Soil-structure interaction for bridges with backwalls

FE-analysis using PLAXIS

Emelie Carlstedt

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Abstract

Bro 2004, BV Bro and the Eurocodes give guidelines for how to consider earth pressure induced by change in temperature and braking forces when designing backwalls. In this thesis those demands are investigated using PLAXIS for evaluation of the earth pressure. The results show that the model in PLAXIS corresponds quite well with the conventions in Bro 2004 and that modelling in PLAXIS gives reliable results. The demand in Bro 2004 that backwalls always shall be designed for passive earth pressure has been found to be pessimistic. In case of long bridges and short backwalls passive earth pressure is most often reached but for shorter bridge lengths in combination with longer backwalls this is almost never the case. It was also found that PLAXIS is sensitive and that the structure of the model and the choice of input are essential. A model in PLAXIS doesn’t make the design more effective but it may be a good tool for analysing the effect of the earth pressure combined with other effects such as the patterns for displacement as well as moment- and force distributions.

Keywords: backwall, passive earth pressure, PLAXIS, interface coefficient
Sammanfattning


Nyckelord: ändskärm, passivt jordtryck, PLAXIS, gränsskikt
Preface

The research reported in this thesis was carried out in cooperation between Grontmij AB in Stockholm and the Division of Structural Design and Bridges at the Royal Institute of Technology (KTH) in Stockholm.

I would like to thank Professor Håkan Sundquist of the Royal Institute of Technology for his scientific advice and for valuable and constructive supervision.

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I would also like to thank Doctor Zein-Eddine Merouani who taught me the fundamentals of soil behaviour and gave me advice concerning PLAXIS so that I was able to understand it better.

Also Mahir Úlker, Ph D student at the Royal Institute of Technology has helped me. I would like to thank him for his interest for my thesis and for his advice concerning modelling.

Great thanks to all the members at the division of Construction at Grontmij AB, Stockholm who have been very helpful and with whom I have shared a lot of enjoyable moments.

Stockholm, December 2008

Emelie Carlstedt
Notations

\[ A \quad \text{Area} \quad [\text{m}^2] \]
\[ A_s \quad \text{Area of reinforcement for bending moment} \quad [\text{m}^2] \]
\[ A_{ss} \quad \text{Area of reinforcement for shear force} \quad [\text{m}^2] \]
\[ E \quad \text{Modulus of elasticity} \quad [\text{N/m}^2] \]
\[ E_{\text{ref}} \quad \text{Secant stiffness in standard drained triaxial test} \quad [\text{N/m}^2] \]
\[ E_{\text{eed}} \quad \text{Tangent stiffness for primary oedometer loading} \quad [\text{N/m}^2] \]
\[ E_{\text{ur}} \quad \text{Unloading/reloading stiffness} \quad [\text{N/m}^2] \]
\[ H \quad \text{Height of the backwall} \quad [\text{m}] \]
\[ I \quad \text{Moment of inertia} \quad [\text{m}^4] \]
\[ K \quad \text{Coefficient of earth pressure} \quad [-] \]
\[ K_a \quad \text{Coefficient of active earth pressure} \quad [-] \]
\[ K_p \quad \text{Coefficient of passive earth pressure} \quad [-] \]
\[ K_0 \quad \text{Coefficient of earth pressure at rest} \quad [-] \]
\[ L \quad \text{Length of the backwall} \quad [\text{m}] \]
\[ L_{\text{contr}} \quad \text{Contributing length} \quad [\text{m}] \]
\[ M \quad \text{Moment} \quad [\text{Nm}] \]
\[ M_A \quad \text{Moment at point A} \quad [\text{Nm}] \]
\[ P \quad \text{Point load} \quad [\text{N}] \]
\[ Q_{\text{lk}} \quad \text{Characteristic load along the bridge} \quad [\text{N}] \]
\[ R_A \quad \text{Shear force at point A} \quad [\text{N}] \]
\[ R_f \quad \text{Failure ratio} \quad [-] \]
\[ R_{\text{inter}} \quad \text{Interface coefficient} \quad [-] \]
\[ T_{e,\text{min}} \quad \text{Minimal air temperature} \quad [\text{°C}] \]
\[ T_{e,\text{max}} \quad \text{Maximal air temperature} \quad [\text{°C}] \]
\[ T_{\text{min}} \quad \text{Minimal air temperature} \quad [\text{°C}] \]
\[ T_{\text{max}} \quad \text{Maximal air temperature} \quad [\text{°C}] \]
\[ \Delta T \quad \text{Change in temperature} \quad [-] \]
\[ \Delta T_N \quad \text{Change in temperature} \quad [-] \]
\[ V_c \quad \text{Contribution of concrete to the shear force capacity} \quad [\text{N}] \]
\[ V_d \quad \text{Shear force} \quad [\text{N}] \]
\[ V_s \quad \text{Contribution of reinforcement to the shear force capacity} \quad [\text{N}] \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>Length of the backwall</td>
<td>[m]</td>
</tr>
<tr>
<td>$c$</td>
<td>Cohesion</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$d$</td>
<td>Thickness</td>
<td>[m]</td>
</tr>
<tr>
<td>$d_{eq}$</td>
<td>Equivalent thickness</td>
<td>[m]</td>
</tr>
<tr>
<td>$d_{tot}$</td>
<td>Effective height for the cross-section of the concrete</td>
<td>[m]</td>
</tr>
<tr>
<td>$f_{bri}$</td>
<td>Bending strength for the concrete</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$f_{y}$</td>
<td>Yield function</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$f_{v}$</td>
<td>Formal shear resistance</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$\bar{f}$</td>
<td>Function of stress</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$g_{i}$</td>
<td>Plastic potential function</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$h$</td>
<td>Depth under the lower end of the backwall</td>
<td>[m]</td>
</tr>
<tr>
<td>$h_{bs}$</td>
<td>Amount of blasted stone</td>
<td>[m]</td>
</tr>
<tr>
<td>$h_{taick}$</td>
<td>Concrete cover</td>
<td>[m]</td>
</tr>
<tr>
<td>$k$</td>
<td>Permeability</td>
<td>[m/day]</td>
</tr>
<tr>
<td>$k_1$</td>
<td>Stiffness</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$k_2$</td>
<td>Coefficient due to height of the concrete</td>
<td>[-]</td>
</tr>
<tr>
<td>$m$</td>
<td>Power law</td>
<td>[-]</td>
</tr>
<tr>
<td>$p$</td>
<td>Pressure</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$p_a$</td>
<td>Active earth pressure</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$p_p$</td>
<td>Passive earth pressure</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$p_{ref}$</td>
<td>Reference pressure</td>
<td>[N/m$^2$]</td>
</tr>
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<td>$p_0$</td>
<td>Earth pressure at rest</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$p_1$</td>
<td>$p_p - p_0$</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$p_t$</td>
<td>Overload</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$q$</td>
<td>Deviatoric stress</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$q_a$</td>
<td>Asymptotic value of the shear strength</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$q_f$</td>
<td>Ultimate deviatoric stress</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$q_{1k}$</td>
<td>Characteristic distributed load at traffic lane one</td>
<td>[N/m$^2$]</td>
</tr>
<tr>
<td>$r_f$</td>
<td>Help parameter for curvature of cracked concrete</td>
<td>[m]</td>
</tr>
<tr>
<td>$r_{os}$</td>
<td>Help parameter for curvature of uncracked concrete</td>
<td>[m]</td>
</tr>
<tr>
<td>$r_s$</td>
<td>Help parameter for curvature of cracked concrete</td>
<td>[m]</td>
</tr>
<tr>
<td>$s$</td>
<td>Distance between centre of reinforcement</td>
<td>[m]</td>
</tr>
<tr>
<td>$x$</td>
<td>Height of compressed zone</td>
<td>[m]</td>
</tr>
<tr>
<td>$x_1$</td>
<td>Distance from the bottom of the backwall</td>
<td>[m]</td>
</tr>
<tr>
<td>$y_{back}$</td>
<td>Height of gravity centre for the backwall</td>
<td>[m]</td>
</tr>
<tr>
<td>$y_{max}$</td>
<td>Maximum deflection</td>
<td>[m]</td>
</tr>
<tr>
<td>$z$</td>
<td>Depth under slab</td>
<td>[m]</td>
</tr>
<tr>
<td>$w$</td>
<td>Weight</td>
<td>[(N/m)/m]</td>
</tr>
<tr>
<td>$w_{c}$</td>
<td>Crack width</td>
<td>[m]</td>
</tr>
<tr>
<td>$w_{l}$</td>
<td>Width of one traffic lane</td>
<td>[m]</td>
</tr>
</tbody>
</table>
\( \alpha \) Expansion coefficient \([1/°C]\)

\( \alpha_1 \) Relation for modulus of elasticity [-]

\( \alpha_2 \) Degree of filling for the volume of the compressed zone [-]

\( \alpha_{Q1} \) Factor for adaptation of the load model [-]

\( \alpha_{l1} \) Factor for adaptation of the load model [-]

\( \beta \) Relative position of the resultant of compression [-]

\( \beta_1 \) Coefficient for long-term load [-]

\( \delta \) Horizontal displacement [m]

\( \delta_A \) Horizontal displacement for passive earth pressure [m]

\( \delta_B \) Horizontal displacement for active earth pressure [m]

\( \varepsilon \) Strain [-]

\( \varepsilon_{cu} \) Failure strain of concrete [-]

\( \varepsilon^e \) Elastic strain [-]

\( \varepsilon^p \) Plastic strain [-]

\( \varepsilon_s \) Strain of reinforcement [-]

\( \varepsilon_{skoll} \) Help parameter for strain [-]

\( \varepsilon_1 \) Vertical strain [-]

\( \phi_{eff} \) Creep factor [-]

\( \varphi \) Angle of friction [°]

\( \varphi' \) Effective angle of friction [°]

\( \varphi_{cv} \) Critical state friction angle [°]

\( \varphi_m \) Mobilised state friction angle [°]

\( \gamma_n \) Coefficient for safety class [-]

\( \gamma_m \) Partial coefficient for concrete [-]

\( \gamma_{ms} \) Partial coefficient for reinforcement [-]

\( \gamma_{ms2} \) Partial coefficient for the modulus of elasticity for reinforcement [-]

\( \gamma_{sat} \) Soil unit weight below phreatic level [N/m³]

\( \gamma_{unsat} \) Soil unit weight above phreatic level [N/m³]

\( \gamma^p \) Function of plastic strains [N/m²]

\( \kappa_{os} \) Curvature for uncracked concrete [1/m]

\( \kappa_s \) Curvature for cracked concrete [1/m]

\( \kappa_1 \) Coefficient for adhesion [-]

\( \lambda_i \) Plastic multiplier [-]

\( \nu \) Poisson’s ratio [-]

\( \rho \) Content of reinforcement [-]

\( \sigma \) Stress [N/m²]

\( \sigma_t \) Allowable tensile stress [N/m²]

\( \sigma' \) Effective stress [N/m²]

\( \sigma_v \) Effective vertical pressure [N/m²]

\( \psi \) Dilatancy angle [°]
\( \psi_m \)  
Mobilised dilatancy angle [°]

\( \xi \)  
Help parameter for shear resistance [-]

\( \xi_w \)  
Crack safety factor [-]

**Indices**

- backwall: parameter defining property of the backwall
- br: parameter defining property of the blasted rock
- deck: parameter defining property of the bridge deck
- moraine: parameter defining property of the moraine
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1 Introduction

1.1 Background

It has been shown in cost comparisons that bridges with backwalls are more cost-efficient than bridges with freestanding abutments and expansion joints. Bridges with backwalls are considered to be more advantageous than bridges with expansion joints since the costs for maintenance is reduced and the riding quality improved.

Bridges with backwalls have on the other hand one problem. The earth pressure against the backwall is crucially important in the design and may cause failure in the embankment behind the bridge (Pétursson, 2000). Bridges with backwalls often require heavy reinforcement since they are designed for passive earth pressure according to the standards given by the Swedish Road Administration (Vägverket). It is therefore beneficial to study the actual earth pressure acting on the structure so that heavily reinforced constructions can be avoided in some cases and in order to make the design more efficient.

