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BEAMFORMING UTILIZING CHANNEL NORM FEEDBACK IN MULTIUSER MIMO SYSTEMS

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ABSTRACT

The problem of beamforming and rate estimation in a multi-user downlink multiple-input multiple-output (MIMO) system with limited feedback and statistical channel information at the transmitter is considered. In order to exploit the spatial properties of the channel, the norm of the channel to each receive antenna is computed. We propose to feed back the largest norm to the transmitter and derive the conditional second and fourth order channel moments in order to design the downlink beamforming weights.

Similar approaches have previously been presented for multi-user multiple-input single-output (MISO) systems. Herein, these techniques are generalized to MIMO systems, by either antenna selection or receive beamforming at the receiver. Two eigenbeamforming strategies are proposed and shown to outperform opportunistic beamforming, based on similar feedback information.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) systems have the potential of increasing the capacity linearly with the number of spatial sub-channels created by the antennas [1, 2]. The spatial diversity induced by multiple antennas may be exploited by transmit beamforming to maximize the signal-to-noise ratio (SNR) for a single beam directed to a single user in each time slot. When several users are available a scheduler may be used to increase the system throughput by scheduling users in time slots in which their channel realization supports a favorable data rate. A scheduling strategy that balances system throughput and user fairness is proportional fair scheduling [3].

The channel capacity with transmit beamforming is maximized if a system has full channel state information (CSI) at the transmitter [4], but the amount of feedback required to fulfill this requirement in a fast fading environment would be prohibitive in many systems. Several schemes with the intention of maintaining high system throughput when only a single variable is fed back per user have been proposed in the literature, e.g., opportunistic beamforming [3] and feedback supported eigenbeamforming [5].

In a system with opportunistic beamforming [3], the base station picks a transmit beamformer at random and each user feeds back the resulting SNR which is exploited by the scheduler. The performance is likely to be good in a system with many users, since there will be users that experience strong SNRs in almost any direction.

The opportunistic approach ignores the channel statistics, but since the SNR becomes known to the transmitter there is no need for rate estimation. Herein, the feedback supported eigenbeamforming strategy presented in [5] is generalized to multiple receive antennas. In this strategy, the receiver feeds back the squared norm of the strongest channel realization among those experienced by its antennas. This information is combined with the available CSI at the transmitter which chooses its beamformer in some clever way, e.g., based on conditional eigenbeamforming. It may also be exploited in spatial division multiple access (SDMA) systems for scheduling purposes [6]. A drawback of using channel norm feedback is that the SNR needs to be estimated at the transmitter, for the purpose of rate adaptation. Herein, the minimum mean square error (MMSE) estimate and its estimation variance are analyzed.

The proposed strategy is analyzed for different receive strategies: antenna selection and receive beamforming. The former approach fits perfectly into a system where the receiver is believed to be equipped with a single antenna. The latter approach is more efficient in terms of exploiting the antenna diversity. It is shown in simulations that both approaches achieve better throughput in a spatially correlated Rayleigh fading environment than the corresponding opportunistic beamforming strategies.

2. SYSTEM MODEL

The system model considers the downlink of a macrocell environment with an elevated base station and several mobile users [7]. The base station is exposed to limited local scattering and is equipped with an array of n_T antennas. Each mobile terminal is surrounded by rich local scattering and is equipped with an array of n_R antennas.

A narrowband frequency flat channel is considered and represented by its symbol sampled complex baseband equivalent. The channel matrix to user k is denoted

$$\mathbf{H}_k = [\mathbf{h}_{k,1} \dots \mathbf{h}_{k,n_R}]^H \in \mathbb{C}^{n_R \times n_T}, \tag{1}$$

and is modeled as Rayleigh fading. The rows are independently and identically distributed $\mathbf{h}_{k,n} \in \mathcal{CN}(\mathbf{0}, \mathbf{R}_k)$ for $n = 1 \dots n_R$, where $\mathbf{R}_k \in \mathbb{C}^{n_T \times n_T}$ is a positive semi-definite covariance matrix. Under the current assumptions, it is reasonable to assume that $\mathbf{R}_k$ is dominated by one or a few eigenmodes, i.e., there is significant spatial correlation. Observe that the merits of eigenbeamforming disappear for channels which are spatially uncorrelated. The received vector $\mathbf{y}_k(t) \in \mathbb{C}^{n_R}$ at user k satisfies

