Long Span Flexible Metal Culverts
Ultimate Load Calculations

by

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Preface

I had never heard of long span flexible metal culverts when Lars Pettersson asked me if I was interested in doing my thesis on them. It didn’t take long for him to convince me that these were interesting types of bridges and that there are good chances that they will be more common in the future. I also got the impression right away that this was a man I could learn a lot from, and that he would give me all the support I needed with the work on this thesis. This proved to be right and I owe him a lot for his commitment and eagerness to help.

I also owe a lot of gratitude to Lars Hansing at ViaCon AB. He has been very helpful whenever I needed something that he could help me with. Another big thanks to ViaCon AB for supporting me with a scholarship during the work with my thesis.

Thanks to Håkan Sundquist for your good advise about the comparison of calculations to the field tests.

I want to thank my colleagues at Skanska Teknik AB for your help and your interest in my thesis. Thanks for the flexibility you’ve shown and for making sure I get around to finishing my masters degree.

Thanks to my family for supporting and encouraging me through all my years of studies. Thank you for letting me do things my way.

Stockholm, April 2007

Johan Hirvi
Abstract

Flexible culverts have been used for more than a hundred years, mostly as water conduits. They can also be used as bridges. For a long time, there were no real models for calculating their strength. In recent years some design methods have been presented. In Sweden they are designed according to “Design of long span flexible metal culverts” by Lars Pettersson and Håkan Sundquist.

The strength of flexible culverts comes through interaction between the culvert and the surrounding soil. The weight of the backfill suppresses the deformations of the culvert. The most common materials for flexible culverts are steel and PVC.

The aim of this thesis was to make a calculation routine in MathCAD according to the third edition of the handbook by Pettersson and Sundquist. This routine would then be used to perform calculations that could be compared to results from full-scale field tests. The effect of some important parameters in the calculations was also to be studied.

A literature review was done about the historical development of flexible culverts and the methods for designing them. Test reports were also studied to be compared to calculations.

The results are presented in a number of graphs and tables, which could be of use for designers of flexible metal culverts, both for preliminary and for detailed design. This report also contains an example calculation, which could be useful if you are not quite sure about what is meant in the handbook by Pettersson and Sundquist.
Sammanfattning av examensarbete ”Rörbroar - Utveckling av dator-program för analys av brottlaster” (Summary in Swedish)

Flexibla kulvertar har använts i över hundra år, främst för att leda vatten. De kan även användas som broar och kallas då rörbroar. Det har länge saknats riktiga beräkningsmodeller för dessa. På senare år har dock beräkningsmetoder presenterats. I Sverige dimensioneras de enligt handboken ”Dimensionering av rörbroar” av Lars Pettersson och Håkan Sundquist.

Rörbroar får sin styrka genom interaktion mellan kulverten och den omgivande jorden. Tyngden av jorden håller tillbaka utböjningar från kulverten. De vanligaste materialen för flexibla kulvertar är stål och PVC.

Målet med examensarbetet var att göra en beräkningsrutin i MathCAD som följer den tredje utgåva av ovan nämnda handbok. Denna rutin skulle sedan användas för att göra beräkningar som skulle kunna jämföras med resultat från fullskaleförsök. Dessutom skulle inverkan av några viktiga parametrar i beräkningarna studeras.

En litteraturstudie genomfördes på den historiska utvecklingen av rörbroar och dess dimensioneringsmetoder. Försöksrapporter har också studerats för att jämföras med beräkningar.

Resultaten presenteras med ett antal grafier och tabeller vilka kan vara till nytta för konstruktörer som ska projektera och/eller dimensionera rörbroar. Denna rapport innehåller även ett beräkningsexempel som kan vara till hjälp om man inte är riktigt säker på vad som menas i handboken.
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Nomenclature

Greek letters

\( \alpha \) Angle defining the cross section of the corrugation [-]

\( \varphi_{\gamma_{s.s}} \) Partial coefficient for the soil in the serviceability limit state [-]

\( \varphi_{\gamma_{s,u}} \) Partial coefficient for the soil in the ultimate limit state [-]

\( \varphi_{\gamma_{t.f}} \) Partial coefficient for the traffic load in the fatigue limit state [-]

