume that the transmitter only wants to transmit a complex valued scalar function $s(t)$ with $E\{|s(t)|^2\} = 1$. Since there are n_T transmit antennas it is possible for the transmitter to choose how $s(t)$ should be transmitted over them. The basic solution would be to just transmit the same signal over all antennas, i.e., $\mathbf{s}(t) = [s(t) \dots s(t)]^T / \sqrt{n_T}$.

Since the antennas in the array have different spatial positions they will create constructive interference in some angular direction and destructive interference in other. This kind of uncontrolled interference is in general bad, but if the transmitter can control the interference it can create constructive interference in the direction of the receiver and use the destructive pattern to reduce the disturbance on other communication links. This strategy is known as *transmit beamforming* and is represented by the beamforming vector $\mathbf{w}_T$, which is chosen to have unit norm ($\|\mathbf{w}_T\|^2 = 1$) so it will not affect the total transmitted signal power. If the available transmission power is P , then the signal model in (2.7) may be expressed as

$$\mathbf{r}(t) = \sqrt{P} \mathbf{H} \mathbf{w}_T s(t) + \mathbf{n}(t). \quad (2.9)$$

Since the transmitted signal is a (complex valued) scalar function, then the desired received signal will most likely also be a (complex valued) scalar function $r(t)$. The

2.4. SIGNAL-TO-NOISE RATIO

receiver has n_R antennas and will receive one signal per antenna that may be used to determine $r(t)$. When considering *receive beamforming* these signals are combined linearly as a weighted sum and may be represented by the vector $\mathbf{w}_R$, which satisfies $\|\mathbf{w}_R\|^2 = 1$. The beamforming vector should be used to, in some sense, maximize to quality of $r(t)$. Using receive beamforming (2.9) becomes

$$r(t) = \sqrt{P}\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T s(t) + \mathbf{w}_R^H \mathbf{n}(t), \quad (2.10)$$

where P is the average transmission power, the channel matrix $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ has independent rows that are $\mathcal{CN}(\mathbf{0}, \mathbf{R}_h)$ and $\mathbf{w}_R^H \mathbf{n}(t) \in \mathcal{CN}(0, N_0)$. Observe that this is an expression with a single input $s(t)$ and single output $r(t)$. Transmit and receive beamforming transforms, in other words, a MIMO system into a SISO system. Since the approach only exploits a single mode of the channel matrix $\mathbf{H}$, beamforming will be especially beneficial when the matrix has a single dominating singular value. Let $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ be the singular value decomposition, then $\mathbf{w}_T$ and $\mathbf{w}_R$ should be chosen as the columns of $\mathbf{V}$ and $\mathbf{U}$ that corresponds to the largest singular value.

Since the purpose of transmit beamforming is to concentrate the transmission in the direction of the receiver, it would be possible to simultaneously transmit to other users located in other directions. This property is exploited in Space Division Multiple Access (SDMA), as described in Section 5.1.

2.4 Signal-to-noise ratio

The signal-to-noise ratio (SNR) is an important quality measure of a communication system (as will be shown in the next section). First consider the case where the channel matrix $\mathbf{H}$ is known. Using the communication model in (2.10), the SNR for a specific channel realization when averaging over the noise realizations (since they will change rapidly as the channel remains almost constant) and the transmitted symbols may be expressed as

$$\text{SNR} = \frac{E\{|\sqrt{P}\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T s(t)|^2\}}{E\{|\mathbf{w}_R^H \mathbf{n}(t)|^2\}} = P \frac{|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2}{E\{|\tilde{n}(t)|^2\}} = \frac{P}{N_0} |\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2, \quad (2.11)$$

where the received noise $\tilde{n}(t)$ has the same distribution as the elements in $\mathbf{n}(t)$. When choosing transmit and receive beamformer, the optimal goal throughout the next three chapters would be to maximize (2.11). If the channel realization is unknown to either the transmitter or the receiver, then it may be better to aim at maximizing the mean SNR. The resulting expression will be

$$\text{SNR} = \frac{P}{N_0} E\{|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2\}, \quad (2.12)$$

where the expectation should be conditioned on all available information.

2.5 Channel capacity

The maximum performance of a communication system may be studied using the *channel capacity* [13]. The capacity is defined as the maximum number of random bits that can be transferred per channel access with an arbitrary low error probability. The Channel Coding Theorem states that there exists at least one code of rate $R \leq C$ which can provide error-free transmission if the code blocks are arbitrarily long. For $R > C$ it can be shown that it is impossible to get a completely error-free transmission. The capacity should be seen as an upper bound for the communication rate that cannot be reached in practice, due to idealized assumption such that perfect channel knowledge and a memoryless channel ($\mathbf{H}$ independent in time).

The capacity of the complex AWGN channel considered in this thesis may be seen as two independent and parallel Gaussian channels. According to [9] the channel capacity for a fixed (deterministic) channel realization will be

$$C = \log_2 (\text{SNR} + 1) = \log_2 \left(\frac{P}{N_0} |\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2 + 1 \right). \quad (2.13)$$

2.5.1 Capacity outage probability

The capacity is the theoretical maximum advisable communication rate. It will be assumed throughout the thesis that the maximum rate $R_{\max}$ is equal to the capacity, i.e., $C = R_{\max}$. This assumption will however only be used when comparing the throughput of different communication strategies, so there is no loss generality in the analysis. The rate function $R(\text{SNR}) = \log_2(\text{SNR} + 1)$ may be replaced by any strictly increasing function without affecting the order of performance in simulations.

It would be advantageous in terms of system throughput to transmit with a rate very close to the current capacity, but since the current SNR in general is unknown to the transmitter it is difficult to determine what rate R to use. With the channel model presented in this chapter there is a positive probability that the gain of $\mathbf{H}$ is very weak, the only rate that is guaranteed to work for all channel realizations is $R = 0$. Therefore the rate will be time-varying.

The current maximum rate may be estimated at the transmitter using an SNR estimate. It is however important to also consider the precision of such an estimation and make necessary adjustment so that the probability of using a rate $\hat{R}$ that is larger than the maximum rate R is very small. The probability of doing such an error is known as the *outage probability*. Let the estimated SNR be denoted $\widehat{\text{SNR}}$, then the estimated rate can be expressed as

$$\hat{R} = \log_2 (\widehat{\text{SNR}} + 1).$$

This estimated rate will in general have a quite large probability of being larger than the currently maximum rate, which should be avoided. The *outage probability* is defined as the probability of such an over-estimation.

The SNR estimation may be modified to satisfy a specific outage probability $\epsilon > 0$. Let the maximum rate given the channel realization be denoted R and

2.6. CHANNEL INFORMATION

assume that the estimation is modified by division by a factor γ . This factor should be determined such that

$$\underbrace{\mathbb{P}\left\{\log_2\left(\frac{\widehat{\text{SNR}}}{\gamma} + 1\right) > R\right\}}_{=\tilde{R}} = \mathbb{P}\left\{\frac{\widehat{\text{SNR}}}{\gamma} > \text{SNR}\right\} < \epsilon.$$

The factor $\gamma > 1$ may typically depend on the statistics of the estimation and may also be a function of $\widehat{\text{SNR}}$. Different strategies of choosing and estimating γ will be considered in the thesis.

In this section it has been assumed that the rate used for communication may take any real value. A hardware implemented system will however only support a limited set of rates corresponding to different signaling schemes (like QAM of different sizes). The set should be known in advance to both the transmitter and the receiver. When the transmitter wants to communicate at a certain rate it chooses the closest smaller rate in the set. Throughout this thesis it will however be assumed that the system supports any rate since focus will be on comparing the performance for an arbitrary set of rates.

2.6 Channel information

This section will discuss to what extent the transmitter and the receiver may estimate the channel and other model parameters, and what amount of feedback that may be used to enhance the estimation. Channel information is crucial for exploiting spatial and multiuser diversity. In order to use beamforming to maximize the throughput, the transmitter and the receiver needs to have some information about the location of the other part. In a multiuser system the base station needs the channel strength to each user in order to schedule users in time and frequency when they are experiencing strong realizations.

2.6.1 Estimation of channel information

If the receiver manages to decode the received signal correctly it may extract two kinds of information. The residual after subtracting the estimated signal part may be used to adaptively estimate the average noise power. The receiver can also easily estimate the overall channel gain $\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T$, which plays an important role in the SNR expression in (2.11).

It is however hard for the receiver to estimate the channel $\mathbf{H}$, since the experienced channel at the receiver will be $\mathbf{H} \mathbf{w}_T$. Even if the transmit beamformer would be known to the receiver, there needs to be a set of different beamformers (during the coherence time of the channel) that together excites all n_T signaling dimensions. This will in general not happen if the beamformers are used to maximize the data throughput. It is therefore a common procedure to construct a *pilot sequence* that can be used for proper channel estimation [14]. This sequence should be known in

advance to both the transmitter and the receiver and may be transmitted whenever partial or full information of the channel realization is needed and previous estimates are regarded as being out-of-date.

A basic pilot sequence can be constructed by fixing the transmitted signal $s(t) = 1$ and transmit it with n_T orthogonal beamformers for $t = 0 \dots n_T - 1$. Consider a MISO channel described by $\mathbf{h}$ and let the beamformers be chosen as the columns of a unitary matrix $\mathbf{U} \in \mathbb{C}^{n_T \times n_T}$, then the received sequence can be expressed as

$$[r(0) \dots r(n_T - 1)] = \sqrt{P} \mathbf{h}^H \mathbf{U} + [n(0) \dots n(n_T - 1)].$$

Hence, the channel vector may be estimated as

$$\hat{\mathbf{h}} = \frac{1}{\sqrt{P}} \mathbf{U} [r(0) \dots r(n_T - 1)]^H,$$

given that $\mathbf{h}$ is approximately constant for $t = 0 \dots n_T - 1$. The use of pilot sequences will of course reduce the overall data throughput, but they have the advantage of only affecting the communication strategy when the pilot is sent. At all other times, transmit beamforming may be used to maximize the received signal energy.

2.6.2 Assumptions

This section will discuss and state assumptions regarding the information about the channel and related variables. For the ease of analysis, it will be assumed throughout the thesis that variables that are easy to estimate will be known without any estimation error.

The transmission power P will naturally be known to the transmitter without estimation. The spectral density of the noise, N_0 , is assumed to be known to both the transmitter and the receiver since it may easily be estimated in by either listen to a silent channel or using the residual in estimations of other parameters. As mentioned in Section 2.2 the spectral density will vary slowly compared to the noise realizations, so it will be easy to estimate it adaptively.

Based on the discussion in the previous section, it is reasonable to assume that both the base station and the mobiles may estimate their received channel realization $\mathbf{H}$ by using a pilot sequence. All usage of channel state information will however increase the computational load on the receiver (which is a problem for mobiles since they are powered with batteries) and each used for the pilot is a symbol that may have carried data. Since the receiver can estimate the instantaneous channel realization, it will be easy for it also estimate the channel statistics, i.e. the covariance matrix $\mathbf{R}_h$, since it changes slowly compared to channel realization. Hence it will be assumed that receiver has knowledge of both the current realization and the statistics. This is known as having full *channel state information at the receiver* (full CSI-R).

The next question is how much channel information that may be estimated by the transmitter. The transmitter cannot measure the channel without information

2.6. CHANNEL INFORMATION

transferred in the opposite direction. There are two common methods of combining uplink and downlink communication on a given frequency band. In Time Division Duplex (TDD) both use the whole bandwidth and can therefore not transmit at the same time. In Frequency Division Duplex (FDD) the bandwidth is divided so that the uplink and downlink have their own bands. With both techniques the channel realizations will in general be independent between the uplink and downlink, but the channel statistics will in many situations still be the same. This is true for TDD since the same channel is used in both direction, at least provided that the time division is faster than the variations in statistics. In FDD the channels are not identical, but if the distance in frequency between the bands is small compared to the carrier frequency it can be argued that the statistics still will be approximately the same [12]. Hence, the channel statistics is assumed to be known at the transmitter.

It is however important to note that the transmitter cannot achieve reliable knowledge of the current realization $\mathbf{H}$ without getting feedback information from the receiver. This is especially true in FDD systems and when the channel realization varies much faster than the time division in TDD systems. Assume that there exists a feedback link for such feedback information which, for simplicity, is error-free. All usage of feedback will reduce the channel availability for data transmission, so it is only reasonable to have a limited feedback strategy which can provide the transmitter with partial channel state information.

There are several types of limited feedback strategies described in literature. The channel matrix may be quantized and the index of the member closest to the current channel realization (in some metric) may be fed back, as discussed in [15]. Many communication strategies don't exploit the entire channel matrix at the transmitter. Then it may be more efficient, in terms of quantization error, to quantize, e.g., the optimal beamforming vector. Such limited feedback are considered in [16] (randomly generated codebook) and [17] (maximum minimum distance between codewords). There are also semi-limited strategies that focus on feedback in multiuser systems, like [18] which suggests that each user should feedback its channel norm. The strongest users are then requested to transmit their full CSI.

While several of the limited strategies above face quantization errors, this thesis will instead use feedback to derive MMSE estimates of the channel statistics using channel norm feedback. The strategies that will be proposed in the following chapters are generalized versions of the one in [19], that considers channel norm feedback in a MISO system. These strategies will be compared to the case without feedback and with feedback of the SNR (provided that it may be predicted at the receiver). Both the channel norm and the SNR are real valued and can in general only be represented by infinitely many bits. Therefore some kind of quantization is needed, as in the codebook attempts mentioned above. If the parameter is slowly varying it is possible to have a quite large quantization space and introduce a Markov chain where the parameter value only can jump a limited number of steps between two feedback occasions. Hence the feedback does not contain the parameter value, but only the location of the next value relative the previous value. This idea will not be further developed in the thesis, but is described in [20].

2.7 Summary

This chapter has derived a narrowband MIMO channel model with AWGN. The model is expressed using the complex baseband equivalent and in discrete form, where the input is a communication symbol. Let the number of transmit antennas be n_T and the number of receive antennas be n_R . When beamforming has been introduced, the resulting model is given in (2.10):

$$r(t) = \sqrt{P} \mathbf{w}_R^H \mathbf{H} \mathbf{w}_T s(t) + \mathbf{w}_R^H \mathbf{n}(t), \quad t \in \mathbb{Z}^+, \quad (2.14)$$

where $s(t), r(t) \in \mathbb{C}$ is the transmitted and the received signal, respectively. The average transmission power P and the transmit beamformer $\mathbf{w}_T \in \mathbb{C}^{n_T}$ are known to the transmitter, while the receive beamformer $\mathbf{w}_R \in \mathbb{C}^{n_R}$ is known to receiver. The receiver may also estimate the channel matrix $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$ without error. The channel matrix has independent and identically distributed rows $\mathbf{h}^H \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_h)$, and the statistics may be estimated without error at both the transmitter and the receiver². The received noise is $\mathbf{w}_R^H \mathbf{n}(t) \in \mathcal{CN}(0, N_0)$ and its statistics may also be estimated without error at the transmitter and the receiver.

There are different time scales involved in the model. The transmitted symbol $s(t)$, the received signal $r(t)$ and the noise realization $n(t)$ will change between each time sample and are therefore denoted as functions of time. The channel realization, and thereby the beamformers, changes much more slowly with time which makes it meaningful to estimate. The statistics change even more slowly over time and will therefore be easy to estimate adaptively.

Observe that the beamforming vectors transform the MIMO channel into a SISO channel. It may therefore be analyzed in the regular way. The instantaneous SNR, given in (2.11), may be expressed as

$$\text{SNR} = \frac{P}{N_0} |\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2, \quad (2.15)$$

and the channel capacity of the system will be $C = \log_2(\text{SNR} + 1)$. The capacity limits the maximum advisable communication rate. The transmitter will have difficulties in estimating the instantaneous SNR at transmitter since the exact channel realization is unknown. Therefore the concept of outage probability was defined as the probability of over-estimating the SNR and thereby the maximum rate.

The chapter was concluded by discussions regarding the feedback between receiver and transmitter. It is assumed there is an error-free feedback link that may transmit a limited amount of information from receiver to transmitter. The thesis will analyze feedback strategies based on channel norm information and compare their performance with strategies without feedback and where the receiver controls the beamformer of the transmitter.

²The channel realization and statistics are known to receiver, so the system is said to have full channel state information at the receiver (full CSI-R). The transmitter has only knowledge of the statistics and is said to have partial channel state information (partial CSI).

Chapter 3

Local scattering channel model

This chapter will describe the model that will be used in the simulations throughout the thesis and motivate the additional assumptions that it is based on. The simulations in later chapters will be used to compare the capacity and the data throughput (with a given outage probability) with different beamforming strategies. Both single user and multiple user scenarios will be considered, as well as environments with and without interference from adjacent cells.

The simulation model is based on the local scattering model, where there are no scattering near the base station, but rich scattering on around and along the way to each mobile. The model will first be described and then used to derive approximative covariance matrices with ULA or UCA at the base station.

3.1 Local scattering model

The simulations are based on the following scattering model, known as the *local scattering model* [5]. The base station is assumed to be fixed at a place elevated from the surroundings, for example placed on the roof top of a building. Hence, there will be no scattering close to the station. The mobiles will on the other hand be able to move around in the environment so the signal between it and the station will be exposed to rich scattering from buildings, vehicles and other objects. Even if the mobile user stands still there will still be objects in the near surrounding that are moving, so the scattering may be assumed to change randomly over time with constant channel statistics. There will be no direct paths between the base station and the mobiles.

In local scattering model, both the base station and the mobile will receive multipath signals that may be described as the sum of signals arriving at different angles from different scatterers. There will however be an important difference in the angular properties between the signals received at the downlink and the uplink. The mobile is surrounded by scatterers so the angle of arrival of the multipath components is assumed to be uniformly distributed. The angular distribution will however be concentrated in the uplink, as outlined in Figure 3.1. The angles are as-

Figure 3.1. Transmission from mobile to base station in an urban environment. There are many scatterers around the mobile, but no scatterers near the base station. The received signal is a sum of signals arriving around a nominal angle θ . The angular spread around θ is modeled as $\mathcal{N}(0, \sigma_a^2)$ and displayed by its standard deviation

summed to be spread as a Gaussian distribution around a line drawn directly between the two terminals. The base station is assumed to be in far-field of the mobile and the scatterers. Hence, the incoming signal from a mobile will be a sum of multiple plane waves (described in Section 1.3). This property will be used when the channel statistics is derived.

When the local scattering model is used in the simulations, the channel realization will be assumed to be independent between each simulated symbol. This will in general be both unreasonable and contra-productive since the usefulness of channel estimations and feedback would disappear. As mentioned previously there will however be a coherence time that defines the time interval needed for the channel to change significantly. The simulated realizations may therefore be seen as samples that have been taken which such a distance that they are approximately independent. The SNR of a realization will represent the mean SNR during the coherence time, since the average noise was used in (2.11). Therefore, the assumption will not degrade the simulation quality, provided that the coherence time is invariant.

3.2 Statistics with the local scattering model

This section will derive an approximative closed form expression for the covariance matrices for ULA and UCA with the local scattering model. Recall that the scatterers are spread so that the reflected signals from the mobile will arrive at the base

3.2. STATISTICS WITH THE LOCAL SCATTERING MODEL

station at angles that are Gaussian distributed around a line drawn from the transmitter to the receiver. It will additionally be assumed that the angular spread $\hat{\theta}_k(t)$ is fairly small, i.e., that the distribution has low variance. Since the mobile antennas are independent it is assumed, without loss of generality, during the derivation that there is only a single transmitting antenna at the mobile.

At time t there are $L(t)$ scatterers around the mobile so the multipath signal may be expressed as

$$\mathbf{r}(t) = \sqrt{P}\mathbf{h}s(t) + \mathbf{n}(t) = \sqrt{\frac{P}{f(r_d)}} \sum_{k=1}^{L(t)} \gamma_k(t) \mathbf{a}(\theta + \tilde{\theta}_k(t)) s(t) + \mathbf{n}(t), \quad (3.1)$$

with θ being the angle of arrival of a transmission directly from the transmitter to the receiver. The factor $\gamma_k(t)$ is the random complex gain and $\theta + \tilde{\theta}_k(t)$ is the angle of arrival of the transmission from the k th reflector, with $\tilde{\theta}_k(t) \in \mathcal{N}(0, \sigma_a^2)$.

The factor $f(r_d)$ describes the transmission power decay, where r_d is the radial distance from the transmitter to the receiver. The simulations will not consider any specific transmission environment (in addition to the local scattering model), so the power decay will be assumed to be inversely proportional to the square radial distance, i.e., $f(r_d) = r_d^2$ for $r_d > 0$. The radial distance is assumed to be measured in some suitable unit such that $r_d = 1$ corresponds to cell boundary (the simulations will only consider cells that are approximately circular).

From (3.1) it may be observed that the channel depends on both the angle of arrival and the radial distance, and that these variables occur in different factors. Hence, two mobiles that are located in the same angular direction θ relative the base station will have proportional properties. If the radial distances to the two mobiles are r_{d1} and r_{d2} , respectively, then their covariance matrices may be expressed as

$$\mathbf{R}_{\mathbf{h}_1} \propto \frac{\tilde{\mathbf{R}}_{\mathbf{h}}(\theta)}{r_{d1}^2} \quad \text{and} \quad \mathbf{R}_{\mathbf{h}_2} \propto \frac{\tilde{\mathbf{R}}_{\mathbf{h}}(\theta)}{r_{d2}^2},$$

where $\tilde{\mathbf{R}}_{\mathbf{h}}(\theta)$ denotes the, in some sense, normalized covariance matrix that contains the joint properties of the mobiles. It will now be shown, using the approach in [6], that this covariance matrix will depend on the array response:

$$\begin{aligned} \tilde{\mathbf{R}}_{\mathbf{h}}(\theta) &\propto E \left\{ \sum_{k=1}^{L(t)} |\gamma_k(t)|^2 \mathbf{a}(\theta + \tilde{\theta}_k(t)) \mathbf{a}^H(\theta + \tilde{\theta}_k(t)) \right\} = \\ &= \sum_{k=1}^{L(t)} E\{|\gamma_k(t)|^2\} \underbrace{E\left\{ \mathbf{a}(\theta + \tilde{\theta}_k(t)) \mathbf{a}^H(\theta + \tilde{\theta}_k(t)) \right\}}_{\stackrel{\text{def}}{=} \mathbf{R}_{\mathbf{a}}(\theta)} \propto \mathbf{R}_{\mathbf{a}}(\theta), \end{aligned}$$

provided that the sum of average channel gain satisfies $\sum_{k=1}^{L(t)} E\{|\gamma_k(t)|^2\} < \infty$ (which it of course will do in practice). Hence, the normalized covariance matrix

may be defined as $\tilde{\mathbf{R}}_{\mathbf{h}}(\theta) = \mathbf{R}_{\mathbf{a}}(\theta)$. The covariance matrices $\mathbf{R}_{\mathbf{a}}^{\text{ULA}}(\theta)$ and $\mathbf{R}_{\mathbf{a}}^{\text{UCA}}(\theta)$ may be approximated analytically using that the variance of the angular spread σ_a^2 typically is small. The complete derivation has been surpassed to Appendix A.2 for the purpose of enhancing the readability of the thesis, but the kh th element of the covariance matrices may (under the given conditions) be expressed as

$$\begin{aligned}\mathbf{R}_{\mathbf{a},kl}^{\text{ULA}}(\theta) &= e^{j\frac{2\pi d}{\lambda_c} \sin(\theta)(k-l)} e^{-\frac{\sigma_a^2}{2} \left(\frac{2\pi d}{\lambda_c} \cos(\theta)(k-l) \right)^2}, \\ \mathbf{R}_{\mathbf{a},kl}^{\text{UCA}}(\theta) &= e^{j\frac{4\pi\alpha}{\lambda_c} \sin(\pi\frac{k-l}{n}) \sin(\pi\frac{k+l-2}{n}-\theta)} e^{-\frac{\sigma_a^2}{2} \left(\frac{4\pi\alpha}{\lambda_c} \sin(\pi\frac{k-l}{n}) \cos(\pi\frac{k+l-2}{n}-\theta) \right)^2},\end{aligned}$$

for ULA and UCA, respectively. These expressions will be used, throughout the thesis, to simulate environments with local scattering where the base station uses either ULA or UCA as antenna structure.