$$\mathbf{y}_k(t) = \mathbf{H}_k \mathbf{x}_k(t) + \mathbf{n}_k(t), \tag{2}$$

where $\mathbf{n}_k(t) \in \mathbb{C}^{n_R}$ is additive white Gaussian noise (AWGN) with zero-mean and covariance matrix $\sigma_k^2 \mathbf{I}$ and $\mathbf{x}_k(t) \in \mathbb{C}^{n_T}$ represents the vector of transmitted signals. The system model in (2) depends on three different time scales. The index $t \in \mathbb{Z}$ denotes the symbol slot on which scale the noise is a white process. The multipath propagation modeled by the channel matrix $\mathbf{H}_k$ changes more slowly and is regarded as semi-constant for a number of symbols. The covariance matrices change even more slowly due to large scale variations, thus the base station is assumed to track the current $\mathbf{R}_k$ and σ_k^2 perfectly. The current channel matrix $\mathbf{H}_k$ is only known to user k.

A beamforming system is considered where the base station maps the scalar signal $s_k(t)$, intended for users k, onto the antenna array using the beamforming vector $\mathbf{w}_{T_k}$:

$$\mathbf{x}_k(t) = \mathbf{w}_{T_k} s_k(t). \tag{3}$$

The signal $s_k(t)$ is normalized to unit mean power and the beamformer satisfies some power constraint $\|\mathbf{w}_{T_k}\|^2 = P$. Two strategies of achieving the scalar receive signal $r_k(t)$ at user k will be considered: antenna selection and receive beamforming.

2.1. Antenna selection

In this strategy, user k listens exclusively to the antenna which experiences the strongest channel realization, in terms of SNR. This antenna is denoted the mth antenna:

$$m = \arg\max_i \frac{|\mathbf{h}_{k,i}^H \mathbf{w}_{T_k}|^2}{\sigma_k^2}. \tag{4}$$

Hence, the signal received by user k may be expressed as

$$r_k(t) = \mathbf{h}_{k,m}^H \mathbf{w}_{T_k} s_k(t) + n_{k,m}(t), \tag{5}$$

where $n_{k,m}(t) \in \mathcal{CN}(0, \sigma_k^2)$ is the noise at the mth receive antenna. Observe that (5) describes the channel of a MISO system whose performance has been improved by exploiting antenna diversity. The resulting SNR is given by

$$\text{SNR}_k = \max_i \frac{|\mathbf{h}_{k,i}^H \mathbf{w}_{T_k}|^2}{\sigma_k^2}. \tag{6}$$

2.2. Receive beamforming

In this strategy, user k combines the received signals linearly over the array. The combining is represented by the beamforming vector $\mathbf{w}_{R_k}$. The signal received by user k may be expressed as

$$r_k(t) = \mathbf{w}_{R_k}^H \mathbf{H}_k \mathbf{w}_{T_k} s_k(t) + \mathbf{w}_{R_k}^H \mathbf{n}_k(t), \tag{7}$$

where $\mathbf{w}_{R_k}^H \mathbf{n}_k(t) \in \mathcal{CN}(0, \sigma_k^2)$ by assuming that the receive beamformer satisfy $\|\mathbf{w}_{R_k}\|^2 = 1$. This assumption is not a restriction in terms of SNR since the scaling affects the signal and noise power equally. The resulting SNR is given by

$$\text{SNR}_k = \frac{|\mathbf{w}_{R_k}^H \mathbf{H}_k \mathbf{w}_{T_k}|^2}{\sigma_k^2}. \tag{8}$$

2.3. System operation

Since the channel has a semi-constant behavior, it is reasonable to exploit feedback for achieving partial CSI at the transmitter. The CSI may be used for efficient beamforming and estimation of the instantaneous SNR, (6) or (8) depending on receive strategy. A proper estimation is crucial for performing accurate scheduling and rate adaptation. The considered system operation is illustrated in Fig. 1.

Each cycle begins with the base station transmitting its pilot sequence over all antennas. Each user k utilizes this information to estimate a variable ρ_k that it feeds back to the base station. This variable will throughout the paper be the squared channel norm to one of the receive antennas, i.e., $\rho_k = \rho_{k,j} = \|\mathbf{h}_{k,j}\|^2$ for some j. The base station exploits this information to estimate the transmit beamformer that maximize the instantaneous SNR for each user k and the corresponding MMSE estimate $\widehat{\text{SNR}}_k(\rho_k) = E\{\text{SNR}_k|\rho_k\}$.

