State duration based Monte Carlo Simulation model with independent failures for Distribution System Reliability Analysis

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Abstract – Increased uncertainty and greater financial stakes in distribution system reliability has brought focus on more efficient and powerful tools to evaluate the risks within distribution system operation. It is quite common that countries or regions that have re-regulated its electricity market apply more heavy handed regulatory approaches for controlling its remaining monopoly, i.e. the grid operations. Applying quality dependent revenue or price caps is the most common way to introduce an environment that somewhat resembles competitive conditions. Traditional methods to evaluate distribution system reliability are usually rather simple and therefore have many advantages due; however, a more complex risk environment for the Distribution System Operators (DSOs) calls for better methods to analyze risks. This paper introduces a basic Monte Carlo Simulation method for distribution system analysis that can be used as an initial simulation method and input to a more powerful and comprehensive Monte Carlo model. Furthermore, the paper also introduces an idea of how to apply a variance reduction technique that resembles importance sampling on the presented Monte Carlo method.

Index Terms — Monte Carlo Simulation, Distribution system reliability, variance reduction.

I. INTRODUCTION

With greater concern for reliability in distribution grids the need for efficient methods for analysing the uncertainty and risks in grid operations is eminent [1]. However, analysing risks using numerical reliability methods is rather complicated even for small systems, and the altitude level of the complexity grows rather quickly to cumbersome proportions as the system grows larger. Using empirical system performance data has the advantage that it is very easy to implement. However, the down-sides weight heavier; first of all, when using empirical data the worst-case scenario might yet be to come and will therefore probably not be included in the risk analysis, second, the empirical data is most often rather limited for distribution systems; hence, the analysis will be likewise, and third, some grids experience

continuous reinvestments making the empirical data not up to date. Therefore Monte Carlo simulation (MCS) techniques are most often used when conducting risk analyses. One of the great advantages with MCS is that, when used correctly, the stochastic behaviour of most systems can be comprehended in a satisfactory way. However, in order to achieve this, the designer of the model must know a great deal about the system at hand.

Distribution systems are typically working perfectly most of the time, and only for a small fraction of the time the supply is totally or partly interrupted. From a simulation viewpoint, this means that a large number of samples will be required to obtain sufficient samples of the rare supply interruption occasions. Consequently, the simulation time will be long even when using modern computers. One way to reduce the simulation time is to apply some kind of variance reduction technique.

Other research activities in this area, such as [2], [3] and [4], has focused of radial systems where one fault can occur at the time, second and third order incidents can be represented using aggregated parameters. Also, MCS techniques applied to distribution systems that include variance reduction are very rare.

This paper introduces a basic state duration based MCS method for distribution system analysis that can be combined with a variance reduction technique that resembles importance sampling. Important to stress in this situation is that importance measure denotes the system's states, and not the component's importance as one might expect. The method, as it is applied in this paper, should be seen as an initial simulation method that either can be used for simple analysis or as input data into a more powerful MCS model that comprehends load point (LP) correlations etc. Furthermore, the paper introduces an idea of how to apply a variance reduction technique resembling importance sampling on the presented MCS. The proposed method is demonstrated on a small test system.

II. BACKGROUND

Traditionally, the main focus within the field of reliability analysis of power systems has been put on the power generation side [5]. The reason for this is that failures in power generation can have devastating consequences both for the customers and the environment. Further, if we look at the area of reliability analysis of power grids, the main effort has been into transmission grids, and the reason for this is somewhat the

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same since the consequences of a single interruption is usually much greater than any interruptions in the distribution grid. However, interruptions in distribution grids are the main cause for customer interruptions [5]. Compared to transmission grids, the distribution grids are rather simple in their structure, which would enable rather accurate methods to be developed. This is however not the case, the methods used for distribution system analysis are rather simplified. The stochastic behaviour of technical equipment is at its toe within distribution system, which might explain the simple methods used, and when trying to explain the uncertainty of distribution grids one can speculate in issues such as [1]:

- Low priority, the Distribution System Operators (DSOs) have historically had small or no earnings by performing in-depth reliability analysis of their distribution grids.
- Maintenance, the schemes for maintenance applied by the DSOs have traditionally been so-called time-based or corrective maintenance schemes. Although, the time-based schemes are mainly based on experience, the underlying analysis can be considered rather simple. Just recently have the DSOs started to implement maintenance schemes based on the system's function and the components' reliability, i.e. RCM.
- Weather sensitive system, distribution grids that mainly consist of overhead lines are very sensitive to weather, hence most of the interruptions could usually be explained as weather interruptions and there is a risk of losing perspective of the interruptions not caused by weather, due to the relatively small amount.

Furthermore, the fact that there has been little money to be made by increasing the quality of supply has made the quality of the reliability data somewhat poor.

III. MONTE CARLO SIMULATION

The aim when developing this sampling approach was primarily to make the simulation time shorter. However, other concerns, originating from the downsides of the method previously applied, have influenced the development and application. First of all, the method should work well for meshed grids that may have a high degree of atomisation and second, the method should be such that it enables the implementation of some kind of variance reduction technique. The latter would enable the method to be even faster.

The idea is to have a continuous time sequence for each sample and to simulate one state and its corresponding duration for each sample. The state of each output, load point, is determined based on initial state of the components connected to the output and the duration of the output's state is determined by the up- and down time cycles of the components affecting the output.

Before getting into what might seem as complicated simulation algorithm, the MCS method will be explained using a small example. If we assume that we want to

apply the method on the small system containing three components and two load points, the system is illustrated in figure 1.

Figure 1. Small illustration system

The components' up-and down-time cycles are generated using uniformly distributed random numbers $u \sim [0,1]$, and the components' inverse distribution functions [6]. After sorting the time incidents when the components switch state chronologically, $T_1 - T_7$ in figure 2, the LPs' state are determined using their corresponding structure function $\phi(\mathbf{x}_i)$ [6]. After that the duration of the LPs' initial states are determined, in this case one up time and one down time sample is generated for LP_1 and LP_2 respectively, i.e., $UT_1 = T_7$ and $DT_2 = T_2$.

Figure 2. Time sequence for small illustration example

The simulation algorithm can be summarized as follows, and the corresponding flow chart is presented in figure 3.

- 1) Start of algorithm, $n = 1$.
- 2) $2 \times m$ random numbers between, $u \sim [0,1]$, m is equal to the number of components in the system.
- 3) Check if it is the first cycle, i.e. $n = 1$.
- 4) The calculations for $n = 1$ and $n \geq 3$ does only differ in the initial test on whether the component is working or not. In this step the time durations of UTs and DTs are generated and assigned to its corresponding component state. Both the UTs and DTs are assumed to be exponentially distributed.
- 5) First, sort the components' state switches in chronological order. Second, identify the time intervals during which the system is defined and in one single state.
- 6) Determine each LP's state during each time interval. The LP's state are determined using their structure function $\phi(\mathbf{x}_j)$ [6], where $\mathbf{x}_j$ is a state vector consisting of the components that affects the state of LP_j . The structure function return a digital state indicator for its output.
- 7) Check if the LPs' state have switched. Y => step 8, N => step 2.
- 8) A log keeps track of which LPs that have switched state at least one time.

- 9) Check if all LPs has switched state at least one time. Y => step 10, N => step 2.
- 10) Log each LP's initial state and the time of the next state switch. This is the simulation sample generated for each LP. Whether it is a UT or DT sample is determined by each LP's initial state. Go to step 1.
- 11) Check, $n < N$ => step 1, else stop. In the simulations conducted in this paper N is between $3 \times 10^5 - 5 \times 10^5$

Figure 3. Simulation algorithm

IV. VARIANCE REDUCTION

Previous research on variance reduction techniques applied on distribution system reliability analysis is quite limited. In this paper an idea of how to implement a method that resembles importance sampling to distribution system reliability analysis is presented and tested.