\( \varphi_{\gamma_{t.f}} \) Partial coefficient for railway load in the fatigue limit state [-]

\( \varphi_{\gamma_{t.s}} \) Partial coefficient for the traffic load in the serviceability limit state [-]

\( \varphi_{\gamma_{t.s,bw}} \) Partial coefficient for railway load in the serviceability limit state [-]

\( \varphi_{\gamma_{t.u}} \) Partial coefficient for the traffic load in the ultimate limit state [-]

\( \varphi_{\gamma_{t.u}} \) Partial coefficient for railway load in the ultimate limit state [-]

\( \varphi_{t} \) Reduction factor for pretension of the bolted connections [-]

\( \gamma_{m} \) Partial coefficient for the material uncertainty of the soil [-]

\( \gamma_{n} \) Partial coefficient for the safety class [-]

\( \eta_{j} \) Calculation parameter [-]

\( \eta_{m} \) Stiffness parameter used in conjunction with judging of the stiffness during installation [m/kN]

\( \lambda_{f} \) Flexibility number which indicates the relative relationship between the stiffness of the pipe and that of the surrounding soil [-]

\( \mu \) Calculation parameter [-]

\( \rho_{1} \) Density for the material to the side of the culvert [kN/m³]

\( \rho_{2} \) Density for the material below the culvert [kN/m³]

\( \rho_{cv} \) Average density for the material within \( h_{c} \) [kN/m³]

\( \rho_{opt} \) Average optimal density for the material within \( h_{c} \) [kN/m³]

\( \xi \) Calculation parameter [-]
### Roman upper case letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>Axis load for the type loads</td>
<td>[kN]</td>
</tr>
<tr>
<td>A_{s,b}</td>
<td>Cross sectional area of the bolts</td>
<td>[m²]</td>
</tr>
<tr>
<td>B</td>
<td>Axis load for the type loads</td>
<td>[kN]</td>
</tr>
<tr>
<td>C</td>
<td>Characteristic structural strength for $2 \times 10^6$ stress alternations</td>
<td>[-]</td>
</tr>
<tr>
<td>D</td>
<td>Diameter of span</td>
<td>[m]</td>
</tr>
<tr>
<td>E</td>
<td>Young’s Modulus</td>
<td>[MPa]</td>
</tr>
<tr>
<td>E_s</td>
<td>Tangent modulus of the soil material in the structural backfill</td>
<td>[MPa]</td>
</tr>
<tr>
<td>H</td>
<td>Vertical distance between the crown of the pipe and the height where the culvert has its largest width</td>
<td>[m]</td>
</tr>
<tr>
<td>N_{cr}</td>
<td>Buckling load for a buried pipe</td>
<td>[kN/m]</td>
</tr>
<tr>
<td>N_s.surr</td>
<td>Normal force in the culvert from the soil up to the crown of the culvert</td>
<td>[kN/m]</td>
</tr>
<tr>
<td>P</td>
<td>Vector in the routine containing the concentrated loads for every load case</td>
<td>[-]</td>
</tr>
<tr>
<td>R</td>
<td>Radius of a circular culvert</td>
<td>[m]</td>
</tr>
<tr>
<td>R</td>
<td>Radii of curvature for the corrugation (used in the MathCAD Routine)</td>
<td>[m]</td>
</tr>
<tr>
<td>R_b</td>
<td>Bottom radius</td>
<td>[m]</td>
</tr>
<tr>
<td>R_c</td>
<td>Corner radius</td>
<td>[m]</td>
</tr>
<tr>
<td>R_{check}</td>
<td>A vector in the routine containing the radii’s to be checked in the lower part of the pipe</td>
<td>[m]</td>
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<tr>
<td>R_P</td>
<td>Relative degree of compaction of the soil according to Standard Proctor</td>
<td>[-]</td>
</tr>
<tr>
<td>R_s</td>
<td>Side radius</td>
<td>[m]</td>
</tr>
<tr>
<td>R_t</td>
<td>Top radius</td>
<td>[m]</td>
</tr>
<tr>
<td>S_{ar}</td>
<td>Reduction factor for load from the overburden</td>
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### Roman lower case letters