Next, the scheduler is used to iteratively transmit data to different users until the feedback is considered outdated and the pilot sequence is transmitted once again.

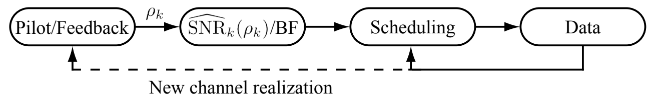


Fig. 1. Schematic system operation at the base station.

2.4. Rate estimation

The Shannon capacity of user k, with the considered model, is $C_k = \log_2(\text{SNR}_k + 1)$ [8] and will be time varying since it depends on the instantaneous SNR_k. The capacity sets a limit on the data rate υ_k which the channel to user k currently can support (in terms of achieving arbitrarily low error probability for arbitrarily long symbol sequences). The supported rate is assumed to be a non-decreasing function $\upsilon_k = \upsilon(\text{SNR}_k)$.

In order to adapt the transmission rate to what is currently supported, the transmitter may use its SNR estimate to derive an estimate $\hat{\upsilon}_k$ of the supported rate. The probability of overestimating the supported rate is denoted as the channel outage probability and should be kept low. In order to control this probability, the rate estimation will herein be calculated as the MMSE SNR estimate $\widehat{\text{SNR}}_k(\rho)$ reduced by a factor α multiplied by the standard deviation of the SNR estimate:

$$\hat{\upsilon}_k = \upsilon\left(\widehat{\text{SNR}}_k(\rho) - \alpha\sqrt{V\{\text{SNR}_k|\rho\}}\right), \tag{9}$$

where $V\{\text{SNR}_k|\rho\}$ is the mean square error of the estimate,

$$V\{\text{SNR}_k|\rho\} = E\{\text{SNR}_k^2|\rho\} - E\{\text{SNR}_k|\rho\}^2, \tag{10}$$

and α is chosen such that some system specified outage probability is satisfied. In simulations, the rate function is chosen to be identical to the capacity, $\upsilon(\text{SNR}_k) = \log_2(\text{SNR}_k + 1)$, but the analysis is independent of this choice.

3. INCORPORATING MULTIPLE RECEIVE ANTENNAS INTO AN EXISTING MISO SYSTEM

This section will show how additional receive antennas may be incorporated into an existing multiple-input single-output (MISO) system with channel norm feedback to exploit antenna diversity without affecting the transmitter. For simplicity, all user indices will be dropped throughout this section. Consider the MISO system described by

$$r(t) = \mathbf{h}^H \mathbf{w}_T s(t) + n(t), \tag{11}$$

with $\mathbf{h} \in \mathcal{CN}(\mathbf{0}, \mathbf{R})$ and feedback of $\rho = \|\mathbf{h}\|^2$. Let $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ be the eigenvalue decomposition of the channel covariance matrix, where $\mathbf{\Lambda}$ is a diagonal matrix with the distinct and strictly positive eigenvalues λ_m ordered arbitrarily. It is proved in [5, 9] that the channel covariance matrix conditioned on the feedback variable is

$$\widehat{\mathbf{R}}(\rho) = E\left\{\mathbf{h}\mathbf{h}^H \middle| \rho = \|\mathbf{h}\|^2\right\} = \mathbf{U}\widehat{\mathbf{\Lambda}}(\rho)\mathbf{U}^H, \tag{12}$$

where $\widehat{\mathbf{\Lambda}}(\rho)$ is a diagonal matrix with the conditional eigenvalues $\widehat{\lambda}_m(\rho)$ calculated from the original eigenvalues using

$$\begin{aligned}\widehat{\lambda}_m(\rho) = \frac{1}{f_{\|\mathbf{h}\|^2}(\rho)} \Bigg[& \frac{\rho e^{-\frac{\rho}{\lambda_m}}}{\lambda_m \prod_{i \neq m}\left(1 - \frac{\lambda_i}{\lambda_m}\right)} + \\ & + \sum_{k \neq m} \frac{e^{-\frac{\rho}{\lambda_m}} - e^{-\frac{\rho}{\lambda_k}}}{\left(1 - \frac{\lambda_k}{\lambda_m}\right)\prod_{i \neq k}\left(1 - \frac{\lambda_i}{\lambda_k}\right)} \Bigg],\end{aligned} \tag{13}$$

where $f_{\|\mathbf{h}\|^2}(\rho)$ is the PDF of $\|\mathbf{h}\|^2$ given by

$$f_{\|\mathbf{h}\|^2}(\rho) = \sum_{k=1}^{n_T} \frac{e^{-\frac{\rho}{\lambda_k}}}{\lambda_k \prod_{i\neq k}\left(1-\frac{\lambda_i}{\lambda_k}\right)}. \tag{14}$$