The focus of this thesis will be road bridges with backwalls. Railway bridges with backwalls are less common due to heavier traffic loads, especially larger braking force which gives rise to larger horizontal movements in the structure. The definition of a bridge with backwalls is presented in the next chapter.

In order to investigate the earth pressure against the backwall a model in PLAXIS has been performed using both Mohr-Coulomb theory and the Hardening Soil model to investigate the behaviour of the soil. The primary focus has been to model the soil as a material with the properties as prescribed by Mohr-Coulomb theory. The rules in Bro 2004 have been used for modeling the temperature change. The results have been compared to the requirements in Bro 2004 and the Eurocodes. The results from the model didn’t differ much from the calculations according to Bro 2004, but the requirements in Bro 2004 were found to be too pessimistic in some cases. The demands in the Eurocode were even more pessimistic but also more difficult to analyze since the demands weren’t that clear.

The model in PLAXIS has also gained knowledge about how the program works and has lead to attention of restrictions and advantages in modeling. One advantage is that the behaviour of the soil can be studied since PLAXIS may show the results in many ways such as patterns for displacements, moment- and force distributions, the stresses in the soil, points for plasticity etcetera.
1.2 Bridge types

This thesis focuses on bridges with backwalls. It is therefore necessary to have an understanding of what characterizes bridges with backwalls and how they function. To start the term slab bridge is presented since most bridges with backwalls are slab bridges but with some differences. Definitions can be found in the publications “Planning of bridges- a handbook” (“Broprojektering- en handbook”) and “Swedish types of bridges” (“Kodförteckning och beskrivning av brotyper”) by the Swedish Road Administration.

1.2.1 Slab bridges

When the available height is restricted the slab bridge is an advantageous solution. The main characteristic of a slab bridge is that the bridge deck is load bearing. The slab is usually made of reinforced concrete or prestressed reinforced concrete. Slab bridges are used for lengths up to 35 m for single spans. Piers may support the slab in case of long spans.

![Figure 1.1: One type of superstructure for slab bridges. (Swedish Road Administration, 2008c)](image)

One type of superstructure for slab bridges is depicted in figure 1.1. It is made with holes to reduce the dead load. An alternative is to make the superstructures homogenous and massive with heavy reinforcement which is a more common method in Sweden.

1.2.2 Slab bridges with backwalls

A slab bridge with backwalls is a common slab bridge in Sweden. Slab bridges with backwalls have the same characteristics as a slab bridge but the length of a bridge with backwalls is longer than for ordinary slab bridges. Slab bridges with backwalls may be as long as 60 -90 m but are often made shorter because of the restraining effect of the temperature. For longer spans regular abutments and expansion joints are used.

One difference between slab bridges and bridges with backwalls is the foundation method. Slab bridges are founded on abutments while bridges with backwalls are continuous over the supports and at the ends since the slab is resting on a pier connected with the embankment and the construction does not have any expansion joints. The principal appearance of a bridge with backwall is depicted in figure 1.2.
1.2. BRIDGE TYPES

The main task of the backwall is to absorb the earth pressures induced by braking forces and temperature change, and the intermediate piers are designed to carry the vertical loads. The height of the backwall is recommended to be at least 2 m by Enquist (Enquist, 1991) and is most often made of concrete. A wingwall may be connected to the backwall to make the transition between the embankment and the ground surface as smooth as possible.

Bridges with backwalls as well as slab frame bridges can be seen as integral bridges in which the superstructure is built together with the abutments to one combined structure. An integral bridge may not have expansion joints in the bridge deck nor between the deck and the abutments, but the bridge may have bearings at intermediate piers. An example of an integral bridge is depicted in figure 1.3. In other countries, for example in the US, the definition is not that strict since they allow both bearings and expansion joints. Comparisons made by Enquist (Enquist, 1991) between integral bridges and bridges with joints and freestanding abutments show that the integral bridges are much more cost-effective.

Foundations for bridges with backwalls may be constructed in several ways. Bridges with raised foundation have the same principal way of operation as bridges with backwalls but the
backwall covers the foundation completely, which prevents the bottom slab from taking horizontal forces. This type of bridge makes it possible to make the construction of the foundation during dry conditions.

Bridges with raised foundations occur frequently in North America and Australia where they are called integral abutment bridges. The backwall of an integral abutment bridge is constructed with piles, which make it possible to exclude the piers. An example of an integral abutment bridge is depicted in figure 1.4.

![Example of an integral abutment bridge](image)

Figure 1.4: Example of an integral abutment bridge. (Collin et al., 2005)

Nilsson (Nilsson, 2008) describes the fact that in case of piles the elongations of the superstructures induce a displacement that gives rise to a moment in the piles, which in time may cause fatigue failure.

Pétursson et al. (Pétursson, 2002) have studied integral abutment bridges in different countries. In Australia a bearing between the end support and the superstructure is used. In North America the end supports are always piled. The longest concrete bridge with integral abutments is 358 m and the longest in steel is 318 m, both built in the US. The acceptable length is not determined from structural calculations but from experience based on expected movement at the bridge ends (from temperature change), which shall be less than 100 mm (50 mm at each end).

In UK integral bridges may be an option for all bridges up to 60 m overall length and skews not exceeding 30° according to Iles (Iles, 2006). Integral bridges have increased significantly between 2000 and 2004 and now account for about half of the total bridge construction in UK. An integral bridge is defined as a bridge with integral abutments and an integral abutment is defined as a bridge abutment connected to the bridge deck without any movements joint for expansion or contraction of the deck. According to the definition above these bridges are integral abutment bridges.

Integral abutment bridges constructed as in North America are now becoming more common in Sweden according to Pétursson (Pétursson, 2000). Collin et al. (Collin, 2005) state that bridges with backwalls connected to steel girders are more cost efficient compared to bridges with conventional abutments and transitional structures.
1.3 Lateral Earth Pressure

Bro 2004 states that backwalls shall be designed for passive earth pressure for all cases. An explanation of the term is presented here. All information below is presented by Cernica (Cernica, 1995) and Hansbo et al. (Hansbo, 1984).

Lateral earth pressure arises when the soil and an adjacent structure is moving relative to one another. If the displacement induced by this movement increases, the earth pressure reaches certain limits. There are three types of pressure, passive, active and at rest pressure.

When the structure is compliant, that means there is no displacement relative to the structure, at rest earth pressure $p_0$ is reached. Since there are no movements in this state there is no failure.

Active earth pressure $p_a$ is reached when the structure is moving away from the soil, see figure 1.5. This reduces pressure.

![Figure 1.5: Active earth pressure. (http://se.geotech.maxit-cms.com/23229)](http://se.geotech.maxit-cms.com/23229)

The opposite of active earth pressure is passive earth pressure $p_p$, which is reached when the structure is moving against the wall, see figure 1.6. The pressure reaches its maximum value.

![Figure 1.6: Passive earth pressure. (http://se.geotech.maxit-cms.com/23229)](http://se.geotech.maxit-cms.com/23229)

The relationship between earth pressure and the movements of the structure is presented by Terzaghi and depicted in figure 1.7.
Figure 1.7: Relationship between earth pressure and the movement of the structure. (Cernica, 1995)

The pressure $p$ is determined by the general equation but with different coefficients of earth pressure

$$ p = K \cdot \sigma_v' $$

(1.1)

where $K$ is the coefficient of earth pressure and $\sigma_v'$ is the effective vertical pressure. The weight density of the soil and possible loads at the surface determines the effective vertical pressure. For clays the shear resistance is added to the pressure.

The coefficient of earth pressure at rest $K_o$ is normally determined by

$$ K_o = 1 - \sin \phi' $$

(1.2)

where $\phi'$ is the effective angle of friction. The active coefficient of earth pressure $K_a$ is determined by

$$ K_a = \frac{1 - \sin \phi'}{1 + \sin \phi'} = \tan^2 \left(45^\circ - \frac{\phi'}{2} \right) $$

(1.3)

The passive coefficient of earth pressure $K_p$ is determined by

$$ K_p = \frac{1 + \sin \phi'}{1 - \sin \phi'} = \tan^2 \left(45^\circ + \frac{\phi'}{2} \right) $$

(1.4)
1.4 Literature review

Bridges with backwalls, integral bridges, and integral abutment bridges have a lot of similarities and are often mixed up. Integral abutment bridges have been studied quite consistently, especially the pile strains. The soil-pile interaction is complex as it contains two co-dependent elements, the flexural pile and the soil that often is inhomogeneous. The moment at the boundary between pile and backwall cannot be verified to be okay by calculations but integral abutment bridges are still a very common solution according to Collin et al. (Collin, 2005).

Pétursson et al. (Pétursson, 2002) state that experience from the United States shows that bridges with integral abutments are increasingly outclassing the traditional bridges with joints, the former being not only less expensive to maintain, but also more affordable to build. The method is also believed to be competitive in other countries. One of the main reasons why integral abutment bridges have not yet become common in Sweden is the difficulty to analyse them. Nilsson (Nilsson, 2008) explains in his thesis that since a conventional elastic analysis fails to explain how the bridge works, the Swedish Road Administration has to approve or reject different solutions and methods for each case. Pétursson et al. (Pétursson, 2002) propose that codes, rules or guidelines for integral abutment bridges are developed in order to simplify the design procedure.

Design of bridges with backwalls has one big question of interest, the effect of the difference in temperature. The difference in temperature between winter and summer is the main reason why movements rise in the structure, and may be as high as 30 mm per end of the bridge. The difference in temperature gives rise to a passive earth pressure trying to make the bridge resume its original condition. Russell et al. (Russell, 1994) describe that these movements can be seen as cracks in the abutment around the girders near the end diaphragm. Differences in temperature between day and night aren’t that big and can be neglected according to Pétursson (Pétursson, 2000).

Thomson (Thomson, 1999) presents five factors that affect the development of the passive earth pressure behind a structure:
- The density of the soil (higher density means higher coefficient for earth pressure)
- The angle of friction between soil and structure (this is restricted by the geometry of the wingwall)
- Type of foundation
- The effect of the constrained filling
- Repeated loading

Many different methods for modelling the soil are suggested, for example by Lehane et al., O’Brien and Pétursson et al. When modelling, the soil is often modelled as a so-called Winkler foundation with elastic springs varying linearly with depth. This method is considered by O’Brien et al. (O’Brien, 1999) to be overly conservative and does not consider the movements at one level within the soil causing changes in stress at other levels according to Lehane (Lehane, 1998). Nilsson (Nilsson, 2008) states that the most common way to represent the soil in FE -models is with springs: simple linear springs, non-linear springs following the behaviour of some soil model or non-linear springs with different test as input data.
Chapter 1. Introduction

Previous studies show different results regarding the effect of the temperature change. Nielsen (Nielsen, 1994) states in his thesis that the coefficient for earth pressure is much smaller than the value suggested in the Swedish standards, hence passive earth pressure is never reached. Davidson et al. (Davidson, 2005) have performed analyses in PLAXIS with the filling modelled as Mohr-Coulomb material in order to define if backwalls shall be designed for passive earth pressure. The results show that backwalls shall be designed for passive earth pressure. In this case the interface coefficient is chosen as rigid, 1,0, compared to the interval recommended by PLAXIS support (PLAXIS, 2008) of 0,1 - 0,2, and the results may therefore be questioned. Nilsson (Nilsson, 2008) found in his thesis that test results compared to theoretical cases show that moments constrain in the backfill material behind the backwalls of integral bridges is between 50- 70% of full moment resistance, regarding deflection and rotation.

It is possible to reduce the horizontal earth pressure by placing an elastic plate (geoplate) on the outer part of the backwall. Nielsen (Nielsen, 1994) describes that the plate absorbs the horizontal deformations caused by temperature movements and the braking force. By reinforcing the soil with a geotextile the settlements in the area close to the backwall can be reduced and heavily reinforced constructions avoided.

The backfill material can be made of sand, gravel or blasted rock. According to Enquist (Enquist, 1991) the choice of material has no effect on the size of the settlements, but it’s known that the compacting and the method of foundation (plates are to be preferred before piles) is crucially important to be able to minimize settlements. A study made by the Swedish Road Administration in 1991 (Enquist, 1991) shows that rather extensive settlements and cracks have been observed for bridges with backwalls, which is due to insufficient compacting of the filling. Olmo Segovia (Olmo Segovia, 2006) is also highlighting the importance of right methods for avoiding damages by stating that the critical part of the construction process is the backfilling. During this process the earth pressure causes significant stresses and internal forces in the construction. Sundquist (Sundquist, 1998) points out that it might be because of the rigorous regulations in the standards that the damages have not been discovered on bridges with backwalls, and that it would be a good idea to investigate this further.