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<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tr>
<td>a</td>
<td>Distance between parallel rows of bolts</td>
<td>[m]</td>
</tr>
<tr>
<td>c</td>
<td>Wave length of the corrugation</td>
<td>[m]</td>
</tr>
<tr>
<td>c&lt;sub&gt;val&lt;/sub&gt;</td>
<td>Wave length of the corrugation (used in the MathCAD Routine)</td>
<td>[m]</td>
</tr>
<tr>
<td>d</td>
<td>Diameter of the bolts</td>
<td>[m]</td>
</tr>
<tr>
<td>d&lt;sub&gt;n&lt;/sub&gt;</td>
<td>Particle size which represents the passed weight of n % on a gradation curve</td>
<td>[mm]</td>
</tr>
<tr>
<td>e&lt;sub&gt;1&lt;/sub&gt;</td>
<td>Distance between parallel rows of bolts in BSK 99 (called a in the routine)</td>
<td>[m]</td>
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<tr>
<td>fatigue</td>
<td>Vector in the routine containing the positions in the load case vectors for the fatigue load cases</td>
<td>[-]</td>
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<tr>
<td>f&lt;sub&gt;buk&lt;/sub&gt;</td>
<td>Plastic yield stress limit for the bolts</td>
<td>[MPa]</td>
</tr>
<tr>
<td>f&lt;sub&gt;rk&lt;/sub&gt;</td>
<td>Characteristic fatigue stress capacity</td>
<td>[MPa]</td>
</tr>
<tr>
<td>f&lt;sub&gt;uk&lt;/sub&gt;</td>
<td>Plastic yield stress limit</td>
<td>[MPa]</td>
</tr>
<tr>
<td>f&lt;sub&gt;yk&lt;/sub&gt;</td>
<td>Elastic yield stress limit</td>
<td>[MPa]</td>
</tr>
<tr>
<td>h</td>
<td>Height of the culvert</td>
<td>[m]</td>
</tr>
<tr>
<td>h&lt;sub&gt;c&lt;/sub&gt;</td>
<td>Height of cover</td>
<td>[m]</td>
</tr>
<tr>
<td>h&lt;sub&gt;c.red&lt;/sub&gt;</td>
<td>Reduced value of the height of cover</td>
<td>[m]</td>
</tr>
<tr>
<td>h&lt;sub&gt;corr&lt;/sub&gt;</td>
<td>Height of the corrugation</td>
<td>[m]</td>
</tr>
<tr>
<td>load&lt;sub&gt;xy&lt;/sub&gt;</td>
<td>Vector in the routine containing the positions and areas for the concentrated loads for every load case</td>
<td>[-]</td>
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<tr>
<td>m&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Length of the straight part of the corrugation</td>
<td>[m]</td>
</tr>
<tr>
<td>n</td>
<td>Number of bolts per meter width of the culvert</td>
<td>[-]</td>
</tr>
<tr>
<td>n&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Number of stress alternations</td>
<td>[-]</td>
</tr>
<tr>
<td>p&lt;sub&gt;traffic&lt;/sub&gt;</td>
<td>Equivalent line load calculated from the traffic load</td>
<td>[kN/m]</td>
</tr>
<tr>
<td>q</td>
<td>Distributed load in the handbook</td>
<td>[kN/m]</td>
</tr>
<tr>
<td>q&lt;sub&gt;b&lt;/sub&gt;</td>
<td>Vector in the routine containing the distributed loads that should be calculated with the numerical method for every load case</td>
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<td>q&lt;sub&gt;handbook&lt;/sub&gt;</td>
<td>Vector in the routine containing the distributed loads that should be calculated with the method in the handbook for every load case</td>
<td>[kN/m]</td>
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<td>Description</td>
<td>Unit</td>
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<td>$q_i$</td>
<td>Vector in the routine containing the distributed loads with an infinite length that should be calculated with the numerical method for every load case</td>
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<tr>
<td>$t$</td>
<td>Thickness of the steel plates</td>
<td>[m]</td>
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<tr>
<td>$x_{\text{divisions}}$</td>
<td>Variable in the routine specifying the number of original calculation points in the x-direction for the numerical method</td>
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<tr>
<td>$y_{\text{divisions}}$</td>
<td>Variable in the routine specifying the number of original calculation points in the x-direction for the numerical method</td>
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PART 1