Thus, the MMSE estimate of the SNR at the transmitter is

$$\widehat{\mathrm{SNR}} = E\left\{\frac{|\mathbf{h}^H\mathbf{w}_T|^2}{\sigma^2}\Big|\rho\right\} = \frac{\mathbf{w}_T^H\widehat{\mathbf{R}}(\rho)\mathbf{w}_T}{\sigma^2}, \tag{15}$$

which is maximized by the beamformer $\mathbf{w}_T$ being the eigenvector corresponding to the largest conditional eigenvalue:

$$\mathbf{w}_T = \sqrt{P}\mathbf{U}_{\cdot,\mathbf{i}}, \text{ where } i = \arg\max_m \widehat{\lambda}_m(\rho). \tag{16}$$

3.1. Exploiting antenna diversity at the receiver

Now consider the case when the receiver has multiple antennas and uses antenna selection. It will first be assumed that the transmitter has knowledge of the number of receive antennas n_R and that antenna selection is used. Then we will show that this information is unnecessary. Assume that the feedback consists of $\boldsymbol{\rho} = [\rho_1 \dots \rho_{n_R}]$, where $\rho_j = \|\mathbf{h}_j\|^2$ is the squared norm of the channel to the jth receive antenna. The MMSE of the SNR at the transmitter is

$$\widehat{\mathrm{SNR}} = \max_j E\{\mathrm{SNR}_j|\boldsymbol{\rho}\} = \max_j \frac{\mathbf{w}_T^H\widehat{\mathbf{R}}(\rho_j)\mathbf{w}_T}{\sigma_k^2}, \tag{17}$$

using the independence between receive channels. It is clear that the SNR estimate is maximized by $\mathbf{w}_T$ being one of the eigenvectors, i.e., the one corresponding to the largest conditional eigenvalue. Intuitively, the expression in (13) is an increasing function in ρ and this may actually be proven analytically using the fact that the distribution is log-concave. As a consequence, the largest conditional eigenvalue will always appear at receive antenna $l = \arg\max_j \rho_j$. Hence, all feedback variables except ρ_l are unnecessary. Observe that the antenna ordering is arbitrary, so the receiver may as well feed back $\rho_{\max} = \max_j \rho_j$. The MMSE of the SNR becomes

$$\widehat{\mathrm{SNR}} = E\left\{\frac{|\mathbf{h}^H\mathbf{w}_T|^2}{\sigma^2}\Big|\rho_{\max}\right\} = \frac{\mathbf{w}_T^H\widehat{\mathbf{R}}(\rho_{\max})\mathbf{w}_T}{\sigma^2}, \tag{18}$$

which is identical to (15) except for a different notation. Hence, the transmitter will disregard that the receiver may be equipped with multiple antennas. The conclusion is that this type of multiple antenna receiver may be incorporated into the described MISO system without affecting the transmit processing. The antenna diversity is still exploited since $\rho_{\max}$ has a distribution with greater probability of being large than that of the distribution in (14).

3.2. SNR estimation variance

The variance of the MMSE SNR estimate in (18) may be calculated according to (10). Recall that i denotes the index of the eigenvector used as transmit beamformer and let $\rho = \rho_{\max}$, for the ease of notation. Then, the first term of (10) may be expressed as

$$E\{\mathrm{SNR}^2|\rho\} = E\left\{\frac{|\mathbf{h}^H\mathbf{w}_T|^4}{\sigma^4}\Big|\rho\right\}, \tag{19}$$

where σ^4 is a constant and $E\{|\mathbf{h}^H\mathbf{w}_T|^4|\rho\}$ is given by [9]