Distribution systems are typically duogeneous system, which means that the outputs have two states, up and down, and that one of the states is by far more common than the other, in this case the up-state. This means that in order to get a sufficient representation of the system's stochastic behavior when applying simple sampling, a significant number of samples are required, hence there is a risk of long simulation times and cumbersome data processing [7]. One way to overcome these negative

aspects of MCS is to apply some kind of variance reduction technique.

Even though variance reduction techniques are convenient to apply in most cases of reliability analysis of distribution systems, the most effective outcome is received when applied to rather safe systems, i.e. low component availability and/or redundant systems. In other words, what we want to know is how modifications of component data affects aggregated parameters and consequently aggregated output and performance indicators. To analyze this, we can start by inspecting the equations commonly used [5] to aggregate the failure rate and repair time for two parallel components, i and j , when we assume component independence:

$$\lambda_{ij} = \frac{\lambda_i \lambda_j (r_i + r_j)}{1 + \lambda_i r_i + \lambda_j r_j} \quad (1)$$

$$r_{ij} = \frac{r_i r_j}{r_i + r_j} \quad (2)$$

Where λ_i and r_i denotes the failure intensity and repair time for component i . Now, assume that we modify the component data in purpose of making the aggregated UT shorter and the DT longer, which would be achieved if the failure rate is increased by a factor x and the repair time is increased by a factor y . i.e.

$$\lambda_m = x \lambda \quad (3)$$

$$r_m = y r \quad (4)$$

where m indicates a modified parameter and furthermore, if we make the assumption that

$$\lambda_i \times r_i \ll 1,$$

and that x, y are such that this relationship is still valid for λ_m and r_m , we can simplify equation (1) by simply disregard the numerator. Hence, we have the following relationships:

$$\lambda_{mi} = x \lambda_i$$

$$r_{mi} = y r_i$$

$$\lambda_{mij} = x^2 y \lambda_{ij}$$

$$r_{mij} = y r_{ij}$$

Similarly, it can be shown that for three parallel components the resulting modification factor for the failure rate is $x^3 y^2$ and just simply y for the repair time. Moreover, the joint failure rate of independent components in series, or aggregated component cluster in series can be calculated as the sum of the failure rates:

$$\lambda_s = \sum_c \lambda_c \quad (5)$$

Where c is the set indicating the single and aggregated components belonging to the minimum cut set of sub set s . This means that the joint failure rate of a modified system can be calculated as:

$$\lambda_{ms} = \sum_c \lambda_{mc} \quad (6)$$

Furthermore, if we assume that all components have comparable failure rates, i.e., have approximately the same magnitude, we know that the failure rate of a single component is much larger than the aggregated failure rate of two or more parallel components. For instance, if λ_i

and λ_j are in the range of 10^{-5} [h⁻¹], than λ_{ij} is at least a factor 10^{-4} smaller [5]. This means that, given the fact that the factor $x \times y$ is not too large, for most distribution systems the following approximation would be valid:

$$\lambda_{ms} \approx x \lambda_s \quad (7)$$

$$r_{ms} = y r_s \quad (8)$$

The question now is how much can we modify the components reliability data without destroying the system's original properties, i.e., how should we choose x and y in order to get a good response in terms of shorter simulation times and still keep the accuracy of the output? The answer to this question will be outlined in the next section.

V. RESULT

In order to test the approximation presented in the previous chapter, reliability analysis simulation were conducted on a small test system, illustrated in figure 4, the component type data is presented in table 1.

Figure 4. Test system for MCS evaluation

TABLE 1. COMPONENT TYPE RELIABILITY DATA

		λ [h ⁻¹]	μ [h ⁻¹]	MTTF [h]	MTTR [h]	A
1	Breaker 40 kV	10^{-5}	0.125	10^5	8	0,99992
2	Breaker 10 kV	10^{-5}	0.125	10^5	8	0,99992
3	Transformer 40/10 kV	10^{-6}	0.0417	10^6	24	0,999976
4	Bus bar 40 kV	5×10^{-6}	0.1	2×10^5	10	0,99995
5	Bus bar 10 kV	10^{-5}	0.125	10^5	8	0,99992
6	Cable 10 kV	2×10^{-5}	0.2	5×10^4	5	0,9999