To summarize, the channel covariance matrix $\mathbf{R}_{\mathbf{h}}$ is inversely proportional to the square distance r_d^2 between the mobile and the base station and proportional to the covariance matrix $\mathbf{R}_{\mathbf{a}}(\theta)$, that depends on the angle of arrival and the angular spread. The exact size of the covariance matrix will be determined in each simulation by defining the average SNR on the cell boundary ($r_d = 1$). This quantity may be expressed as $P\mathbf{R}_{\mathbf{h},ii}/N_0$, where $\mathbf{R}_{\mathbf{h},ii}$ is some element on the diagonal of $\mathbf{R}_{\mathbf{h}}$ (they will all be equal).

3.3 Summary

This chapter has presented the local scattering model that will be used purely in simulations throughout the thesis. The model assumes that there are no scatterers close to the base station, while the mobile is surrounded by a rich amount of scatterers. The cells are assumed to be circular in the simulations and distances will be measured in some suitable unit, so that each cell will have unit radius.

An important consequence is shown in Figure 3.1, namely that the multipath signal received at the base station will have limited angular spread. By modeling this spread as Gaussian distributed it was observed that the channel covariance matrix will be proportional to the power decay and a covariance matrix that depends purely on the antenna structure and the variance of the angular spread. Such covariance matrices were presented for cases with either ULA or UCA at the base station. The exact size of the covariance matrix will be determined in each simulation by defining the average SNR on the cell boundary.

The covariance matrix will play an important role in the downlink beamforming that will be considered in the following chapters. The matrix contains the average spatial properties of the channel and may therefore be exploited by the base station to direct its beamformer in the direction that gives the largest average SNR.

Chapter 4

Single-user MIMO downlink

This chapter will consider the downlink of a MIMO communication system with a single base station and a single mobile user. The analysis will be based on the assumptions summarized in Section 2.7. The aim of this chapter is to derive optimal beamforming strategies (in the sense of mean SNR maximization) for different types of channel norm feedback. The derivations will focus on deriving unbiased SNR estimations conditioned on the feedback information and study their statistics. The strategies will be compared to the upper bound (derived with full CSI at both the transmitter and the receiver) in simulations that both consider the mean SNR capacity and the real performance for a specific capacity outage probability.

A practical implementation of the beamforming strategies considered in this chapter may be described schematically as follows. Recall the assumption that the channel realization will be constant for some amount of symbol times and denote the length of this period as a *cycle*. Each cycle will consists of four steps:

- **Pilot transmission.** The base station transmits the pilot sequence.
- **Channel estimation and feedback.** The receiving mobile use the pilot to calculate the feedback information. This information is immediately sent back to the base station.
- **Estimation and beamforming.** The base station uses the feedback information to estimate the SNR and choosing a transmit beamformer that will maximize it. The base station chooses a communication rate that will fulfill the capacity outage probability (given the SNR estimation and its statistics).
- **Data transmission.** The base station uses the remaining symbol times to data transmission. The receiver fixates its receive beamformer such that its SNR estimation will be maximized.

The next section will introduce and analyze the feedback strategies considered in this chapter. Then the mean SNR capacity of these strategies will be compared in a simulation, and finally the problem of modifying the SNR estimation at the transmitter to satisfy a certain outage probability will be addressed. The solution

will involve derivations of the estimation variance. The chapter will be concluded by a performance comparison of the strategies under somewhat fair conditions.

4.1 Optimal beamforming

This section will derive beamforming strategies that maximize the SNR and the communication rate when the base station just transmits a single beam. It is important to note that such strategies will not be optimal from a capacity point of view. The overall capacity would be achieved by transmitting several simultaneous beams and thereby exploiting all spatial dimensions. Throughout the chapter the word 'optimal' will however denote a strategy that maximizes the SNR given that each transmitter only transmits a single beam.

Recall the expressions for the instantaneous SNR in (2.11) and the average SNR in (2.12). Observe that the factor P/N_0 cannot be altered using beamforming and will therefore only affect the scaling of the SNR. Hence, the analysis will without loss in generality focus on maximizing $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$, either the instantaneous value or in mean.

First the optimal beamforming with full CSI at both the transmitter and the receiver will be described. Then the optimal beamforming with full CSI-R will be analyzed for four feedback cases: no feedback, feedback of $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$, feedback of $\|\mathbf{h}_i\|^2$ for all rows of the channel matrix and feedback of $\max_i \|\mathbf{h}_i\|^2$.

4.1.1 The upper bound

An upper bound of the SNR performance of the communication system may be derived by assuming that both the transmitter and the receiver have full channel state information. For a given channel matrix $\mathbf{H}$, the squared norm $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$ should be maximized. The optimal beamformers may be derived using the singular value decomposition of the channel matrix. Let the decomposition be denoted

$$\mathbf{H} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H,$$

where $\mathbf{U} \in \mathbb{C}^{n_R \times n_R}$ and $\mathbf{V} \in \mathbb{C}^{n_T \times n_T}$ are unitary matrices. The matrix $\mathbf{\Sigma} \in \mathbb{C}^{n_R \times n_T}$ has the singular values on the diagonal in a non-increasing order, while the other elements are zero. Hence, the optimal choice of beamformers would be the columns associated with the largest singular value:

$$\mathbf{w}_T = \mathbf{V}_{:,1}, \quad \mathbf{w}_R = \mathbf{U}_{:,1}. \quad (4.1)$$

This choice satisfies the unit norm constraint and under other norm constraints the beamformers $\mathbf{w}_T$ and $\mathbf{w}_R$ needs only to be scaled properly. Using the beamformers in (4.1), the maximum value of the channel norm will be the largest singular value of $\mathbf{H}$ and may be expressed as

$$|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2 = \|\mathbf{H}\|_2^2,$$

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where $\|\cdot\|_2$ denotes the spectral norm. Observe that the choice of transmit beamformer fulfills

$$\mathbf{H}\mathbf{w}_T = \mathbf{U}\boldsymbol{\Sigma}_{:,1} = \mathbf{U}_{:,1}\|\mathbf{H}\|_2,$$

and since $\mathbf{U}$ is unitary this leads to the conclusion that

$$\mathbf{w}_R = \mathbf{U}_{:,1} = \frac{\mathbf{H}\mathbf{w}_T}{\|\mathbf{H}\|_2} = \frac{\mathbf{H}\mathbf{w}_T}{\|\mathbf{H}\mathbf{w}_T\|}. \quad (4.2)$$

Hence, the receive beamformer will just be vector that matches the experienced channel $\mathbf{H}\mathbf{w}_T$ by pointing in the same direction. The received signal power therefore also be expressed as

$$|\mathbf{w}_R^H \mathbf{H}\mathbf{w}_T|^2 = \left(\frac{(\mathbf{H}\mathbf{w}_T)^H \mathbf{H}\mathbf{w}_T}{\|\mathbf{H}\mathbf{w}_T\|} \right)^2 = \|\mathbf{H}\mathbf{w}_T\|^2.$$

To summarize, an upper bound to the SNR maximizing beamforming has been derived by assuming full CSI at both the transmitter and the receiver. The optimal transmit and receive beamformer are given in (4.1) and the corresponding SNR can be expressed as

$$\text{SNR} = \frac{P}{N_0} \|\mathbf{H}\mathbf{w}_T\|^2 = \frac{P}{N_0} \|\mathbf{H}\|_2^2. \quad (4.3)$$

4.1.2 Optimal beamforming with full CSI-R without feedback

The communication model considered in the thesis has only full CSI-R, in contrary to the upper bound which was based on full CSI at both the receiver and the transmitter. The transmitter will however have knowledge of the channel statistics.

Since the receive beamformer cannot introduce any new energy to the signal, the best transmission strategy for the transmitter would be to maximize the total received signal power $\|\sqrt{P}\mathbf{H}\mathbf{w}_T\|^2$ over the receive antennas. In this case without feedback the transmitter has no knowledge the current realization, but it may maximize the average received signal power $E\{\|\sqrt{P}\mathbf{H}\mathbf{w}_T\|^2\}$ by choosing $\mathbf{w}_T$ properly. Recall that the rows of the channel $\mathbf{H}$ may be expressed $\mathbf{h}_i^H \in \mathcal{CN}(0, \mathbf{R}_h)$. Then

$$\begin{aligned} E\{\|\sqrt{P}\mathbf{H}\mathbf{w}_T\|^2\} &= E\left\{P \sum_{i=1}^{n_R} |\mathbf{h}_i^H \mathbf{w}_T|^2\right\} = P \sum_{i=1}^{n_R} \mathbf{w}_T^H E\{\mathbf{h}_i \mathbf{h}_i^H\} \mathbf{w}_T \\ &= P n_R \mathbf{w}_T^H \mathbf{R}_h \mathbf{w}_T. \end{aligned} \quad (4.4)$$

Observe that all covariance matrices are hermitian and let the eigenvalue decomposition be

$$\mathbf{R}_h = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H,$$

where $\boldsymbol{\Lambda}$ is a diagonal matrix with the eigenvalues $\lambda_1 \dots \lambda_{n_T}$ ordered in non-increasing order and $\mathbf{U} \in \mathbb{C}^{n_T \times n_T}$ is a unitary matrix with the corresponding

eigenvectors as columns. Hence, the expression in (4.4) will be maximized when $\mathbf{w}_T$ is chosen as the eigenvector belonging to the eigenvalue with largest norm, i.e.,

$$\mathbf{w}_T = \mathbf{U}_{:,1}. \quad (4.5)$$

This strategy is known as *eigenbeamforming* [21]. The choice of transmit beamformer makes $\mathbf{w}_T^H \mathbf{R}_h \mathbf{w}_T = \lambda_1$ in (4.4).

The next question is how the receiver should choose its beamformer optimally. The received signal over the antenna array will be $\mathbf{H}\mathbf{w}_T s(t) + \mathbf{n}(t)$ and the direction of $\mathbf{H}\mathbf{w}_T$ may easily be estimated at the receiver (either directly or via estimation of $\mathbf{H}$). In order to maximize $|\mathbf{w}_R^H \mathbf{H}\mathbf{w}_T|^2$, the receive beamforming vector should be chosen parallel to $\mathbf{H}\mathbf{w}_T$:

$$\mathbf{w}_R = \frac{\mathbf{H}\mathbf{w}_T}{\|\mathbf{H}\mathbf{w}_T\|}. \quad (4.6)$$

Hence, the average received signal power may be expressed as

$$E\{|\mathbf{w}_R^H \mathbf{H}\mathbf{w}_T|^2\} = E\left\{\left(\frac{(\mathbf{H}\mathbf{w}_T)^H \mathbf{H}\mathbf{w}_T}{\|\mathbf{H}\mathbf{w}_T\|}\right)^2\right\} = E\{\|\mathbf{H}\mathbf{w}_T\|^2\}.$$

To summarize, the optimal transmit and receive beamformer with full CSI-R is given by (4.5) and (4.6), respectively. The corresponding SNR may be expressed as

$$\text{SNR} = \frac{P}{N_0} E\{\|\mathbf{H}\mathbf{w}_T\|^2\} = \frac{P}{N_0} n_R \lambda_1. \quad (4.7)$$

Observe the similarities with the expressions in (4.2) and (4.3) for the upper bound.

4.1.3 Optimal beamforming with full CSI-R and $|\mathbf{w}_R^H \mathbf{H}\mathbf{w}_T|^2$ feedback

There is an important drawback of with beamforming strategy derived in the previous section that should be apparent if it would be used in practice. The transmit beamformer was used to maximize the average SNR, but since there are no feedback information the transmitter will never have any knowledge of the instantaneous SNR. This is a large problem since the instantaneous SNR controls the channel capacity. The lack of a proper estimate of the instantaneous SNR will significantly reduce the data throughput when considering a specific outage probability, as will be shown in the end of the chapter.

The problem of SNR estimation at the transmitter may be overcome by introducing some kind of feedback. This section will consider feedback of $\varrho = |\mathbf{w}_R^H \mathbf{H}\mathbf{w}_T|^2$. This information may easily be used to calculate the instantaneous SNR, see (2.11). The receiver cannot, without additional feedback, persuade the transmitter to choose a specific beamformer so the transmit beamformer $\mathbf{w}_T$ will be chosen independently of ϱ , i.e., in exactly the same way as in (4.5). As a consequence the optimal receive beamformer will still be given by (4.6). The only difference compared to the case without feedback is that the current SNR will now be known to the transmitter.

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It will be shown in simulations that this method performs well. It would in fact be difficult to find a more effective strategy that only includes feedback of a single variable. The feedback strategy will however be suboptimal in situations where the transmit beamformer cannot be predicted by the receiver. This may for example be the case when the transmitter uses the channel information from several users to transmit several simultaneous beams (as in SDMA, see Section 5.1). This will motivate the study of other feedback strategies in the thesis.

4.1.4 Optimal beamforming with full CSI-R and $\|\mathbf{h}_i\|^2, \forall i$, feedback

This beamforming strategy is based on feedback of the squared norm $\rho_i = \|\mathbf{h}_i\|^2$ of the current channel realization to each receive antenna, $i = 1 \dots n_R$. This is in other words the squared norm of each row in the channel matrix $\mathbf{H}$. This strategy has been analyzed in [19] and [8] for the special case with one receive antenna, but as will be shown here their results can easily be extended for multiple receive antennas. Observe that the feedback will be of n_R variables, which may be a major drawback if n_R is large (this will motivate the strategy considered in the next section).

Let the feedback information be described by the vector $\boldsymbol{\rho} = [\rho_1 \dots \rho_{n_R}]$. In order to maximize the average SNR the transmit beamformer $\mathbf{w}_T$ should be chosen to maximize the total expected received signal power $E\{\|\sqrt{P}\mathbf{H}\mathbf{w}_T\|^2|\boldsymbol{\rho}\}$ over the receive antenna array. This expression may be simplified as

$$\begin{aligned} E\left\{\|\sqrt{P}\mathbf{H}\mathbf{w}_T\|^2|\boldsymbol{\rho}\right\} &= E\left\{P\|\mathbf{H}\mathbf{w}_T\|^2|\boldsymbol{\rho}\right\} = E\left\{P\sum_{k=1}^{n_R}|\mathbf{h}_k^H\mathbf{w}_T|^2|\boldsymbol{\rho}\right\} \\ &= P\mathbf{w}_T^H\left(\sum_{k=1}^{n_R}E\{\mathbf{h}_k\mathbf{h}_k^H|\rho_k\}\right)\mathbf{w}_T. \end{aligned} \quad (4.8)$$

The expression includes a sum of channel covariance matrices conditioned on the squared channel norm to each receive antenna. A closed form expression for these covariance matrices are given as a theorem in [19] and will now be described. Let $\mathbf{R}_h = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ be the eigenvalue decomposition of the channel covariance matrix. The matrix $\mathbf{\Lambda}$ is diagonal with the eigenvalues $\lambda_1 \dots \lambda_{n_T}$ ordered with strictly decreasing norm. The derivation in [19] requires that $\lambda_i \neq \lambda_j, i \neq j$, but this is not a much of a limitation in real applications since the eigenvalues will be random real valued variables. The conditional channel covariance matrix is given by

$$\hat{\mathbf{R}}_h(\rho) = E\left\{\mathbf{h}\mathbf{h}^H|\rho = \|\mathbf{h}\|^2\right\} = \mathbf{U}\hat{\mathbf{\Lambda}}(\rho)\mathbf{U}^H, \quad (4.9)$$

where $\hat{\mathbf{\Lambda}}(\rho)$ is a diagonal matrix with the eigenvalues $\hat{\lambda}_m(\rho)$ calculated from the original eigenvalues using

$$\hat{\lambda}_m(\rho) = \frac{1}{f_{\|\mathbf{h}\|^2}(\rho)} \left[\frac{\rho e^{-\frac{\rho}{\lambda_m}}}{\lambda_m \prod_{i \neq m} \left(1 - \frac{\lambda_i}{\lambda_m}\right)} + \sum_{k \neq m} \frac{e^{-\frac{\rho}{\lambda_m}} - e^{-\frac{\rho}{\lambda_k}}}{\left(1 - \frac{\lambda_k}{\lambda_m}\right) \prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)} \right], \quad (4.10)$$

where $f_{\|\mathbf{h}\|^2}(\rho)$ is the PDF of $\|\mathbf{h}\|^2$ given by

$$f_{\|\mathbf{h}\|^2}(\rho) = \sum_{k=1}^{n_T} \frac{e^{-\frac{\rho}{\lambda_k}}}{\lambda_k \prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)}. \quad (4.11)$$

Using this, the SNR in (4.8) may be expressed as

$$P \mathbf{w}_T^H \left(\sum_{i=1}^{n_R} E\{\mathbf{h}_i \mathbf{h}_i^H | \rho_i\} \right) \mathbf{w}_T = P \mathbf{w}_T^H \mathbf{U} \left(\sum_{i=1}^{n_R} \hat{\Lambda}(\rho_i) \right) \mathbf{U}^H \mathbf{w}_T \quad (4.12)$$

and as a consequence the optimal transmit beamformer is the eigenvector i corresponding to the largest sum of conditional eigenvalues:

$$\mathbf{w}_T = \mathbf{U}_{\cdot, \mathbf{k}}, \text{ where } k = \arg \max_m \sum_{i=1}^{n_R} \hat{\lambda}_m(\rho_i). \quad (4.13)$$

By the same reason as in the previous sections the optimal receive beamformer is given by

$$\mathbf{w}_R = \frac{\mathbf{H} \mathbf{w}_T}{\|\mathbf{H} \mathbf{w}_T\|}. \quad (4.14)$$

To summarize, the optimal transmit and receive beamformer with full CSI-R and feedback of the squared norm of the channel realization to each receive antenna is given by (4.13) and (4.14), respectively. The corresponding mean SNR may be expressed as

$$\text{SNR} = \frac{P}{N_0} E\{\|\mathbf{H} \mathbf{w}_T\|^2 | \boldsymbol{\rho}\} = \frac{P}{N_0} \sum_{i=1}^{n_R} \hat{\lambda}_k(\rho_i), \quad (4.15)$$

with k chosen such that the sum is maximized. Once again, the SNR will not be known to the transmitter when choosing the communication rate. It will however be shown that the feedback information also is sufficient to calculate a good SNR estimate.

4.1.5 Optimal beamforming with full CSI-R and $\max_i \|\mathbf{h}_i\|^2$ feedback

There is a main difference between two previous beamforming strategies, namely the amount of feedback information. The first method exploits $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$, which is a single real valued variable, while the second method feedback the set $\boldsymbol{\rho} = [\rho_1 \dots \rho_{n_R}]$, $\rho_i = \|\mathbf{h}_i\|^2$ that contain n_R real valued variables. Therefore a reduced version of the second strategy will also be considered in this section. It will be based on feedback of $\max_i \|\mathbf{h}_i\|^2$ and this choice may be motivated as follows.

Consider the special case where the receiver uses *antenna selection* instead of optimally combining them with receive beamforming, i.e., the receiver only considers the signal received on the antenna with the currently highest SNR:

$$\max_i \text{SNR}_i = \max_i \frac{P}{N_0} |\mathbf{h}_i^H \mathbf{w}_T|^2.$$

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Now assume that the transmitter get $\boldsymbol{\rho} = [\rho_1 \dots \rho_{n_R}]$ as feedback information, as in the previous strategy. Recall that all rows of $\mathbf{H}$ are identically distributed and let $\hat{\mathbf{R}}_{\mathbf{h}}(\rho_i) = \mathbf{U}\hat{\mathbf{\Lambda}}(\rho_i)\mathbf{U}^H$ denote the eigenvalue decomposition of the conditional covariance matrix. In order to achieve maximum average link quality the transmit beamformer should be chosen such that $\max_i E\{\text{SNR}_i|\boldsymbol{\rho}\}$ is maximized:

$$\begin{aligned} \max_i E\{\text{SNR}_i|\boldsymbol{\rho}\} &= \max_i \frac{P}{N_0} E\{|\mathbf{h}_i^H \mathbf{w}_T|^2|\boldsymbol{\rho}\} \\ &= \max_i \frac{P}{N_0} \mathbf{w}_T^H \hat{\mathbf{R}}_{\mathbf{h}}(\rho_i) \mathbf{w}_T = \max_i \frac{P}{N_0} \mathbf{w}_T^H \mathbf{U} \hat{\mathbf{\Lambda}}(\rho_i) \mathbf{U}^H \mathbf{w}_T. \end{aligned} \quad (4.16)$$

It is clear that $\mathbf{w}_T$ should be chosen as one of the eigenvectors. It is intuitive that the conditional eigenvector expression in (4.10) is an increasing function in ρ , and it may actually be proved analytically using that the distribution is log-concave. Since the rows of $\mathbf{H}$ are identically distributed, the largest conditional eigenvalue will always appear at row $j = \arg \max_i \rho_i$. Hence, all feedback information in $\boldsymbol{\rho}$ except $\rho_j = \max_i \|\mathbf{h}_i\|^2$ is unnecessary.

This argument somewhat justifies the choice of $\max_i \|\mathbf{h}_i\|^2$ as feedback information. Antenna selection may be used as an alternative to beamforming to reduce the computational demands on the receiver, but it will inevitably result in a lower capacity since the information received on the other antennas is just discarded. Therefore the general case with both transmit and receive beamforming will be considered, which also makes the performance of this feedback strategy more comparable with the previous strategies.

First, observe that the ordering of the rows in $\mathbf{H}$ has never been used in the analysis in the thesis. Hence the transmitter may, without loss of generality, assume that $\max_i \|\mathbf{h}_i\|^2 = \rho_1$. Then the conditional eigenvalues for the covariance matrix of this row may be calculated according to (4.10). The next important observation is that even if $\rho_i, i = 2 \dots n_R$, is unknown to the transmitter it is known that $0 \leq \rho_i \leq \rho_1$. Therefore the eigenvalues of the covariance matrices of these rows be conditioned on this information. Let the conditional covariance matrix of these rows be denoted

$$\hat{\mathbf{R}}_{\mathbf{h}}(0 \leq \rho_i \leq \rho_1) = E\left\{\mathbf{h}\mathbf{h}^H \middle| 0 \leq \rho_i \leq \rho_1\right\} = \mathbf{U}\hat{\mathbf{\Lambda}}(0 \leq \rho_i \leq \rho_1)\mathbf{U}^H, \quad (4.17)$$

where $\hat{\mathbf{\Lambda}}(0 \leq \rho_i \leq \rho_1)$ is a diagonal matrix with the conditional eigenvalues denoted $\hat{\lambda}_m(0 \leq \rho_i \leq \rho_1), m = 1 \dots n_T$. These may be expressed as

$$\begin{aligned} \hat{\lambda}_m(0 \leq \rho_i \leq \rho_1) &= E\{\hat{\lambda}_m(\rho_i) | 0 \leq \rho_i \leq \rho_1\} \\ &= \frac{1}{\int_0^{\rho_1} f_{\|\mathbf{h}\|^2}(\varrho) d\varrho} \times \\ &\quad \left[\lambda_m - \frac{(\lambda_m + \rho_1)e^{-\frac{\rho_1}{\lambda_m}}}{\prod_{i \neq m} \left(1 - \frac{\lambda_i}{\lambda_m}\right)} + \sum_{k \neq m} \frac{\lambda_k e^{-\frac{\rho_1}{\lambda_k}} - \lambda_m e^{-\frac{\rho_1}{\lambda_m}}}{\left(1 - \frac{\lambda_k}{\lambda_m}\right) \prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)} \right], \end{aligned} \quad (4.18)$$

with the normalization factor

$$\int_0^{\rho_1} f_{\|\mathbf{h}\|^2}(\varrho) d\varrho = 1 - \sum_{k=1}^{n_T} \frac{e^{\rho_1/\lambda_k}}{\prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)}. \quad (4.19)$$

These expressions should be seen as contributions made by the thesis, but the complete derivation has been surpassed to Appendix A.3.1 for the purpose of enhancing the readability of the thesis.