$$E\{|\mathbf{h}^H\mathbf{w}_T|^4|\rho\} = \frac{2}{f_{\|\mathbf{h}\|^2}(\rho)}\sum_{j\neq i}\frac{\lambda_j e^{-\frac{\rho}{\lambda_j}}}{\left(1-\frac{\lambda_j}{\lambda_i}\right)^2\prod_{l\neq j}\left(1-\frac{\lambda_l}{\lambda_j}\right)} + \frac{\left(\rho-\sum_{j\neq i}\frac{\lambda_j}{1-\frac{\lambda_j}{\lambda_i}}\right)^2 + \sum_{j\neq i}\frac{\lambda_j^2}{\left(1-\frac{\lambda_j}{\lambda_i}\right)^2}}{f_{\|\mathbf{h}\|^2}(\rho)\lambda_i\prod_{l\neq i}\left(1-\frac{\lambda_l}{\lambda_i}\right)}e^{-\frac{\rho}{\lambda_i}}, \tag{20}$$

where λ_j are the eigenvalues of the covariance matrix $\mathbf{R}$ and $f_{\|\mathbf{h}\|^2}(\rho)$ is given in (14). The second term of the expression in (10) is given by (18). Hence, the estimation variance is

$$V\{\mathrm{SNR}|\rho\} = \frac{E\{|\mathbf{h}^H\mathbf{w}_T|^4|\rho\} - (\mathbf{w}_T^H\widehat{\mathbf{R}}(\rho)\mathbf{w}_T)^2}{\sigma^4}, \tag{21}$$

with $E\{|\mathbf{h}^H\mathbf{w}_T|^4|\rho\}$ given in (20).

4. EXPLOITING CHANNEL NORM FEEDBACK

The advantages of the previous scheme, in terms of compatibility with the previously proposed MISO processing and the simple receiver complexity, comes with the drawback of not fully exploiting the antenna diversity. By using receive beamforming and taking this into account in the transmitter, the following scheme will achieve a higher SNR performance. For simplicity, all user indices will be dropped throughout this section.

Consider the system model in (7) for the case when the receiver feeds back $\rho_{\max} = \max_j \rho_j$, with $\rho_j = \|\mathbf{h}_j\|^2$. Observe that the ordering of the receive antennas is arbitrary so the transmitter may, without loss of generality, assume that $\rho_{\max} = \rho_1$. Next, observe that even though ρ_j, $\forall j \neq 1$, are unknown to the transmitter they are known to satisfy $0 \leq \rho_j \leq \rho_1$. Let the conditional covariance matrix of these rows be denoted

$$\widehat{\mathbf{R}}(0\leq\rho_j\leq\rho_1) = E\left\{\mathbf{h}\mathbf{h}^H\Big|0\leq\rho_j\leq\rho_1\right\}, \; j\neq 1. \tag{22}$$

A closed form expression of $\widehat{\mathbf{R}}(0\leq\rho_j\leq\rho_1)$, and especially its eigenvalues, is given by Theorem 1.

4.1. Computing the conditional covariance matrix

Theorem 1. *Let $\mathbf{h}\in\mathcal{CN}(\mathbf{0},\mathbf{R})$ and let $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$ be the eigenvalue decomposition of $\mathbf{R}$. The matrix $\mathbf{\Lambda}$ is diagonal with distinct and strictly positive eigenvalues λ_j in arbitrary order. If the realizations $\mathbf{h}$ satisfies $0\leq\|\mathbf{h}\|^2\leq\rho_1$, then the conditional covariance matrix $\widehat{\mathbf{R}}(0\leq\rho\leq\rho_1)$ with $\rho = \|\mathbf{h}\|^2$ may be expressed as*

$$\widehat{\mathbf{R}}(0\leq\rho\leq\rho_1) = \mathbf{U}\widehat{\mathbf{\Lambda}}(0\leq\rho\leq\rho_1)\mathbf{U}^H.$$

Here $\widehat{\mathbf{\Lambda}}(0\leq\rho\leq\rho_1)$ is a diagonal matrix with the elements

$$\widehat{\lambda}_m(0\leq\rho\leq\rho_1) = \frac{1}{\int_0^{\rho_1} f_{||\mathbf{h}||^2}(\varrho)d\varrho}\Bigg[\lambda_m + \sum_{k\neq m}\frac{\lambda_k e^{-\frac{\rho_1}{\lambda_k}} - \lambda_m e^{-\frac{\rho_1}{\lambda_m}}}{\left(1-\frac{\lambda_k}{\lambda_m}\right)\prod_{i\neq k}\left(1-\frac{\lambda_i}{\lambda_k}\right)} - \frac{(\lambda_m+\rho_1)e^{-\frac{\rho_1}{\lambda_m}}}{\prod_{i\neq m}\left(1-\frac{\lambda_i}{\lambda_m}\right)}\Bigg],$$

where the normalization factor given by

$$\int_0^{\rho_1} f_{||\mathbf{h}||^2}(\varrho)d\varrho = 1 - \sum_{k=1}^{n_T}\frac{e^{\rho_1/\lambda_k}}{\prod_{i\neq k}\left(1-\frac{\lambda_i}{\lambda_k}\right)}. \tag{23}$$