The test system is a rather small double fed distribution system with four load points. The system has a high degree of redundancy and is considered to be fully automated, which means that even though the operation is conducted as if it is a radial grid, the system can be considered to be meshed since the switching times are very short. The open circles on line L2 indicate that is not usually in use but can be switched in if necessary. The system's component data is presented in table 1. The data of the test system is fictitious but has been inspired from real values, as those in [8]

The simulations were conducted for five different values of x and y , i.e. $x, y = \{1, 2, 4, 10, 20\}$. Choosing $x, y = 1$ means that the inputs are unmodified, i.e., this choice corresponds to simple sampling. The MC-algorithm use to simulate the system reliability is the one described in figure 3. Furthermore, for all combinations of x and y 25 different seeds were used for the random number generator, this due to two reasons, to give a better idea of the estimation of the estimated parameters and reproducibility. To give some kind of instruments to compare simulations on a system level we introduce a variance coefficient, Cv . The variance coefficient used here is simply the sum of the square root of the variance of the expected expectation for each output data, ie down times and up times, divided with its expected value:

$$Cv = \sum_{i=1}^4 \left(\frac{\sqrt{Var[m_{UT_i}]} + \sqrt{Var[m_{DT_i}]} }{m_{UT_i} + m_{DT_i}} \right) \quad (9)$$

Where the variance of the estimated expectation is determined as:

$$Var[m_{Z_i}] = \frac{Var[Z_i]}{n_{Z_i}} \quad (10)$$

Where Z_i is either the simulated up or down time i , m is the mean value of Z_i and n_{Z_i} is the number of samples of Z_i . The total number of sample in each simulation is 300000.

Table 2 and 3 shows the simulation result for the LPs' up- and down times respectively. The results are from the previous specified configurations of x and y , from simulation for 10 different seeds to the random number generator. The label "interval" in table 2 and 3 notes the percentile interval, $I\%$, for the maximum and minimum value:

$$I\% = \frac{m_{Z \max} - m_{Z \min}}{m_{Z \text{average}}}$$

where Z_i is either the simulated up- or down time. The "deviation" label in denotes the percentile deviant of the average simulated value from the numerically calculated value. What we can see in table 2 and 3 is that first of all that the up time estimation is much better than the down time estimation. This is due the relatively small number of down time sample. Second, low values of x and y result in significant deviations for the down time. This can also be explained by the few samples generated. Third, high values of x and y result in significant deviation of the up times. The reason for this is the simplifications described in the previous section; for large numbers of x and y equation (7) is no longer valid.