Thomson (Thomson, 1999) has found that compacting backfill behind a retaining wall or bridge abutment has the effect of inducing residual stresses within the soil. The residual stresses are due to the mobilization of friction angle at the wall to backfill interface. The residual stresses remain only when the wall/abutment is completely rigid (zero displacement either differentially along the wall height or with respect to a stationary plane.) Neither the geometry of the wingwalls nor the abutment foundation type has any effect on lateral earth pressure due to compaction of the fill. Compacting of the backfill has a significant effect on the pressures developed as the wall/abutment displaces. The analysis is based on the assumption that the filling is properly compacted so that no changes in time have to be considered. The effect of the compaction is however neglected as most often in design.

The soil has been modeled using Mohr-Coulombs theory. Research of a steel box culvert by Olmo Segovia (Olmo Segovia, 2006) indicate that linear elastic theory doesn’t represent the real behaviour of the soil and that theory of Mohr-Coulomb present results from PLAXIS with better correspondence to the field measurements than Hardening Soil model. On the other hand the Hardening Soil model gives results with bigger accuracy, but the demand for more parameters makes it more complicated. Bolteus (Bolteus, 1984) points out the necessity
to simplify the behaviour of the soil in order to receive useful and reliable information, why Mohr-Coulombs theory is preferred.

Regarding PLAXIS as a design tool even a simple 2D Finite Element model can be efficiently used to analyze the behaviour of the bridge. This is verified by measurements in the thesis by Nilsson (Nilsson, 2008). In the end of 2005, the Swedish Road Administration ordered an Integral Bridge as a part of a research and development program, the bridge over Leduån River. Nilsson et al. (Nilsson, 2007) show that the preliminary results indicate that rather simple FE-models suitable for design practice may be used to calibrate unknown parameters of the soil under traffic load.

1.5 Aim and scope of work

The aim of this thesis is to study how earth pressure introduced by change in temperature against the backwall could be taken into account when designing the backwall. The aim is thus to study the problem in a FEM-program and to compare the results with the requirements in Swedish standards and Eurocodes.

The analyses has been carried out using PLAXIS to study the behaviour of the lateral earth pressure induced by the change in temperature and braking force so that a more rationalized design method may be found. This thesis also aims at investigating if PLAXIS is a rational tool for design that may give more advantageous results for bridges with backwalls than calculations according to the standards given by the Swedish Road Administration. This is done by evaluation of the results and by examination of the disadvantages and advantages of the program.
2 Current design method

2.1 Standards

Requirements for bridge design are given by the Swedish Road Administrations publication, Bro 2004. The same requirements are valid for rail bridges and the complementing demands in BV Bro published by the Swedish Railway Administration should also be followed. Since road bridges are the focus of this thesis the requirements for rail bridges will not be given further attention after this chapter.

The standards called Eurocode are constituted by nine chapters describing the European standards for different kinds of structures such as timber, concrete, composite, steel etc. The expectation is that these standards will be valid from December 2010 (the Eurocodes will replace the Swedish standards for concrete and steel et al.).

2.1.1 Bro 2004

According to Bro 2004 backwalls shall be designed for passive earth pressure both in serviceability limit state and ultimate limit state. Table 21-1 in Bro 2004 gives the characteristics for different materials.

A common way of designing is to combine the effect of the earth pressure, the earth pressure induced by structure moving against the soil (may be caused by change in temperature and/or braking force) and the surcharge load for retaining structures etc. This method will be compared to the demand of passive earth pressure as stated above.

The braking force is a horizontal force of 200 kN for bridges with lengths up to 10 m, 500 kN for lengths up to 40 m and 800 kN for bridges equal to or longer than 170 m. Lengths in between these lengths can be interpolated rectilinear. The length of the bridge is the distance between adjacent joints that do not transfer horizontal forces.

The change in temperature \( \Delta T \) varies according to which part of Sweden the structure will be built and the values for the maximum \( T_{\text{max}} \) and the minimum air temperature \( T_{\text{min}} \) can be seen in appendix A.

The change in temperature is given by
\[
\Delta T = T_{\text{max}} - T_{\text{min}}
\] (2.1)

The expansion coefficient for concrete \( \alpha \) is \( 1 \cdot 10^{-5} \, (1/°C) \).

Earth pressure against the backwall caused by temperature change and braking force is determined by the horizontal displacement of the structure moving against soil \( \delta \) and given by

\[
p = p_0 \text{ if } \delta = 0
\]
\[
p = p_0 + c_1 \cdot \delta \cdot \frac{200}{H} \cdot p_1 \text{ if } 0 < \delta < \frac{H}{200}
\]
\[
p = p_0 + c_1 \cdot p_1 \text{ if } \delta \geq \frac{H}{200}
\] (2.2)

where

\[
p_1 = p_p - p_0
\] (2.3)

and \( H \) is the height of the backwall.

\( c_1 \) is 1 when the effect of the earth pressure is unfavourable, e.g. increased temperature makes the concrete restraint, and 0,5 when the effect of the earth pressure is favourable, e.g. the effect on an intermediate support when the braking force is transmitted to the filling.

The surcharge load is caused by a temporary load on the road adjacent the structure, normally traffic load. The intensity of the surcharge load \( p_1 \) is 20 kN/m\(^2\) calculated for a width up to 6 m and 10 kN/m\(^2\) for the rest of the width. The horizontal pressure can be calculated according to

\[
p = K \cdot p_1
\] (2.4)

where \( K \) is the coefficient for earth pressure at rest.

For load combinations the passive earth pressure shall be multiplied with the load factor \( \psi \gamma \) for ultimate limit state. For each load combination a maximum of four variable actions are included. The load giving the biggest deformation shall be given the bigger value for the load factor. All loads shall be multiplied with their respective load factor (table 22-1 in Bro 2004) and added together. The sum should be at least 1,0.

According to Sundquist (Sundquist, 2008) friction between the backwall and the soil may be included if the effect is favourable. The friction is however most often neglected since the effect of including it in the analysis only improves the results a few percent and the calculations are considered to be time consuming. (Koskinen, 2008)

Bro 2004 also provides a few requirements of the appearance of a concrete backwall. The thickness should be at least 200 mm. The distance between the bottom of the adjacent slab and the bottom of the backwall should be at least 0,60 m. The distance between the area of the
slope in front of the backwall and the bottom of the backwall shall be at least 1.0 m perpendicular to the area of the slope due to erosion.

### 2.1.2 BV Bro

For rail bridges some additions to the requirements in Bro 2004 are presented.

The traction and braking forces have to be taken into account. The traction force has the size of 30 kN/m, but maximum 1000 kN in total and the braking force has the size of 27 kN/m, but maximum 5400 kN in total. In case of several tracks the braking force or traction force is applied at one track and traction force at all the other tracks. An evenly distributed load given in table BV 21.2241 in BV Bro calculates the increased earth pressure caused by structure moving against earth. The load shall be spread according to figure 2.1.

![Figure 2.1: Distribution of load in case of sleeper in ballast. (Swedish Rail Administration, 2008)](image)

The horizontal movements caused by braking force and traction force in load combination V: A for serviceability limit state is limited by
- 80 mm for bridge with expansion joints in the track
- 5.0 mm for bridge with welded tracks and ballast through the hole structure
- 5.0 mm for track with sleeper on concrete or without expansion joints in the track
The horizontal movement of the end of the bridge shall also be limited to $H/200$ for load combination IV (including temperature).

The surcharge load is calculated based on a traffic load presented in table 2.1. At the depth of $h$ m under the lower edge of the rail the surcharge load is evenly distributed along a width of $h + 2.25$ m placed centrically over the centreline (load spread by the relation 4:1). If it is more unfavourable the surcharge load is distributed along a width of $2h + 2.25$ m (load spread by the relation 1:1). In case of more than one track the surcharge load is restricted to be spread along a width of maximum 2.25 m against the adjacent track. The horizontal pressure is calculated according formula 2.4 but with the coefficient of earth pressure is taken as the coefficient at rest or the active coefficient.
Chapter 2. Current design method

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Table 2.1: Distributed load for calculation of the surcharge load.

Loads shall be multiplied with the respective load factors $\psi \gamma$ for load combinations according to table BV 22-1 in BV Bro for ultimate limit state in the same way as for road bridges.

### 2.1.3 Eurocode

A braking force $Q_{ik}$ along the length of the bridge at the height of the pavement shall be included and may not exceed 900 kN for road bridges. It is calculated as

$$Q_{ik} = 0.6 \alpha_{Q1} (2Q_{ik}) + 0.10 \alpha_{q1} q_{ik} w_1 L$$

$$180 \alpha_{Q1} \cdot 10^3 \leq Q_{ik} \leq 900 \cdot 10^3$$

(2.5)

where $\alpha_{Q1}$ and $\alpha_{q1}$ are factors for adaptation of the load model, $q_{ik}$ is the characteristic distributed load at traffic lane one, $w_1$ is the width of one traffic lane and $L$ is the observed length of the superstructure.

The change in temperature $\Delta T_N$ is based on the maximal $T_{e,\text{max}}$ and minimal air temperatures $T_{e,\text{min}}$ that are given in national maps for isotherms.

$$\Delta T_N = T_{e,\text{max}} - T_{e,\text{min}}$$

(2.6)

This is the same method as the one presented in Bro 2004.

The expansion coefficient for concrete $\alpha$ is $10 \cdot 10^{-6} (1/°C)$, which is the same value as the value given in Bro 2004.

For normally consolidated soil, at rest conditions should normally be assumed in the ground behind a retaining structure if the movement of the structure is less than $5 \cdot 10^{-4} \cdot H$ ($H$ is the height of the backwall). So if the movement is larger than $H/2000$ passive earth pressure shall be assumed for a structure moving against the soil and active earth pressure shall be assumed for a structure moving away from the soil.
2.2 Example

This section presents a calculated example for a road bridge using Bro 2004. The same data are also used for the model in PLAXIS so that the two methods can be compared.

According to the requirements in Bro 2004 the backwall is designed for earth pressure caused by temperature change and braking force. The effect of the braking force is well known but the movement caused by the braking force is difficult to determine. The size of the movement caused by change in temperature is relatively easy to determine but instead the mobilisation of the earth pressure due to this movement is not well known. Due to this uncertainty the temperature change has been given more attention than the braking force. The method of design given in Bro 2004 is accepted and adopted by designers.

In this example the bridge has one span and the total bridge length have been set to 18 m. The total height of the backwall has been set to 2 m and the width of the backwall to 0,7 m.

2.2.1 Earth pressure

The earth pressure is the earth pressure at rest which is calculated as

\[ p = K_0 \cdot \gamma_{br} \cdot z \text{ kN/m}^2 \]  

(2.7)

Values are presented in Bro 2004. The coefficient for earth pressure at rest is 0,34 for blasted rock and the weight density \( \gamma_{bs} \) is 18 kN/m\(^3\). \( z \) is the depth under the ground surface.
\[ p = 0.34 \cdot 18 \cdot 0.7 \cdot z = 4.28 \cdot z \text{ kN/m} \]  \hspace{1cm} (2.8)

The backwall can be seen as a cantilever connected with the slab. At the topside of the bridge deck the earth pressure is zero and at the height of the centre line of the bridge deck the pressure has reached a small value. In this case the centre line of the bridge deck is used as a reference point for a pressure equal to zero. This technique is very common among designers and reduces the earth pressure negligibly. Figure 2.3 shows the difference in pressure between these techniques.

The distribution of the earth pressure \( p \) caused by change in temperature is depicted in figure 2.4 along with the reaction force and the moment.