*Introduction and Literature Review*
1 Introduction

1.1 Background
Long span flexible metal culverts are accepted for use within field of the Swedish Road Administration, (Vägverket), and the Swedish Rail Administration, (Banverket). The design should be done according to "Design of long span flexible metal culverts", Rapport 58, Brobyggnad 2000, 2nd edition, 2002. Lars Pettersson, Skanska, and Håkan Sundquist, KTH, are the authors of that report. As they were revising the report for the third edition I was approached by Lars Pettersson to make a routine in MathCAD and compare the calculations to a number of full-scale field tests carried out in Sweden.

1.2 Aims and Scope
The first objective of this thesis is to develop a routine in MathCAD for the design of long span flexible metal culverts according to the report by Pettersson & Sundquist. The routine should be easy to use with the help of the mentioned report. It should be as automatic as possible without restricting its usefulness. There should be no limitation to the number of loads that can be applied. The routine should suggest a value or a choice of values for the parameters used in the formulae, but the user should also be able to replace the suggested values with his or her own data.

Another aim of this report is to investigate how well the theory describes the reality. In this case it means modeling bridges that have been field-tested and comparing the results from the calculations to the results measured in the field tests. A study of parameters in the design method will also be performed.

The calculation routine will not cover anything that is not covered in the report by Pettersson & Sundquist. It will not cover the design of bridges for railways. The studied bridges will be the ones in full-scale field tests performed in Sweden, where a failure load has been determined as an axis force. Five such tests have been performed in the so-called Enköping test series and four in the so-called Järpås test series. In this report results from three of the Enköping- and two of the Järpås tests are used.

1.3 Software Used
All the calculations for this master’s thesis have been made in MathCAD 13.1 from Mathsoft Engineering & Education, Inc. It is a powerful math program with an interface similar to a word processor. It is capable of performing numeric calculations as well as find symbolic solutions and to present them in a nice way. Another strength is its possibility to handle units. It is possible to link it to other applications, like using spread sheets from excel inside MathCAD.

AutoDesk Architectural Desktop (ADT) 2006 have been used to create figures used in MathCAD and in this report. It is very easy to copy figures from ADT to Word or MathCAD.

Microsoft Excel 2000 has been used to create the graphs and tables in this report. The results from the MathCAD calculations have simply been saved in excel sheets where the final presentations have been made.

Microsoft Word 2000 has been used to write this report.
2 Long Span Flexible Metal Culverts

A culvert is defined as a conduit used to enclose a flowing body of water. This is also the most common use for culverts – providing a pathway for an open stream where it meets an artificial barrier such as a road. But culverts are also being used as bridges or tunnels, when a road passes under a railway for example. Culverts can be made of many different materials, but steel, polyvinyl chloride (PVC) and concrete are the most common. 

Flexible metal culverts refer to corrugated sheet metal forming a pipe or an arch, which achieves its load carrying capacity by interaction with the surrounding soil. When a load is applied at the top of the culvert, the top sinks and the sides move outwards. If the pressure from the surrounding soil is large enough, this movement is restricted, and the culvert walls will be subjected to a compressive thrust (ConnDOT Drainage Manual 2000). Long span flexible metal culverts means culverts with a span larger than 2 m, which is the span which defines a bridge in Sweden (Pettersson and Sundquist 2003). The largest span known to the author on a bridge of this type is about 24 m, built in Canada.

![Figure 2:1](http://encyclopedia.thefreedictionary.com)

**Figure 2:1** Vertical loads deform the structure. (ConnDOT Drainage Manual 2000)

2.1 History

Flexible culverts have been used since the end of the 19th century. In the beginning they were built in factories as corrugated steel pipes, and over the years a good knowledge of safe structures for spans up to approximately 2400 mm was achieved. In 1931 a structural plate pipe with larger corrugation was developed. It was assembled at site, and made it possible to increase the diameters. Such structures have been built with diameters up to 8 m and arch spans up to 18 m. (AISI 2002)