When the conditional eigenvalues are known, the transmit beamformer should be chosen as the eigenvector k that maximizes

$$\frac{P}{N_0} E\{\|\mathbf{H}\mathbf{w}_T\|^2 | \rho_1 = \max_i \|\mathbf{h}_i\|^2\} = \frac{P}{N_0} \left(\hat{\lambda}_k(\rho) + (n_R - 1) \hat{\lambda}_k(0 \leq \rho_i \leq \rho_1) \right),$$

which may be expressed as

$$\mathbf{w}_T = \mathbf{U}_{\cdot k}, \text{ where } k = \arg \max_m \hat{\lambda}_m(\rho) + (n_R - 1) \hat{\lambda}_m(0 \leq \rho_i \leq \rho_1). \quad (4.20)$$

By the same reason as in the previous sections the optimal receive beamformer is given by

$$\mathbf{w}_R = \frac{\mathbf{H}\mathbf{w}_T}{\|\mathbf{H}\mathbf{w}_T\|}. \quad (4.21)$$

To summarize, the optimal transmit and receive beamformer with full CSI-R and feedback of the largest squared norm of the channel realizations to the receive antennas is given by (4.20) and (4.21), respectively. The corresponding mean SNR may expressed as

$$\text{SNR} = \frac{P}{N_0} E\{\|\mathbf{H}\mathbf{w}_T\|^2 | \max_i \|\mathbf{h}_i\|^2\} = \frac{P}{N_0} \left(\hat{\lambda}_k(\rho) + (n_R - 1) \hat{\lambda}_k(0 \leq \rho_i \leq \rho_1) \right) \quad (4.22)$$

with k chosen such that the sum is maximized. The second term may be seen as the gains in average performance of using receive beamforming instead of antenna selection.

4.2 Mean SNR capacity simulation

The mean instantaneous SNR with the some different beamforming strategies will be evaluated in this simulation. Observe that the result of the simulation should not be confused with the actual mean performance, which demands the instantaneous SNR to be estimated at the transmitter. This simulation will in other words show the upper bound on the capability of the strategies. The actual performance (for a given outage probability) will be simulated in the end of the chapter.

The simulation was performed with two different transmit antenna structures: ULA and UCA. In both cases the number of transmit antennas, as well as the number of receive antennas, was fixed at four. The distance between adjacent

4.2. MEAN SNR CAPACITY SIMULATION

antenna elements at the transmitter was $\lambda_c/2$, where λ_c is the carrier wavelength. This gives $d = \lambda_c/2$ for ULA and the radius to be $\alpha = \lambda_c/(2\sqrt{2})$ for UCA. In both cases the ratio between transmission power and noise density was chosen such that $P\mathbf{R}_{\mathbf{h},ii}/N_0 = 1$ on the cell boundary, following the notation given in Section 3.2.

The simulation considers a single mobile at the cell boundary that is moving in the angular direction, while the transmitter is fixed. The angles were chosen uniformly in the interval $0 \leq \theta \leq \pi$, which due to the array symmetry contains all information. The standard deviation of the angular spread was 15 degrees. The performance will be measured in terms of instantaneous mean SNR over 50,000 realizations per angle of transmission.

The five beamforming strategies described in the previous section will be evaluated in the simulation. In addition to that the following will also be considered: The first strategy chooses the transmit beamformer randomly as a vector with uncorrelated variables that are $\mathcal{CN}(0, 1)$. The vector is then normalized such that $\|\mathbf{w}_T\|^2 = 1$ is satisfied. The other strategy will just fixate the transmit beamformer in the direction $\mathbf{w}_T = [1 \dots 1]^T / \sqrt{n_T}$. In both these case the receiver will, as in all the other cases, chose its beamformer according to the expression in (4.6).

4.2.1 Result

The simulation results are shown Figure 4.1, where (a) shows the mean SNR capacity over the 10% strongest channel realizations (in terms of $\|\mathbf{H}\|_F$) and (b) shows the mean SNR capacity over all realizations.

It was expected that the case with feedback of $\|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T\|^2$ should perform equally with the method without feedback, since the feedback only affects the SNR knowledge at the transmitter. It is however interesting that also the two other feedback strategies give exactly the same result, realization for realization, as the case without feedback. This is not true in general, but [8] states that it is true for strong channel realizations. The explanation in this simulation is probably that the largest eigenvalue λ_1 of the channel covariance matrix is significantly larger than the other eigenvalues. Hence, the largest conditional eigenvalue will always be $\hat{\lambda}_1(\cdot)$ and therefore the three methods will choose the same eigenvector as transmit beamformer. By apparent reasons these four strategies have been summarized in the figure as a single curve named *eigenbeamforming*.

The first observation is that the difference in mean instantaneous SNR between knowing the exact channel realization at the transmitter and just knowing the statistics is small, especially in UCA and for some angles in ULA. The ULA seems in fact to achieve the maximum performance for angles close to $\pi/2$, which may be interpreted as the optimal beamformer being almost deterministic in this direction. As mentioned in the beginning of the simulation there is an important difference between the upper bound and the strategies based on eigenbeamforming: The transmitter in the latter case chooses beamformer based on the statistics and has no exact knowledge of the instantaneous SNR without receiving the information through the feedback link. Hence the transmitter can perhaps not choose the transmission rate

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(a) Mean SNR capacity over the 10% strongest channel realizations.

(b) Mean SNR capacity over all channel realizations.

Figure 4.1. The mean SNR capacity for different angles of transmission with seven different transmit beamforming strategies are compared, but the four eigenbeamforming strategies coincide completely. The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

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R optimally, so the real difference in performance may be considerably larger, which will be shown in the end of the chapter. Methods for SNR estimation, with and without feedback, will be considered in the next sections.

The second observation is that a correctly chosen beamformer gives at least a 5 dB higher mean SNR capacity compared to just choosing the beamformer at random. The random strategy has however the same mean performance in all directions for both ULA and UCA, but there seems to be a small variation in ULA when considering the 10% strongest realizations.

As one would expect from the structure of ULA, the fixed beamformer is comparable with the eigenbeamforming when the angle of transmission is zero, i.e., when the direct path to the receiver is orthogonal to the array. For angles far from zero the performance is poor and for UCA the beamformer is unsuitable for all angles since the antennas are located such that they will create destructive interference if the received signals are not shifted in phase.

If the mean SNR capacity over the 10% strongest realizations is compared with the mean over all realizations, there is an increase of 2-3 dB in performance of for all strategies. The behavior is otherwise similar in all cases, but it is important to note that the difference between the upper bound and the eigenbeamforming is reduced. The comparison shows typically the benefit of exploiting diversity.

When it comes to comparing the performance of ULA and UCA the first question is whether a comparison of the results in this simulation would be fair. The answer is not trivial. The distance between adjacent antennas was chosen equally in both arrays, but the maximum distance between two antennas is larger in ULA which may be an advantage. It is clear from the simulation that UCA provides a better mean capacity than ULA for most angles of transmission. In special cases where the possible angles of transmission are restricted to a certain circle sector, the ULA may however provide the best capacity.

4.3 Estimation of SNR at the transmitter

There are several reasons why it is important to estimate the instantaneous SNR at the transmitter. As described in Section 2.5.1, the SNR needs to be estimated in order to choose a communication rate that may provide sufficiently low outage probability. When systems with multiple mobile users are considered in the next chapters the set of instantaneous SNR for all users may be used to exploit multiuser diversity, i.e., to schedule mobiles when they experience good channels.

Four eigenbeamforming strategies have been considered in the chapter: without feedback, with feedback of $\varrho = |\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$, with feedback of $\rho_i = \|\mathbf{h}_i\|^2$ for each receive antenna and with feedback of $\max_i \rho_i$. While the second strategy is based on providing the transmitter with knowledge of the SNR, it needs to be estimated in the other strategies. The obvious MMSE estimate of the instantaneous SNR would

be the mean SNR given by (4.7), (4.15) and (4.22), respectively, for the three cases:

$$\begin{aligned}\widehat{\text{SNR}}(\mathbf{R}_h) &= \frac{P}{N_0} n_R \lambda_1, \\ \widehat{\text{SNR}}(\mathbf{R}_h, \rho) &= \frac{P}{N_0} \sum_{i=1}^{n_R} \hat{\lambda}_k(\rho_i), \\ \widehat{\text{SNR}}(\mathbf{R}_h, \max_i \rho_i) &= \frac{P}{N_0} \hat{\lambda}_k(\max_i \rho_i) + (n_R - 1) \hat{\lambda}_k(0 \leq \rho \leq \max_i \rho_i).\end{aligned}\tag{4.23}$$

These are proper estimators since P and N_0 have been assumed to be known. Their performance will now be evaluated in simulations and then analyzed in more detail.

4.3.1 Simulation

The simulation was performed with the same parameters as in the previous simulation. It is based on 19 uniformly spread angles of transmission between 0 and $\pi/2$ with 30,000 realizations for each angle. All realizations were sorted by increasing squared channel norm $\|\mathbf{H}\|_F^2$ and those with near integer value (closer than 0.1 from an integer) were picked out and formed a set for further analysis.

The correct and the estimated SNR for each set was calculated and compared in two ways: estimation bias and standard deviation: These were calculated as

$$\frac{\sum_k \text{SNR}_k - \widehat{\text{SNR}}_k}{\sum_k \text{SNR}_k} \quad \text{and} \quad \sqrt{\frac{\sum_k |\text{SNR}_k - \widehat{\text{SNR}}_k|^2}{\sum_k |\text{SNR}_k|^2}}, \quad \text{respectively.}$$

The result of the simulation is shown in Figure 4.2 (observe the difference in scaling for the three strategies). The performance of the estimator without feedback, Figure 4.2a, seems to be bad. The estimator is unbiased so the integral of the bias curve should be zero, but as expected the SNR will be overestimated for weak channels and underestimated for strong channels. The bias curve crosses zero as the mean deviation reaches its minimal value. Channels of such strength happens to have a mean SNR near the overall mean SNR, but generally speaking the estimator seems to be unusable for most channel realizations. It may however be important to note that a relative estimation error of, for example, 100% only will result in a communication rate error of 1 bit (for large SNR).

The performance of the estimator with feedback of $\max_i \|\mathbf{h}_i\|^2$, Figure 4.2b, shares the unwanted overestimation of weak channels and underestimation of strong channels (in terms of $\|\mathbf{H}\|_F^2$) with the previous strategy. This is due to that $\|\mathbf{H}\|_F^2$ is not completely known to the transmitter. The estimator is still unbiased (by design) when averaging over all realization, and it will also be unbiased when averaging over a set of realizations with a constant $\max_i \rho_j$. Both the bias and the standard deviation are much smaller with this strategy compared to the case without feedback. The standard deviation is also significantly decreasing with increasing channel strength.

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(a) Bias and standard deviation for SNR estimation without feedback.

(b) Bias and standard deviation for SNR estimation with feedback of $\max_i \|\mathbf{h}_i\|^2$.

(c) Bias and standard deviation for SNR estimation with feedback of $\|\mathbf{h}_i\|^2$ for all i .

Figure 4.2. Relative SNR estimation bias and standard deviation given in percentage for three different feedback strategies. The simulation is based on 19 different angle of transmission between 0 and $\pi/2$ with 30,000 realizations each. Realization with near integer squared norm $\|\mathbf{H}\|_F^2$ were used to produce the figure. The percentiles (10/30/50/70/90) are shown as vertical lines. The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

The performance of the estimator with feedback of $\|\mathbf{h}_i\|^2$ for all rows of $\mathbf{H}$, Figure 4.2c, is the best in comparison with the previous two strategies. The simulation results are actually quite good. The estimator is unbiased for all sets of realizations with a specific channel strength $\|\mathbf{H}\|_F^2$, since this value is known to the transmitter. The standard deviation is just a fraction of the case without feedback and is a factor 2 smaller than with feedback of $\max_i \|\mathbf{h}_i\|^2$. The standard deviation is also significantly decreasing with increasing channel strength.

The performance of ULA and UCA (with the given antenna distances) may also be compared, but it should be kept in mind that the two antenna arrays are perhaps not fairly comparable (see Section 4.2 for a discussion). In the case without feedback, the UCA seems to be slightly better for mediocre channel strengths, but worse for weak channel realizations. In the case with feedback of $\max_i \|\mathbf{h}_i\|^2$ there are no significant differences, but in the case with feedback of $\|\mathbf{h}_i\|^2$, for all i , the standard deviation slope is steeper for UCA than for ULA. Hence, ULA gives a better SNR estimation than UCA when the channel realization is weak while the opposite is true for good channels. Since it is desirable to exploit diversity to avoid transmitting data over weak channel realization it seems like that UCA is the better choice of array structure.

4.4 Quality analysis of the SNR estimation

This section will analyze the statistical properties of SNR estimation at the transmitter. In order to achieve reliable communication it is important to have a low outage probability. The error probability will be correlated with the estimation statistics and will therefore be analyzed in this section.

4.4.1 SNR statistics with full CSI-R

This section will derive a closed form expression for the estimation variance for the case without feedback. The transmit and receive beamformers are given by (4.5) and (4.6), respectively, and the instantaneous SNR may be expressed as

$$\text{SNR} = \frac{P}{N_0} |\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T| = \frac{P}{N_0} \|\mathbf{H} \mathbf{w}_T\|^2,$$

where $\mathbf{w}_T$ is the eigenvector corresponding to the largest eigenvalue in $\mathbf{R}_h$. Using the SNR estimate $\widehat{\text{SNR}}(\mathbf{R}_h)$ in (4.23), the estimation variance may be expressed as

$$V\{\text{SNR}\} = E\left\{(\text{SNR} - \widehat{\text{SNR}}(\mathbf{R}_h))^2\right\} = \left(\frac{P}{N_0}\right)^2 E\left\{\|\mathbf{H} \mathbf{w}_T\|^4\right\} - \widehat{\text{SNR}}^2(\mathbf{R}_h),$$

where the equality follows from that the estimator is unbiased. The elements within each row of $\mathbf{H}$, $\mathbf{h}_i$, are correlated, but by using the eigenvalue decomposition of the covariance matrix $\mathbf{R}_h = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H$ the vectors may be expressed as

$$\mathbf{h}_i = \mathbf{U} \mathbf{v}_i,$$

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where $\mathbf{v}_i \in \mathbb{C}^{n_T}$ is a vector with uncorrelated variables with the covariance matrix $\mathbf{\Lambda}$. Recall that the transmit beamformer was chosen such that $\mathbf{U}^H \mathbf{w}_T = [1 \ 0 \dots 0]^T$. Let the first element in $\mathbf{v}_i$ be denoted $v_{i,1} \in \mathcal{CN}(0, \lambda_1)$ and recall from (2.8) that the magnitude of a complex Gaussian variable is Rayleigh distributed. Then

$$\begin{aligned} E \{ \|\mathbf{H} \mathbf{w}_T\|^4 \} &= E \left\{ \left(\sum_{i=1}^{n_R} |\mathbf{h}_i^H \mathbf{w}_T|^2 \right)^2 \right\} \\ &= \sum_{i=1}^{n_R} E \{ |v_{i,1}^*|^4 \} + \sum_{i=1}^{n_R} \sum_{j \neq i} E \{ |v_{i,1}^*|^2 \} E \{ |v_{j,1}^*|^2 \}, \end{aligned}$$

where the variance $E \{ |v_{i,1}^*|^2 \} = E \{ v_{i,1} v_{i,1}^* \} = \lambda_1$ and

$$E \{ |v_{i,1}^*|^4 \} = \int_0^\infty x^4 \frac{2x}{\lambda_1} e^{-x^2/(\lambda_1)} dx = \int_0^\infty \frac{t^2}{\lambda_1} e^{-t/(\lambda_1)} dt = [\text{part.int.}] = 2\lambda_1^2.$$

Hence, the variance may be expressed as

$$\begin{aligned} E \left\{ \left(\text{SNR} - \widehat{\text{SNR}}(\mathbf{R}_h) \right)^2 \right\} &= \left(\frac{P}{N_0} \right)^2 (2n_R + n_R(n_R - 1) - n_R^2) \lambda_1^2 \\ &= \left(\frac{P}{N_0} \right)^2 n_R \lambda_1^2. \end{aligned} \tag{4.24}$$

4.4.2 SNR statistics with full CSI-R and $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$ feedback

As noted when the strategy was introduced it will provide the transmitter with SNR knowledge. Hence the variance will be zero and the PDF will be given by

$$f_{\text{SNR}}(x) = \begin{cases} 1, & x = \frac{P}{N_0} |\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2, \\ 0, & \text{otherwise.} \end{cases}$$

4.4.3 SNR statistics with full CSI-R and $\|\mathbf{h}_i\|^2, \forall i$, feedback

This section will derive a closed form expression for the estimation variance for the case with feedback of $\boldsymbol{\rho} = [\rho_1 \dots \rho_{n_R}]$, $\rho_i = \|\mathbf{h}_i\|^2$. The transmit and receive beamformers are given by (4.13) and (4.14), respectively, and the instantaneous SNR may be expressed as

$$\text{SNR} = \frac{P}{N_0} |\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T| = \frac{P}{N_0} \|\mathbf{H} \mathbf{w}_T\|^2,$$

where $\mathbf{w}_T$ is the eigenvector corresponding to the eigenvalue λ_k of $\mathbf{R}_h$ that maximizes $\sum_{i=1}^{n_R} \hat{\lambda}_k(\rho_i)$. Using the SNR estimate $\widehat{\text{SNR}}(\mathbf{R}_h, \boldsymbol{\rho})$ in (4.23), the estimation variance may be expressed as

$$V \{ \text{SNR} | \boldsymbol{\rho} \} = \left(\frac{P}{N_0} \right)^2 E \{ \|\mathbf{H} \mathbf{w}_T\|^4 | \boldsymbol{\rho} \} - \widehat{\text{SNR}}^2(\mathbf{R}_h, \boldsymbol{\rho}),$$

where the equality follows from that the estimator is unbiased. The elements within each row of $\mathbf{H}$, $\mathbf{h}_i|\rho_i$, are correlated but by using the eigenvalue decomposition of the covariance matrix $\widehat{\mathbf{R}}_{\mathbf{h}}(\rho_i) = \mathbf{U}\widehat{\mathbf{\Lambda}}(\rho_i)\mathbf{U}^H$ the vector may be expressed as

$$\mathbf{h}_i|\rho_i = \mathbf{U}\mathbf{v}_i|\rho_i,$$

where $\mathbf{v}_i|\rho_i \in \mathbb{C}^{n_T}$ is a vector with uncorrelated variables with the covariance matrix $\widehat{\mathbf{\Lambda}}(\rho_i)$. The variables are no longer complex Gaussian, but still zero-mean since

$$E\{\mathbf{v}_i|\rho_i\} = E\{|\mathbf{v}_i|e^{j\phi_i}|\rho_i\} = E\{|\mathbf{v}_i||\rho_i\} \underbrace{E\{e^{j\phi_i}\}}_{=0} = 0.$$

Recall that the transmit beamformer was chosen such that

$$\mathbf{U}^H \mathbf{w}_T = [\underbrace{0 \dots 0}_{k-1 \text{ st}} \ 1 \ \underbrace{0 \dots 0}_{n_T-k \text{ st}}]^T.$$

Let the k th element in $\mathbf{v}_i$ be denoted $v_{i,k}$ and its variance will be $\widehat{\lambda}_k(\rho_i)$. Then

$$\begin{aligned} E\{\|\mathbf{H}\mathbf{w}_T\|^4|\boldsymbol{\rho}\} &= E\left\{\left(\sum_{i=1}^{n_R} |\mathbf{h}_i^H \mathbf{w}_T|^2\right)^2 \middle| \boldsymbol{\rho}\right\} \\ &= \sum_{i=1}^{n_R} E\{|v_{i,k}^*|^4|\rho_i\} + \sum_{i=1}^{n_R} \sum_{j \neq i} E\{|v_{i,k}^*|^2|\rho_i\} E\{|v_{j,k}^*|^2|\rho_j\}, \end{aligned}$$

where the variance $E\{|v_{i,k}^*|^2|\rho_i\} = E\{v_{i,k} v_{i,k}^*|\rho_i\} = \widehat{\lambda}_k(\rho_i)$ and the conditional fourth order moment of $|v_{i,k}|$ is derived in [8]:

$$\begin{aligned} E\{|v_{i,k}|^4|\rho_i\} &= \frac{\left(\rho_i - \sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}}\right)^2 + \sum_{j \neq k} \frac{\lambda_j^2}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2}}{f_{\|\mathbf{h}\|^2}(\rho_i) \lambda_k \prod_{l \neq k} \left(1 - \frac{\lambda_l}{\lambda_k}\right)} e^{-\frac{\rho_i}{\lambda_k}} \\ &\quad + \frac{2}{f_{\|\mathbf{h}\|^2}(\rho_i)} \sum_{j \neq k} \frac{\lambda_j e^{-\frac{\rho_j}{\lambda_j}}}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2 \prod_{l \neq j} \left(1 - \frac{\lambda_l}{\lambda_j}\right)}, \end{aligned} \quad (4.25)$$

where λ_j are the eigenvalues of the covariance matrix $\mathbf{R}_{\mathbf{h}}$. Observe that

$$\sum_{i=1}^{n_R} \sum_{j \neq i} \widehat{\lambda}_k(\rho_i) \widehat{\lambda}_k(\rho_j) - \left(\sum_{i=1}^{n_R} \widehat{\lambda}_k(\rho_i)\right)^2 = -\sum_{i=1}^{n_R} \widehat{\lambda}_k^2(\rho_i).$$

Then the variance may be expressed as

$$E\left\{\left(\text{SNR} - \widehat{\text{SNR}}(\mathbf{R}_{\mathbf{h}}, \boldsymbol{\rho})\right)^2 \middle| \boldsymbol{\rho}\right\} = \left(\frac{P}{N_0}\right)^2 \sum_{i=1}^{n_R} \left(E\{|v_{i,k}|^4|\rho_i\} - \widehat{\lambda}_k^2(\rho_i)\right), \quad (4.26)$$

with $E\{|v_{i,k}|^4|\rho_i\}$ given in (4.25).