The moment \( M_A \) and the shear force \( R_A \) at point A has been calculated according to the table presented by Sundquist (Sundquist, 2007) along with the maximum deflection \( y_{\max} \)

\[
R_A = \frac{p \cdot L}{2} = \frac{8.56 \cdot 2}{2} = 8.6 \text{ kN/m}
\]

\[
M_A = \frac{p \cdot L^2}{3} = \frac{8.56 \cdot 2^2}{3} = 11.4 \text{ kNm/m}
\]

\[
y_{\max} = y_B = \frac{11p \cdot L^4}{120EI} = \frac{11 \cdot 8.56 \cdot 10^3 \cdot 2^4}{120 \cdot 33 \cdot 10^9 \cdot 0.7^3} = 0.013 \text{ mm}
\]  \hspace{1cm} (2.9)

\( L \) is the length of the bridge, \( E \) is the modulus of elasticity for concrete and \( I \) is the moment of inertia for the concrete.
The distributed load can be seen as a point load \( P \) if multiplied with the length of the beam. The stiffness \( k_1 \) has been calculated as

\[
k_1 = y^{-1} (P = p \cdot L = 1) = \frac{120EI}{11L^3} = \frac{120 \cdot 33 \cdot 10^9 \cdot 0.7^3}{11 \cdot 2^3} = 1,286 \text{ GPa}
\]  

(2.10)

The stiffness is constant regardless of load or pressure.

### 2.2.2 Temperature change

The analysis has been performed for the area of Stockholm, which is why the temperatures for this area are used. Figure A.1 and A.2 in Appendix A present the values of the maximal and the minimal air temperature \( T_{\text{max}} \) and \( T_{\text{min}} \) for the area of Stockholm

\[
T_{\text{max}} = 36 \degree C
\]

\[
T_{\text{min}} = -29 \degree C
\]  

(2.11)

The difference in temperature has been calculated

\[
\Delta T = T_{\text{max}} - T_{\text{min}} = 36 - (-29) = 65 \degree C
\]  

(2.12)

The difference in temperature causes the bridge to expand when warm and contract when cold, equally on both sides of the bridge. The contributing length of the bridge is therefore half of the total bridge length and the coefficient for length expansion is \( \alpha = 1,0 \cdot 10^{-5} \text{ (1/\degree C)} \) for concrete. The movement caused by the change in temperature \( \delta \) has been calculated

\[
\delta = L_{\text{cont}} \cdot \Delta T \cdot \alpha = \frac{18}{2} \cdot 65 \cdot 1,0 \cdot 10^{-5} = 6,0 \text{ mm}
\]  

(2.13)

\( L_{\text{cont}} \) is the contributing length.

The displacement has been compared to

\[
H = \frac{2}{200} = 10 \text{ mm}
\]  

(2.14)

Since \( 0 < \delta < \frac{H}{200} \) and the effect of the earth pressure is unfavourable the earth pressure has been calculated as

\[
p = p_0 + c_1 \cdot \delta \cdot \frac{200}{H} \cdot p_1 = \left( K_0 + c_1 \cdot \delta \cdot \frac{200}{H} \cdot (K_p - K_0) \right) \cdot \gamma_{br} \cdot z
\]  

(2.15)

The coefficient for passive earth pressure is 5,83 for blasted rock according to Bro 2004.
Chapter 2. Current design method

\[ p = \left( 0,34 + 1 \cdot 0,006 \cdot \frac{200}{2} \cdot (5,83 - 0,34) \right) \cdot 18 \cdot z = 65,4 \cdot z \text{ kN/m}^2 \]  \hspace{1cm} (2.16)

The pressure at the bottom of the backwall has been calculated

\[ p = 65,4 \cdot 2 \cdot 0,7 = 91,56 \text{ kN/m} \]  \hspace{1cm} (2.17)

The earth pressure due to temperature change has the same distribution as the earth pressure, see figure 2.4. The moment, shear force and deflection have therefore been calculated using the same method as for the earth pressure.

\[ R_\lambda = \frac{p \cdot L}{2} = \frac{8,56 \cdot 2}{2} = 8,6 \text{ kN/m} \]

\[ M_\lambda = \frac{p \cdot L^2}{3} = \frac{8,56 \cdot 2^2}{3} = 11,4 \text{ kNm/m} \]  \hspace{1cm} (2.18)

\[ y_{\text{max}} = y_B = \frac{11p \cdot L^4}{120EI} = \frac{11 \cdot 8,56 \cdot 10^9 \cdot 2^4}{120 \cdot 33 \cdot 10^9 \cdot 0,7^3} = 0,013 \text{mm} \]

2.2.3 Braking force

The braking force has been interpolated between the values given by Bro 2004 to 507 kN. The earth pressure is increased to balance this force. The resultant of the earth pressure should have the same size as the braking force, but is distributed along the width of the bridge. The width is assumed to be 7,5 m, which results in an earth pressure of 67,6 kN/m at the bottom of the backwall. The earth pressure has the same appearance as in figure 2.4.

The moment, shear force and deflection have been calculated

\[ R_\lambda = \frac{q \cdot L}{2} = \frac{67,6 \cdot 2}{2} = 67,6 \text{ kN/m} \]

\[ M_\lambda = \frac{q \cdot L^2}{3} = \frac{67,6 \cdot 2^2}{3} = 90,1 \text{kNm/m} \]  \hspace{1cm} (2.19)

\[ y_{\text{max}} = y_B = \frac{11q \cdot L^4}{120EI} = \frac{11 \cdot 67,6 \cdot 10^9 \cdot 2^4}{120 \cdot 33 \cdot 10^9 \cdot 0,7^3} = 0,105 \text{mm} \]

2.2.4 Surcharge load

The surcharge load is evenly distributed along the backwall and can be seen in picture 2.5.
2.2. Example

Figure 2.5: Structural system for the backwall in case of evenly distributed load.

The pressure has been calculated

\[ p = K \cdot p_t = 0.34 \cdot 20 = 6.8 \text{kN/m}^2 \]
\[ p = 6.8 \cdot 0.7 = 4.76 \text{kN/m} \]  \hspace{1cm} (2.20)

The shear force, the moment and the deflection caused by the surcharge load has been calculated according to the tables presented by Sundquist (Sundquist, 2007)

\[ R_A = p \cdot L = 4.76 \cdot 2 = 9.52 \text{kN/m} \]
\[ M_A = \frac{p \cdot L^2}{2} = \frac{4.76 \cdot 2^2}{2} = 9.52 \text{kNm/m} \]  \hspace{1cm} (2.21)
\[ y_{\text{max}} = \frac{p L^4}{8EI} = \frac{4.76 \cdot 10^3 \cdot 2^4}{8 \cdot 33 \cdot 10^9 \cdot 0.7^3} = 0.01 \text{mm} \]

The effect of the earth pressure, the temperature change, the braking force and the surcharge load has been summed up in a total with the following results

\[ R_{\text{Atot}} = 8.6 + 91.6 + 67.6 + 9.52 \approx 177.3 \text{kN/m} \]
\[ M_{\text{Atot}} = 11.4 + 122.1 + 90.1 + 9.52 \approx 233.1 \text{kNm/m} \]  \hspace{1cm} (2.22)
\[ y_{\text{max tot}} = 0.013 + 0.142 + 0.105 + 0.01 = 0.27 \text{mm} \]

These results have been compared to the case of passive earth pressure acting on the backwall as stated in Bro 2004

\[ p = K_p \cdot \gamma_{\text{br}} \cdot z \text{kN/m}^2 = 5.83 \cdot 18 \cdot 2 \cdot 0.7 = 146.9 \text{kN/m} \]  \hspace{1cm} (2.23)

The coefficient for passive earth pressure is 5.83 for blasted rock according to Bro 2004. The moment, shear force and deflection have been calculated.
\[ R_A = \frac{p \cdot L}{2} = \frac{146.9 \cdot 2}{2} = 146.9 \text{kN/m} \]
\[ M_A = \frac{p \cdot L^2}{3} = \frac{146.9 \cdot 2^2}{3} = 195.9 \text{kNm/m} \]
\[ v_{\text{max}} = v_B = \frac{11 \cdot p \cdot L^4}{120EI} = \frac{11 \cdot 146.9 \cdot 10^3 \cdot 2^4}{120 \cdot 33 \cdot 10^9 \cdot 0.7} = 0.23 \text{ mm} \]

(2.24)

2.2.5 Ultimate limit state

The reinforcement has been calculated in Mathcad based on the results above. The concrete has been assumed to be of quality C32/40 and the reinforcement of quality B500B.

Calculations have been performed according to rules in BBK 04 by Boverket and can be found in appendix B1.

Reinforcement Φ25 s330 for example have been found to be suitable for resisting the bending moment and shear links Φ8 s100 may be appropriate to compensate for shear force in ultimate limit state.

2.2.6 Serviceability limit state

Calculations have been performed according to rules in BBK 04 and can be found in appendix B2.

The amount of reinforcement calculated in serviceability limit state was found to be considerably larger than the amount calculated in ultimate limit state. The required amount has been found to be for example Φ25 s125 for the bending moment. The amount of reinforcement to compensate for shear force calculated in ultimate limit state is still valid.

It is important to observe that the calculated reinforcement only applies for the backwall and is not supposed to be used for the bridge deck, not even the bridge deck closest to the backwall. The area of transition is complicated and deserves a separate investigation. This area is usually made with reinforcement shifted into the bridge deck. The bridge deck is designed according to the earth pressure calculated by Bro 2004.
3 Detailed analysis method

3.1 The model

Analyses have been performed using PLAXIS, which is a well-recognized two- and three dimensional program. PLAXIS uses the Finite Element Method (FEM), which is a method that is used to approximate the solution of a problem. This is done by dividing the model into elements that then can be given different properties such as support conditions, loads, material properties etc. With help of adequate equilibrium equation the problem may be solved.

PLAXIS has been developed specifically for the analysis of deformation and stability in geotechnical engineering projects and make it possible to consider many important properties of the soil that other programs using FEM do not include.

The mesh can be either 6 or 15 node triangular elements and the model can be meshed using five levels of global coarseness with different average element size: very coarse, coarse, medium, fine and very fine. Lines or points can be refined with a local element size factor of 0.5 for the selected item.

PLAXIS makes it possible to carry out the analysis in a number of phases so that features can be added or removed in different stages.

The effect of the ground water level is considered making the calculations of the effective stresses in the soil correct, but the effect of changing the ground water level have not been examined in this thesis.

It is important to remember that the model in PLAXIS may have built-in uncertainties. This means that an easier model to start with is a good idea.

3.1.1 Beam elements

The backwall have been modelled using beam elements that are defined by a flexural rigidity $EI$ and axial stiffness $EA$. From these two parameters an equivalent thickness $d_{eq}$ is calculated from the equation

$$d_{eq} = \sqrt{\frac{12 EI}{EA}}$$  \hspace{1cm} (3.1)
Beam elements have three degrees of freedom per node: two translational degrees of freedom and one rotational degree of freedom. The beam elements are based on Mindlin’s beam theory. This theory allows for beam deflection due to shearing as well as bending. In addition, the element can change length when an axial force is applied. Beam elements can become plastic if a prescribed maximum bending moment or maximum axial force is reached.

![Figure 3.1: Position of nodes and stress points in a 3-node and a 5-node beam element. (PLAXIS Manual, 2004)](image)

Bending moments and axial forces are evaluated from the stresses at the stress points. A 3-node beam element contains two pairs of Gaussian stress points whereas a 5-node beam element contains four pairs of stress points. The position of these points can be seen in figure 3.1.

### 3.1.2 Interface elements

Interfaces are used to evaluate the interaction between the structure and the soil. These are defined with a strength reduction factor $R_{\text{inter}}$ that models the roughness of the interaction. The interface is in general weaker and more flexible than the associated soil layer why the strength reduction factor is smaller than 1.0. Hence a rigid interface is assigned the value 1.0 and that means that the backwall moves along with the soil. A smoother interface has a value over 0.0 and means that the soil and backwall moves more independent of each other. A low interface coefficient about 0.1 - 0.2 for a weak interface (complete smoothness can not be achieved) is recommended by PLAXIS (PLAXIS, 2008) based on experience.

Schweiger (Schweiger, 2007) presents analyses performed in PLAXIS made with the Hardening Soil Model that clearly shows the significant influence of the parameter $R_{\text{inter}}$. Compared to a Mohr-Coulomb model high differences are obtained for the displacements. It is obvious from theses results that input parameters for modeling wall/soil interaction have to be chosen very carefully, which is however a difficult task because the elastic stiffness of an interface is not a well defined mechanical property.