Proof. The eigenvalue $\widehat{\lambda}_m(0 \le \rho_i \le \rho_1)$ may be expressed as

$$\widehat{\lambda}_m(0 \le \rho \le \rho_1) = E\{\widehat{\lambda}_m(\rho)\big|0 \le \rho \le \rho_1\} = \int_0^{\rho_1} \widehat{\lambda}_m(\rho) \frac{f_{||\mathbf{h}||^2}(\rho)}{\int_0^{\rho_1} f_{||\mathbf{h}||^2}(\varrho) d\varrho} d\rho,$$

which may be integrated directly, using (13) and (14). □

4.2. Computing the MMSE SNR estimate

The SNR maximizing receive beamformer will match the channel as

$$\mathbf{w}_R = \frac{\mathbf{H}\mathbf{w}_T}{\|\mathbf{H}\mathbf{w}_T\|}. \tag{24}$$

The resulting MMSE estimate of the SNR at the transmitter may be calculated as

$$\begin{aligned} \widehat{\mathrm{SNR}} &= E\Big\{\frac{|\mathbf{w}_R^H \mathbf{H}\mathbf{w}_T|^2}{\sigma^2}\Big|\rho_1 = \max_j \|\mathbf{h}_j\|^2\Big\} \\ &= E\Big\{\frac{|\mathbf{h}_1^H \mathbf{w}_T|^2}{\sigma^2}\Big|\rho_1\Big\} + \sum_{j=2}^{n_R} E\Big\{\frac{|\mathbf{h}_j^H \mathbf{w}_T|^2}{\sigma^2}\Big|0 \le \rho_j \le \rho_1\Big\} \\ &= \frac{\mathbf{w}_T^H \big(\widehat{\mathbf{R}}(\rho_1) + (n_R - 1)\widehat{\mathbf{R}}(0 \le \rho_j \le \rho_1)\big)\mathbf{w}_T}{\sigma^2}, \end{aligned} \tag{25}$$

which will be maximized by the beamformer $\mathbf{w}_T$ being the eigenvector corresponding to the largest sum of conditional eigenvalues:

$$\mathbf{w}_T = \sqrt{P}\mathbf{U}_{\cdot,\mathbf{i}}, \tag{26}$$

where $i = \arg\max_m \widehat{\lambda}_m(\rho) + (n_R - 1)\widehat{\lambda}_m(0 \le \rho_j \le \rho_1)$. Observe that the second term of (25) corresponds to the performance gain compared to the antenna selection approach.

4.3. SNR estimation variance

The variance of the MMSE SNR estimate in (25) may be calculated according to (10). Recall that i denotes the index of the eigenvector used as transmit beamformer and observe that the independence between rows of the channel $\mathbf{H}$ gives that

$$\begin{aligned} V\big\{\mathrm{SNR}|\rho_1 = \max_j \rho_j\big\} &= V\{\mathrm{SNR}|\rho_1\} \\ &\quad + (n_R - 1)V\{\mathrm{SNR}|0 \le \rho_j \le \rho_1\}, \end{aligned} \tag{27}$$

where the first term corresponds to the variance of the signal received at the antenna with the strongest channel norm ρ_1. This variance is the same as in the antenna selection case and is given by (20) and (21). The second term of (27) may be expressed as

$$\begin{aligned} V\{\mathrm{SNR}|0 \le \rho_j \le \rho_1\} &= \frac{E\left\{|\mathbf{h}^H \mathbf{w}_T|^4 \big| 0 \le \rho_j \le \rho_1\right\}}{\sigma^2} \\ &\quad - \big(\mathbf{w}_T^H \widehat{\mathbf{R}}(0 \le \rho_j \le \rho_1)\mathbf{w}_T\big)^2, \end{aligned} \tag{28}$$

where $E\left\{|\mathbf{h}^H \mathbf{w}_T|^4 \big| 0 \le \rho_j \le \rho_1\right\}$ is given by Theorem 2. Observe that the increase in MMSE SNR estimate with receive beamforming comes with an increase in uncertainty in the estimation.