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4.4.4 SNR statistics with full CSI-R and $\max_i \|\mathbf{h}_i\|^2$ feedback

This section will derive a closed form expression for the estimation variance for the case with feedback of $\max_i \rho_i = \max_i \|\mathbf{h}_i\|^2$. The transmit and receive beamformers are given by (4.20) and (4.21), respectively, and the instantaneous SNR may be expressed as

$$\text{SNR} = \frac{P}{N_0} |\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T| = \frac{P}{N_0} \|\mathbf{H} \mathbf{w}_T\|^2,$$

where $\mathbf{w}_T$ is the k th eigenvector corresponding to the eigenvalue of $\mathbf{R}_h$ that maximizes $\hat{\lambda}_k(\rho) + (n_R - 1)\lambda_k(0 \leq \rho_i \leq \rho_1)$. Using the SNR estimate $\widehat{\text{SNR}}(\mathbf{R}_h, \max_i \rho_i)$ in (4.23), the estimation variance may be expressed as

$$V\{\text{SNR} \big|_{\max_i \rho_i}\} = \left(\frac{P}{N_0}\right)^2 E\{\|\mathbf{H} \mathbf{w}_T\|^4 \big|_{\max_i \rho_i}\} - \widehat{\text{SNR}}^2(\mathbf{R}_h, \max_i \rho_i), \quad (4.27)$$

where the equality follows from that the estimator is unbiased. Without loss of generality it is assumed that $\max_i \rho_i = \rho_1$. The elements in each of the vectors $\mathbf{h}_1|_{\rho_1}$ and $\mathbf{h}_i|(0 \leq \rho_i \leq \rho_1)$, $i > 1$, are correlated, but by using the eigenvalue decomposition of the covariance matrix $\mathbf{R} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H$ the vectors may be expressed as

$$\mathbf{h}_1|_{\rho_1} = \mathbf{U} \mathbf{v}_1|_{\rho_1},$$

$$\mathbf{h}_i|(0 \leq \rho_i \leq \rho_1) = \mathbf{U} \mathbf{v}_i|(0 \leq \rho_i \leq \rho_1), \quad i > 1,$$

where $\mathbf{v}_1|_{\rho_1} \in \mathbb{C}^{n_T}$ is a vector with uncorrelated zero-mean variables with the diagonal covariance matrix $\hat{\mathbf{\Lambda}}(\rho_1)$ from (4.9). The vectors $\mathbf{v}_i|(0 \leq \rho_i \leq \rho_1)$, $i > 1$, are independent and identically distributed with uncorrelated zero-mean elements and the diagonal covariance matrix $\hat{\mathbf{\Lambda}}(0 \leq \rho_i \leq \rho_1)$. It is important to note that they are no longer complex Gaussian vectors. Recall that the transmit beamformer was chosen such that

$$\mathbf{U}^H \mathbf{w}_T = [\underbrace{0 \dots 0}_{k-1 \text{ st}} \ 1 \ \underbrace{0 \dots 0}_{n_T-k \text{ st}}]^T.$$

Let the k th element in $\mathbf{v}_i$ be denoted $v_{i,k}$ and its variance will be $\hat{\lambda}_k(\rho_1)$, $i = 1$, or $\hat{\lambda}_k(0 \leq \rho \leq \rho_1)$, $i > 1$. Then the variance in (4.27) may be expressed as

$$\begin{aligned} & \left(\frac{P}{N_0}\right)^2 E\left\{\|\mathbf{H} \mathbf{w}_T\|^4 \big|_{\max_i \rho_i}\right\} - \widehat{\text{SNR}}^2(\mathbf{R}_h, \max_i \rho_i) = \\ & = \left(\frac{P}{N_0}\right)^2 E\left\{\left(\sum_{i=1}^{n_R} |\mathbf{h}_i^H \mathbf{w}_T|^2\right)^2 \big|_{\max_i \rho_i}\right\} - \left(\frac{P}{N_0} E\left\{\sum_{i=1}^{n_R} |\mathbf{h}_i^H \mathbf{w}_T|^2 \big|_{\max_i \rho_i}\right\}\right)^2 \\ & = \left(\frac{P}{N_0}\right)^2 \left(E\{|v_{1,k}|^4|_{\rho_1}\} - \hat{\lambda}_k^2(\rho_1)\right) + \\ & + \left(\frac{P}{N_0}\right)^2 (n_R - 1) \left(E\{|v_{2,k}|^4|_{0 \leq \rho_2 \leq \rho_1}\} - \hat{\lambda}_k^2(0 \leq \rho_2 \leq \rho_1)\right) \end{aligned} \quad (4.28)$$

where it was used in the last step that all $\mathbf{v}_i, 2 \leq i \leq n_R$ are identically distributed. The term $E \{|v_{1,k}|^4 | \rho_i\}$ is given in (4.25), while $E \{|v_{2,k}|^4 | 0 \leq \rho_2 \leq \rho_1\}$ remains to be calculated. This quantity may be expressed as

$$\begin{aligned}
 E \{|v_{i,k}|^4 | 0 \leq \rho_i \leq \rho_1\} &= \frac{1}{\int_0^{\rho_1} f_{\|\mathbf{h}\|^2}(\varrho) d\varrho} \times \\
 &\left[\frac{\left(\lambda_k - \sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}}\right)^2 - \left(\rho_1 + \lambda_k - \sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}}\right)^2 e^{-\frac{\rho_1}{\lambda_k}}}{\lambda_k \prod_{l \neq k} \left(1 - \frac{\lambda_l}{\lambda_k}\right)} + \right. \\
 &\left. + \frac{\left(\lambda_k + \sum_{j \neq k} \frac{\lambda_j^2}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2}\right) \left(1 - e^{-\frac{\rho_1}{\lambda_k}}\right)}{\lambda_k \prod_{l \neq k} \left(1 - \frac{\lambda_l}{\lambda_k}\right)} + 2 \sum_{j \neq k} \frac{\lambda_j^2 \left(1 - e^{-\frac{\rho_1}{\lambda_j}}\right)}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2 \prod_{l \neq j} \left(1 - \frac{\lambda_l}{\lambda_j}\right)} \right]. \tag{4.29}
 \end{aligned}$$

with $\int_0^{\rho_1} f_{\|\mathbf{h}\|^2}(\varrho) d\varrho$ given in (4.19) and λ_i being the unconditional eigenvalues of $\mathbf{R}_h$. The expression in (4.29) should be seen as a contribution made by the thesis, but the complete derivation has been surpassed to Appendix A.3.2 for the purpose of enhancing the readability of the thesis.

4.5 Reliable communication rate

In order to achieve a reliable communication link it is important to avoid capacity outage, i.e., to avoid choosing a rate at the transmitter that is higher than the current channel conditions allow. The capacity depends on the instantaneous SNR, while the rate must be based on the SNR estimate of the transmitter. An overestimated SNR will result in an unreliable communication rate.

Since the probability density of the SNR estimate typically is spread on both sides of the correct SNR the probability of overestimating the SNR will be quite large. Therefore this section will consider methods of modifying the SNR estimate such that the probability of overestimating the SNR, the outage probability, may be controlled. The key observation is that it is much better to underestimate the SNR, than overestimating it. The following two methods will be considered:

$$\begin{aligned}
 \widehat{\text{SNR}}_{\text{new}} &= (1 - \alpha) \widehat{\text{SNR}}, \\
 \widehat{\text{SNR}}_{\text{new}} &= \widehat{\text{SNR}} - \beta \sqrt{V(\widehat{\text{SNR}})}. \tag{4.30}
 \end{aligned}$$

The factors α and β may be considered as functions that depend on the same information as the estimate $\widehat{\text{SNR}}$. Throughout the thesis the outage probability will be 5%, i.e., that

$$\mathbb{P} \left\{ \widehat{\text{SNR}}_{\text{new}} > \text{SNR} \right\} = 0.05.$$

Observe that the SNR, the estimation and the standard deviation of the estimation all contain the factor P/N_0 , so the α and β needed to achieve the given outage probability is independent of the transmit signal power and noise power. The values will

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also be independent of the distance. The variables may however still be dependent on the number of signaling antennas and the statistics in different directions, but it would be desirable to find a method with parameters that is fairly independent of the statistics and current realization. The values of α and β can be allowed to depend on the number of transmit and receive antennas, n_T and n_R , since these may be considered as constant in a communication system.

In the case without feedback the estimate $\widehat{\text{SNR}}(\mathbf{R}_h)$ and its variance is given in (4.23) and (4.24), respectively. The second method will give the new estimate

$$\widehat{\text{SNR}}_{\text{new}}(\mathbf{R}_h) = \frac{P}{N_0} n_R \lambda_1 - \beta \sqrt{\left(\frac{P}{N_0}\right)^2 n_R \lambda_1^2} = \left(1 - \frac{\beta}{\sqrt{n_R}}\right) \frac{P}{N_0} n_R \lambda_1,$$

which is the same as in the first method with $\alpha = \beta/\sqrt{n_R}$. Hence, only the first method will be considered in this case.

4.5.1 Simulation

This simulation will evaluate the two methods for achieving reliable communication described above. The values of α and β will be estimated numerically for different antenna situations. Two different antennas structures, ULA and UCA, and four different sets of antennas were considered: $n_T = n_R = 2$; $n_T = 4, n_R = 2$; $n_T = n_R = 4$ and $n_T = 8, n_R = 4$. Observe that ULA and UCA is the same in the first case, except from a rotation of coordinate system. The simulation was performed with the same parameters as in the previous two simulations.

The simulation consists of two parts. The first is based on 20,000 realizations of each of the 19 angles of transmission spread uniformly between 0 and $\pi/2$. In the second part the angle of transmission is fixed at 60 degrees and the realizations are grouped based on $\|\mathbf{H}\|_F^2$. Each group has approximately integer squared norm and contains 4,000 realizations that have been generated with a Monte Carlo simulation.

In both parts the correct and estimated SNR was sorted such that the approximately smallest α and β that gives an outage probability less than 0.05 could be picked out. Estimates of α and β were calculated for each angle/norm. In the first part the average over all angles where also calculated, since it would be desirable to find a single value that provides approximately the same outage probability for all angles.

In some directions the difference between the correct and estimated SNR were very small, as well as the variance, which gave numerical problems. When considering the first method the correct value of α in such situations would be close to zero, so it was defined to be zero. The value of β in the second method doesn't matter much since it is multiplied with a very small standard deviation. Hence it was excluded from the calculation of the overall β . All points that gave numerical problems have been left out in the simulation figures.

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(a) Estimated α -value with 2 antenna transmitter and 2 antenna receiver.

(b) Estimated α -value with 4 antenna transmitter and 2 antenna receiver.

(c) Estimated α -value with 4 antenna transmitter and 4 antenna receiver.

(d) Estimated α -value with 8 antenna transmitter and 4 antenna receiver.

Figure 4.3. Estimated α -value for different angles of transmission and feedback strategies that gives an outage probability of 0.05 when the SNR estimate is modified using the first method in (4.30): $(1 - \alpha)\text{SNR}$. Two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

4.5. RELIABLE COMMUNICATION RATE

(a) Estimated β -value with 2 antenna transmitter and 2 antenna receiver.

(b) Estimated β -value with 4 antenna transmitter and 2 antenna receiver.

(c) Estimated β -value with 4 antenna transmitter and 4 antenna receiver.

(d) Estimated β -value with 8 antenna transmitter and 4 antenna receiver.

Figure 4.4. Estimated β -value for different angles of transmission and feedback strategies that gives an outage probability of 0.05 when the SNR estimate is modified using the second method in (4.30): $\text{SNR} - \beta\sqrt{V(\text{SNR})}$. Two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

4.5.2 Result

The result of the first part is shown in Figure 4.3 and 4.4. For the case without feedback the value of α is independent of the angle of transmission. Observe that the value is the same for both ULA and UCA, but more importantly note how large α is (even if it reduces when the number of antennas increases). In the $\mathbf{H} \in \mathbb{C}^{4 \times 4}$ case the simulations gives $\alpha \approx 0.657$, which would result in a 2/3 loss in SNR performance compared to the optimal rate.

When considering the first method and the strategy with feedback of $\|\mathbf{h}_i\|^2$ for each row of $\mathbf{H}$ recall the simulation results in Section 4.2. The UCA (with more than two transmit antennas) is expected to give the same optimal α for all angles, since the previous simulation shows that the mean SNR is the same for all angles. For ULA, the result is the opposite. The previous simulations show that the estimation variance goes to zeros for angles close to $\pi/2$ and hence an α close to zero would be desirable for these angles, which is proved by the simulation. For other angles other values of α is needed to get an outage probability of 0.05, so the first method do not fulfill the requirement of being independent of the angle.

For the strategy with feedback of $\max_i \|\mathbf{h}_i\|^2$ the behavior is similar. For UCA (with more than two transmit antennas) the optimal α is constant over the angles, while it is changing for ULA. The value of α is however significantly larger, which is especially clear for UCA, so the previous strategy will give a better SNR performance. Also note that α no longer reaches zero for angles close to $\pi/2$.

The second method seems to have somewhat better properties than the first method. For UCA both feedback strategies give a constant β over all angles. This is not exactly the case for ULA, but the α -curve is smoother than with the first method. In the $\mathbf{H} \in \mathbb{C}^{4 \times 4}$ case the differences is small enough to use the average β -value in later simulations. The case with feedback of $\max_i \|\mathbf{h}_i\|^2$ gives the smallest value in all of the simulations, but this is not equivalent to that the method should give the best performance. The variance of this strategy is significantly larger than for the opposing strategy.

The result for second part is shown in Figure 4.5 and 4.6, where percentiles (10%, 30% 50%, 70%, 90%) are shown as vertical lines to provide some information of the occurrence of different channel strengths.

The first method shows large differences in performance for different channel norms. The behavior is noticeable for all strategies and comparable between ULA and UCA. The problem seems to reduce with increasing number of transmit antennas, especially for strong realization and UCA. The second method has much better properties when considering the strategy with feedback of $\|\mathbf{h}_i\|^2$ for all rows of $\mathbf{H}$. For this case the β -value seems to be flat over the channel strength for both ULA and UCA. The same is unfortunately not true for the strategy with feedback of $\max_i \|\mathbf{h}_i\|^2$. In a communication system where multiuser diversity is expected users will usually transmit when their channels are strong, so perhaps a smaller β than the average over all angles can be used and still having an outage probability of 0.05.

4.5. RELIABLE COMMUNICATION RATE

(a) Estimated α -value with 2 antenna transmitter and 2 antenna receiver.

(b) Estimated α -value with 4 antenna transmitter and 2 antenna receiver.

(c) Estimated α -value with 4 antenna transmitter and 4 antenna receiver.

(d) Estimated α -value with 8 antenna transmitter and 4 antenna receiver.

Figure 4.5. Estimated α -value for different values of the channel squared norm $\|\mathbf{H}\|_F^2$ and feedback strategies that gives an outage probability of 0.05 when the SNR estimate is modified using the first method in (4.30). The percentiles (10/30/50/70/90) are shown as vertical lines and two different transmit antenna are considered: ULA (left column) and UCA (right column).

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(a) Estimated β -value with 2 antenna transmitter and 2 antenna receiver.

(b) Estimated β -value with 4 antenna transmitter and 2 antenna receiver.

(c) Estimated β -value with 4 antenna transmitter and 4 antenna receiver.

(d) Estimated β -value with 8 antenna transmitter and 4 antenna receiver.

Figure 4.6. Estimated β -value for different values of the squared channel norm $\|\mathbf{H}\|_F^2$ and feedback strategies that gives an outage probability of 0.05 when the SNR estimate is modified using the second method in (4.30). The percentiles (10/30/50/70/90) are shown as vertical lines and two different transmit antenna are considered: ULA (left column) and UCA (right column).

4.6. PERFORMANCE COMPARISON WITH RELIABLE COMMUNICATION

To summarize, the first method only gives a constant α -value for all angles with UCA, provided that $n_T > 2$. The value does however change with increasing channel norm. The second method gives a constant β -value for all angles with UCA. The performance for ULA is better with the second method than with the first method, but the β -value is not entirely constant over the angles. For both antenna structures, β changes with the channel norm when it is unknown to the transmitter, so it could be interesting to also consider strategies with feedback of $\|\mathbf{H}\|_F^2$. Both methods have in other words important drawbacks, but they will still be used in the next section to compare the feedback strategies under somewhat fair conditions.

4.6 Performance comparison with reliable communication

This section will conclude the chapter by comparing the performance of the different beamforming strategies with reliable communication. In all cases the outage probability was chosen to be 0.05, or zero when the instantaneous SNR was completely known to the transmitter. The simulation was performed with the same prerequisites as in Section 4.2, in order to make the mean SNR capacity in Figure 4.1 comparable with this simulation. The number of realizations for each of the 19 uniformly spread angles of transmission from 0 to $\pi/2$ were 50,000.

The upper bound was calculated by assuming full CSI at both the transmitter and the receiver. In the strategy with feedback of $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$ the SNR will be known to transmitter, so the concepts of outage probability may be ignored. For the strategies with feedback of $\|\mathbf{h}_i\|^2$ for all rows of $\mathbf{H}$ or feedback of $\max_i \|\mathbf{h}_i\|^2$ the second method in (4.30) was used to get reliable communication for ULA, while for UCA both the first and the second method was used. For the case without feedback the two methods coincide. The values of α and β were derived numerically in the simulations in the previous section.

4.6.1 Result

The average estimated SNR are shown in Figure 4.7 and may be compared to the mean SNR previously shown in Figure 4.1. In part (a) of the figure the performance when averaging over the 10% strongest realizations (in terms of $\|\mathbf{H}\|_F$) is shown, while (b) shows the performance when averaging over all realizations.

Since the strategy with feedback of $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$ achieves the capacity of eigenbeamforming it will also provide the performance closest to the upper bound. The case without feedback gives on the other hand really poor performance. There is a mean loss of 5 dB compared to the upper bound and the loss is 7 dB when considering the strongest realizations, due to the fact that the channel strength is completely unknown to the transmitter.

When the feedback consists of $\|\mathbf{h}_i\|^2$ for all rows of $\mathbf{H}$ the performance is much better, especially when considering UCA where the performance is the same in all directions and just 0.5 dB less than the upper bound and the difference reduces with

CHAPTER 4. SINGLE-USER MIMO DOWNLINK

(a) Mean estimated SNR over the 10% strongest channel realizations.

(b) Mean estimated SNR over all channel realizations.

Figure 4.7. The mean estimated SNR for different angles of transmission with seven different transmit beamforming strategies are compared with an outage probability of 0.05. The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

4.7. SUMMARY

growing channel norm. With ULA the difference compared to the upper bound is 2-3 dB for some angles and close to zero for other angles.

When the feedback consists of $\max_i \|\mathbf{h}_i\|^2$ the performance is 2-3 dB better than without feedback, but is still 2-4 dB less than the upper bound for all angles. In comparison with the previous feedback strategy it is also significantly worse for all angles in UCA and for most angles in ULA. It is interesting that this strategy does not reach the upper bound for angles close to $\pi/2$ in ULA, in contrary to all other feedback strategies.

If the mean estimated SNR over the 10% strongest realizations is compared with the mean over all realizations, there is an increase of 2-3 dB in performance of for all strategies except the one without feedback. The strategies behaves similarly in both cases, but it is important to note that the difference between the upper bound and feedback of $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$ or $\|\mathbf{h}_i\|^2$, for all rows of $\mathbf{H}$, is reduced, while the gap to the other two strategies increases.

The mean performance of the first and second method in (4.30) may also be compared. The first method is best when averaging over all realizations, but for the strongest realizations the opposite is true (and the difference gets larger as the channel norm increases).

To summarize, there is a clear difference in performance between the different feedback strategies. The strategy with feedback of $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$ is closest to the upper bound, but the strategy $\|\mathbf{h}_i\|^2$ for all rows of $\mathbf{H}$ are quite close, especially for the case with UCA. The strategy with feedback of $\max_i \|\mathbf{h}_i\|^2$ has clearly a lower performance, but performs still much better than the case without feedback.

4.7 Summary

This chapter has presented and analyzed three feedback strategies based on the channel norm, as well as the upper bound and the special case without any feedback. The feedback information consist of $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$, $\|\mathbf{h}_i\|^2$ for all rows of $\mathbf{H}$ and $\max_i \|\mathbf{h}_i\|^2$, respectively, for the three feedback strategies. The optimal transmit and receive beamformers (in sense of maximizing the SNR for a single beam) have been derived for each strategy. The MMSE estimate (and its estimation variance) of the instantaneous SNR conditioned on the information available at the transmitter has also been derived for each strategy. The main analytical contribution of this chapter is the closed form expressions for the SNR estimate and its variance for the strategy with $\max_i \|\mathbf{h}_i\|^2$ feedback.

The first simulation shows that all strategies have an average instantaneous SNR close to the upper bound. It is also interesting to note that all strategies (with and without feedback) give exactly the same result, with the considered simulation model. The difference will instead be in the statistics of the SNR estimate at the transmitter. This estimation is crucial when the transmitter decides which communication rate that will be used, since the probability of choosing a rate that leads to outage should be kept small.

In order to reduce the outage probability, the SNR estimation at the transmitter needs to be modified. Two methods have been considered: multiplying the estimate by a constant factor (smaller than one) or subtracting the standard deviation of the current estimate multiplied by a constant factor. The former method is less computationally demanding, but it only works properly with UCA. For ULA it seems hard to find a value of the factor that gives the same outage probability independently of the angle of transmission. The latter method seems to be the most efficient when considering the 10% strongest channel realizations, which would be useful in a system with multiuser diversity. It also seems to be less dependent on the angle of transmission. In comparison between the two methods, the one based on the statistics seems to be the best choice. It should however be kept in mind that even if these results seem to be reasonable, they are based on the simulation environment and not the analytical model.

The chapter is concluded by a simulation that compares the performance of the different feedback strategies. It was expected that the strategy with feedback of $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$ would be closest to the upper bound since the instantaneous SNR will become entirely known to the transmitter. This strategy, however, has the drawback of assuming that the transmit beamformer may be estimated at the receiver before transmission, which may be an unwanted assumption in a multiuser communication system. The strategy with feedback of $\|\mathbf{h}_i\|^2$ for all rows of $\mathbf{H}$ are in many cases quite close to the previous strategy. This occurs for all angles of transmission with UCA and for some angles with ULA. The strategy with feedback of only the maximum squared channel norm, $\max_i \|\mathbf{h}_i\|^2$, has a clearly lower performance, but has the advantage of only using a single feedback variable.

Finally, it may be observed from the simulations that UCA features many good properties and seems to have an overall better performance than ULA. The benefits are related to its circular symmetry, while the main drawback of ULA is that it only provides a performance similar or better than UCA for a small interval of angles. This drawback may however be turned into an advantage if the mobiles may only occur in a limited sector.

Chapter 5

Multi-user MIMO downlink

This chapter will consider the downlink of a MIMO communication system with a single base station and multiple mobile users. The analysis and system description presented in the previous chapter are still valid, provided that only a single user may transmit simultaneously over the same frequency band. First, some different multiple-user communication schemes and the concept of scheduling will be reviewed. The chapter will be concluded by a multiuser TDMA simulation that evaluates the gain of multiuser diversity in a system using some of the strategies from the previous chapter.

5.1 Multiuser communication schemes

As described earlier, the common architecture of a wireless communication system is based on that the area is divided into cells (with somewhat fuzzy borders), each served by a base station. Hence all mobile users located in the same cell communicate with the same base station and share the same transmission medium. The base station must be able to separate the transmission from the mobiles, so they cannot transmit in exactly the same way from the same spatial location. There are several multiple access schemes used for this purpose. The common strategies are summarized below.

Time division multiple access

Several users can transmit on the same frequency band if their transmissions are non-overlapping in time. This technique is known as *Time division multiple access* (TDMA) and divides the time into *timeslots*. Only one user is allowed to transmit during a timeslot and active users are assigned to slots in an often periodical manner.

It is important to note that a timeslot defines the time interval when the receiver expects to receive data. Since the mobile users are located at different distances and may move around freely, there is a great need of synchronization. This can be achieved by not allowing the mobile to transmit for its entire timeslot, i.e., there are silent *guard intervals* in the beginning and the end of the slot. The receiver

informs the transmitter when the transmission moves into one of these intervals, so the timing can be adjusted.

TDMA transmissions are non-continuous so it is possible to assign timeslots based on demand. There is however a few drawbacks with TDMA, like reduction in data rate due to synchronization and the pulsating power envelope that may interfere with other devices.