The earliest “strength tests” performed on corrugated steel pipes were various loads applied to unburied pipes. Later laboratory “sand box” and hydraulic tests were made. At Iowa State College and the University of North Carolina fill loads were measured on buried pipe and on their foundations in 1913. In 1923, large-scale field tests were run on the Illinois Central Railroad by American Railway Engineering Association (AREA), measuring dead loads. That was the first time it was shown that flexible culverts and the compacted soil could work as a composite structure. Their measurements showed that the culvert only carried 60 % of the dead load, and the rest was carried by the soil. The first “Handbook of Drainage and
Construction Products” was published by ARMC O, the American Rolling Mill Company, in 1941. (AISI 1971, http://contech-cpi.com)

The concept of a thin compression ring supported by soil pressures was introduced in the 1940’s. This provided a path to rational design criteria. Further research was made at Utah State University 1967 to 1970. This led to a more refined design approach with greater accuracy. (AISI 1971)

The D.B. culvert, a structural plate pipe under almost 61 m of fill, was a research project by the State of California in 1975. It was drastically under designed with a much lower capacity than the load effect and expected to fail, which it did. The performance data it provided was important for the development and verification of new design tools. (AISI 2002, Bacher and Kirkland 1985)

Since then, design procedures have been developed with the help of finite element methods. The Soil-Culvert Interaction (SCI) Method, by J. M. Duncan, is one of them. It uses design graphs and formulas based on finite element analyses (FEA). (AISI 2002)

In Sweden, from the mid 1980’s design of long span flexible culverts was done with help of simple diagrams and standard drawings, covering two types of profiles – low-rise culverts and vertical ellipses. They were made for spans up to about 5 m.

About the same time, work started to develop a design method that would be able to consider different soil materials and different heights of cover as well as larger spans. Several full-scale tests were performed, and the design method was presented in the year 2000. The Swedish Road Administration, (Vägverket), and the National Rail Administration, (Banverket), accepted the method in their codes Bro 2002 and BV Bro respectively. After the introduction of the box culvert to the Swedish market in 2002, full-scale tests were made, which led to the 3rd edition of the handbook, published in 2006.
2.2 Culvert Profiles

There are a number of different profiles for this type of culverts. The 3rd edition of the handbook by Pettersson and Sundquist, from here on referred to as the handbook, covers seven different profiles, which are illustrated in the following figures.

**Figure 2:3** A. Circular pipe of constant radii.

**Figure 2:4** B. Arch with a single top radius.
Figure 2:5 C. Horizontal ellipse.

Figure 2:6 D. Vertical ellipse.
Figure 2:7 E. Pipe arches and low-rise culverts (with three radii’s).

Figure 2:8 F. Arches comprised of metal sheets with three different radii.

Figure 2:9 G. Box culvert.
The box culvert can also be designed with reinforcement sheets in the crown or in the corners according to the following figure.

![Figure 2:10 Box culvert with reinforcement steel plate sheets.]

2.3 Pros and Cons for Flexible Metal Culverts

There are many factors to consider when choosing the design for a bridge. Obviously there are economical considerations, short term and long term. The production time, including design and construction, could be a deciding factor, need for maintenance should not be neglected. Flexibility for future redesign and, depending on the location of the bridge, architectonical values might be important.

2.3.1 Advantages

<table>
<thead>
<tr>
<th>Feature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production time</td>
<td>Less design and construction time compared to other types of bridges.</td>
</tr>
<tr>
<td>Economy</td>
<td>An economic alternative.</td>
</tr>
<tr>
<td>Continuity</td>
<td>Roads can be built continuously over the bridge since the metal culvert has a soil overfill. No need for bridge approaches which are sensitive for settlements.</td>
</tr>
<tr>
<td>Flexibility</td>
<td>The crossings can easily be widened if the culvert is extended at the ends.</td>
</tr>
<tr>
<td>Appearance</td>
<td>A very natural look can be achieved because of the soil overfill.</td>
</tr>
<tr>
<td>Maintenance</td>
<td>Very little maintenance is required.</td>
</tr>
</tbody>
</table>