4.4. Computing the conditional fourth order moment

Theorem 2. *Let* $\mathbf{h} \in \mathcal{CN}(\mathbf{0}, \mathbf{R})$ *and let the distinct and strictly positive eigenvalues of* $\mathbf{R}$ *be denoted* λ_j*. Let* $\mathbf{w}_T$ *denote the ith eigenvector of* $\mathbf{R}$*. If the realizations* $\mathbf{h}$ *satisfies* $0 \le \|\mathbf{h}\|^2 \le \rho_1$*, then the conditional fourth order moment of* $\mathbf{h}^H \mathbf{w}_T$ *is given by*

$$\begin{aligned} E\left\{|\mathbf{h}^H \mathbf{w}_T|^4 \big| 0 \le \rho \le \rho_1\right\} &= \frac{1}{\int_0^{\rho_1} f_{||\mathbf{h}||^2}(\varrho) d\varrho} \times \\ &\left[\frac{\left(\lambda_i - \sum_{j \ne i} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_i}}\right)^2 - \left(\rho_1 + \lambda_i - \sum_{j \ne i} \frac{\lambda_j}{1 - \frac{\lambda_j}{\lambda_i}}\right)^2 e^{-\frac{\rho_1}{\lambda_i}}}{\prod_{l \ne i}\left(1 - \frac{\lambda_l}{\lambda_i}\right)} \right. \\ &+ \frac{\left(\lambda_i^2 + \sum_{j \ne i} \frac{\lambda_j^2}{\left(1 - \frac{\lambda_j}{\lambda_i}\right)^2}\right)\left(1 - e^{-\frac{\rho_1}{\lambda_i}}\right)}{\prod_{l \ne i}\left(1 - \frac{\lambda_l}{\lambda_i}\right)} \\ &\left. + 2\sum_{j \ne i} \frac{\lambda_j^2 \left(1 - e^{-\frac{\rho_1}{\lambda_j}}\right)}{\left(1 - \frac{\lambda_j}{\lambda_i}\right)^2 \prod_{l \ne j}\left(1 - \frac{\lambda_l}{\lambda_j}\right)} \right], \end{aligned}$$

where $\rho = \|\mathbf{h}\|^2$ *and the normalization factor is given in (23).*

Proof. The conditional fourth order moment may be expressed as

$$\begin{aligned} &E\{|\mathbf{h}^H \mathbf{w}_T|^4 \big| 0 \le \rho \le \rho_1\} \\ &= \int_0^{\rho_1} E\left\{|\mathbf{h}^H \mathbf{w}_T|^4 \big| \rho\right\} \frac{f_{||\mathbf{h}||^2}(\rho)}{\int_0^{\rho_1} f_{||\mathbf{h}||^2}(\varrho) d\varrho} d\rho, \end{aligned} \tag{29}$$

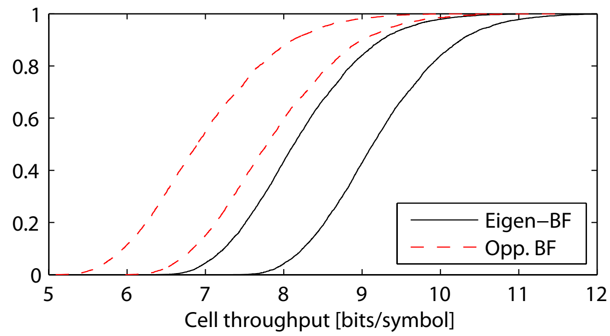
which may be integrated directly, using (14) and (20). □