Frequency division multiple access

When the available frequency bandwidth is wide, it can be divided into several smaller frequency bands that can be used for independent transmissions. In *Frequency division multiple access* (FDMA) this idea is exploited to allow several users to transmit simultaneously. Each user is assigned an own frequency band and may therefore transmit continuously. As a consequence the need of time synchronization that limits the performance of TDMA is avoided, but there are instead requirements on frequency synchronization.

A new and theoretically important case of FDMA is Orthogonal Frequency Division Multiple Access (OFDMA). The scheme is based on OFDM, where the bandwidth is divided into many narrowband channels and the carriers are chosen orthogonal to each other. The set of carriers can then be assigned to different users. The number of subchannels is fixed in a system, but users can be assigned different amounts of them based their rate requirements.

Code division multiple access

The purpose of the method known as *Code division multiple access* (CDMA) is that each mobile user should be able to transmit simultaneously using the whole bandwidth. This is achieved by coding each transmission such that it becomes orthogonally. Hence, many users can share the time and frequency domain and still be possible to separate, but they may create interference.

The coding can be described as a modulation with a sequence known as the *spreading code*. One of the key advantages of CDMA comes from that this code is a pseudonoise sequence and will be quite orthogonal to the spreading code of other users (and to time-shifts of it own). Therefore, the limited interference that each user experience will be similar to Gaussian noise. This creates a demand on power control - the transmit power should not be higher than necessary since it will increase the noise on all other simultaneous transmissions.

Space division multiple access

If there are multiple antennas on the transmitter these can be used for beamforming. It has been described earlier that the transmit beamformer can creating constructive interference in some directions and destructive interference in other. This property is exploited in *Space division multiple access* (SDMA) to transmit several simultaneous signals in different directions. The transmitter will of course need to know the

5.2. SCHEDULING

directions of each user, but this information may be estimated from the channel covariance matrix (especially when the angular spread is small).

In SDMA the cell is divided into subareas and only a single user can transmit simultaneously in each area at the same time. It is important to note that the beam achieved with transmit beamforming never will be completely concentrated in the direction of the receiver. There will always be some energy leakage that creates interference on users in adjacent areas.

5.1.1 Real multiple access implementations

The four multiple access methods described above may be combined in a single communication system. The GSM system is a combination of TDMA and FDMA, which makes it possible to assign different frequency bands to a user at different times. The assignment of slots in such a system can be performed randomly, but it would be more effective to use a *scheduler* that assign time-frequency slots to users when their channels are strong. The third generation mobile system UMTS is, on the other hand, based on wideband CDMA, but the uplink and downlink needs to be separated in either time (TDD) or frequency (FDD) to avoid large interference.

5.2 Scheduling

In a multi-user communication scheme there is often a maximum number of allowed simultaneous users. To support additional users there is need for a *scheduler* that divides time, frequency, spreading codes and cell space between the users - based on some of the multiple access techniques described above. Since it is desirable to minimize the overall transmission power the task of the scheduler is to, in some sense, schedule users when their channels are strong. The channel quality may be measured in the currently maximum reliable transmission rate.

The optimal scheduling from a power consumption point of view would be to constantly communicate with those that currently have the highest SNR, i.e., supports the highest rate. This strategy will be called *maximum throughput scheduling* and is a simplified version of the strategy analyzed in [22]. The drawback is that users that are closer to base station in average will have a better SNR and will therefore get a much higher average data throughput and shorter intervals between each transmission slot. A scheduling method with, in the long term, more fair properties is the *proportional fair scheduling* described in [23] and here below.

5.2.1 Proportional fair scheduling

In order to make the description easier, assume that a TDMA-based system with K active users is used. Instead of just assigning each time slot to the user that supports the highest rate, this scheduling scheme will at time slot m also consider the average throughput $T_k[m]$ of the users $k = 1 \dots K$ in an exponentially weighted window with time constant t_w .

Let $R_k[m]$ be the maximum reliable data rate that the channel of user k supports during time slot m given the current SNR estimate. Then the scheduling algorithm will assign the slot to the user k^* that maximizes the ratio between data rate and average throughput:

$$k^* = \arg \max_k \frac{R_k[m]}{T_k[m]}.$$

The average throughput is updated between each time slot as

$$T_k[m+1] = \begin{cases} \left(1 - \frac{1}{t_w}\right) T_k[m] + \frac{1}{t_w} R_k[m], & k = k^*, \\ \left(1 - \frac{1}{t_w}\right) T_k[m], & k \neq k^*. \end{cases} \quad (5.1)$$

It can be shown that this algorithm in the long term is fair in proportion to the mean maximum rate of each user, see [23] for details. The method doesn't directly consider any latency requirements, but when a weak user has waited for a long time to get a time slot ($T_k[m]$ becomes small) it will probably get one when its channel is particularly strong relative to its own channel conditions.

5.3 Throughput and multiuser diversity with TDMA

In this section the properties of multiuser diversity will be studied in a communication system with one base station and multiple mobiles. The performance of the strategies from the previous chapter will be simulated for different amount of users. The simulation will consider both maximum throughput scheduling and proportional fair scheduling (with a window length of $4 \times \text{Number of mobiles}$ realizations) in a TDMA system.

The simulation was performed in a system with four transmit and equally many receive antennas. Both ULA and UCA were considered at the base station and the standard deviation of the angular spread was fixed at 15 degrees. The distance between adjacent antenna elements at the transmitter was $\lambda_c/2$, where λ_c is the carrier wavelength. The ratio between transmission power and noise density was chosen such that $PR_{h,ii}/N_0 = 1$ on the cell boundary, following the notation given in Section 3.2. For the purpose of comparing the multiuser gain, three numbers of users were considered: 1, 4 and 20 users. The users were placed at random with equal probability anywhere in a circle with unit radius, but restricted to $0.2 \leq r_d \leq 1$ to avoid unreasonable large energies.

To reduce the dependence on the user positions, 500 scenarios were considered with 150 realizations in each scenario, which unfortunately wasn't enough to get completely smooth curves. The realizations were used to estimate CDF:s for the mean throughput (per user and symbol) and cell throughput (per symbol). The same realizations were used for all the five strategies presented in previous chapter.

In the case of proportional fair scheduling the average throughput $T_k[m]$ in (5.1) needs to be initialized close to the average value to avoid transient behaviors that affect the performance. It was initialized as the statistically average throughput of

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Figure 5.1. The CDF of user mean throughput/cell throughput for different beamforming strategies in a cell with 1 randomly located user ($0.2 \leq r_d \leq 1$). The performance with maximum throughput and proportional fair scheduling coincides, as well as the cell and user throughput. The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

the user, divided with the number of users and multiplied by 1.25 to represent the increase in performance due to multiuser diversity. Then $2 \times \text{Number of mobiles}$ realizations were used to correct the value before the realizations used as measurements were taken (there remaining transient was neglected).

5.3.1 Result

The CDF of mean throughput per user and the cell throughput is shown in Figure 5.1 (1 user, both schedulers), Figure 5.2-5.3 (4 users, maximum throughput and proportional fair, respectively) and Figure 5.4-5.5 (20 users, maximum throughput and proportional fair, respectively). The figures do in other words show the probability of getting a smaller or equal throughput.

The multiuser diversity gain is very clear from the simulations: The upper bound on cell throughput for 1 user is in the interval $[3.6 \ 8]$, for 4 users in $[4.3 \ 8]$ and for 20 users in $[5.6 \ 8]$. It is interesting to note that maximum cell throughput seems not increase, but the diversity gives an increased minimal expected cell throughput so the average cell throughput will increase with the number of users. Another interesting observation, that coincides with have been observed earlier, is that the intervals are somewhat smaller for UCA than for ULA.

The difference between maximum throughput and proportional fair scheduling

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(a) CDF of the mean throughput per users and symbol time (in a cell with 4 users).

(b) CDF of the cell throughput per symbol time (in a cell with 4 users).

Figure 5.2. The user mean throughput and cell throughput for different beamforming strategies in a cell with 4 randomly located users ($0.2 \leq r_d \leq 1$) and maximum throughput scheduling. The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

5.3. THROUGHPUT AND MULTIUSER DIVERSITY WITH TDMA

(a) CDF of the mean throughput per users and symbol time (in a cell with 4 users).

(b) CDF of the cell throughput per symbol time (in a cell with 4 users).

Figure 5.3. The user mean throughput and cell throughput for different beamforming strategies in a cell with 4 randomly located users ($0.2 \leq r_d \leq 1$) and proportional fair scheduling. The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

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(a) CDF of the mean throughput per users and symbol time (in a cell with 20 users).

(b) CDF of the cell throughput per symbol time (in a cell with 20 users).

Figure 5.4. The user mean throughput and cell throughput for different beamforming strategies in a cell with 20 randomly located users ($0.2 \leq r_d \leq 1$) and maximum throughput scheduling. The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

5.3. THROUGHPUT AND MULTIUSER DIVERSITY WITH TDMA

(a) CDF of the mean throughput per users and symbol time (in a cell with 20 users).

(b) CDF of the cell throughput per symbol time (in a cell with 20 users).

Figure 5.5. The user mean throughput and cell throughput for different beamforming strategies in a cell with 20 randomly located users ($0.2 \leq r_d \leq 1$) and proportional fair scheduling. The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

is what may have been expected. With the former scheduler there will be users that get a mean throughput close to zero and there will be users that get really high mean throughput. The CDF for the latter scheduler is steeper and guarantees a user mean throughput larger than zero, but to the cost of a lower cell throughput.

With maximum throughput scheduling the strategy without feedback assign a constant rate to each user and will therefore be highly unfair since the statistically strongest user will get all attention. This weakness disappear when the proportional fair method is used, but in both cases the cell throughput for the case without feedback is from 1.5 to 3 bits lower than the upper bound (depending on the number of user) and the gap seems to be growing with the number of users.

When comparing the cell throughput of the three strategies with feedback there is a clear order of performance. The strategy based on $|\mathbf{w}_R^H \mathbf{H} \mathbf{w}_T|^2$ has the best performance, while the strategy with feedback of $\|\mathbf{h}_i\|^2$ for all rows of $\mathbf{H}$ naturally gives a higher bit rate than with feedback of just $\max_i \|\mathbf{h}_i\|^2$. The difference between these two strategies is small when the number of users is few, but increases with the number of users.

The fairness of this comparison can be somewhat questioned since the reliable SNR estimations were calculated as in a system with only a single user. With multiuser diversity the channel distribution of the scheduled user will be different such that the probability of having a strong realization increases. This will probably allow slightly higher communication rates since the risk overestimation reduces. The result would be an even larger gain of exploiting multiuser diversity.

5.4 Summary

This chapter has reviewed the common methods of incorporating multiple users in a communication system. A scheduler may be used to divide time (TDMA) and/or frequency (FDMA) between different users. Users may also transmit simultaneously at the same frequency band by either coding their transmission orthogonally (CDMA) or exploiting different spatial dimensions (SDMA).

The thesis considers transmission over narrowband channels. Observe that OFDMA (orthogonal frequency division multiple access) may be used to divide a wider frequency band into several narrowband subchannels. Multiuser diversity may then be exploited on each subchannel by using either SDMA or TDMA. The latter one has been considered in this chapter due to its theoretical simplicity. Two schedulers have been review: Maximum throughput and Proportional fair. It has been shown that while the former scheduler maximizes the cell throughput by constantly transmitting to the user with the strongest channel, the latter scheduler guarantees some amount of throughput even for mobiles far from the base station but to the cost of a significant reduction in cell throughput.

The chapter was concluded by a TDMA simulation that shows the significant gain in cell throughput achieved by multiuser diversity.

Chapter 6

Multi-cell MIMO downlink

This chapter will generalize the results from the previous chapters to the case with multiple cells. The important assumption is that the downlink communication occurs simultaneously on the same frequency band in all cells. Hence, each receiver will not only be disturbed by noise but also by the interfering transmissions. First the communication model in (2.10) will be extended to include inter-cell interference. Then it will be shown that the channel capacity now will be a function of the signal-to-interference-and-noise ratio (SINR).

Under the assumption that the base stations may not cooperate, the optimal receive beamforming strategy will be derived and compared with two suboptimal strategies (with full CSI) that reduces the complexity at the receiver. Then the available channel information will be discussed and the assumptions will be used to suggest three partially new feedback strategies. These will be analyzed and compared under somewhat fair conditions.

6.1 Channel model with inter-cell interference

The communication model considered in the previous chapter needs to be modified for two main reasons. The first is that there are several cells and each of them has one downlink channel. The most important reason is however that several cells may transmit simultaneously over the same frequency band, which will create inter-cell interference.

Assume that base station i transmits the signal $s_i(t)$ using the beamformer $\mathbf{w}_{T_i}$. The receiving mobile is denoted $\mathcal{I}(i)$ and uses the receive beamformer $\mathbf{w}_{R_{\mathcal{I}(i)}}$. Let $r_{\mathcal{I}(i)}(t)$ denote the received signal at the mobile and let the channel matrix from station j to the receiver of station i be denoted $\mathbf{H}_{\mathcal{I}(i),j}$. The received signal may then be expressed as

$$r_{\mathcal{I}(i)}(t) = \sqrt{P} \mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i} s_i(t) + \mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{n}_{\mathcal{I}(i)}(t) + \sqrt{P} \sum_{j \neq i} \mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j} s_j(t), \quad (6.1)$$

where $\mathbf{n}_{\mathcal{I}(i)}(t) \in \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I})$ is AWGN and P is the average transmission power, which for simplicity is assumed to be the same for all stations (the problem of power control will not be considered in the thesis). The number of interfering base stations is finite, not only due to the difficulty of manufacturing an infinite number of base stations but also since the transmission power decays with distance. Base stations located at great distances from a receiver will naturally have such low impact and random behavior that they may be modeled as a disturbance included in the noise term.

6.2 Signal-to-interference-and-noise ratio

In Section 2.4 the signal-to-noise ratio (SNR) was defined as a quality measure and the tight connection to the maximum reliable communication rate was shown in Section 2.5. In this section the corresponding results will be derived for the case with inter-cell interference.

The generalization of SNR in a multi-cell system is the *signal-to-interference-and-noise* ratio (SINR). For the purpose of simplicity, the base stations are assumed to transmit each symbol simultaneously, which is known as *burst synchronization*. The instantaneous SINR when transmitting an arbitrary symbol from base station i to mobile $\mathcal{I}(i)$, and when averaging over the noise realizations (since they will change rapidly as the channel remains almost constant), may be expressed as

$$\begin{aligned} \text{SINR}_{\mathcal{I}(i)} &= \frac{E\{|\sqrt{P}\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i} s_i(t)|^2\}}{E\{|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{n}_{\mathcal{I}(i)}(t) + \sqrt{P} \sum_{j \neq i} \mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j} s_j(t)|^2\}} \\ &= \frac{P|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i}|^2}{N_0 + P \sum_{j \neq i} |\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j}|^2}, \end{aligned} \quad (6.2)$$

where the equality follows from that the noise is white and that the signal $s_j(t)$ from different base stations are independent. It will next be shown that the SINR of the multi-cell channel defines the channel capacity in the same way as the SNR did in the single-cell case.

6.2.1 Channel capacity

The channel capacity of the communication model in (6.1) may be derived in a similar way as in Section 2.5. Observe that since the channel matrices have been modeled with having complex Gaussian elements, the interference will have the same type of distribution. Hence the interference plus noise will be complex Gaussian distributed so the channel capacity may still be derived according to [9]. The capacity may therefore be expressed as

$$C_{\mathcal{I}(i)} = \log_2(\text{SINR}_{\mathcal{I}(i)} + 1) = \log_2 \left(\frac{P|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i}|^2}{N_0 + P \sum_{j \neq i} |\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j}|^2} + 1 \right). \quad (6.3)$$

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It is clear that the SINR will play the same important role in this chapter as the SNR has done in the previous chapters. If the SINR is maximized, by choosing appropriate beamformers, the maximum reliable communication rate will also be maximized under the given conditions.

6.2.2 SINR estimation at the transmitter

Since the capacity is a function of the instantaneous SINR, it is necessary for the transmitter to estimate SINR in order to choose a communication rate that achieves high throughput with a limited outage probability. This section will discuss the problem of SINR estimation at the transmitter.

It will be assumed throughout this chapter that the base stations cannot cooperate when choosing transmit beamformers. As a consequence, the transmit beamformer $\mathbf{w}_{T_i}$ will be chosen to maximize $\text{SINR}_{\mathcal{I}(i)}$, i.e., the influence on all other communication links is ignored by the transmitter. It would of course be possible to connect all base stations to a *central resource manager* that receives all channel information available at all base stations as feedback and maximize the sum SINR over all cells (perhaps with some fairness condition) by choosing appropriate beamformers. This will however increase the beamforming complexity and create feedback delays between the stations and the central resource manager. According to [24] there is a growing interest of placing the intelligence on each base station of the reasons just mentioned. This will justify the assumption of independent transmit beamforming at the base stations and the focus in the thesis will instead be on the choice of receive beamformers.

Since the transmit beamformer $\mathbf{w}_{T_i}$ only occurs in the numerator of (6.2) the problem of choosing the one that maximizes the expression will be very similar to maximizing the SNR in the previous chapters. The receive beamformer $\mathbf{w}_{R_{\mathcal{I}(i)}}$ that maximizes (6.2) may however depend on both the transmit channel $\mathbf{H}_{\mathcal{I}(i),i}$ and the interference structure. This will make it harder for the transmitter i to predict the receive beamformer's influence on the SINR.

If the current channel matrices, transmit beamformers of $j \neq i$ and the receive beamformer is (at least partly) unknown to transmitter i , then it may be necessary for the transmitter to maximize and estimate the mean SINR. The resulting expression will be

$$\text{SINR}_i = \frac{PE\{|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i}|^2\}}{N_0 + P \sum_{j \neq i} E\{|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j}|^2\}}, \quad (6.4)$$

where the expectation should be conditioned on all available information.

6.3 Channel information

It has been assumed in Section 2.6 that receiver $\mathcal{I}(i)$ has knowledge of both the current channel matrix $\mathbf{H}_{\mathcal{I}(i),i}$ from the closest base station and the channel statistics.

The transmitting base station has however only knowledge of the channel statistics. In this chapter it would also be necessary to make assumptions regarding the channels from all interfering base stations to the receiver $\mathcal{I}(i)$, i.e., $\mathbf{H}_{\mathcal{I}(i),j}$ for $j \neq i$, and the transmit beamformer used by these stations.

First recall the assumption that the base stations may not cooperate. This leads to the conclusion that the transmit beamformers $\mathbf{w}_{T_j}, j \neq i$, will be unknown to base station i . The mobiles will naturally have independent spatial locations, and hence only base station i and its receiver $\mathcal{I}(i)$ will be able to predict the transmit beamformer $\mathbf{w}_{T_i}$. All other mobiles and base stations may however base their behavior on the statistical properties of the beamforming. Assume that the angular directions of the users are uniformly distributed. Then the first two moments of the transmit beamformers may be approximated as

$$\begin{aligned}\mathbf{W}_{\mu_1}^{(n_T)} &\approx E\{\mathbf{a}(\theta)\}/\sqrt{n_T} \\ \mathbf{W}_{\mu_2}^{(n_T)} &\approx E\{\mathbf{a}(\theta)\mathbf{a}^H(\theta)\}/n_T,\end{aligned}\tag{6.5}$$

where $\mathbf{a}(\theta)$ denotes the array response with n_T antennas of the transmitter and the signaling power is assumed to be equally spread over the antennas. The validity of this model and the importance of using such a detailed model instead of just approximating $\mathbf{W}_{\mu_2}^{(n_T)}$ by a diagonal matrix will be shown in the following short simulation.

Consider the scenario in Figure 6.1a, where the transmitting base station i and the receiving mobile $\mathcal{I}(i)$ are fixed and interference is studied for different directions θ of the interfering base station j . The interfering base station may use its beamformer to, with equal probability, concentrate the transmission in any direction. The transmitter will use eigenbeamforming and the receiver is assumed to use its beamformer purely to maximize this signal power, as in previous chapters.

Assume that the receiver may estimate $\mathbf{H}_{\mathcal{I}(i),j}$, while the specific beamformer used by the interferer is unknown. The receiver would like to estimate $E\{\mathbf{H}_{\mathcal{I}(i),j}\mathbf{w}_{T_j}\mathbf{w}_{T_j}^H\mathbf{H}_{\mathcal{I}(i),j}^H\}$ (and perhaps send it back to the transmitter). Two models for the average interference power given by the transmit beamformer will be compared. The first is given in (6.5) and the second approach is to model the beamformer as a normalized identically distributed white Gaussian vector. The simulation is based on 90 different angles between 0 and $\pi/2$ and the mean interference power is averaged over 2,000 realizations per angle. The simulation considers 4 antenna ULA (left) and UCA (right) at the transmitter and interferer, while the receiver may have any antenna structure with 4 antennas. The transmit antenna distance is $\lambda_c/2$ and the standard deviation of the angular spread is 15 degrees.

The relative deviation between the estimated and the correct mean interference power is calculated in the simulation and presented in Figure 6.1b for different angles of the interfering base station. It is clear from the simulation that the approximative array response model of the transmit beamformers is good, while the random beamforming model shows large deviations.

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(a) A scenario with fixed transmitter and receiver, while the interferer will be studied for different angles.

(b) The relative deviation between the estimated and the correct mean interference power for different angle of interferer, following the coordinate system in (a).

Figure 6.1. The importance of a detailed transmit beamforming model is shown using the scenario in (a). The relative deviation when estimating the mean interference power using either antenna response or a white Gaussian vector is shown in (b). The simulation considers 4 antenna ULA (left) and UCA (right) at the transmitter and interferer, while the receiver may have any antenna structure with 4 antennas.

The array responses for ULA and UCA given in (1.1) and (1.2), respectively, seems to have no closed form expression for the first and second moments (when $n_T > 1$), but they may be pre-calculated numerically since they will be constant in a given communication system.

6.3.1 Interfering channels

The channel from the interfering base stations to the mobile $\mathcal{I}(i)$ will be independent of the channel from the interfering base stations to base station i . Therefore only the mobile that experiences the interference will be able to estimate it. This section will discuss the amount of interfering channel information that it would be reasonable to assume that the receiver may estimate in practice.

This chapter will instead concentrate on the case when the interfering channel realizations are partially known to the mobile. Throughout the chapter it will be assumed that the mobile may use the residual of the received signal to adaptively estimate the matrix in the denominator of the SINR expression in (6.2) averaged over all possible beamformers. Let this matrix be denoted $\mathbf{R}_{\mathbf{IN}}^{\mathcal{I}(i)}$ and observe that it may be expressed as

$$\mathbf{R}_{\mathbf{IN}}^{\mathcal{I}(i)} = E \left\{ N_0 \mathbf{I} + P \sum_{j \neq i} \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j} \mathbf{w}_{T_j}^H \mathbf{H}_{\mathcal{I}(i),j}^H \right\} = N_0 \mathbf{I} + P \sum_{j \neq i} \mathbf{H}_{\mathcal{I}(i),j} \mathbf{W}_{\mu_2}^{(n_T)} \mathbf{H}_{\mathcal{I}(i),j}^H. \quad (6.6)$$

To summarize, the transmit beamformers used by the interfering base stations are unknown, but their first two moments may be estimated as in (6.5) and are assumed to be known to all communication units. The channel realization and statistics of the interference is unknown to the transmitter, but may be partially estimated at the mobile. Each mobile is assumed to estimate the average interference-plus-noise power over all transmit beamformers given in (6.6) without error. In this chapter, both the transmitter and the receiver are said to have partial channel state information (partial CSI).

6.4 Optimal and suboptimal beamforming with full CSI

The word 'optimal' will be used throughout this chapter for denoting beamforming strategies that maximizes the SINR of the channel, but it should be kept in mind that there are other strategies with for example several beams that are more efficient in terms of overall capacity.