2.3.2 Disadvantages

<table>
<thead>
<tr>
<th>Feature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Span width</td>
<td>The maximum span width, although undefined, is relatively low. The largest span in the world is 24 m.</td>
</tr>
<tr>
<td>Height</td>
<td>The construction height, the vertical distance between the road within the culvert and the road above the culvert, may be bigger than for other types of bridges.</td>
</tr>
<tr>
<td>Appearance</td>
<td>They might be considered less beautiful than ordinary bridges.</td>
</tr>
</tbody>
</table>

3 Soil

As previously mentioned, the soil is very important for flexible metal culverts. Without it, in some cases the culverts are just strong enough to carry the load of the people putting the corrugated steel plates together. Still, the finished bridge with the soil above and around the culvert is able to support a railway or a road with heavy traffic.
It is very important that the soil properties are chosen carefully. It is not necessary to use the same backfill material all around the culvert, but the compaction of each layer of backfill is crucial. The capacity of the bridge depends heavily on the interaction between culvert and soil. (Pettersson and Sundquist 2006)

During the process of backfilling, the culvert can easily deform. The deformations should be monitored so that the design shape doesn’t get distorted. Each layer of backfill should not exceed 200 mm after compaction, but this measurement can vary depending on the packing method. The layers should be placed uniformly on both sides of the culvert. (AISI 2002)

In the handbook the stiffness parameter for the culvert is defined as the relative relationship between the stiffness of the pipe and that of the surrounding soil. This stiffness parameter is then used to calculate the sectional forces in the culvert. The tangent modulus of the soil material in the structural backfill, $E_s$, is calculated from some key properties of the soil. It is then multiplied to the third power of the free span of the culvert to get the soil stiffness.

### 3.1 Key Properties

The following properties determine the tangent modulus, which in turn determine the soil stiffness. Of course, a higher value for the soil stiffness means that the soil will spread the effects of the concentrated loads more effectively and the structure will be stronger.

- **Density**
  - Different materials can be used in different layers of the backfill. There are three values for the density that affect the soil stiffness; the mean value of the densities above the culvert, to the side of the culvert, and the density of the material below the culvert.

- **Particle size distribution**
  - $d_{10}$, $d_{50}$ and $d_{60}$ in mm.

- **Angle of friction**
  - The angle of friction for the soil, in degrees.

- **Degree of compaction**
  - The relative degree of compaction, in the units Standard Proctor $R_{P}^{std}$. It should be at least 95%. (AISI 2002)

### 4 Steel Profiles

There are a number of standard steel profiles, commonly referred to as corrugated plates, for flexible metal culverts. The corrugations are circular arcs connected by tangents. In the handbook, formulae and cross-section parameters are presented for four types of steel profiles.
Figure 4:1 125x26 Plate profile, measurements.

Figure 4:2 150x50 Plate profile, measurements.

Figure 4:3 200x55 Plate profile, measurements.
5 Traffic Loads

The way traffic is modelled in the Swedish codes is by use of equivalent load cases and type loads. The equivalent load cases are described in VV Bro 2004 chapter 21.22. The type loads are described in Publication 1998:78 by (Vägverket). These loads are used to find the worst possible load case that will act on the bridge during its service lifetime. This gives a large safety margin and provides designers with an effective design process. It would simply be impossible to design bridges by modelling every possible vehicle that may pass over the bridge during its lifetime.

5.1 Traffic Loads in the Handbook

The method used in the handbook is slightly different. Instead of calculating bending moments and forces from the live load directly, it uses an equivalent line load, which yield the same vertical stresses at the level of the crown of the pipe as the live load itself. This is what the Soil-Culvert Interaction Method (SCI, by Duncan) is based on. It simply converts the actual live load by using two equations by Boussinesq, one for the vertical stresses at a certain depth (vertically under the load) caused by a line load to a quasi-infinite elastic body, and the corresponding equation for a point load.

What is done is actually to break the problem down into smaller pieces. First you are just interested in what stresses occur in the soil (at the depth equal to the height of cover) from a certain load case. Then you convert it to the equivalent line load (which gives exactly the same stresses at the crown of the pipe) and apply it to the culvert.

5.2 Calculating The Equivalent Line Load

A drawback of the SCI method is that there is no equation that gives the location of the point where the maximum stresses occur in the soil. This point has to be found with a numerical method. This means that you will never find a point that has larger vertical stresses than the worst point, but it is possible that you do not find the worst point. In other words, this part of the method introduces a risk of being non-conservative.