TABLE 2. SIMULATED INTERVAL AND DEVIATION FOR THE LOAD POINTS' UP TIMES

x	y		Up Time			
			LP 1	LP 2	LP 3	LP 4
1	1	Interval	0,71%	0,82%	0,55%	0,49%
		Deviation	-0,07%	-0,07%	-0,13%	-0,05%
2	1	Interval	0,62%	0,63%	0,54%	0,49%
		Deviation	-0,18%	-0,12%	0,10%	0,00%
4	1	Interval	0,76%	0,66%	0,65%	0,39%
		Deviation	-0,42%	-0,40%	0,00%	-0,10%
10	1	Interval	0,52%	0,80%	0,61%	0,35%
		Deviation	-1,15%	-1,17%	-0,12%	-0,18%
20	1	Interval	0,67%	0,47%	0,24%	0,49%
		Deviation	-2,19%	-2,15%	-0,22%	-0,20%
1	2	Interval	0,48%	0,56%	0,42%	0,79%
		Deviation	-0,01%	0,04%	0,05%	0,07%
2	2	Interval	0,48%	0,53%	0,67%	0,36%
		Deviation	-0,35%	-0,38%	-0,09%	-0,02%
4	2	Interval	0,45%	0,72%	0,76%	0,42%
		Deviation	-0,68%	-0,67%	-0,03%	0,01%
10	2	Interval	0,50%	0,72%	0,66%	0,52%
		Deviation	-2,31%	-2,30%	-0,19%	-0,33%
20	2	Interval	0,57%	0,54%	0,64%	0,40%
		Deviation	-4,46%	-4,51%	-0,42%	-0,59%
1	4	Interval	0,60%	0,66%	0,58%	0,55%
		Deviation	-0,42%	-0,41%	0,04%	-0,04%
2	4	Interval	0,65%	0,48%	0,56%	0,70%
		Deviation	-0,95%	-0,90%	-0,11%	-0,09%
4	4	Interval	0,65%	0,56%	0,32%	0,66%
		Deviation	-1,77%	-1,73%	-0,15%	-0,20%
10	4	Interval	0,61%	0,86%	0,87%	0,71%
		Deviation	-4,35%	-4,40%	-0,40%	-0,48%
20	4	Interval	0,40%	0,60%	0,42%	0,56%
		Deviation	-8,89%	-8,86%	-0,79%	-0,96%
1	10	Interval	0,68%	0,52%	0,66%	0,30%
		Deviation	-0,99%	-1,07%	-0,16%	-0,10%
2	10	Interval	0,67%	0,45%	0,53%	0,52%
		Deviation	-2,31%	-2,30%	-0,16%	-0,24%
4	10	Interval	0,45%	0,65%	0,63%	0,55%
		Deviation	-4,45%	-4,53%	-0,48%	-0,57%
10	10	Interval	0,84%	0,91%	0,51%	0,50%
		Deviation	-10,96%	-10,92%	-0,85%	-1,23%
20	10	Interval	0,47%	0,51%	0,87%	0,47%
		Deviation	-20,50%	-20,49%	-1,87%	-2,64%
1	20	Interval	0,57%	0,43%	0,51%	0,68%
		Deviation	-2,21%	-2,13%	-0,26%	-0,31%
2	20	Interval	0,84%	0,53%	0,49%	0,49%
		Deviation	-4,50%	-4,54%	-0,37%	-0,49%
4	20	Interval	0,57%	0,43%	0,62%	0,53%
		Deviation	-8,77%	-8,79%	-0,79%	-1,05%
10	20	Interval	0,55%	0,63%	0,60%	0,35%
		Deviation	-20,69%	-20,75%	-1,98%	-2,63%
20	20	Interval	0,39%	0,42%	0,63%	0,81%
		Deviation	-52,72%	-52,75%	-14,94%	-16,32%