As mentioned above the performance maximization is a more complex problem in a system with inter-cell interference. While the SNR expression only contains the transmit and receive beamformers of the current base station and mobile, the SINR expression also depends on the transmit beamformer of all other base stations - even if the dependence decreases with distance.

This section will first consider the optimal beamforming strategy when considering full CSI. The optimal strategy will be denoted as the upper bound and may be

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seen as optimal weighting between using the receive beamformer to maximizing the numerator and minimizing the denominator of (6.4), i.e., maximizing the received signal power and minimizing the power of the interference.

Two suboptimal receive beamforming strategies will also be considered and compared with the upper bound. These will be based on the following two extremes: to use the receive beamformer to either maximize the received signal power or minimize the received interference power. One reason of actually using these suboptimal strategies would be to reduce the complexity at the receiver.

6.4.1 The upper bound

As an upper bound to the choice of receive beamformer, assume that the channel matrices, transmit beamformers, and average transmit and noise power in (6.2) are known to the receiver. By dividing the numerator and denominator with P and by putting $\mathbf{w}_{R_{\mathcal{I}(i)}}$ outside the brackets the SINR may be expressed as

$$\text{SINR}_{\mathcal{I}(i)} = \frac{\mathbf{w}_{R_{\mathcal{I}(i)}}^H (\mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i} \mathbf{w}_{T_i}^H \mathbf{H}_{\mathcal{I}(i),i}^H) \mathbf{w}_{R_{\mathcal{I}(i)}}}{\mathbf{w}_{R_{\mathcal{I}(i)}}^H \left(\frac{N_0}{P} \mathbf{I} + \sum_{j \neq i} \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j} \mathbf{w}_{T_j}^H \mathbf{H}_{\mathcal{I}(i),j}^H \right) \mathbf{w}_{R_{\mathcal{I}(i)}}}. \quad (6.7)$$

The maximization of this scalar quotient is known as *maximal ratio combining* and will now be shown to be a generalized eigenvalue problem. First introduce the notation $\mathbf{A}_{\mathcal{I}(i)} = \mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i} \mathbf{w}_{T_i}^H \mathbf{H}_{\mathcal{I}(i),i}^H$ and $\mathbf{B}_{\mathcal{I}(i)} = \frac{N_0}{P} \mathbf{I} + \sum_{j \neq i} \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j} \mathbf{w}_{T_j}^H \mathbf{H}_{\mathcal{I}(i),j}^H$. Then the SINR expression in (6.7) may be rewritten as

$$\mathbf{w}_{R_{\mathcal{I}(i)}}^H (\mathbf{A}_{\mathcal{I}(i)} - \text{SINR}_{\mathcal{I}(i)} \mathbf{B}_{\mathcal{I}(i)}) \mathbf{w}_{R_{\mathcal{I}(i)}} = 0, \quad (6.8)$$

which is equivalent to $\mathbf{w}_{R_{\mathcal{I}(i)}}$ being the eigenvector corresponding to the zero eigenvalue of $\mathbf{A}_{\mathcal{I}(i)} - \text{SINR}_{\mathcal{I}(i)} \mathbf{B}_{\mathcal{I}(i)}$, i.e., $(\mathbf{A}_{\mathcal{I}(i)} - \text{SINR}_{\mathcal{I}(i)} \mathbf{B}_{\mathcal{I}(i)}) \mathbf{w}_{R_{\mathcal{I}(i)}} = 0$. It may also be expressed as a generalized eigenvalue problem

$$\mathbf{A}_{\mathcal{I}(i)} \mathbf{w}_{R_{\mathcal{I}(i)}} = \lambda \mathbf{B}_{\mathcal{I}(i)} \mathbf{w}_{R_{\mathcal{I}(i)}},$$

where $\mathbf{w}_{R_{\mathcal{I}(i)}}$ should be chosen as the eigenvector corresponding to largest eigenvalue $\lambda_{\max}$. This will result in $\text{SINR}_{\mathcal{I}(i)} = \lambda_{\max}$.

The matrix $\mathbf{B}_{\mathcal{I}(i)}$ will in general be non-singular (otherwise the noise to signal power N_0/P would be identically zero at some receive antenna). Hence, the generalized eigenvalue problem may easily be converted into a regular eigenvalue problem

$$\mathbf{B}_{\mathcal{I}(i)}^{-1} \mathbf{A}_{\mathcal{I}(i)} \mathbf{w}_{R_{\mathcal{I}(i)}} = \lambda \mathbf{w}_{R_{\mathcal{I}(i)}},$$

or perhaps in a more numerically stable form by using the cholesky factorization $\mathbf{B}_{\mathcal{I}(i)} = \mathbf{V}_{\mathcal{I}(i)} \mathbf{V}_{\mathcal{I}(i)}^H$:

$$\mathbf{V}_{\mathcal{I}(i)}^{-1} \mathbf{A}_{\mathcal{I}(i)} \mathbf{V}_{\mathcal{I}(i)}^{-H} \tilde{\mathbf{w}}_{R_{\mathcal{I}(i)}} = \lambda \tilde{\mathbf{w}}_{R_{\mathcal{I}(i)}},$$

where $\tilde{\mathbf{w}}_{R_{\mathcal{I}(i)}} = \mathbf{V}_{\mathcal{I}(i)}^H \mathbf{w}_{R_{\mathcal{I}(i)}}$.

6.4.2 Received signal power maximization

If the receive beamformer is used to maximize the received signal power, i.e., treating the interference as if it was white noise, the problem formulation will be reduced to the same as in the previous chapters. The numerator in (6.4) should be maximized and the transmit beamformer $\mathbf{w}_{T_i}$ may be treated as known to the receiver since the receiver will have at least as much channel information as the transmitter (this wouldn't be true if the base stations were allowed to cooperate). The optimal receive beamformer should be parallel to the experienced channel $\mathbf{H}_{\mathcal{I}(i),i}\mathbf{w}_{T_i}$:

$$\mathbf{w}_{R_{\mathcal{I}(i)}} = \frac{\mathbf{H}_{\mathcal{I}(i),i}\mathbf{w}_{T_i}}{\|\mathbf{H}_{\mathcal{I}(i),i}\mathbf{w}_{T_i}\|}. \quad (6.9)$$

6.4.3 Minimization of interference power

Now consider a strategy of using the receive beamformer to minimize the interference power, which corresponds to minimizing the denominator of the SINR expression. Recall the denominator from the first step of (6.2):

$$\begin{aligned} E\{|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{n}_{\mathcal{I}(i)}(t) + \sqrt{P} \sum_{j \neq i} \mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j} s_j(t)|^2\} = \\ = N_0 + PE\{|\sum_{j \neq i} \mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j} s_j(t)|^2\}. \end{aligned}$$

The sum may be rewritten as a matrix multiplication

$$\sum_{j \neq i} \mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),j} \mathbf{w}_{T_j} s_j(t) = \mathbf{w}_{R_{\mathcal{I}(i)}}^H \bar{\mathbf{H}}_{j \neq i} \bar{\mathbf{w}}_{T_{j \neq i}},$$

with the composite channel matrix $\bar{\mathbf{H}}_{j \neq i} = [\mathbf{H}_{i,1} \dots \mathbf{H}_{i,i-1} \mathbf{H}_{i,i+1} \dots]$ and the composite transmit signal $\bar{\mathbf{w}}_{T_{j \neq i}} = [\mathbf{w}_{T_1}^T s_1(t) \dots \mathbf{w}_{T_{i-1}}^T s_{i-1}(t) \mathbf{w}_{T_{i+1}}^T s_{i+1}(t) \dots]^T$. The receiver has no information about $\bar{\mathbf{w}}_{T_{j \neq i}}$. The receiver may however choose a beamformer that minimizes $\mathbf{w}_{R_{\mathcal{I}(i)}}^H \bar{\mathbf{H}}_{j \neq i}$. Let the number of interfering base stations be $n < \infty$. Then the singular value decomposition of the composite channel matrix $\bar{\mathbf{H}}_{j \neq i} \in \mathbb{C}^{n_R \times n \cdot n_T}$ may be expressed as $\bar{\mathbf{H}}_{j \neq i} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H$, where $\mathbf{U} \in \mathbb{C}^{n_R \times n_R}$ and $\mathbf{V} \in \mathbb{C}^{n \cdot n_T \times n \cdot n_T}$ are unitary matrices. The matrix $\mathbf{\Sigma}$ has the same dimension as $\bar{\mathbf{H}}_{j \neq i}$ and has the singular values on the main diagonal in non-increasing order, while the other elements are zero. The optimal choice of receive beamformer would be the column of $\mathbf{U}$ that corresponds to the smallest singular value:

$$\mathbf{w}_{R_{\mathcal{I}(i)}} = \mathbf{U}_{:,n_R}. \quad (6.10)$$

If $n \cdot n_T < n_R$ or if any singular value is zero, the influence of the interference will disappear completely. In general there will be some remaining interference power.

6.4. OPTIMAL AND SUBOPTIMAL BEAMFORMING WITH FULL CSI

Figure 6.2. Two lattices with hexagonal cells and with different frequency reuse factor: (a) 1, (b) $1/3$. The middle cell (which will be considered in simulations) and its six closest cells that are using the same frequency band are shaded. The shaded area corresponds to the region where users may be placed randomly in simulations.

6.4.4 Mean capacity simulation

The mean capacity with the two suboptimal strategies will be compared with the upper bound based on maximum ratio combining. Both the transmitter and the receiver are assumed to have full CSI. Hence the transmitter will use the right singular vector corresponding to the largest singular value of the channel as transmit beamformer. The receive beamforming strategies of maximum ratio combining, received signal power maximization and minimization of interference power (from the previous three sections) will be considered. Observe that the result of the simulation should not be confused with the actual mean performance, which demands the instantaneous SINR to be estimated at the transmitter.

In order to reduce the inter-cell interference, many communications systems divide the bandwidth in X parts and only permits transmission over one of them in each cell. The purpose is to avoid that adjacent cells uses the same frequency band, which will reduce the inter-cell interference since the power decays with distance. The frequency *reuse factor* is defined as $1/X$ and describes the fraction of the whole bandwidth used in each cell. When choosing reuse factor the decrease in bandwidth should be compared to the increase in throughput per frequency band and base station (observe that each band should use X times more power in order to make the overall transmit power constant). If a system should favor from a reuse factor of $1/X$ there should be an increase in performance of at least a factor X , otherwise the overall performance will not be increased.

This simulation will consider two reuse factors: 1 and 1/3. The corresponding cell networks are shown in Figure 6.2, where the six closest cells that use the same frequency band as the middle cell are shaded. The interference in the middle cell will also depend on cells that are more far away, but this part of the interference is neglected in the simulation since their influence will be significantly smaller.

The two cell networks consist of hexagonal cells that precisely fit a circle of unit radius. For the ease of simulation, each cell is assumed to have exactly 20 users. These are placed randomly and uniformly in each circle, but restricted to $0.2 \leq r_d \leq 1$ to avoid unreasonable large energies (as shown by the shaded area in Figure 6.2). In order to reduce the dependence of the user positions 200 scenarios were considered with 100 realizations in each scenario. The two previously described scheduling methods, maximum throughput and proportional fair, will be considered. The initialization of the latter method was performed as in Section 5.3.

The scheduler of each base station uses the instantaneous SINR in (6.2), but observe that this expression contains the transmit beamformer of other base stations. Even if each base station in the simulation is assumed to have full CSI it would be unreasonable for them to take other base stations' simultaneous scheduling choices into account in the own scheduler. Hence, the dominator of (6.2) will be replaced with average over all interfering beamformers, as in (6.6).

The simulation considers both ULA and UCA with four antennas at the transmitter, four receive antennas at the receiver, the antenna separation $\lambda_c/2$ and a standard deviation of the angular spread of 15 degrees.

6.4.5 Result

The simulation results for reuse factor 1 is shown in Figure 6.3 and 6.4 with maximum throughput and proportional fair scheduling, respectively. The results for reuse factor 1/3 are shown in Figures 6.5 and 6.6 with Maximum throughput and Proportional fair scheduling, respectively. The user mean throughput and cell throughput will depend on the average SNR at the cell boundary. Two extremes are considered in this simulation: $PR_{\mathbf{h},ii}/N_0 = 1$ (solid) and $PR_{\mathbf{h},ii} \gg N_0$ (dashed).

It is clear from simulations that none of the suboptimal strategies come especially close to the optimal solution, so the extra complexity of the optimal receive beamforming (maximum ratio combining) may be worthwhile. It is interesting to note that the strategy that minimizes the interference power performs better than the strategy that maximizes the signal power when $PR_{\mathbf{h},ii} \gg N_0$ for all situations (and specially with maximum throughput scheduling). With $PR_{\mathbf{h},ii}/N_0 = 1$ the two strategies are quite comparable with maximum throughput scheduling, while it is better to maximize the signal power with proportional fair scheduling - especially with a reuse factor of 1/3.

Another interesting observation is that while the capacity in a system without interference would grow logarithmically to infinity with the transmit power, the increase of signal power will only result in suppression of the influence of noise and thereby a limited increase in performance. The gain in capacity of increasing the

6.4. OPTIMAL AND SUBOPTIMAL BEAMFORMING WITH FULL CSI

(a) CDF of the mean capacity per users and symbol (20 users and reuse factor 1).

(b) CDF of the cell capacity per symbol (20 users and reuse factor 1).

Figure 6.3. The user mean capacity throughput and cell capacity throughput for optimal and suboptimal beamforming strategies with maximum throughput scheduling. The frequency reuse factor is 1, as in Figure 6.2a, and each cell contains 20 randomly located users ($0.2 \leq r_d \leq 1$). The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

(a) CDF of the mean capacity per users and symbol (20 users and reuse factor 1).

(b) CDF of the cell capacity per symbol (20 users and reuse factor 1).

Figure 6.4. The user mean capacity throughput and cell capacity throughput for optimal and suboptimal beamforming strategies with proportional fair scheduling. The frequency reuse factor is 1, as in Figure 6.2a, and each cell contains 20 randomly located users ($0.2 \leq r_d \leq 1$). The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

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(a) CDF of the mean capacity per users and symbol (20 users and reuse factor 1/3).

(b) CDF of the cell capacity per symbol (20 users and reuse factor 1/3).

Figure 6.5. The user mean capacity throughput and cell capacity throughput for optimal and suboptimal beamforming strategies with maximum throughput scheduling. The frequency reuse factor is 1/3, as in Figure 6.2a, and each cell contains 20 randomly located users ($0.2 \leq r_d \leq 1$). The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

(a) CDF of the mean capacity per users and symbol (20 users and reuse factor 1/3).

(b) CDF of the cell capacity per symbol (20 users and reuse factor 1/3).

Figure 6.6. The user mean capacity throughput and cell capacity throughput for optimal and suboptimal beamforming strategies with proportional fair scheduling. The frequency reuse factor is 1/3, as in Figure 6.2a, and each cell contains 20 randomly located users ($0.2 \leq r_d \leq 1$). The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

6.5. FEEDBACK STRATEGIES WITH PARTIAL CSI

transmitted signal power will be larger in a system with a small reuse factor since there is more to gain in suppressing the noise when the interference is small. The capacity differences between the two reuse factors for the upper bound is about 1.5 bits, which is a significant increase but enough to support a choice of the smaller factor from an overall capacity point of view.

To summarize, the suboptimal strategy of minimizing the received interference power is better than maximizing the received signal power in most cases and especially when the noise power is significantly smaller than the transmitted signal power. The interference cancellation will be even more efficient with increasing number of antennas. For the given number of antennas, both strategies are however clearly less effective than the optimal receive beamforming. In the given environment there seems to be nothing to gain in overall performance by exploiting other approaches than universal frequency reuse (1/1). The capacity of a system with inter-cell interference will have an upper bound as the transmit power increases, in contrary to the case without interference where it will grow unbounded.

6.5 Feedback strategies with partial CSI

This section will introduce and analyze three feedback strategies for the purpose of estimation and maximizing the average SINR at the receiver. In the first strategy the receiver just feeds back its current SINR estimate, while the second and third method feed back $\|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$ and the current sum of mean interference and noise power (which will be approximated as zero in the third method) for different criterion on $\mathbf{w}_{R_{\mathcal{I}(i)}}$.

In contrary to the beamforming strategies considered in the previous chapters, the three strategies presented in this section are not claimed to be completely optimal and SINR estimators will in fact be unbiased. The transmit beamformers are based on eigenbeamforming, but there will be approximations in the two latter strategies that will them non-optimal. The receive beamformers will be chosen to maximize the SINR using maximum ratio combining, but some of the difficulties and lack of information will be solved by approximations. The strategies may, in some sense, be considered as approximatively optimal given the feedback strategies and the channel information assumptions from the previous sections.

6.5.1 Beamforming with partial CSI and SINR feedback

This beamforming strategy is based on eigenbeamforming at the transmitter. The receiver predicts the transmit beamformer and feeds back its estimation of the instantaneous SINR. Such a strategy will demand that the transmit beamformer can be predicted at the receiver, so it may be difficult to generalize it to situations where several beams are transmitted (like for example SDMA). For the special case considered in this chapter it may be difficult to find a more efficient strategy that only includes a single feedback variable.

The receiver cannot, without feedback of something more than its SINR estimation, persuade the transmitter to choose a specific beamformer. Hence, the transmit beamformer $\mathbf{w}_{T_i}$ will be chosen according to the channel statistics to maximize the mean received signal power $E\{\|\mathbf{H}_{\mathcal{I}(i),i}\mathbf{w}_{T_i}\|^2\}$ which, as shown earlier, results in

$$\mathbf{w}_{T_i} = \mathbf{U}_{\cdot,1}, \quad (6.11)$$

where $\mathbf{U}_{\cdot,1}$ is eigenvector corresponding to the largest eigenvalue of the channel covariance matrix $\mathbf{R}_{\mathbf{h}}^{\mathcal{I}(i),\mathbf{i}}$.

The receive beamformer $\mathbf{w}_{R_{\mathcal{I}(i)}}$ that maximizes the SINR should be determined according to the generalized eigenvalue problem described in Section 6.4.1, but unfortunately the interference and noise matrix in the denominator of the SINR expression in (6.7) is only known in average. Since the influence of this matrix will be suppressed it may however be a valid approximation to replace the term with its mean value. The resulting approximative SINR expression, when averaging over the interfering transmit beamformers in the denominator, will be

$$\begin{aligned} \text{SINR}_{\mathcal{I}(i)} &\approx \frac{\mathbf{w}_{R_{\mathcal{I}(i)}}^H (P\mathbf{H}_{\mathcal{I}(i),i}\mathbf{w}_{T_i}\mathbf{w}_{T_i}^H\mathbf{H}_{\mathcal{I}(i),i}^H)\mathbf{w}_{R_{\mathcal{I}(i)}}}{\mathbf{w}_{R_{\mathcal{I}(i)}}^H E\{N_0\mathbf{I} + P\sum_{j \neq i} \mathbf{H}_{\mathcal{I}(i),j}\mathbf{w}_{T_j}\mathbf{w}_{T_j}^H\mathbf{H}_{\mathcal{I}(i),j}^H\}\mathbf{w}_{R_{\mathcal{I}(i)}}} \\ &= P \frac{\mathbf{w}_{R_{\mathcal{I}(i)}}^H (\mathbf{H}_{\mathcal{I}(i),i}\mathbf{w}_{T_i}\mathbf{w}_{T_i}^H\mathbf{H}_{\mathcal{I}(i),i}^H)\mathbf{w}_{R_{\mathcal{I}(i)}}}{\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{R}_{\mathbf{IN}}^{\mathcal{I}(\mathbf{i})} \mathbf{w}_{R_{\mathcal{I}(i)}}}. \end{aligned} \quad (6.12)$$

This expression will be maximized by receive beamformer being equal to eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_{\mathbf{IN}}^{-1}(\mathbf{H}_{\mathcal{I}(i),i}\mathbf{w}_{T_i}\mathbf{w}_{T_i}^H\mathbf{H}_{\mathcal{I}(i),i}^H)$. Let its eigenvalue decomposition be denoted $\mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$, where $\mathbf{\Lambda}$ is a diagonal matrix with the eigenvalues ordered in non-increasing order and $\mathbf{V}$ is a unitary matrix with the corresponding eigenvectors as columns. The receive beamformer may be denoted

$$\mathbf{w}_{R_{\mathcal{I}(i)}} = \mathbf{V}_{\cdot,1} \quad (6.13)$$

and the estimated SINR will be the largest eigenvalue

$$\widehat{\text{SINR}}_{\mathcal{I}(i)} = \max_k \lambda_k \left(\mathbf{R}_{\mathbf{IN}}^{\mathcal{I}(\mathbf{i})^{-1}} (P\mathbf{H}_{\mathcal{I}(i),i}\mathbf{w}_{T_i}\mathbf{w}_{T_i}^H\mathbf{H}_{\mathcal{I}(i),i}^H) \right), \quad (6.14)$$

where $\lambda_k(\cdot)$ is a function that will give the k th eigenvalue (in some arbitrary order).

To summarize, this section has derived a suboptimal strategy with full CSI-R of the transmit channel, knowledge of the average noise and interference power $\mathbf{R}_{\mathbf{IN}}^{\mathcal{I}(\mathbf{i})}$ given in (6.6) and with feedback of the SINR estimation of the receiver. The transmit and receive beamformers are given in (6.11) and (6.13), respectively, and the SINR estimate that will be known to both the receiver and the transmitter is given in (6.14).

6.5. FEEDBACK STRATEGIES WITH PARTIAL CSI

6.5.2 Beamforming with partial CSI and $\|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$ and

$\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{R}_{\text{IN}}^{\mathcal{I}(i)} \mathbf{w}_{R_{\mathcal{I}(i)}} \text{ feedback}$

A drawback of the strategy with the SINR feedback considered in the previous section is the lack of spatial information. The transmitter will receive information of what the SINR would be with (unconditional) transmit eigenbeamforming, but cannot distinguish between, for example, a strong channel realization and weak influence from noise and interference. This section will consider feedback of two variables: $\|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$ that describes the current channel strength and $\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{R}_{\text{IN}}^{\mathcal{I}(i)} \mathbf{w}_{R_{\mathcal{I}(i)}}$ that is the average interference and noise power. This may allow the transmitter to also exploit information from other sources like, for example, other mobiles. The discussion in this section will lead to two strategies, based on different usage of $\mathbf{w}_{R_{\mathcal{I}(i)}}$ in the feedback information.

The purpose of using $\|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$ as feedback variable is that it contains spatial information that may be exploited to calculate a conditional covariance matrix $\hat{\mathbf{R}}_{\mathbf{h}}^{\mathcal{I}(i),i}(\rho)$ and use conditional transmit eigenbeamforming. Recall from Section 6.4.1 that the optimal receive beamformer is calculated from a generalized eigenvalue problem known as maximum ratio combining. The derivation did unfortunately assume that transmit beamformer was known in advance, which stand in contrary to the purpose of this feedback strategy. Since the beamformer cannot be predicted at the receiver, there will only be approximate solutions. Hence, the receiver may as well use some $\mathbf{w}_{R_{\mathcal{I}(i)}}$ in the feedback information and then use some other beamformer that will maximize its simultaneous SINR estimation when it is actually receiving data. This section will consider two different choices of $\mathbf{w}_{R_{\mathcal{I}(i)}}$ used for calculating the feedback information:

Method 1

This method will focus on using a receive beamformer in the feedback information that minimizes the sum of mean interference and noise power. Hence, the receive beamformer $\mathbf{w}_{R_{\mathcal{I}(i)}}$ will be the eigenvector corresponding to the smallest eigenvalue of $\mathbf{R}_{\text{IN}}^{\mathcal{I}(i)}$. This choice of beamformer would suppress much of the interference, but not entirely. It will however be assumed that

$$\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{R}_{\text{IN}}^{\mathcal{I}(i)} \mathbf{w}_{R_{\mathcal{I}(i)}} \approx N_0.$$

The value of N_0 is already known to the transmitter and therefore the amount of feedback information may be reduced to only include $\|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$.