Pettersson and Sundquist use the formula for a point load when calculating the equivalent line load, but the loads in the Swedish bridge codes are actually pressures over small areas (the area of a tire in contact with the road, which is simplified to a rectangle). There are expressions by Boussinesq for the pressure vertically below a corner of a rectangular load. By
using superposition with positive and negative loads, these equations give the stresses at a certain depth at any point of interest (Enochsson, O. Hejll, A. and others 2002). This should lead to better accuracy than to model the pressures as concentrated loads.

\[
\begin{align*}
\sigma_z &\leftarrow 0 \text{Pa} \quad \text{if } \sigma_z < 0 \text{ Pa} \quad \text{neg} = 0 \\
\sigma_z &\leftarrow \text{0Pa} \quad \text{if } \sigma_z < 0 \text{Pa} \quad \text{neg} \neq 0 \\
\end{align*}
\]

**Figure 5:1** Using superposition when modelling the load as rectangles.

A quick comparison between the two methods show that while modelling the load as point loads always lead to positive values for the vertical stresses (as long as the depth is not zero), modeling the same load as a rectangle sometimes lead to negative values. This should be closer to the truth – when you push the soil down at one place, it can be pushed up outside of the loaded zone.
Calculation point at the corner of the load:

\[
P := 150\text{kN} \quad q := \frac{P}{0.2 \cdot 2.25} - 2
\]

\[
z := 0.5\text{m} \quad a := 0.2\text{m} \quad b := \frac{2.25}{2}\text{m} \quad s := \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 + z^2}
\]

\[
\sigma_{z,b} := \text{boussinesq}\ (q, a, b, 0, 0, z, 1) \quad \boxed{\sigma_{z,b} = 37.383\text{kPa}}
\]

\[
\sigma_{z,P} := \frac{3 \cdot P \cdot z^3}{2\pi \cdot s^5} \quad \boxed{\sigma_{z,P} = 36.063\text{kPa}}
\]

Calculation point at a distance from the corner of the load:

\[
x := 2\text{m} \quad y := 1\text{m} \quad \sigma_{z,b} := \text{boussinesq}\ (q, a, b, x, y, z, 1) \quad \boxed{\sigma_{z,b} = -1.134\text{kPa}}
\]

\[
s := \sqrt{\left(\frac{x - a}{2}\right)^2 + \left(\frac{y - b}{2}\right)^2 + z^2} \quad \sigma_{z,P} := \frac{3 \cdot P \cdot z^3}{2\pi \cdot s^5} \quad \boxed{\sigma_{z,P} = 0.286\text{kPa}}
\]

**Figure 5:2** *The difference between modelling the same load as a point load or as a distributed load.*
PART 2

MathCAD Routine for Calculations
6 Design

As a part of the work with this thesis a routine in MathCAD that performs the required calculations for the design of a flexible metal culvert has been made. It is based on the 3rd edition of the handbook by Pettersson and Sundquist, with one exception. Instead of modelling the tire pressures as point loads they are modelled as rectangle loads, as described in part 5.2 of this thesis.

The routine is made so that when you have given the input data, you only have to check if the design criteria are met. If not, you have to modify the design and make a new calculation with the new input data. Performing the calculations for a low-rise culvert shows the routine and the design procedure.

6.1 Input Data

In this section the input data required to perform the calculations with the MathCAD routine are described.

Figure 6:1 Measurements for the culvert in the example.
6.1.1 Geometry of the Culvert

The first thing you need to define is which type of culvert profile it is. The different types are defined by upper case letters from A to G corresponding to the figures 1.3 A to 1.3 G in the handbook. Then you need to provide a number of dimensions; the height, H, the height of cover, h_c, the diameter of span, D, the top radius R_t, the side radius, R_s, the bottom radius, R_b, and the corner radius, R_c, all defined as in the handbook. The radii’s in the lower part of the pipe should be placed in the vector called R_check.

\[
\text{Define the geometry}
\]