TABLE 3. SIMULATED INTERVAL AND DEVIATION FOR THE LOAD POINTS' DOWN TIMES

x	y		Down Time			
			LP 1	LP 2	LP 3	LP 4
1	1	Interval	38,61%	49,19%	34,23%	26,33%
		Deviation	-0,74%	-2,42%	2,99%	4,74%
2	1	Interval	20,21%	25,88%	19,24%	18,59%
		Deviation	3,88%	-2,68%	6,18%	2,53%
4	1	Interval	23,72%	22,39%	15,54%	16,32%
		Deviation	-0,21%	0,60%	4,14%	4,65%
10	1	Interval	14,84%	8,64%	7,82%	10,07%
		Deviation	2,53%	0,60%	5,98%	4,21%
20	1	Interval	8,39%	8,46%	5,43%	7,14%
		Deviation	1,77%	1,63%	7,23%	5,64%
1	2	Interval	57,00%	50,39%	23,57%	36,75%
		Deviation	0,57%	-0,14%	5,08%	2,16%
2	2	Interval	18,81%	14,95%	8,95%	12,77%
		Deviation	-0,56%	-0,59%	6,59%	3,70%
4	2	Interval	10,68%	12,79%	9,49%	10,05%
		Deviation	0,94%	2,12%	7,08%	6,26%
10	2	Interval	9,49%	9,90%	5,56%	9,84%
		Deviation	0,99%	-1,11%	7,06%	6,17%
20	2	Interval	3,99%	2,99%	4,61%	3,52%
		Deviation	0,00%	0,29%	7,53%	5,27%
1	4	Interval	20,22%	15,50%	12,83%	13,96%
		Deviation	-0,06%	-0,97%	3,88%	4,46%
2	4	Interval	17,27%	13,95%	7,56%	8,01%
		Deviation	5,21%	1,26%	6,72%	5,47%
4	4	Interval	11,88%	11,23%	7,90%	6,48%
		Deviation	0,74%	1,33%	7,41%	4,85%
10	4	Interval	9,31%	7,23%	2,25%	3,01%
		Deviation	1,15%	1,11%	7,68%	6,41%
20	4	Interval	5,25%	3,27%	3,65%	3,18%
		Deviation	-0,57%	-0,95%	8,35%	6,80%
1	10	Interval	9,97%	9,17%	7,95%	8,20%
		Deviation	0,89%	0,37%	5,39%	5,75%
2	10	Interval	6,18%	9,60%	7,65%	8,67%
		Deviation	1,51%	0,98%	6,98%	6,68%
4	10	Interval	7,17%	4,95%	6,73%	3,64%
		Deviation	-0,20%	0,45%	6,98%	5,27%
10	10	Interval	4,75%	4,14%	1,79%	4,84%
		Deviation	-0,58%	-0,33%	9,30%	6,76%
20	10	Interval	2,71%	2,21%	1,12%	2,06%
		Deviation	-1,02%	-0,69%	11,76%	9,16%
1	20	Interval	5,92%	7,43%	4,26%	8,78%
		Deviation	1,41%	1,69%	7,36%	6,97%
2	20	Interval	5,96%	3,54%	3,82%	4,69%
		Deviation	0,23%	0,89%	7,53%	5,78%
4	20	Interval	3,94%	5,32%	2,90%	3,58%
		Deviation	0,02%	-0,33%	8,18%	6,38%
10	20	Interval	2,83%	2,91%	3,22%	2,05%
		Deviation	-2,06%	-1,67%	11,35%	8,60%
20	20	Interval	1,95%	2,26%	1,52%	1,66%
		Deviation	-13,67%	-13,61%	7,23%	3,19%

Table 4 shows the average number of down time samples generated for each configuration of x and y , the total number of samples in each simulation is 300 000. Also in table 4 is the average systems' variation coefficient, C_v . First of all we can notice that small number of down time samples generated for small numbers of x and y . What we can see is that, C_v , seems the decrease the more down time samples that are generated, which is quite natural looking at equation (9). This does not however mean that result is sufficiently accurate compared to the original system. This is due to the fact that large values of x and y , changes the system properties and consequently the expected value of the simulation so much that we cannot use them. Thus, one have to combine the content of table 2, 3 and 4 to find which configuration of x and y that is optimal for this particular system.

TABLE 4. NUMBER OF DOWN TIME SAMPLES AND VARIANCE COEFFICIENT, C_v , FOR THE CHOSEN CONFIGURATIONS OF x AND y .

x	y	LP1	LP2	LP3	LP4	C_v
1	1	62	67	150	116	0,42
2	1	125	120	289	229	0,32
4	1	249	257	578	469	0,22
10	1	640	641	1483	1178	0,15
20	1	1278	1282	2971	2352	0,11
1	2	125	126	305	231	0,32
2	2	247	249	586	470	0,22
4	2	515	516	1198	938	0,16
10	2	1281	1284	2925	2340	0,10
20	2	2545	2532	5822	4634	0,08
1	4	248	250	586	465	0,22
2	4	505	510	1179	935	0,17
4	4	1005	1009	2365	1873	0,12
10	4	2561	2560	5811	4664	0,08
20	4	5171	5173	11519	9253	0,06
1	10	641	634	1487	1169	0,15
2	10	1261	1266	2909	2312	0,11
4	10	2518	2525	5832	4625	0,08
10	10	6501	6517	14343	11530	0,05
20	10	13291	13288	28098	22699	0,04
1	20	1263	1247	2916	2328	0,11
2	20	2506	2541	5784	4648	0,08
4	20	5127	5119	11504	9230	0,06
10	20	13353	13299	28220	22747	0,04
20	20	27633	27629	53827	43912	0,03