Recall that the channel matrix $\mathbf{H}_{\mathcal{I}(i),i}$ consists of rows $\mathbf{h}_i^H \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\mathbf{h}}^{\mathcal{I}(i),i})$. Observe that since the receive beamformer used when determining $\rho = \|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$ was independent of $\mathbf{H}_{\mathcal{I}(i),i}$, then

$$\tilde{\mathbf{h}} = \mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i} \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\mathbf{h}}^{\mathcal{I}(i),i}).$$

The important consequence is that the conditional covariance matrix $\widehat{\mathbf{R}}_{\mathbf{h}}^{\mathcal{I}(i),i}(\rho)$ may be calculated as in Section 4.1.4. Let the eigenvalue decomposition of the conditional covariance matrix be denoted

$$\widehat{\mathbf{R}}_{\mathbf{h}}(\rho) = \mathbf{U}\widehat{\mathbf{\Lambda}}(\rho)\mathbf{U}^H,$$

where $\widehat{\mathbf{\Lambda}}(\rho)$ is a diagonal matrix with the eigenvalues $\widehat{\lambda}_m(\rho)$ calculated from the original eigenvalues using (4.10). Then the mean received signal power may be estimated at the transmitter as

$$E\{|P\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i}|^2\} = P\mathbf{w}_{T_i}^H \widehat{\mathbf{R}}_{\mathbf{h}}(\rho) \mathbf{w}_{T_i} = P\mathbf{w}_{T_i}^H \mathbf{U} \widehat{\mathbf{\Lambda}}(\rho) \mathbf{U}^H \mathbf{w}_{T_i}$$

and will be maximized by

$$\mathbf{w}_{T_i} = \mathbf{U}_{\cdot,k}, \text{ where } k = \arg \max_m \widehat{\lambda}_m(\rho). \quad (6.15)$$

The transmitter will then estimate the SINR as

$$\widehat{\text{SINR}}_i = \frac{P\widehat{\lambda}_k(\rho)}{\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{R}_{\mathbf{IN}}^{\mathcal{I}(i)} \mathbf{w}_{R_{\mathcal{I}(i)}}}, \quad (6.16)$$

with $\rho = \|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$ and the conditional eigenvalues calculated according to (4.10). The receiver will finally use its beamformer to maximize the SINR using maximum ratio combining, i.e., choosing $\mathbf{w}_{R_{\mathcal{I}(i)}}$ as the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_{\mathbf{IN}}^{-1}(\mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i} \mathbf{w}_{T_i}^H \mathbf{H}_{\mathcal{I}(i),i}^H)$.

Method 2

This method will focus on using a receive beamformer in the feedback information that maximizes $\mathbf{R}_{\mathbf{IN}}^{-1}(\mathbf{H}_{\mathcal{I}(i),i} \mathbf{H}_{\mathcal{I}(i),i}^H)$, i.e., that solves the maximum ratio combining problem when the influence of the transmit beamformer is neglected. This method seems to be quite similar to the previous method, but now the value of $\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{R}_{\mathbf{IN}}^{\mathcal{I}(i)} \mathbf{w}_{R_{\mathcal{I}(i)}}$ will not be known to the transmitter. There is another important difference, namely that the receive beamformer used in the feedback is dependent on the current channel realization. Therefore the expression of $\widehat{\mathbf{R}}_{\mathbf{h}}^{\mathcal{I}(i),i}(\rho)$ used in the previous method will no longer be valid, but it may still be an acceptable approximation. This will be shown with the following short simulation.

Consider the case when the receiver uses the left singular vector corresponding to the largest singular value of $\mathbf{H}_{\mathcal{I}(i),i}$ as receive beamformer $\mathbf{w}_{R_{\mathcal{I}(i)}}$. The MISO channel $\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}$ will no longer be complex Gaussian, but the error of approximating the conditional covariance matrix $\widehat{\mathbf{R}}_{\mathbf{h}}^{\mathcal{I}(i),i}(\rho)$ (for $\rho = \|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$) with the approach in Section 4.1.4 will be evaluated in this simulation. The correct conditional covariance matrix has been estimated using 10,000 samples (generated with Monte Carlo simulation) for each integer value of ρ . It is compared with the

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approximative approach in an environment with a 4 antenna UCA with antenna distance $\lambda_c/2$ at the transmitter and a receiver with 4 antennas at a transmission angle of 30 degrees. The standard deviation of the angular spread was 15 degrees.

The results are shown in Figure 6.7. The left part, (a), shows the largest singular value of the difference between the correct and estimated covariance matrices (normalized by the norm of the covariance matrix), while right part, (b), shows the ratio between the correct and estimated eigenvalues of $\widehat{\mathbf{R}}_{\mathbf{h}}^{\mathcal{I}(i),i}(\rho)$. The simulation shows clearly that the approximation is good for large values of ρ . The largest eigenvalue is slightly overestimated for weak realizations, but otherwise correct. The two smallest eigenvalues seems to be correct independently of the realization strength, but it should be noted that these eigenvalues are very small. The second largest eigenvalue is underestimated and there seems to be a bias even for strong realizations. This problem may be solved by introducing a model of how this eigenvalue should be modified, but this will not be considered in the thesis since only the largest eigenvalue will be exploited in the beamforming. The conclusion is that the approximative model is good, especially for strong channel realizations (which are common in a system with multiuser diversity). This method will consider receive beamformers that are less correlated with the channel and these are believed to give an even better approximation.

Hence, the feedback variable $\rho = \|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$ may be used to approximate the conditional covariance matrix as

$$\widehat{\mathbf{R}}_{\mathbf{h}}^{\mathcal{I}(i),i}(\rho) \approx \mathbf{U} \widehat{\mathbf{\Lambda}}(\rho) \mathbf{U}^H,$$

where $\widehat{\mathbf{\Lambda}}(\rho)$ is a diagonal matrix with the eigenvalues $\widehat{\lambda}_m(\rho)$ calculated from the original eigenvalues using (4.10). Then the mean received signal power may be estimated at the transmitter as

$$E\{|P\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i}|^2\} = P\mathbf{w}_{T_i}^H \widehat{\mathbf{R}}_{\mathbf{h}}(\rho) \mathbf{w}_{T_i} = P\mathbf{w}_{T_i}^H \mathbf{U} \widehat{\mathbf{\Lambda}}(\rho) \mathbf{U}^H \mathbf{w}_{T_i}$$

and will be maximized by

$$\mathbf{w}_{T_i} = \mathbf{U}_{\cdot, \mathbf{k}}, \text{ where } k = \arg \max_m \widehat{\lambda}_m(\rho). \quad (6.17)$$

The transmitter will then estimate the SINR as

$$\widehat{\text{SINR}}_i = \frac{P\widehat{\lambda}_k(\rho)}{\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{R}_{\mathbf{IN}}^{\mathcal{I}(i)} \mathbf{w}_{R_{\mathcal{I}(i)}}}, \quad (6.18)$$

with $\rho = \|\mathbf{w}_{R_{\mathcal{I}(i)}}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$ and the conditional eigenvalues calculated according to (4.10). The receiver will finally use its beamformer to maximize the SINR using maximum ratio combining, i.e., choosing $\mathbf{w}_{R_{\mathcal{I}(i)}}$ as the eigenvector corresponding to the largest eigenvalue of $\mathbf{R}_{\mathbf{IN}}^{-1}(\mathbf{H}_{\mathcal{I}(i),i} \mathbf{w}_{T_i} \mathbf{w}_{T_i}^H \mathbf{H}_{\mathcal{I}(i),i}^H)$.

Figure 6.7. The error of estimating the conditional covariance matrix $\hat{\mathbf{R}}_\mathbf{h}(\rho)$, for $\rho = \|\mathbf{w}_T^H \mathbf{H}\|^2$, using the approach in Section 4.1.4 when the receive beamformer is the left singular vector corresponding to the largest singular value of $\mathbf{H}$. (a) shows the largest singular value of the difference between the covariance (normalized by the norm of the covariance matrix), while (b) shows the ratio between correct and estimated eigenvalues of $\hat{\mathbf{R}}_\mathbf{h}^{\mathcal{I}^{(i)},i}(\rho)$. The percentiles (10/30/50/70/90) are shown as vertical lines.

6.6 Performance comparison with reliable communication

This section will conclude the chapter by comparing the three feedback strategies introduced in Section 6.5 with the upper bound from Section 6.4.1 under somewhat fair conditions.

The multi-cell environment considered in the simulation has a frequency reuse factor of 1 and may be described by Figure 6.2a. For the ease of simulation, each cell is assumed to have exactly 20 users. These are placed randomly and uniformly in each circle, but restricted to $0.2 \leq r_d \leq 1$ to avoid unreasonable large energies. In order to reduce the dependence of the user positions 200 scenarios were considered with 100 realizations in each scenario. The simulation considers both ULA and UCA with four antennas at the transmitter, four receive antennas at the receiver, the antenna separation $\lambda_c/2$ and a standard deviation of the angular spread of 15 degrees. The ratio between transmitted power and noise spectral density was fixed at $P\mathbf{R}_{\mathbf{h},ii}/N_0 = 1$ on the cell boundary, following the notation given in Section 3.2.

The two previously described scheduling strategies, maximum throughput and proportional fair, will be considered. The average user throughput of the latter method was initialized by using the mean from the previous scenario and then corrected by 40 realizations before the real scheduling began to reduce the transient

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behavior. The exponential window was of length 40.

The fairness of the comparison is achieved by demanding an outage probability of 0.05. Recall the approach in Section 4.5 where the SINR estimate was reduced to fit the outage probability by using either a multiplicative factor or by subtracting a factor multiplied by the standard deviation. The latter approach will not be considered in this comparison since the statistics of the interference is unknown to the transmitter. Hence, the outage probability will be achieved by finding the α such that

$$\mathbb{P}\left((1 - \alpha)\widehat{\text{SINR}} > \text{SINR}\right) = 0.05.$$

The α -value is derived numerically as in Section 4.5, with the only exception that communication environment described above is used.

6.6.1 Result

The CDF of the user mean throughput and cell throughput is shown in Figure 6.8 and Figure 6.9 for the case with maximum throughput and proportional fair scheduling, respectively.

It is quite expected that the strategy with feedback of the SINR estimate would provide the highest performance. There is a gap of 0.5-1 bits (depending on the scheduler) between the upper bound and this strategy. The two other strategies have CDF:s of the cell throughput that are much wider in the case with maximal throughput scheduling and may give both lower and as high performance as SINR feedback. With proportional fair scheduling there is a clear gap to previous strategy. It is interesting to note that the two latter feedback strategies give almost the same performance. The strategy that uses $\mathbf{w}_R$ from maximum ratio combining in the feedback gives slightly better throughput, but this strategy will on the other hand demand feedback of two variables.

The upper bound is almost the same for both ULA and UCA, but there are differences in performance between the two antenna arrays. All three strategies give a noticeable better performance with UCA. The difference between the two scheduling methods is what would have been expected: Proportional fair scheduling guarantees a larger minimal user mean throughput but the cost of 1 bit of cell throughput, i.e., almost 1/4 of the throughput.

To summarize, the strategy with feedback of the SINR has a higher guaranteed cell throughput than the other methods, but these may give the same performance with maximum throughput scheduling while there is a clear difference with considering proportional fair scheduling. The two strategies that are based on feedback of $\|\mathbf{w}_{R(i)}^H \mathbf{H}_{\mathcal{I}(i),i}\|^2$ are very similar in performance. The version that uses $\mathbf{w}_R$ from maximum ratio combining in the feedback performs slightly better, but to the cost of one additional feedback variable.

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(a) CDF of the mean throughput per users and symbol time (in a cell with 20 users).

(b) CDF of the cell throughput per symbol time (in a cell with 20 users).

Figure 6.8. The user mean throughput and cell throughput for different beamforming strategies with maximum throughput scheduling. The environment is given in Figure 6.2a and each cell contains 20 randomly located users ($0.2 \leq r_d \leq 1$). The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

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(a) CDF of the mean throughput per users and symbol time (in a cell with 20 users).

(b) CDF of the cell throughput per symbol time (in a cell with 20 users).

Figure 6.9. The user mean throughput and cell throughput for different beamforming strategies with proportional fair scheduling. The environment is given in Figure 6.2a and each cell contains 20 randomly located users ($0.2 \leq r_d \leq 1$). The number of transmit and receive antennas is four and two different transmit antenna structures are considered: ULA (left column) and UCA (right column).

6.7 Summary

This chapter has generalized the analytical model from the previous chapters to the case with multiple cells that communicate simultaneously at the same frequency band. In this situation the downlink signal will not only be corrupted by white noise, but also correlated interference. It was shown in the beginning of the chapter that the signal-to-interference-and-noise ratio (SINR) will have the same effect on the channel capacity as the SNR had in a single-cell environment.

The occurrence of interference introduces several estimation problems, since it is unreasonable to assume that instantaneous SINR may be estimated either at the base station or the mobiles. The mobile may however estimate the covariance matrix of interference plus noise, averaged over all possible transmit beamformers at the interfering stations.

It is assumed throughout the chapter that the base stations may not cooperate in their beamforming, which leads to the fact that their beamforming decisions will be very similar to the single-cell case. The receiver is however exposed to a quite different task, namely to maximizing the SINR by finding a balance between maximizing the signal power and suppressing the interference. The optimal receive beamforming is given by maximum ratio combining, which corresponds to the solution of a generalized eigenvalue problem. This approach has been compared to the two suboptimal extremes: maximizing the signal power and minimizing the interference power. It is clear from the simulations that these two suboptimal approaches give a significant lower performance. It is however interesting to notice that the interference minimizing method gives a much better result when the base station transmits with a signal power much larger than the noise power. This effect will probably be more legible as the number of receive antennas increases.

The chapter has also introduced three feedback strategies that have been compared under somewhat fair conditions. The first strategy provides the best performance and is based on feedback of the SINR. The two other strategies are based on the idea that the receiver may use another beamformer in the feedback information than it will actually use when transmissions are received. The first of these strategies feedbacks the channel norm when the receive beamformer is fixed to minimize the interference. By assuming that the remaining interference will be small and that the resulting MISO channel will have the same distribution as rows of the original channel matrix, the transmitter may use the conditional covariance matrix as in the previous chapters. The second of these strategies uses a receive beamformer based on maximum ratio combining in the feedback. This strategy will only give approximately the same analysis as the previous strategy and the remaining interference power needs also to be fed back. The latter method gives a slightly better performance, but not enough to justify the additional feedback variable.

Chapter 7

Summary

The thesis has reviewed and motivated a model of a cellular narrowband MIMO communication system with beamforming, additive white Gaussian noise and possibly interference from other cells. The channels from the base station to each receive antenna at the mobile are assumed to be independent and identically distributed complex Gaussian variables. The thesis has also considered a simulation model based on local scattering that has been used in simulations throughout the thesis for ULA and UCA as antenna structure at the base station.

Several feedback strategies for the downlink have been analyzed to produce optimal beamforming in terms of maximizing the SNR and thereby the capacity for the case when the base station only transmits a single beam. The feedback strategies are based on the channel norm and most of them may be used in a SDMA system. Two of the strategies are MIMO generalizations of the channel norm supported eigenbeamforming strategy introduced for MISO systems in [19]. The minimum mean-square error SNR estimate and its variance have been analyzed at the transmitter and should be seen as major contributions by the thesis. All feedback strategies mentioned in the thesis were first analyzed in the single-cell case and then generalized to allow multiple users and use scheduling to exploit multiuser diversity by schedule users when they experience particular strong channel realizations.

For the multi-cell case, with intercell interference, the base stations were assumed to act independently of each other. As a consequence, the transmit beamforming problem will be similar to that of the single-cell case. The receive beamformer will however affect both the signal and the interference, which makes the problem a bit more complex. Three approaches have been analyzed and compared, namely the maximizing signal power, minimizing interference power, and performing the SINR maximizing combination. These were used to propose, analyze and compare three new feedback strategies influenced by those used in the single-cell case.

7.1 Conclusions

The analysis of narrowband channels is important in the development of MIMO-OFDM, which is believed to be the foundation of the fourth generation of mobile telecommunication systems. The channel of such systems may be modeled for very specific and complex environments, but the thesis focused on analyzing general properties. Therefore the Rayleigh fading model has been adopted for its relevance in environments with rich scattering and its relative simple mathematical structure. The assumption of independent and identically distributed channels to each mobile antenna proved to be valuable in the generalization of previously known MISO strategies.

The thesis has considered downlink strategies based on limited feedback of channel norms. This information has been exploited at the transmitter to solve several related problems: SNR/SINR and rate estimation, transmit beamforming and scheduling. The rate estimation problem may be avoided by feedback of the overall channel norm for fixated beamformers. Simulations show that this approach is the most efficient (among those considered in the thesis) in terms of cell throughput for the case when only a single beam is transmitted by each base station. The approach is however limited to situations where the transmit beamformer may be estimated at the receiver. The other strategies studied in the thesis permit cleverer beamforming and their concepts are quite straightforward to generalize to a SDMA system, which seems to be the option of interest in future multiuser communication.

Two particularly interesting strategies are introduced by the thesis and analyzed for the single-cell case. The first strategy is based on feedback of the squared channel norm to each receive antenna, while the second strategy only feedback the largest squared norm. Both strategies are based on exploiting eigenbeamforming based on the feedback conditional channel covariance matrices. The SNR (and its variance) is estimated at the transmitter based on MMSE estimation and the corresponding rate estimate is used for scheduling. The former method will naturally provide a better performance due to the larger amount of feedback. The simulations show that the majority of the increase in performance between the first method and the approach without feedback is achieved by the second method. The difference will of course depend on the number of antennas and the simulation environment.

In the multi-cell case with intercell interference, the optimal receive beamformer in terms of SINR is given by a generalized eigenvalue problem that optimally combines signal maximization and interference suppression. It is shown in simulations that the suboptimal strategy of only suppressing the interference is considerably better in high SNR situations than only maximizing the signal power, while they are quite comparable in low SNR. The approach provides a significantly lower capacity than the optimal solution, but it is shown in the concluding multi-cell simulation that it may be advantageous to use this information in the feedback. When the receive beamformer is fixated and independent of the channel realization, the squared norm of the remaining channel may be treated as in the single-cell case.

Two methods of achieving reliable communication in terms of avoiding over-

7.2. FUTURE WORK

estimation of the supported rate have also been evaluated. The first is based on multiplying the estimate by a constant factor (smaller than one), while the second subtracts the standard deviation of the current estimate multiplied by a constant factor. The constant factors of these methods are chosen so that a specific outage probability is satisfied. Simulations show that the somewhat more computational demanding latter method has better properties in terms of that the factor is quite independent of the angular direction to the receiver and it slightly more efficient in terms of cell throughput in a system with multiuser diversity.

Finally, it may be noted that UCA has shown several advantages compared to ULA, which only seems to be attractive when the region of allowed users is limited to a quite thin sector.

7.2 Future work

The communication system considered in the thesis may be further analyzed in several directions. Some of these are summarized below:

- **SDMA.** The base stations in the thesis are restricted to transmitting only a single simultaneous beam. It has been noted several times in the thesis that such an approach will not be optimal in terms of maximizing the overall capacity. The capacity may however be achieved by exploiting all spatial dimension by incorporating SDMA with the different feedback strategies.
- **Feedback of $\|\mathbf{H}\|_F^2$.** The thesis considers strategies based on feedback of the squared channel norm to each receive antenna. This strategy was simplified by only demanding feedback of the maximum of them. Another obvious simplification would be to consider feedback of $\|\mathbf{H}\|_F^2$. Both these approaches have the advantage of only demanding feedback of a single variable.
- **More careful multi-cell communication.** The feedback strategies proposed and evaluated in the multi-cell chapter are quite simplified. It would be interesting to evaluate the possibility of developing strictly optimal strategies given some feedback information. It would also be interesting to analyze the variance of the SNR estimators of the three strategies with the purpose of enhancing the throughput for a given outage probability.
- **Uplink communication.** The analysis and simulations have only considered the downlink of communication channel, but with some adjustments it seems like some of the analysis will still be valid for the uplink.
- **Evaluation in specific simulation environments.** The strategies presented in the thesis have been evaluated in a quite simple and unspecific simulation environment. It would be interesting to evaluation the performance in other environment, perhaps used for simulations in industry.

Appendix A

Complementary theory and derivations

A.1 Complex Gaussian distribution

Most of the analysis in the thesis is performed in the complex baseband equivalent to a passband AWGN channel. This motivates the interest of complex generalizations of common random distributions, especially regarding the Gaussian distribution. This appendix will introduce a special form of the Complex Gaussian distribution, based on [2].

A random variable $x = x_R + ix_I$, where $x_R, x_I \in \mathbb{R}$, is complex Gaussian if x_R and x_I are Gaussian distributed. A special type of these complex Gaussian variables will be used throughout the thesis, namely such that are circular symmetric. These satisfies that x and $e^{j\theta}x$ are identically distributed for all $\theta \in \mathbb{R}$. By examining the first and second moment of the real and imaginary part of $e^{j\theta}x$, it is seen that the circular symmetry is only satisfied if x_R and x_I are independent and identically distributed zero-mean Gaussian variables. These requirements are exactly those used throughout the thesis when complex Gaussian variables are used.

The following notation is used for a circular symmetric complex Gaussian variable: $x \in \mathcal{CN}(0, \sigma^2)$ if and only if $x = x_R + ix_I$, with the independent variables $x_R, x_I \in \mathcal{N}(0, \sigma^2/2)$.

As a consequence of the circular symmetry the variable $x \in \mathcal{CN}(0, \sigma^2)$ may also be expressed as $x = |x|e^{j\phi}$, where $|x| \in R(\sigma/\sqrt{2})$, i.e., Rayleigh distributed, and independent of the phase ϕ that is uniformly distributed over $[0, 2\pi)$.

A.2 Covariance matrices with the local scattering model

This appendix will derive approximative closed form expressions for

$$\mathbf{R}_{\mathbf{a}}(\theta) = E \left\{ \mathbf{a}(\theta + \tilde{\theta}_k(t)) \mathbf{a}^H(\theta + \tilde{\theta}_k(t)) \right\},$$

with arbitrarily θ and $\tilde{\theta}_k \in \mathcal{N}(0, \sigma_a^2)$. Two types of array response structures will be considered, namely ULA and UCA. It will be assumed that the angular spread $\tilde{\theta}$ is quite small.