5. PERFORMANCE EVALUATION

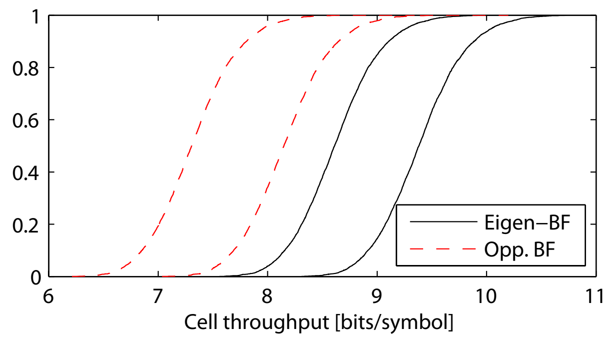
This section will compare the performance of the proposed strategies with opportunistic beamforming, assuming the same conditions in terms of the amount of feedback and the receive strategy. The evaluation considers a circular cell of radius R with uniformly distributed users in the area $0.2R \le r \le R$. The elevated base station is equipped with an eight-antenna uniform circular array (UCA) with half a wavelength antenna separation, while the mobiles are equipped with four arbitrarily structured antennas. There is no scattering close to the base station, while the mobiles are exposed to rich scattering. The scatterers are Gaussian distributed in the xy-plane and will be evaluated for different amount of angular spread as seen from the base station. The signal power is assumed to decay as $1/r^4$ and the mean SNR of a user at the cell boundary is 10 dB.

The evaluation consists of 5000 scenarios per angular spread, where each represents a unique setup with randomly fixated mobiles and constant channel statistics. The average cell throughput is calculated over 500 channel realizations, where each is used for transmission for 4 time slots. The randomized beamformer with the opportunistic approach is fixed during each realization, so the feedback is equal to the proposed strategies.

Proportional fair scheduling is used with a time scale of the average throughput at 100 scheduling decisions. The rates used by the proposed strategies are estimated as in (9), with the outage probability set at approximately 5% in the simulation.



(a) Lightly loaded cell (4 users).



(b) Moderately loaded cell (20 users).

Fig. 2. The CDF (over scenarios) of the cell throughput in a system with a transmitting eight-antenna UCA and four receive antennas. Opportunistic beamforming is compared to the proposed feedback supported eigenbeamforming for different receive strategies: antenna selection and receive beamforming (increasing performance).

5.1. Simulation results

The simulation results for an angular spread of 15 degrees are summarized in Fig. 2 for 4 and 20 users in (a) and (b), respectively. The cumulative distribution function (CDF) of the cell throughput (over the scenarios) is plotted for the proposed channel norm supported eigenbeamforming and opportunistic beamforming with two different receive strategies: antenna selection and receive beamforming.

It is observed that the proposed strategies have a significantly higher throughput than opportunistic beamforming in all cases. The difference is especially clear in the case with a moderate number of users, which indicates that the proposed strategies exploit the spatial dimensions more efficiently, without having to rely on multi-user diversity to this end.

Fig. 3 shows the mean cell throughput as a function of angular spread. When considering antenna selection the proposed strategy outperforms opportunistic beamforming for a wide range of angle spreads and with receive beamforming it becomes overall superior.

6. CONCLUSIONS

Closed form expressions for the second and fourth order channel statistics, conditioned on an upper limit of the current squared channel norm, have been presented. These are, together with the conditional channel statistics for a given squared channel norm, shown to be important in the study of downlink MIMO communication with limited channel norm feedback. Herein, the channel norm to each receive antenna is computed and the largest is fed back to the transmitter to support beamforming, scheduling and rate estimation.

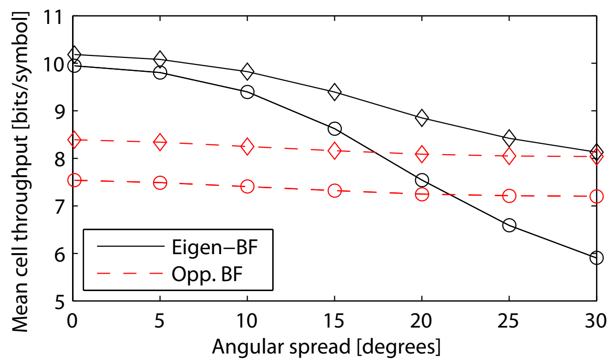


Fig. 3. The mean cell throughput for different angular spreads in a system with a transmitting eight-antenna UCA and four receive antennas. The proposed channel norm supported eigenbeamforming with antenna selection (circles) or receive beamforming (diamonds) is compared to the corresponding opportunistic beamforming.

Two simple strategies, with either antenna selection or beamforming at the receiver, that exploits the spatial channel conditions are proposed. These are based on eigenbeamforming and exploit the conditional channel statistics to derive closed form expressions of the MMSE SNR estimates and its variance. With a proportional fair scheduler, both strategies are shown to outperform the corresponding opportunistic beamforming, without increasing neither the feedback level nor the computational load on the receiver. The proposed strategies are, in comparison, especially efficient in exploiting multi-user diversity.

7. REFERENCES

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