![Figure 3.2: Distribution of nodes and stress points in interface elements and their connection to soil elements. (PLAXIS Manual, 2004)](image)
Figure 3.2 shows how interface elements are connected to soil elements. For 15-node soil elements the corresponding interface elements are defined by five pair of nodes and for 6-node elements the numbers of interface elements are three.

Each interface has a “virtual thickness” which is an imaginary dimension used to define the material properties of the interface. The virtual thickness is calculated as the virtual thickness factor times the average element size. The element size is determined by the global coarseness of the mesh and the default value of the virtual thickness factor is 0.1.

Corners in stiff structures and an abrupt change in boundary condition may lead to high peaks in the stresses and strains. Using additional interface elements as in figure 3.3 solves the problem. These elements will enhance the flexibility of the finite element mesh and will thus prevent non-physical stress results.

Figure 3.3: Flexible corner points with improved stress results. (PLAXIS Manual, 2004)

3.1.3 Soil behaviour

Mohr-Coulomb theory

Soil and rock tend to behave in a highly non-linear way under load. The well-known Mohr-Coulomb model can be considered as a first order approximation of real soil behaviour. This elastic plastic model requires five basic input parameters, namely a Young’s modulus $E$, a Poisson’s ratio $\nu$, a cohesion $c$, a friction angle $\phi$, and a dilatancy angle $\psi$. The stress as a function of strain for a Mohr-Coulomb model can be seen in figure 3.4.

Figure 3.4: Perfect elastic plastic model. (PLAXIS Manual, 2004)

The total strain $\varepsilon$ is calculated as the sum of the elastic strain $\varepsilon^e$ and the plastic strain $\varepsilon^p$

$$\varepsilon = \varepsilon^e + \varepsilon^p$$ (3.2)
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The stress $\sigma'$ can then be calculated using Hooke's law

$$\sigma' = E \cdot \varepsilon^e = E \cdot (\varepsilon - \varepsilon^p)$$

(3.3)

Mohr-Coulomb is a model that uses a yield function $f$ to evaluate whether or not plasticity occurs. When the yield function $f = 0$ the material is plastic. Plastic potential functions $g_1, g_2$ etc. are also used so that the dilatancy is not overestimated. The plastic strain may be calculated as

$$\varepsilon^p = \lambda_1 \frac{\partial g_1}{\partial \sigma} + \lambda_2 \frac{\partial g_2}{\partial \sigma} + \ldots \text{etc}$$

(3.4)

$\lambda$ is the plastic multiplier. For pure elastic behaviour $\lambda$ is zero and in case of plastic behaviour $\lambda$ is positive.

$$\lambda = 0 \text{ for } f < 0 \text{ or } \frac{\partial f}{\partial \sigma} \cdot E \cdot \varepsilon \leq 0 \text{ (Elasticity)}$$

$$\lambda > 0 \text{ for } f = 0 \text{ or } \frac{\partial f}{\partial \sigma} \cdot E \cdot \varepsilon \geq 0 \text{ (Plasticity)}$$

(3.5)

The full Mohr-Coulomb yield condition consists of six yield functions formulated in terms of principal effective stresses

$$f_{1a} = \frac{1}{2} \left( \sigma'_2 - \sigma'_3 \right) + \frac{1}{2} \left( \sigma'_2 + \sigma'_3 \right) \sin \varphi - c \cos \varphi \leq 0$$

$$f_{1b} = \frac{1}{2} \left( \sigma'_3 - \sigma'_2 \right) + \frac{1}{2} \left( \sigma'_3 + \sigma'_2 \right) \sin \varphi - c \cos \varphi \leq 0$$

$$f_{2a} = \frac{1}{2} \left( \sigma'_3 - \sigma'_1 \right) + \frac{1}{2} \left( \sigma'_3 + \sigma'_1 \right) \sin \varphi - c \cos \varphi \leq 0$$

$$f_{2b} = \frac{1}{2} \left( \sigma'_1 - \sigma'_3 \right) + \frac{1}{2} \left( \sigma'_1 + \sigma'_3 \right) \sin \varphi - c \cos \varphi \leq 0$$

$$f_{3a} = \frac{1}{2} \left( \sigma'_1 - \sigma'_2 \right) + \frac{1}{2} \left( \sigma'_1 + \sigma'_2 \right) \sin \varphi - c \cos \varphi \leq 0$$

$$f_{3b} = \frac{1}{2} \left( \sigma'_2 - \sigma'_1 \right) + \frac{1}{2} \left( \sigma'_2 + \sigma'_1 \right) \sin \varphi - c \cos \varphi \leq 0$$

(3.6)

The effective stresses are the solution of a triaxial test when all shear stress components are zero. They are arranged in algebraic order

$$\sigma'_1 \leq \sigma'_2 \leq \sigma'_3$$

(3.7)

In addition to the yield functions, six plastic potential function are defined for the Mohr-Coulomb model
\[ g_{1a} = \frac{1}{2} (\sigma'_2 - \sigma'_3) + \frac{1}{2} (\sigma'_2 + \sigma'_3) \sin \psi \]
\[ g_{1b} = \frac{1}{2} (\sigma'_3 - \sigma'_2) + \frac{1}{2} (\sigma'_3 + \sigma'_2) \sin \psi \]
\[ g_{2a} = \frac{1}{2} (\sigma'_3 - \sigma'_1) + \frac{1}{2} (\sigma'_3 + \sigma'_1) \sin \psi \]
\[ g_{2b} = \frac{1}{2} (\sigma'_1 - \sigma'_3) + \frac{1}{2} (\sigma'_1 + \sigma'_3) \sin \psi \]
\[ g_{3a} = \frac{1}{2} (\sigma'_1 - \sigma'_2) + \frac{1}{2} (\sigma'_1 + \sigma'_2) \sin \psi \]
\[ g_{3b} = \frac{1}{2} (\sigma'_2 - \sigma'_1) + \frac{1}{2} (\sigma'_2 + \sigma'_1) \sin \psi \]

(3.8)

PLAXIS also defines a tension cut-off which makes sure that the soil does not sustain more tensile stress than it can if \( c > 0 \).

\[
\begin{align*}
  f_4 &= \sigma'_1 - \sigma_1 \leq 0 \\
  f_5 &= \sigma'_2 - \sigma_1 \leq 0 \\
  f_6 &= \sigma'_3 - \sigma_1 \leq 0
\end{align*}
\]

(3.9)

\( \sigma_1 \) is the allowable tensile stress, often taken as zero.

**Hardening Soil theory**

Another method of modelling the soil is to use Hardening Soil theory. Instead of using a bi-linear curve as in the Mohr-Coulomb theory this model uses a hyperbolic stress-strain curve to model a perfectly plastic behaviour, see figure 3.5.

![Hyperbolic model](image)

Figure 3.5: Hyperbolic model.

Hardening Soil model is not only based on the principal stresses but can be expanded due to plastic straining. Shear hardening describes the behaviour where irreversible strains rise due
Chapter 3. Detailed analysis method

to primary deviatoric loading and compression hardening describes the irreversible plastic strains due to primary compression in oedometer loading and isotropic loading.

The Hardening Soil model is a more advanced method for modelling the soil but it doesn’t include all that many parameters as the Mohr-Coulomb model. The Hardening Soil model is characterised by the following parameters:
- $E_{50}^{ref}$ is the secant stiffness in standard drained triaxial test which has the same value of Young’s modulus as the modulus used for the Mohr-Coulomb model.
- $E_{oed}^{ref}$ is the tangent stiffness for primary oedometer loading. PLAXIS Manual (PLAXIS, 2004) prescribes that the oedometer modulus is set equal to the reference modulus.
- $E_{ur}^{ref}$ is the unloading/reloading stiffness which is set equal to the reference modulus times three.
- The power law $m$ is the amount of stress dependency.
- Parameters for characterising the failure according to the Mohr-Coulomb model $c$, $\phi$, $\psi$ are also used.

The basic idea for formulation of the Hardening-Soil model is the hyperbolic relationship between the vertical strain, $\varepsilon_1$, and the deviatoric stress, $q$, in primary triaxial loading. The relationship is depicted in figure 3.6.

Figure 3.6: Hyperbolic stress-strain relation in primary loading for a standard drained triaxial test. (PLAXIS Manual, 2004)

Standard drained triaxial test tend to yield curves that can be described by:

$$-\varepsilon_1 = \frac{1}{2E_{50}} \frac{q}{1 - q/q_a} \text{ for } q < q_f$$

(3.10)

$q_a$ is the asymptotic value of the shear strength and the parameter $E_{50}$ is defined as

$$E_{50} = E_{50}^{ref} \left( \frac{c \cos \phi - \sigma'_3 \sin \phi}{c \cos \phi + p^{ref} \sin \phi} \right)^m$$

(3.11)

where $p^{ref}$ is the reference pressure, which by default is set to 100 kPa.

The stiffness modulus $E_{ur}$ is calculated in the same way.
\[ E_{ur} = \frac{E_{ur}^{\text{ref}} (c \cos \varphi - \sigma_3 \sin \varphi)}{c \cos \varphi + p_{\text{ref}} \sin \varphi} \]  

The quantity \( q_a \) is given by

\[ q_a = \frac{q_f}{R_f} \]  

(3.13)

\( R_f \) is the failure ratio, which by default is set to 0.9 and the ultimate deviatoric stress, \( q_f \) is defined as

\[ q_f = (c \cot \varphi - \sigma_3) \frac{2 \sin \varphi}{1 - \sin \varphi} \]  

(3.14)

When \( q = q_f \), the failure criterion is satisfied and perfectly plastic yielding occurs as described by the Mohr-Coulomb model.

The plastic strains are calculated from a yield function \( f \)

\[ f = \bar{f} - \gamma^p \]  

(3.15)

where \( \bar{f} \) is a function of stress and \( \gamma^p \) is a function of plastic strains:

\[ \bar{f} = \frac{1}{E_{\text{so}}} \frac{q}{1 - q / q_a} E_{ur} - \frac{2q}{E_{ur}} \]  

\[ \gamma^p = -2 \epsilon_1^p - \epsilon_v^p \approx -2 \epsilon_1^p \]  

(3.16)

The superscript \( p \) is used to define plastic strains.

There is a relation between \( \dot{\epsilon}_v^p \) and \( \dot{\gamma}^p \)

\[ \dot{\epsilon}_v^p = \sin \psi_m \dot{\gamma}^p \]  

(3.17)

where \( \psi_m \) is the mobilised dilatancy angle.

For \( \sin \varphi_m < 3/4 \sin \varphi \):
\[ \psi_m = 0 \]

For \( \sin \varphi_m \geq 3/4 \sin \varphi \) and \( \psi > 0 \):
\[ \sin \psi_m = \max \left( \frac{\sin \varphi_m - \sin \varphi_{cv}}{1 - \sin \varphi_m \sin \varphi_{cv}}, 0 \right) \]  

(3.18)

For \( \sin \varphi_m \geq 3/4 \sin \varphi \) and \( \psi \leq 0 \):
\[ \sin \psi_m = \psi \]  

If \( \varphi = 0 \):
\[ \psi_m = 0 \]

\( \varphi_{cv} \) is the critical state friction angle, being a material constant independent of density and \( \varphi_m \) is the mobilised friction angle.
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\[
\sin \varphi_m = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3 - 2c \cot \varphi}
\]

\[
\sin \varphi_{cv} = \frac{\sin \varphi - \sin \psi}{1 - \sin \varphi \sin \psi}
\]

(3.19)

It may be added that the material contracts when \( \varphi_m < \varphi_{cv} \) and that dilatancy occurs when \( \varphi_m > \varphi_{cv} \).

3.2 Example

The background for the model in PLAXIS is shown in figure 3.7. Because of symmetry half the bridge has been modeled, with a length of 9 m. In order to get good results an adequate amount of soil to the left of the bridge is required, 15 m has been chosen in this example. The height of the backwall has been set to 2 m and a support has been placed 2 m from the backwall to define a pier at this point. Blasted rock with a depth of 3 m supports the backwall left of the backwall and has been chosen with the inclination of 45° 1 m below the bridge deck. Below the blasted rock 5 m of moraine has been modeled. A fictive material to the right of the backwall is required in the first time step.