A.2.1 Derivation of $\mathbf{R}_a$ with ULA

This section will derive an expression for $\mathbf{R}_a(\theta)$ for ULA. The derivation is inspired by the approach in [5]. Recall the array response given in (1.1) and observe that

$$\begin{aligned}\mathbf{R}_a(\theta) &= E \left\{ \mathbf{a}_{\text{ULA}}(\theta + \tilde{\theta}) \mathbf{a}_{\text{ULA}}^H(\theta + \tilde{\theta}) \right\} \\ &= \int_{-\infty}^{\infty} p(\tilde{\theta}, \sigma) \left[e^{j0} \dots e^{j \frac{2\pi d}{\lambda_c} \sin(\theta + \tilde{\theta}) \cdot (n-1)} \right]^T \left[e^{-j0} \dots e^{-j \frac{2\pi d}{\lambda_c} \sin(\theta + \tilde{\theta}) \cdot (n-1)} \right] d\tilde{\theta},\end{aligned}$$

where $p(\tilde{\theta}, \sigma)$ is the Gaussian density function for the angular spread. For small angles β the following approximation is reasonable

$$\begin{aligned}\sin(\alpha + \beta) &= \sin(\alpha) \cos(\beta) + \sin(\beta) \cos(\alpha) \\ &\approx \sin(\alpha) + \beta \cos(\alpha)\end{aligned}$$

and since it has been assumed that $\tilde{\theta}$ typically is small, the k th element of $\mathbf{a}_{\text{ULA}}(\theta + \tilde{\theta})$ may be approximated as

$$e^{j \frac{2\pi d}{\lambda_c} \sin(\theta + \tilde{\theta}) \cdot (k-1)} \approx e^{j \frac{2\pi d}{\lambda_c} [\sin(\theta) + \tilde{\theta} \cos(\theta)] (k-1)}.$$

Using this, the kl th element in $\mathbf{a}_{\text{ULA}}(\theta + \tilde{\theta}) \mathbf{a}_{\text{ULA}}^H(\theta + \tilde{\theta})$ can be expressed as

$$e^{j \frac{2\pi d}{\lambda_c} [\sin(\theta) + \tilde{\theta} \cos(\theta)] (k-1)} e^{-j \frac{2\pi d}{\lambda_c} [\sin(\theta) + \tilde{\theta} \cos(\theta)] (l-1)} = e^{j \frac{2\pi d}{\lambda_c} [\sin(\theta) + \tilde{\theta} \cos(\theta)] (k-l)}.$$

The kl th element of the covariance matrix may now be expressed as

$$\begin{aligned}\mathbf{R}_{kl} &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_a^2}} e^{-\frac{\tilde{\theta}^2}{2\sigma_a^2}} e^{j \frac{2\pi d}{\lambda_c} [\sin(\theta) + \tilde{\theta} \cos(\theta)] (k-l)} d\tilde{\theta} \\ &= e^{j \frac{2\pi d}{\lambda_c} \sin(\theta) (k-l)} e^{-\frac{\sigma_a^2}{2} \left(\frac{2\pi d}{\lambda_c} \cos(\theta) (k-l) \right)^2} \\ &\quad \times \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_a^2}} e^{-\frac{1}{2\sigma_a^2} \left(\tilde{\theta} - j \frac{2\pi d \sigma_a^2}{\lambda_c} \cos(\theta) (k-l) \right)^2} d\tilde{\theta},\end{aligned}$$

where the remaining integral can be interpreted as being over a Gaussian density function. Hence, the integral will be equal to 1 and the kl th element of the covariance matrix may be expressed as

$$\mathbf{R}_{kl} = e^{j \frac{2\pi d}{\lambda_c} \sin(\theta) (k-l)} e^{-\frac{\sigma_a^2}{2} \left(\frac{2\pi d}{\lambda_c} \cos(\theta) (k-l) \right)^2}. \quad (\text{A.1})$$

A.2.2 Derivation of $\mathbf{R}_a$ with UCA

This section will derive an expression for $\mathbf{R}_a(\theta)$ for UCA. The derivation is a slightly modified version of the one provided in [5]. Recall the array response given in (1.1)

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and observe that

$$\begin{aligned}\mathbf{R}_a(\theta) &= E \left\{ \mathbf{a}_{\text{UCA}}(\theta + \tilde{\theta}) \mathbf{a}_{\text{UCA}}^H(\theta + \tilde{\theta}) \right\} \\ &= \int_{-\infty}^{\infty} p(\tilde{\theta}, \sigma) \left[e^{-j \frac{2\pi\alpha}{\lambda_c} \cos(\theta + \tilde{\theta})} \dots e^{-j \frac{2\pi\alpha}{\lambda_c} \cos(\theta + \tilde{\theta} - 2\pi \frac{n-1}{n})} \right]^T \\ &\quad \times \left[e^{j \frac{2\pi\alpha}{\lambda_c} \cos(\theta + \tilde{\theta})} \dots e^{j \frac{2\pi\alpha}{\lambda_c} \cos(\theta + \tilde{\theta} - 2\pi \frac{n-1}{n})} \right] d\tilde{\theta},\end{aligned}$$

where $p(\tilde{\theta}, \sigma)$ is the Gaussian density function for the angular spread. For small angles β the following approximation is reasonable

$$\begin{aligned}\cos(\alpha + \beta) &= \cos(\alpha) \cos(\beta) - \sin(\beta) \sin(\alpha) \\ &\approx \cos(\alpha) - \beta \sin(\alpha)\end{aligned}$$

and since it has been assumed that $\tilde{\theta}$ typically is small, the k th element of $\mathbf{a}_{\text{UCA}}(\theta + \tilde{\theta})$ may be approximated as

$$\begin{aligned}e^{-j \frac{2\pi\alpha}{\lambda_c} \cos(\theta + \tilde{\theta} - 2\pi \frac{k-1}{n})} &\approx e^{-j \frac{2\pi\alpha}{\lambda_c} [\cos(\theta - 2\pi \frac{k-1}{n}) - \tilde{\theta} \sin(\theta - 2\pi \frac{k-1}{n})]} \\ &= e^{-j \frac{2\pi\alpha}{\lambda_c} [\cos(2\pi \frac{k-1}{n} - \theta) + \tilde{\theta} \sin(2\pi \frac{k-1}{n} - \theta)]}.\end{aligned}$$

Using this, the kl th element in $\mathbf{a}_{\text{UCA}}(\theta + \tilde{\theta}) \mathbf{a}_{\text{UCA}}^H(\theta + \tilde{\theta})$ can be expressed as

$$\begin{aligned}e^{-j \frac{2\pi\alpha}{\lambda_c} [\cos(2\pi \frac{k-1}{n} - \theta) + \tilde{\theta} \sin(2\pi \frac{k-1}{n} - \theta)]} e^{j \frac{2\pi\alpha}{\lambda_c} [\cos(2\pi \frac{l-1}{n} - \theta) + \tilde{\theta} \sin(2\pi \frac{l-1}{n} - \theta)]} &= \\ e^{-j \frac{4\pi\alpha}{\lambda_c} [-\sin(2\pi \frac{k-l}{2n}) \sin(2\pi \frac{k+l-2}{2n} - \theta) + \tilde{\theta} \sin(2\pi \frac{k-l}{2n}) \cos(2\pi \frac{k+l-2}{2n} - \theta)]} &= \end{aligned}$$

The kl th element of the covariance matrix can now expressed as

$$\begin{aligned}\mathbf{R}_{kl} &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_a^2}} e^{-\frac{\tilde{\theta}^2}{2\sigma_a^2}} e^{-j \frac{4\pi\alpha}{\lambda_c} [-\sin(2\pi \frac{k-l}{2n}) \sin(2\pi \frac{k+l-2}{2n} - \theta) + \tilde{\theta} \sin(2\pi \frac{k-l}{2n}) \cos(2\pi \frac{k+l-2}{2n} - \theta)]} d\tilde{\theta} \\ &= e^{j \frac{4\pi\alpha}{\lambda_c} \sin(2\pi \frac{k-l}{2n}) \sin(2\pi \frac{k+l-2}{2n} - \theta)} e^{-\frac{1}{2\sigma_a^2} \left(\frac{4\pi\alpha\sigma_a^2}{\lambda_c} \sin(2\pi \frac{k-l}{2n}) \cos(2\pi \frac{k+l-2}{2n} - \theta) \right)^2} \\ &\quad \times \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_a^2}} e^{-\frac{1}{2\sigma_a^2} \left(\tilde{\theta} + j \frac{4\pi\alpha\sigma_a^2}{\lambda_c} \sin(2\pi \frac{k-l}{2n}) \cos(2\pi \frac{k+l-2}{2n} - \theta) \right)^2} d\tilde{\theta},\end{aligned}$$

where the remaining integral can be interpreted as being over a Gaussian density function. Hence, the integral will be equal to 1 and the kl th element of the covariance matrix may be expressed as

$$\mathbf{R}_{kl} = e^{j \frac{4\pi\alpha}{\lambda_c} \sin(\pi \frac{k-l}{n}) \sin(\pi \frac{k+l-2}{n} - \theta)} e^{-\frac{\sigma_a^2}{2} \left(\frac{4\pi\alpha}{\lambda_c} \sin(\pi \frac{k-l}{n}) \cos(\pi \frac{k+l-2}{n} - \theta) \right)^2}. \quad (\text{A.2})$$

A.3 Conditional statistics with limited norm

This appendix contains complete derivations regarding some quite extensive results regarding the conditional statistics of $\mathbf{h} \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_h)$, provided that $0 \leq \|\mathbf{h}\|^2 \leq \rho_1$. These derivations should be seen as main contribution by the thesis, but have been gathered here to enhance readability of the rest of the thesis.

A.3.1 Conditional covariance matrix

This section will derive a closed form expression for the conditional covariance matrix of $\mathbf{h} \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_{\mathbf{h}})$, given that $0 \leq \|\mathbf{h}\|^2 \leq \rho_1$. The derivation will use the following result, proved in a Lemma in [8]:

$$\sum_{i=1}^n \frac{1}{\lambda_i^m \prod_{j=1, j \neq i}^n \left(1 - \frac{\lambda_j}{\lambda_i}\right)} = \begin{cases} 1, & m = 0, \\ 0, & m = 1 \dots n, \end{cases} \quad \lambda_i > 0, \quad \lambda_i \neq \lambda_j, i \neq j. \quad (\text{A.3})$$

Let $\mathbf{R}_{\mathbf{h}} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ denote the unconditional eigenvalue decomposition of the covariance matrix, where $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \dots, \lambda_{n_T}\}$ is diagonal with the eigenvalues in arbitrary order. The eigenvalues are assumed to be unique. Let $\rho = \|\mathbf{h}\|^2$, then conditional covariance matrix $\hat{\mathbf{R}}_{\mathbf{h}}(0 \leq \rho \leq \rho_1)$ may then be expressed as

$$\hat{\mathbf{R}}_{\mathbf{h}}(0 \leq \rho \leq \rho_1) = E\left\{\mathbf{h}\mathbf{h}^H \middle| 0 \leq \rho \leq \rho_1\right\} = \mathbf{U}\hat{\mathbf{\Lambda}}(0 \leq \rho \leq \rho_1)\mathbf{U}^H, \quad (\text{A.4})$$

where $\hat{\mathbf{\Lambda}}(0 \leq \rho \leq \rho_1)$ is a diagonal matrix with the conditional eigenvalues denoted $\hat{\lambda}_m(0 \leq \rho \leq \rho_1), m = 1 \dots n_T$. These eigenvalues may be expressed using notation from [19] that previously has been stated in (4.10) and (4.11):

$$\hat{\lambda}_m(0 \leq \rho \leq \rho_1) = E\{\hat{\lambda}_m(\rho) | 0 \leq \rho \leq \rho_1\} = \int_0^{\rho_1} \hat{\lambda}_m(\rho) \frac{f_{\|\mathbf{h}\|^2}(\rho)}{\int_0^{\rho_1} f_{\|\mathbf{h}\|^2}(\varrho) d\varrho} d\rho, \quad (\text{A.5})$$

with $\hat{\lambda}_m(\rho)$ and $f_{\|\mathbf{h}\|^2}(\rho)$ given in (4.10) and (4.11), respectively. The normalization factor $\int_0^{\rho_1} f_{\|\mathbf{h}\|^2}(\varrho) d\varrho$ can be calculated as

$$\begin{aligned} \int_0^{\rho_1} f_{\|\mathbf{h}\|^2}(\varrho) d\varrho &= \left[- \sum_{k=1}^{n_T} \frac{e^{\varrho/\lambda_k}}{\prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)} \right]_0^{\rho_1} \\ &= \sum_{k=1}^{n_T} \frac{1}{\prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)} - \sum_{k=1}^{n_T} \frac{e^{\rho_1/\lambda_k}}{\prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)} \\ &= 1 - \sum_{k=1}^{n_T} \frac{e^{\rho_1/\lambda_k}}{\prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)}, \end{aligned} \quad (\text{A.6})$$

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where (A.3) was used in the last step and λ_i is the unconditional eigenvalues. The main integral in (A.5) may then be calculated as

$$\begin{aligned}
& \int_0^{\rho_1} \hat{\lambda}_m(\rho) f_{\|\mathbf{h}\|^2}(\rho) d\rho = \\
& = \int_0^{\rho_1} \frac{\rho e^{-\frac{\rho}{\lambda_m}}}{\lambda_m \prod_{i \neq m} \left(1 - \frac{\lambda_i}{\lambda_m}\right)} + \sum_{k \neq m} \frac{e^{-\frac{\rho}{\lambda_m}} - e^{-\frac{\rho}{\lambda_k}}}{\left(1 - \frac{\lambda_k}{\lambda_m}\right) \prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)} d\rho \\
& = \left[-\frac{\rho e^{-\frac{\rho}{\lambda_m}} + \lambda_m e^{-\frac{\rho}{\lambda_m}}}{\prod_{i \neq m} \left(1 - \frac{\lambda_i}{\lambda_m}\right)} + \sum_{k \neq m} \frac{-\lambda_m e^{-\frac{\rho}{\lambda_m}} + \lambda_k e^{-\frac{\rho}{\lambda_k}}}{\left(1 - \frac{\lambda_k}{\lambda_m}\right) \prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)} \right]_0^{\rho_1} \quad (\text{A.7}) \\
& = \frac{\lambda_m(1 - e^{-\frac{\rho_1}{\lambda_m}}) - \rho_1 e^{-\frac{\rho_1}{\lambda_m}}}{\prod_{i \neq m} \left(1 - \frac{\lambda_i}{\lambda_m}\right)} + \sum_{k \neq m} \frac{\lambda_m(1 - e^{-\frac{\rho_1}{\lambda_m}}) - \lambda_k(1 - e^{-\frac{\rho_1}{\lambda_k}})}{\left(1 - \frac{\lambda_k}{\lambda_m}\right) \prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)} \\
& = \lambda_m - \frac{(\lambda_m + \rho_1)e^{-\frac{\rho_1}{\lambda_m}}}{\prod_{i \neq m} \left(1 - \frac{\lambda_i}{\lambda_m}\right)} + \sum_{k \neq m} \frac{\lambda_k e^{-\frac{\rho_1}{\lambda_k}} - \lambda_m e^{-\frac{\rho_1}{\lambda_m}}}{\left(1 - \frac{\lambda_k}{\lambda_m}\right) \prod_{i \neq k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)},
\end{aligned}$$

where the last equality follows from putting the terms without exponential factors outside the brackets in the numerator and using (A.3).

To summarize, the covariance matrix $\mathbf{R}_h = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ conditioned on that $0 \leq \|h\|^2 \leq \rho_1$ may be expressed as $\hat{\mathbf{R}}_h(0 \leq \rho \leq \rho_1) = \mathbf{U}\hat{\mathbf{\Lambda}}(0 \leq \rho \leq \rho_1)\mathbf{U}^H$, where $\hat{\mathbf{\Lambda}}(0 \leq \rho \leq \rho_1)$ is a diagonal matrix with the conditional eigenvalues $\hat{\lambda}_m(\rho)$ given by (A.5), (A.6) and (A.7).

A.3.2 Conditional fourth order moments

This section will derive the conditional fourth order moment

$$E \{ |v_k|^4 | 0 \leq \|\mathbf{h}\|^2 \leq \rho_1 \}$$

for elements of the white vector $\mathbf{v} = [v_1 \dots v_{n_T}]^T = \mathbf{U}^H \mathbf{h}$, where $\mathbf{h} \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_h)$ and $\mathbf{R}_h = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ is the eigenvalue decomposition. The matrix $\mathbf{\Lambda} = \text{diag} \{ \lambda_1, \dots, \lambda_{n_T} \}$ is diagonal with the eigenvalues in arbitrary order. The eigenvalues are assumed to be unique. The fourth order moment may be expressed using notation, given in [19], that has previously been stated in (4.11) and (4.25):

$$E \{ |v_k|^4 | 0 \leq \rho \leq \rho_1 \} = \int_0^{\rho_1} E \{ |v_k|^4 | \rho \} \frac{f_{\|\mathbf{h}\|^2}(\rho)}{\int_0^{\rho_1} f_{\|\mathbf{h}\|^2}(\varrho) d\varrho} d\rho, \quad (\text{A.8})$$

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where $\rho = \|\mathbf{h}\|^2$. The normalization factor $\int_0^{\rho_1} f_{\|\mathbf{h}\|^2}(\varrho) d\varrho$ was calculated in (A.6), while the main integral may be calculated as

$$\begin{aligned}
 & \int_0^{\rho_1} E \{ |v_k|^4 | \rho \} f_{\|\mathbf{h}\|^2}(\rho) d\rho = \\
 & = \int_0^{\rho_1} \frac{\left(\rho - \sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}} \right)^2 + \sum_{j \neq k} \frac{\lambda_j^2}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2}}{\lambda_k \prod_{l \neq k} \left(1 - \frac{\lambda_l}{\lambda_k}\right)} e^{-\frac{\rho}{\lambda_k}} + \sum_{j \neq k} \frac{2\lambda_j e^{-\frac{\rho}{\lambda_j}}}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2 \prod_{l \neq j} \left(1 - \frac{\lambda_l}{\lambda_j}\right)} d\rho \\
 & = \frac{\left[-\left(\rho_1 - \sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}} \right)^2 - 2\lambda_k \left(\rho_1 - \sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}} \right) - 2\lambda_k^2 - \sum_{j \neq k} \frac{\lambda_j^2}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2} \right] e^{-\frac{\rho_1}{\lambda_k}}}{\lambda_k \prod_{l \neq k} \left(1 - \frac{\lambda_l}{\lambda_k}\right)} \\
 & + \frac{\left(\sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}} \right)^2 - 2\lambda_k \sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}} + 2\lambda_k^2 + \sum_{j \neq k} \frac{\lambda_j^2}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2}}{\lambda_k \prod_{l \neq k} \left(1 - \frac{\lambda_l}{\lambda_k}\right)} + \\
 & + 2 \sum_{j \neq k} \frac{\lambda_j^2 \left(1 - e^{-\frac{\rho_1}{\lambda_j}}\right)}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2 \prod_{l \neq j} \left(1 - \frac{\lambda_l}{\lambda_j}\right)} \\
 & = \frac{\left(\lambda_k - \sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}} \right)^2 - \left(\rho_1 + \lambda_k - \sum_{j \neq k} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_k}} \right)^2 e^{-\frac{\rho_1}{\lambda_k}}}{\lambda_k \prod_{l \neq k} \left(1 - \frac{\lambda_l}{\lambda_k}\right)} + \\
 & + \frac{\left(\lambda_k + \sum_{j \neq k} \frac{\lambda_j^2}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2} \right) \left(1 - e^{-\frac{\rho_1}{\lambda_k}}\right)}{\lambda_k \prod_{l \neq k} \left(1 - \frac{\lambda_l}{\lambda_k}\right)} + 2 \sum_{j \neq k} \frac{\lambda_j^2 \left(1 - e^{-\frac{\rho_1}{\lambda_j}}\right)}{\left(1 - \frac{\lambda_j}{\lambda_k}\right)^2 \prod_{l \neq j} \left(1 - \frac{\lambda_l}{\lambda_j}\right)}. \tag{A.9}
 \end{aligned}$$

Observe that the first two terms of the last expression have the same denominator and have only been separated due to space limitations.

To summarize, the conditional fourth order moment $E \{ |v_k|^4 | 0 \leq \|\mathbf{h}\|^2 \leq \rho_1 \}$ may be calculated as (A.9) divided by (A.6).

Bibliography

- [1] C. Capps, “Near field or far field?,” *Electrical Design News (EDN)*, 2001.
- [2] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.
- [3] J. Winters, “Optimum combining in digital mobile radio with cochannel interference,” *IEEE Journal on Selected Areas in Communications*, vol. 2, no. 4, pp. 528–539, 1984.
- [4] K. Yu, M. Bengtsson, and B. Ottersten, “MIMO channel models,” in *Smart Antennas – State-of-the-Art*. Hindawi Publishing Corporation, 2004.
- [5] R. Stridh, *Smart Antennas in Wireless Networks: System Issues and Performance Limits*, Ph.D. thesis, Royal Institute of Technology (KTH), 2003.
- [6] B. Völcker, M. Bengtsson, and B. Ottersten, “Spatially spread sources in antenna array processing,” in *Adaptive Antenna Arrays: Trends and Applications*, pp. 394–419. Springer Verlag, 2004.
- [7] V. Kühn, *Wireless Communication over MIMO Channels: Applications to CDMA and Multiple Antenna Systems*, Wiley, 2006.
- [8] D. Hammarwall, M. Bengtsson, and B. Ottersten, “Acquiring partial CSI for spatially selective transmission by instantaneous channel norm feedback,” Submitted to *IEEE Trans. on Signal Processing*.
- [9] T.M. Cover and J.A. Thomas, *Elements of Information Theory*, Wiley, 1991.
- [10] J.W. Lindeberg, “Eine neue herleitung des exponentialgesetzes in der wahrscheinlichkeitsrechnung,” *Mathematische Zeitschrift*, vol. 15, pp. 211–225, 1922.
- [11] S.L. Zabell, “Alan Turing and the Central limit theorem,” *Mathematical Association of America*, vol. 102, no. 6, pp. 483–494, 1995.
- [12] M. Bengtsson and B. Ottersten, “Optimal and suboptimal transmit beamforming,” in *Handbook of Antennas in Wireless Communications*. CRC Press, 2001.

BIBLIOGRAPHY

- [13] C.E. Shannon, "A mathematical theory of communication," *Bell System Technical Journal*, vol. 27, no. 6, pp. 379–423, 623–656, 1948.
- [14] R. Negi and J. Cioffi, "Pilot tone selection for channel estimation in a mobile OFDM system," *IEEE Trans. on Consumer Electronics*, vol. 4, no. 3, pp. 1122–1128, 1998.
- [15] D.J. Love, R.W. Heath Jr., W. Santipach, and M.L. Honig, "What is the value of limited feedback for MIMO channels?," *IEEE Communications Magazine*, vol. 42, no. 10, pp. 54–59, 2004.
- [16] A.D. Dabbagh and D.J Love, "On the tradeoff between feedback rate and capacity loss in limited feedback MIMO systems," *IEEE Vehicular Technology Conference*, vol. 2, pp. 856–860, 2005.
- [17] B. Mondal and R.W. Heath Jr., "An upper bound on SNR for limited feedback MIMO beamforming systems," in *IEEE Information Theory Workshop*, 2004, pp. 408–412.
- [18] D. Gesbert, L. Pittman, and M. Kountouris, "Transmit correlation-aided scheduling in multiuser MIMO networks," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, 2006, vol. 4, pp. 249–252.
- [19] D. Samuelsson, M. Bengtsson, and B. Ottersten, "Improved multiuser diversity using smart antennas with limited feedback," in *Proc. European Signal Processing Conference*, 2005.
- [20] K. Huang, B Mondal, W. Heath, and J.G. Andrews, "Markov models for limited feedback MIMO systems," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, 2006, vol. 4, pp. 9–12.
- [21] C. Brunner, W. Utschick, and J.A. Nossek, "Exploiting the short-term and long-term channel properties in space and time: Eigenbeamforming. Concepts for the BS in WCDMA," *European Trans. on Telecommunications*, vol. 12, pp. 365–378, 2001.
- [22] R. Knopp and P.A. Humblet, "Information capacity and power control in single-cell multiuser communications," in *Proc. IEEE International Conference on Communications*, 1995, vol. 1, pp. 331–335.
- [23] P. Viswanath, D.N.C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. on Information Theory*, vol. 48, no. 6, pp. 1277–1294, 2002.
- [24] T. Ren and R.J. La, "Downlink beamforming algorithms with inter-cell interference in cellular networks," *IEEE Trans. on Wireless Communications*, vol. 5, no. 10, pp. 2814–2823, 2006.