Table 2 - 4 show that the model works for mean values, and the variance coefficient decreases for some values of x and y with maintained mean values and decreased simulation deviation. Converging mean value is however not the same as working all together since the mean value has no information about distribution of the variable. So, in order to analyze how well the distribution of the simple sampled system align with simulations of system that have been modified the distribution functions, $F_i(Z_i)$, for the up- and down times must be compared. Z denotes any up or down time output i . From table 2 - 4 three cases were selected, $\{x,y\} = \{1,1\}, \{4,4\}, \{10,2\}$. The number of samples for each simulation, N , were set to 50 000 and

500 000, in order to compare the effect of variance reduction. Figure 5 – 8 show the simulated distribution function of the down times for the three sets of x and y . Over all, the simulated down time distribution functions seems to align well for the simple sampled system and its importance sampled counterpart. The simulation were the total number of sample is 500 000 show better alignment since more samples of the simple sampled down time is generated. However, the variance reduced samples show sufficient representation even the total number of samples were as few as 50 000. The distribution functions for the down times of LP 1 and LP2 differ to some degree even when the total number of samples were 500 000, but that deviation can be linked to the rather few samples generated in for the simple sample system for those particular cases.

Figure 5. Simulated distribution function, for DT of LP1, $F_1(DT_1)$.

Figure 6. Simulated distribution function, for DT of LP2, $F_2(DT_2)$.

Figure 7. Simulated distribution function, for DT of LP_3 , $F_3(DT_3)$.

Figure 8. Simulated distribution function, for DT of LP_4 , $F_4(DT_4)$.

Figure 9 – 12 show the simulated distribution function of the up times for the three sets of x and y . The alignment for the up time distribution function is very good and the three lines look like one for both the simulated series.

Figure 9. Simulated distribution function, for UT of LP_1 , $F_5(UT_1)$.

Figure 10. Simulated distribution function, for UT of LP_2 , $F_6(DT_2)$.

Figure 11. Simulated distribution function, for UT of LP_3 , $F_7(UT_3)$.

Figure 12. Simulated distribution function, for UT of LP_4 , $F_8(UT_4)$.

VI. CONCLUSIONS AND DISCUSSION

Based on the results presented in the previous chapter the following conclusions are made:

- State-duration based MCS are relatively easy to implement and to overview.
- The simulation technique enables the person analyzing the data to complement and to modify the input parameters in order to achieve the required degree of accuracy.
- The simulation algorithm is quick enough so that the

output data can be used as inputs to a more powerful and extensive MCS model, in which features such as LP correlations can be included, such as copula models.

- In order to apply the technique of simplified importance sampling there are a number of system-parameters that have to be met such as component homogeneity, small failure rates, rather short repair times, mainly serial connections, to make it work.
- If these system-parameters are met, small modifications of the components' input data can be made without compromising the specific stochastic system behavior.
- By modifying the input data within the tolerance levels significant simulation time can be saved, for this particular system the total number of samples was reduced to 1/10.
- The general strategy to set the modification parameters, x and y , is to numerically study the system and to make simple sensitivity analyses of the selected system, maybe even simulate a small part of the system and check the accuracy.

As mentioned earlier, simulating the load points' up and down time distribution is a appropriate way the achieve input data for a more powerful MCS model, such as a copula model were complicated feature such as load point dependence can be comprehended. Furthermore, the introduced variance reduction technique modifies the system parameters so that the only way undo the changes would be to modify the components data back before it is put into context of the system. This would however not lead to any improvement and nothing would have been won. However, when considering the basic design of most distribution systems, its operation scheme and consider the time saved by applying the introduced sampling method one can conclude that in most cases these modifications are quite ok, if not perfectly OK. One should however be very careful when analyzing systems that are meshed in its main structure. What one can do in such cases is to divide the system into sub system and use different sampling schemes for the different parts.

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