The model have to be prevented to move by boundary conditions, the lower edge of the moraine has been set as fixed in both directions and the right and the left borders have been set as fixed in x-direction. Point 8 has been set as fixed in both directions since two displacements directed reverse each other act at this point making it impossible for this point to move. The moment is therefore zero at this point. An interface has been defined between the backwall and the soil.

![Figure 3.7: The studied model.](image)

Figure 3.7: The studied model.
3.2.1 Modeling the soil

The filling has been restricted to blasted rock, a very common material for filling for bridges with backwalls especially in the area around Stockholm. It has been modeled using Mohr-Coulomb theory, but an analysis with the Hardening Soil model has also been performed. Used values for both soil models are presented in table 3.1 and 3.2.

\( \gamma_{\text{sat}} \) is the soil unit weight below phreatic level and \( \gamma_{\text{unsat}} \) is the soil unit weight above phreatic level. The weight density for both saturated and unsaturated blasted rock is given in Bro 2004, along with the angle of friction for blasted rock. The weight density for water is added to the saturated weight density according to PLAXIS Manual (PLAXIS, 2004). The weight density for moraine along with angle of friction can be found in “Slab foundation” (“Plattgrundläggning”) by the Swedish Geotechnical Institute and “Determination of soil parameters” (“Bestämning av jords hållfasthets- och deformationsegenskaper”) by the Swedish Road Administration. The modulus of elasticity for both moraine and blasted rock can also be found in these handbooks, but the modulus of elasticity for moraine may vary.

PLAXIS Manual (PLAXIS, 2004) suggests that the angle of dilatancy \( \psi \) for cohesionless soils can be calculated as (if the angle of friction is equal to or larger than 30°)

\[
\psi = \phi - 30^\circ
\]  
(3.20)

Some values can not be found in literature and have to be chosen. As a first approximation the permeability \( k \) of the soil has been chosen to have the same size both in the direction of \( x \) and of \( y \) and has been chosen to 1,0 for both blasted rock and moraine. The cohesion \( c \) has been set to 1 kPa for the blasted rock and 5 kPa for the moraine. A higher cohesion indicates a higher content of clay. The value of Poisson’s ratio ranges between 0 and 0,5. 0,25 has been chosen as a first approximation for both materials. The power law \( m \) is higher (1,0) for softer materials. 0,5 has therefore been chosen.

<table>
<thead>
<tr>
<th></th>
<th>Blasted rock</th>
<th>Moraine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material model</td>
<td>Mohr-Coulomb</td>
<td>Mohr-Coulomb</td>
</tr>
<tr>
<td>Material type</td>
<td>Drained</td>
<td>Drained</td>
</tr>
<tr>
<td>( \gamma_{\text{unsat}} )</td>
<td>18 kN/m³</td>
<td>18 kN/m³</td>
</tr>
<tr>
<td>( \gamma_{\text{sat}} )</td>
<td>21 kN/m³</td>
<td>20 kN/m³</td>
</tr>
<tr>
<td>( k )</td>
<td>1 m/day</td>
<td>1 m/day</td>
</tr>
<tr>
<td>( E )</td>
<td>50000 kN/m²</td>
<td>20000 kN/m²</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0,25</td>
<td>0,25</td>
</tr>
<tr>
<td>( c )</td>
<td>1 kN/m²</td>
<td>5 kN/m²</td>
</tr>
<tr>
<td>( \phi )</td>
<td>45°</td>
<td>35°</td>
</tr>
<tr>
<td>( \psi )</td>
<td>15°</td>
<td>5°</td>
</tr>
</tbody>
</table>

Table 3.1: Input values for the soils for Mohr-Coulomb theory.

<table>
<thead>
<tr>
<th></th>
<th>Blasted rock</th>
<th>Moraine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material model</td>
<td>Hardening Soil</td>
<td>Hardening Soil</td>
</tr>
<tr>
<td>Material type</td>
<td>Drained</td>
<td>Drained</td>
</tr>
<tr>
<td>( \gamma_{\text{unsat}} )</td>
<td>18 kN/m³</td>
<td>18 kN/m³</td>
</tr>
</tbody>
</table>
Chapter 3. Detailed analysis method

<table>
<thead>
<tr>
<th>( \gamma_{sat} )</th>
<th>21</th>
<th>20</th>
<th>kN/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>1</td>
<td>1</td>
<td>m/day</td>
</tr>
<tr>
<td>( E_{ref,50} )</td>
<td>50000</td>
<td>20000</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( E_{ref,\text{sed}} )</td>
<td>50000</td>
<td>20000</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( E_{ref,\text{ur}} )</td>
<td>150000</td>
<td>60000</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( m )</td>
<td>0,5</td>
<td>0,5</td>
<td></td>
</tr>
<tr>
<td>( c )</td>
<td>1</td>
<td>5</td>
<td>kN/m²</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>45</td>
<td>35</td>
<td>°</td>
</tr>
<tr>
<td>( \psi )</td>
<td>15</td>
<td>5</td>
<td>°</td>
</tr>
</tbody>
</table>

Table 3.2: Input values for the soils for Hardening Soil theory.

A parameter \( R_{\text{inter}} \) relating the strength in the interface to the strength of the soil shall also be defined for the interface between the blasted rock and the backwall. If the adhesion is very stiff this parameter is chosen to be rigid and is set equal to 1. Since the interface is considered to be smooth this parameter has been set to be as low as possible, 0,1.

Filling at the inner side of the backwall may also give rise to earth pressure against the backwall but has been neglected because it is much smaller than the earth pressure at the opposite side of the backwall.

3.2.2 Modeling the structure

The backwall and the bridge deck have been restricted to be made of concrete and have been modeled as linear elastic beams. The height of the backwall has been set to 2 m and the width of the backwall to 0,7 m. The thickness of the bridge deck has also been set to 0,7 m. Input values are presented in table 3.3. Grid lines have been used to model the structure and care has been taken to the thickness. Only the concrete slab and the backwall have been included in the analyses why the grid line doesn’t become very complicated. No wing walls have been considered in the analyses. Wing walls may make the structure stiffer and a 3D model would enable observation of this effect. The grid lines can be seen in figure 3.8 as dotted lines.

![Figure 3.8: Grid lines for the case when the only parts participating in the analysis are the concrete slab and the backwall.](image)

Modeling the backwall at the level of the grid line and the upper border of the soil at the same level reduces the depth of the soil with 0,35 m. This amount of soil has however been neglected because it is considered to have a very small effect.
The weight \( w \) of the superstructure and the backwall has been set equal to zero due to the construction method. The weight of the superstructure and the backwall don’t affect the pressures in the soil since the filling is put into place after the deformations of superstructure and the backwall have reached equilibrium. This means that no additional stresses rise in the filling.

\[
\begin{array}{|c|c|c|}
\hline
 & \text{Backwall} & \text{Bridge deck} \\
\hline
E & 33 & 33 & \text{GPa} \\
A & 0,7 & 0,7 & \text{m}^2 \\
I & 0,02858 & 0,02858 & \text{m}^4 \\
w & 0 & 0 & (\text{kN/m})/\text{m} \\
\nu & 0 & 0 & - \\
\hline
\end{array}
\]

Table 3.3: Input values for the backwall and for the bridge deck.

\( E \) is the modulus of elasticity, \( A \) is the area and \( I \) the moment of inertia.

### 3.2.3 Modeling in time steps

The model requires a surface between the bridge deck and the moraine in order to function correct. Since there is not any in reality this has been solved by using a fictive material with very low density so that the stresses that rise in the moraine below can be neglected. Low values for other parameters for the fictive material have also been chosen so that the effect of the fictive material is minimized.

The calculation has then been performed in three stages. In the first stage the fictive material has been included, see figure 3.9.

Figure 3.9: First time step.

In the second stages the fictive material has been removed, see figure 3.10.
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In the third stage the effect of temperature change has been modeled, see figure 3.11. The effect of the temperature has been modeled as a prescribed displacement of the size calculated according to the method in Bro 2004. This makes it possible to compare the moment, the shear force and the deflection calculated by the model with the values calculated according to Bro 2004.

Figure 3.10: Second time step.

Figure 3.11: Third time step.
4 Results and discussions

The model presented above has been run to compare the earth pressure induced by change in temperature calculated by PLAXIS to the earth pressure according to the requirements in Bro 2004 and in Eurocode. The moment at the top of the backwall is the main object for comparison.

4.1 Mesh sensitivity study

A study of convergence has been performed in order to check if the results converge for finer meshes. Convergence means that the results by PLAXIS can be trusted. Table 4.1 shows the results presented as the moment at the top of the backwall for different meshes for a displacement of 6 mm and figure 4.1 shows the results in a plot. A refined line mesh is a mesh with twice as many node points around the line of interest which in this case is the backwall.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>M (kNm/m)</th>
<th>Mesh number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>-124,0</td>
<td>1</td>
</tr>
<tr>
<td>Coarse+refined line</td>
<td>-127,5</td>
<td>2</td>
</tr>
<tr>
<td>Medium</td>
<td>-125,6</td>
<td>3</td>
</tr>
<tr>
<td>Medium+refined line</td>
<td>-120,2</td>
<td>4</td>
</tr>
<tr>
<td>Fine</td>
<td>-120,5</td>
<td>5</td>
</tr>
<tr>
<td>Fine+refined line</td>
<td>-112,9</td>
<td>6</td>
</tr>
<tr>
<td>Very fine</td>
<td>-122,7</td>
<td>7</td>
</tr>
<tr>
<td>Very fine+refined line</td>
<td>-114,7</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 4.1: Moment at the top of the backwall for different meshes.
Chapter 4. Results and discussions

Figure 4.1: Plot of the moments for different meshes.

From figure 4.1 it is clear that the meshes with a refined line diverge from the other values. Compared to the calculations according to Bro 2004 which gives a moment of 122.1 kNm/m, the maximum standard deviation is 7.5 % when the refined line meshes are included. The basic idea is that a finer mesh should give more reliable result, but this doesn’t seem to be the case for this model. Why the model behaves in this way has not been investigated further and may be an area of research. A plot that doesn’t include the refined line meshes is presented below in figure 4.2. Note that the mesh numbers have been changed so that the coarse mesh respond to mesh number 1, the medium mesh respond to mesh number 2 etc.

Figure 4.2: Plot of the moments for a coarse, a medium, a fine and a very fine mesh.

The moment at the top of the backwall in figure 4.2 all lies in a narrow interval. Compared to the calculations according to Bro 2004 these results only have a standard deviation of 2.8 %. The very fine mesh with a moment of 122.7 kNm/m has been chosen as the result of the PLAXIS calculation.
4.2 Without temperature effect

In this chapter the results for the first and second stage of the model are shown in figure 4.3 to 4.6. In these stages the effect of temperature change has not been considered.

Figure 4.3: Deformed mesh for the first stage scaled up 500 times.

Since no material may disappear the blasted rock is deformed downwards when the bridge deck is bent upwards. This is due to the low density of the fictive material which easily may be deformed and is pushed upwards by the blasted rock at the right of the backwall.
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Figure 4.4: Distribution of the moment along the backwall at the first stage.

The moment at the top of the backwall is at the first stage 37,8 kNm/m. The moment should be zero in case of blasted rock at both sides of the backwall so that the pressure from the left side would be equal to the pressure from the right side.

Figure 4.5: Deformed mesh for the second stage scaled up 500 times.
The moment at the top of the backwall is at the second stage 37.6 kNm/m. The moment at this stage is nearly the same as in the first stage. The difference in moment is due to the fictive material at the right of the backwall. But since the first stage includes the fictive material only the second stage is of interest.

Since the second stage does not include any displacements it would be reasonable to assume that at rest pressure is valid for this stage. PLAXIS presents the horizontal effective stresses for this stage. In figure 4.7 the horizontal effective stresses along a line behind the backwall is presented. This line has been picked by hand and figure 4.7 therefore also shows the stresses some meters below the backwall. In figure 4.8 only the stresses along the backwall is presented.
Figure 4.7: Horizontal effective stresses for the second stage presented by PLAXIS. (Redrawn from PLAXIS.)

Figure 4.8: Horizontal effective stresses for the second stage in numbers.

The stress at the bottom of the backwall (y-coordinate 6) can not be calculated correctly by PLAXIS since this point is singular. Instead the same slope along the total length of the backwall would be realistic. The curve in figure 4.9 has been interpolated in this way. (The singular point also effects the moment that becomes higher than it should be due to the higher stress.)
These effective normal stresses calculated by PLAXIS may be compared to the horizontal earth pressure behind the backwall as presented in figure 4.10. The horizontal earth pressure has been calculated for at rest pressure according to Bro 2004 and can be found in equation 2.11 and 2.12.

The curves in figure 4.9 and 4.10 have the same slope but PLAXIS presents a curve that is shifted 5 kN/m² to the left. This difference is due to the interface elements connecting the backwall with the blasted rock. The stress defining the interface elements is difficult to determine but causes probably the stress displacement shown in figure 4.10.
4.3 Temperature effect

Figure 4.11 and 4.12 shows the deformed mesh and the moment along the backwall at the third stage of the model. In this stage a prescribed displacement of 6 mm has been added to model the change in temperature. The prescribed displacement gives rise to a larger bending moment at the top of the backwall but the shape of the moment curve is still the same.

When applying the load for temperature change the backwall is deflected, see figure 4.11. The deflection reduces the moment some so that the backwall may rotate. According to Nielsen (Nielsen, 1994) this effect can be neglected. This was confirmed in analysis since an increased flexural rigidity of the bridge deck only had a small effect on the moment. Deflection of the backwall doesn’t reduce the moment substantially.
4.3. **TEMPERATURE EFFECT**

Figure 4.12: Distribution of the moment along the backwall at the third stage.

The moment at the top of the backwall in the third stage is 122.7 kN/m² for a prescribed displacement of 6 mm. This shall be compared to the moment due to temperature change according to Bro 2004. The distribution of the moment due to temperature change is given by the formula

\[
M(x) = \frac{qL^2}{6} \left( 1 - \frac{x_1}{L} \right)^2 \left( 2 + \frac{x_1}{L} \right)
\]  

(4.1)

where \(x_1\) is the distance from the bottom of the backwall.

The distribution is shown in figure 4.13 and corresponds well to the moment calculated by PLAXIS.
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Figure 4.13: Distribution of moment according to Bro 2004.

The stress along the backwall for the third stage is presented in figure 4.14 and with numbers in figure 4.15.

Figure 4.14: Horizontal effective stresses for the third stage presented by PLAXIS. (Redrawn from PLAXIS.)
4.4. Relation between displacement and moment

In the previous chapter a prescribed displacement of 6 mm was checked. Other prescribed displacements are also interesting to investigate in case of longer or shorter bridge decks. The relation between the prescribed displacement and the moment respectively the force behind the backwall (calculated per meter width) are presented in figure 4.16 and 4.17.

Figure 4.15: Horizontal effective stresses for the third stage in numbers.

The effect of the singular point at the bottom of the backwall can also be seen at this stage.
Figure 4.16: Comparison of the moment calculated by PLAXIS at the top of the backwall and the moment according to Bro 2004.

Figure 4.17: Comparison of the force behind the backwall calculated by PLAXIS and the force according to Bro 2004.
The moment calculated by PLAXIS coincide with the moment suggested by Bro 2004 at the point where the displacement is 6 mm and the force coincide with the force suggested by Bro 2004 at the point where the displacement is around 7 mm. The values calculated by PLAXIS are higher than the values suggested by Bro 2004 when the displacement is smaller the value at the intersection. Where the displacements are larger than the relation $H/200$ (10 mm) Bro 2004 prescribes that the moment respectively the force stabilise at the limit of passive pressure, but this can not be verified by the model since the force respectively the moment continue to grow even after this point. Since the model is performed as a linear-elastic model all displacements larger than 18 mm leads to a collapse in the model.

It can be concluded that the results from the model correspond quite well to the rules in Bro 2004. According to Bro 2004 backwalls shall be designed for passive earth pressure but this is a pessimistic demand, especially for short bridges with high backwalls. For such a case the limit for passive pressure is high and at the same time the displacement is low so that passive pressure never is reached.

The Eurocodes prescribes that passive pressure shall be used for a movement larger than $H/2000$. This demand is even more pessimistic than the requirements in Bro 2004. Since no method of calculation has been found the result is not analysed further in this thesis.

### 4.5 Parameters that affect the result

A study to investigate what parameters affect the results has been performed. By changing the examined parameter the moment at the top of the backwall can be compared to the reference moment for a prescribed displacement of 6 mm, 122,7 kNm/m.

The permeability $k$ of the soil has been found to have no effect on the bending moment even for a large interval, 0,001 to 1000 m/day.

The weight density for unsaturated moraine $\gamma_{\text{unsat}}$ has been changed in the interval of 15 to 20 kN/m$^3$ and the weight density for saturated moraine $\gamma_{\text{sat}}$ has been changed in the interval of 20 to 25 kN/m$^3$. Changing the weight densities of the moraine has almost no effect on the moment as well as changing the angle of friction for the moraine $\phi_{\text{moraine}}$, the angle of dilatancy for the moraine $\psi_{\text{moraine}}$ and the cohesion for the moraine $c_{\text{moraine}}$. The angle of friction for the moraine have been examined for the interval $25^\circ$ to $45^\circ$, the angle of dilatancy for the moraine the interval $1^\circ$ to $15^\circ$ and the cohesion for the moraine the interval 0,2 to 10 kPa. Poisson’s ratio for moraine $\nu_{\text{moraine}}$ has been examined for the interval 0 to 0,4 and was found to have some influence on the bending moment.

Parameters for the blasted rock that have been found to have some effect on the moment are the dilatancy $\psi_{\text{br}}$, Poisson’s ratio $\nu_{\text{br}}$ and the cohesion for the blasted rock $c_{\text{br}}$. Poisson’s ratio for the blasted rock has been checked for the interval 0 to 0,4, the dilatancy for the blasted rock the interval $1^\circ$ to $15^\circ$ and the cohesion the interval 0,2 to 5 kPa.

A change of Young’s modulus for the backwall $E_{\text{backwall}}$ in the interval 20 to 50 GPa has a very small effect on the bending moment.

Parameters that affect the moment significant are the modulus of elasticity for the moraine $E_{\text{moraine}}$, the amount of blasted rock $h_{\text{br}}$, the interface coefficient $R_{\text{inter}}$, the thickness of
the backwall $d_{\text{backwall}}$, the thickness of the bridge deck $d_{\text{deck}}$, and the modulus of elasticity for the bridge deck $E_{\text{deck}}$. These parameters are discussed in the next chapter.

4.5.1 The interface coefficient, $R_{\text{inter}}$

The interface coefficient has been set to 0.1 according to the recommendations from PLAXIS support (PLAXIS, 2008). This value may be set as high as 1 in case of a sand/concrete interface with higher surface roughness and the interface coefficient can’t be less than 0.01. Figure 4.18 shows how the moment changes with a changed value of the interface coefficient.

![Figure 4.18: Variation of the moment due to a changed interface coefficient.](image)

The moment may deviate up to 19.7% for a change of interface coefficient. The effect of the interface coefficient has the biggest effect between 0.01 and 0.4. This means that the choice of interface coefficient is very important and must be chosen carefully. Since this parameter can not be measured in a simple way it is recommended that the interface coefficient is examined for a suitable interval for the current model.

Measurements of soil pressure at site may be used to calibrate the calculations for modelling so that the interface coefficient may be given a better approximation.

4.5.2 The modulus of elasticity for moraine, $E_{\text{moraine}}$

The reference value for the modulus of elasticity for the moraine was set to 20,000 kN/m$^2$. According to handbooks this value may vary between 10,000 kN/m$^2$ and 50,000 kN/m$^2$. The result for different modulus of elasticity for the moraine is presented in figure 4.19.
4.5. PARAMETERS THAT AFFECT THE RESULT

Figure 4.19: Variation of the moment due to change in modulus of elasticity for the moraine.

The deviation in moment may be 9.9% for a change in modulus of elasticity for the moraine. It could therefore be a good idea to do an in-situ test to measure the modulus of elasticity for the moraine.

A higher modulus of elasticity for the moraine can be seen as a stiffer reaction (springs) against the backwall so that the moment at the top of the backwall is reduced.

4.5.3 The amount of blasted rock $h_{br}$

The reference height of the blasted rock was set to 2 m. The amount of the blasted rock cannot be varied much since the upper level of the blasted rock always must be the same as the level of the bridge deck. The height of the blasted rock has been varied in the interval 2 m to 4 m. The variation of the moment is depicted in figure 4.20.
Chapter 4. Results and discussions

4.5.4 The thickness of the backwall, $d_{\text{backwall}}$

The thickness of the backwall was set to 0.7 m as a reference value. Bro 2004 prescribes that a backwall must have a thickness over 0.2 m. Figure 4.21 shows how the moment changes with different thicknesses of the backwall.

Figure 4.21: Variation of the moment due to different thicknesses of the backwall.
Choosing the thickness of the backwall as 0.2 m gives a standard deviation of 36.1%. The thickness of the backwall is of great importance but is often known for the current case.

A thickness of the backwall under 0.4 m gives rise to a large deflection. A thickness between 0.4 and 0.7 m is most economical according to the conditions used for this model.

### 4.5.5 The thickness of the bridge deck, $d_{\text{deck}}$ or the modulus of elasticity for the bridge deck, $E_{\text{deck}}$

The thickness of the bridge deck was set to 0.7 m as a reference value. Thicknesses as low as 0.2 m and as high as 1.5 m have been studied. Figure 4.22 shows how the moment changes with different thicknesses of the bridge deck. The same behaviour has been found for changed modulus of elasticity for the bridge deck.

![Figure 4.22: Variation of the moment due to different thicknesses of the bridge deck.](image)

Choosing the thickness of the bridge deck as 0.2 m gives a standard deviation of 63.2%. A thickness between 0.2 and 0.6 m gives a lower moment than the reference moment. This can be explained by the fact that the superstructure gets bent at in this interval and the earth pressure then gets reduced (as well as the moment). The moment doesn’t change much in the interval between 0.7 and 1.5 m. A thickness of 0.7 m is therefore a good choice.

From figure 4.20 it can also be concluded that a high value of the flexural rigidity (thick deck or/and a high modulus of elasticity) makes the deck so stiff that the behaviour of the backwall can be seen as a cantilever. A lower value of the flexural rigidity makes the backwall behave like a spring.

The thickness of the bridge deck is of great importance but is often chosen before modelling due to other requirements. The same is valid for the modulus of elasticity for the bridge deck.
4.6 Hardening Soil model

A test using the Hardening Soil model has been performed for a prescribed displacement of 6 mm. The results are shown in figure 4.23 and 4.24.

Figure 4.23: Deformed mesh for the third stage using the Hardening Soil model scaled up 100 times.
Figure 4.24: Distribution of the moment along the backwall for the third stage using the Hardening Soil model.

The moment at the top of the backwall when a prescribed displacement of 6 mm is used is 112.6 kNm/m. This value differ 8.2 % from the reference moment and the Hardening Soil model therefore has a lower correspondence with the moment calculated according to Bro 2004 than the Mohr-Coulomb theory. Further studies may be performed to analyze the differences in behaviour between the Hardening Soil model and Mohr-Coulomb theory and to study what parameters affect the result for a model with Hardening Soil theory.
5 Conclusions

The model in PLAXIS has been found to correspond quite well with the requirements in Bro 2004. The moments calculated by PLAXIS all lie in a narrow interval for different meshes except for refined line meshes. Why the refined line meshes deviate from the other meshes should be investigated further. The model behaves as assumed since it displays the at rest pressure for a case without impact of temperature and when a temperature change is included in the model the curve for moment corresponds very well with the curve calculated according to Bro 2004 for the same temperature. From these results it can be concluded that modelling in PLAXIS gives reliable results.

There is however one important difference between the model and the requirements in Bro 2004; the model shows almost linearly growing curves for the moment and the force compared to the curves according to Bro 2004 that stabilise at the level of passive pressure after $H/200$. This means that the demand in Bro 2004 that backwalls shall be designed for passive earth pressure is pessimistic for the investigated type of geometry since the displacement $H/200$ almost never is reached for these conditions. The demand is extra pessimistic for short bridges with high backwalls. The demand in the Eurocode that prescribes that passive pressure shall be used for a movement larger than $H/2000$ is even more pessimistic but since no method of calculation has been found this has not been analysed further in this thesis.

There are some parameters that may affect the results considerable; if possible the amount of blasted rock, the modulus of elasticity for the bridge deck and the thickness of the bridge deck as well as the thickness of the backwall shall be determined before modelling in order to avoid irresolute earth pressure. An in-situ test to measure the modulus of elasticity for the moraine is also suggested. The interface coefficient must be chosen very carefully and it is recommended that this parameter is studied for the current case. If soil pressure is measured at site calibrated calculations may be performed in order to approximate the interface coefficient. Since there is not much research concerning this parameter this is an important area of research in order to make PLAXIS a more reliable and more frequently used tool.

Using Hardening Soil theory for modelling the soil has been found to give less correspondence to the requirements in Bro 2004 than using Mohr-Coulomb theory. The choice of soil model should be considered for the current case. A more advanced soil model may have the advantage of the parameters converging more quickly than for Mohr-Coulomb theory. Since the Hardening Soil model doesn’t include all that many parameters as Mohr-Coulomb theory there may on the other hand be larger divergences for the parameters used in Hardening Soil theory. The impact of the interface coefficient may have a larger effect on the model when using the Hardening Soil model as pointed out by Schweiger (Schweiger, 2007) and may be an area of research.
Chapter 5. Conclusions

Finally, a model in PLAXIS doesn’t make the design of backwalls more effective but it may be a good tool for analysing the earth pressure in combination with other effects such as patterns for displacements as well as moment- and force distributions that are the prime objectives for a designer. Another advantage is that PLAXIS shows the correct behaviour of the earth pressure which makes it a trustable tool. PLAXIS is also easy to handle. Even though PLAXIS has some defects the advantages make PLAXIS a frequently used tool for geotechnical problems compared to other FEM-programs.
Bibliography


Russell Henry G. and Gerken Lee J. (1994). *Jointless Brigdes- the Knowns and the Unknowns*. Concrete Inter


Sundquist H. (2007). *Beam and frame structures*. Royal Institute of Technology (KTH), Stockholm, Sweden


A  Maximal and minimal air temperature

Figure A.1: \( T_{\text{min}} \). (Swedish Road Administration, 2008a)
Figure A.2: $T_{\text{max}}$. (Swedish Road Administration, 2008a)
B Calculations of reinforcement

The total moment and the total force are used to calculate the amount of reinforcement. Calculations are performed and presented in Mathcad (Mathcad, 2008). Parameters are defined in the notations.

B.1 Ultimate limit state

Safety class
\[ \gamma_n := 1.2 \]
Safety factors
\[ \gamma_m := 1.5 \]
\[ \gamma_{ms} := 1.15 \]
\[ \gamma_{ms2} := 1.05 \]

Concrete
\[ \varepsilon_{cu} := 0.25 \times 10^{-3} \]
\[ f_{cck} := 30.5 \text{MPa} \]
\[ f_{ctk} := 2.00 \text{MPa} \]
\[ E_{ck} := 33.0 \times 10^9 \]

Reinforcement
\[ \phi := 25 \text{mm} \]
\[ f_{yk} := 500 \text{MPa} \]
\[ E_{sk} := 200 \text{GPa} \]

Dimensions
\[ b := 1 \text{m} \]
\[ d := 0.7 \text{m} \]

Concrete cover
\[ h_{täck} := (45 + 10) \text{mm} \]

Total height
\[ d_{tot} := d - h_{täck} - \frac{\phi}{2} \]
Chapter B. Calculations of reinforcement

Moment \( M_{Atot} := 233.1 \text{kN} \cdot \text{m} \)

Shear force \( R_{Atot} := 177.3 \text{kN} \)

**Reinforcement to restrict bending**

\( \alpha_2 := 0.8 \)
\( \beta := 0.4 \)

\[ x := d \left( \frac{1}{2} \beta - \sqrt{\frac{1}{4 \beta^2} - \frac{M_{Atot}}{\alpha_2 \beta \cdot f_{cc} \cdot b \cdot d_{tot}^2}} \right) \]

It must be checked that

\[ \varepsilon_s \geq \varepsilon_{skoll} \]

\[ 1 - \frac{x}{d_{tot}} \cdot \varepsilon_{cu} \]

\[ \varepsilon_{skoll} := \frac{f_{st}}{E_s} \]

The amount of reinforcement may be calculated as

\[ A_s := \begin{cases} \alpha_2 \cdot b \cdot x \cdot \frac{f_{cc}}{f_{st}} & \text{if } \varepsilon_s \geq \varepsilon_{skoll} \\ \alpha_2 \cdot b \cdot x \cdot \frac{f_{cc}}{\varepsilon_s \cdot E_s} & \text{if } \varepsilon_s < \varepsilon_{skoll} \end{cases} \]

\[ A_s = 1.146 \times 10^{-3} \text{ m}^2 \]

\( \phi25 \text{s330} \) is appropriate.

**Reinforcement compensating for the effect of the shear force**

\[ \rho := \frac{A_s}{b \cdot d} \]

\[ \xi := 1.3 - 0.4 \frac{d}{m} \]

\[ f_v := \xi \cdot (1 + 50 \cdot \rho) \cdot 0.30 f_{ct} \]

\[ V_c := f_v \cdot b \cdot d \]
0. B.2 SERVICEABILITY LIMIT STATE

\[ V_d \leq V_c + V_s \]
\[ V_d := R_{Atot} \]
\[ V_s := V_d - V_c \]
\[ A_{ss} := \frac{V_s}{0.9 \cdot d \cdot f_{st}} \]

\[ A_{ss} = 3.513 \times 10^{-4} \text{ m} \]

\( \phi \leq 100 \) may be appropriate

B.2 Serviceability limit state

Safety class
\[ \gamma_n := 1.2 \]

Safety factors
\[ \gamma_{ms} := 1.5 \]
\[ \gamma_{ms2} := 1.05 \]

Concrete
\[ f_{ctk} := 2.00 \times 10^6 \]
\[ E_{ck} := 33.0 \times 10^9 \]
\[ E_c := \frac{E_{ck}}{\gamma_{ms2} \gamma_n} \]
\[ f_{ct} := \frac{f_{ctk}}{\gamma_n \gamma_{ms}} \]
\[ \varepsilon_{cu} := 0.25 \times 10^{-3} \]

Reinforcement
\[ \phi := 0.02 \]
\[ E_{sk} := 200 \times 10^9 \]
\[ E_s := \frac{E_{sk}}{\gamma_{ms2} \gamma_n} \]

Creep factor
\[ \phi_{eff} := 2 \]
\[ \alpha_1 := \frac{E_s}{E_c (1 + \phi_{eff})} \]

Dimensions
\[ b := 1 \quad d := 0.7 \]

Tolerance
10mm

Concrete cover
\[ h_{täck} := (45 + 10) \times 10^{-3} \]

Total height
\[ d_{tot} := d - h_{täck} = \frac{\phi}{2} \times 63 \]
Chapter B. Calculations of reinforcement

\[ A_{\text{back}} := b \cdot d \]
\[ I_{\text{back}} := \frac{b \cdot d^3}{12} \]
\[ y_{\text{back}} := \frac{d}{2} \]

Life time category: L100
Class of exposure: XD3
Crack safety factor: \( \xi_w := 1.8 \)
Crack width: \( w_k := 0.15 \times 10^{-3} \)
Axial force: \( N_A := 0 \)
Moment: \( M_{\text{Atot}} := 233.1 \times 10^3 \)

**Checked if cracked**

Not cracked if \[ \sigma \leq f_{\text{cbt}} \]

\[ k_2 := 0.6 + \frac{0.4}{d_{\text{tot}}} \]
\[ 1 \leq k_2 \leq 1.4 \quad \text{ok} \]
\[ k_2 = 1.049 \]

\[ f_{\text{cbt}} := k_2 \frac{f_{\text{ct}}}{\xi_w} \]
\[ f_{\text{cbt}} = 6.472 \times 10^5 \]

\[ \sigma := \frac{-N_A}{A_{\text{back}}} + \frac{M_{\text{Atot}}}{I_{\text{back}}} \left( \frac{y_{\text{back}}}{2} \right) \]
\[ \sigma = 1.427 \times 10^6 \]

\( \sigma > f_{\text{cbt}} \quad \text{Cracked!} \)

**Iteration to find the area which fulfills the crack criterion**

\( \beta_1 := 0.5 \)
\( \kappa_1 := 0.8 \)
iteration :=
\[
\begin{align*}
    B_2 \text{ SERVICEABILITY LIMIT STATE} \\
    x_S & \leftarrow 0.3d_{\text{tot}} \\
    A_S & \leftarrow 1000 \times 10^{-6} \\
    \nu & \leftarrow 1 - \frac{\beta_1}{2.5K_1} \frac{f_{\text{cbt}}}{\sigma} \\
    & \text{if} \left( 1 - \frac{\beta_1}{2.5K_1} \frac{f_{\text{cbt}}}{\sigma} \right) \geq 0.4 \\
    & 0.4 \ \text{otherwise} \\
\end{align*}
\]

while \( A_S \)
\[
\begin{align*}
    \text{while } x_S \\
    z & \leftarrow d_{\text{tot}} - \frac{x_S}{3} \\
    T & \leftarrow \frac{M_{\text{Atot}} + |N_A|d_{\text{tot}}}{z} \\
    D & \leftarrow T - |N_A| \\
    \sigma_c & \leftarrow 2T \frac{x_S}{\sigma} \\
    \sigma_s & \leftarrow D \frac{A_s}{\sigma} \\
    x_{ny} & \leftarrow d_{\text{tot}} \frac{\alpha_1}{\sigma_s} \\
    & \text{if} \left| x_s - x_{ny} \right| \leq 0.01 \times 10^{-3} \\
    x_s & \leftarrow x_{ny} \\
    \rho_r & \leftarrow \frac{A_s}{2(h_\text{täck} + \frac{\phi}{2})} \\
    \varepsilon_2 & \leftarrow d_{\text{tot}} - \left( h_\text{täck} + \frac{\phi}{2} \right) - x_{ny} \\
    \varepsilon_1 & \leftarrow d_{\text{tot}} + \left[ \left( h_\text{täck} + \frac{\phi}{2} \right) - x_{ny} \right] \\
    \kappa_2 & \leftarrow 0.125 \left( 1 + \frac{\varepsilon_2}{\varepsilon_1} \right) \\
    s_{rm} & \leftarrow 50 \times 10^{-3} + K_1K_2 \frac{\phi}{\rho_r} \\
    w_{kny} & \leftarrow 1.7v \frac{\sigma_s}{E_{sk}} s_{rm} \\
    & \text{if} \left| w_{kny} - w_k \right| \leq 0.002 \times 10^{-3} \\
    A_S & \leftarrow A_S - 10 \times 10^{-6} \\
\end{align*}
\]

\[
\begin{pmatrix}
    x_{ny} \\
    A_S
\end{pmatrix}
\]
Chapter B. Calculations of reinforcement

\[ x := \text{iteration}_0 \quad \text{m} \quad \quad x = 92 \text{mm} \]

\[ A_s := \text{iteration}_1 \cdot \text{m}^2 \quad \quad A_s = 3910 \text{mm}^2 \]

\( \phi \geq 25 \) s125 is appropriate.

**Deflection**

Curvature for cracked concrete

\[ r_f := \frac{E_c \cdot I_{\text{back}}}{M_{\text{Atot}}} \quad \quad r_s := \frac{d}{\varepsilon_{\text{cu}}} \]

\[ \kappa_s := \frac{1}{r_f} + \frac{1}{r_s} \]

Curvature for uncracked concrete

\[ \Delta \varepsilon_{\text{cs}} := 1.25 \varepsilon_{\text{cu}} - 0.75 \varepsilon_{\text{cu}} \]

\[ r_{os} := \frac{d}{\Delta \varepsilon_{\text{cs}}} \quad \quad \kappa_{os} := \frac{1}{r_{os}} \]

\[ \kappa_s \geq \kappa_{os} \]

Check if

\[ \kappa_s = 6.685 \times 10^{-4} \]

\[ \kappa_{os} = 1.786 \times 10^{-4} \quad \quad \text{ok} \]