<table>
<thead>
<tr>
<th>Culvert profile:</th>
<th>Possible values are &quot;A&quot;, &quot;B&quot;,..., &quot;G&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>profile := &quot;E&quot;</td>
<td></td>
</tr>
<tr>
<td>H := 3.02m</td>
<td></td>
</tr>
<tr>
<td>h_c := 1m</td>
<td></td>
</tr>
<tr>
<td>D := 6.05m</td>
<td></td>
</tr>
<tr>
<td>R_t := 3.052m</td>
<td></td>
</tr>
<tr>
<td>R_s := R_t</td>
<td></td>
</tr>
<tr>
<td>R_b := 6.459m</td>
<td></td>
</tr>
<tr>
<td>R_c := 1.308m</td>
<td></td>
</tr>
<tr>
<td>R_check := [R_b, R_c]</td>
<td>Vector with different radii to be checked in the lower part of the culvert. If there are no such radii, just put it equal to ( R_t ) but in vector form.</td>
</tr>
</tbody>
</table>

Figure 6:2 Geometry input in the MathCAD routine.

6.1.2 Properties of the Corrugated Plates

The data needed for the steel plates is the thickness, t, the height of the corrugation, h_corr, the wavelength, c (named c_val in the routine), and the radii of curvature, R. You also need to know some material properties for the steel; Young’s modulus, E, the yield stress \( f_yk \), and the ultimate stress \( f_{uk} \).

<table>
<thead>
<tr>
<th>Steel profile</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>t := 5mm</td>
<td></td>
</tr>
<tr>
<td>h_corr := 55mm</td>
<td></td>
</tr>
<tr>
<td>c_val := 200mm</td>
<td></td>
</tr>
<tr>
<td>R := 53mm</td>
<td></td>
</tr>
<tr>
<td>E := 200GPa</td>
<td></td>
</tr>
<tr>
<td>( f_yk ) := 315MPa</td>
<td></td>
</tr>
<tr>
<td>( f_{uk} ) := 470MPa</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6:3 Input for the corrugated plates.

6.1.3 Data for the Bolts

Geometrical input for the bolts are the diameter, d, the cross sectional area, \( A_{sb} \), the number of bolts per meter length of the culvert, n, and the distance between parallel rows of bolts, a. The plastic yield stress limit, \( f_{ruk} \), and a reduction factor for pretension, \( \varphi_n \), is also needed. Using pretension increases the fatigue load capacity; therefore the capacity has to be reduced for bolts with little or no pretension. Pretension is not used for the bolts in flexible metal culverts. For the fatigue state the number of stress alternations, \( n_t \), and the characteristic structural strength for \( 2 \times 10^6 \) stress alternations, C, are needed. This is found in BSK 99, table B3:2.
6.1.4 Partial Coefficients

The partial coefficients are taken from the code used for designing the bridge. In Sweden there are different codes for railway bridges than for other bridges. The railway bridges are designed according to the codes from (Banverket), while other bridges are designed according to the codes from (Vägverket). Because of this; the routine in MathCAD has different sets of partial coefficients. This is a preparation for a future expansion of the routine to cover railway bridges.

The input needed is the safety class of the bridge, \( \gamma_n \), and the partial coefficients for the serviceability-, ultimate- and fatigue limit states. They are also different for the soil and the traffic loads, and as previously mentioned; railway loads could replace the traffic loads. Subscript s.s in the routine stand for soil serviceability state, t.s for traffic serviceability state, the add-on .bv stands for (Banverket) meaning the railway load.

\[
\begin{align*}
\gamma_n & := 1.1 \\
\phi_{\gamma_{s.s}} & := \begin{pmatrix} 1.1 \\ 0.9 \end{pmatrix} \quad \phi_{\gamma_{t.s}} := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \phi_{\gamma_{t.s.bv}} := \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
\phi_{\gamma_{s.u}} & := \begin{pmatrix} 1 \\ 0.9 \end{pmatrix} \quad \phi_{\gamma_{t.u}} := \begin{pmatrix} 1.5 \\ 0 \end{pmatrix} \quad \phi_{\gamma_{t.u.bv}} := \begin{pmatrix} 1.4 \\ 0 \end{pmatrix} \quad \phi_{\gamma_{t.f}} := 1.0 \quad \phi_{\gamma_{t.f.bv}} := 0.8
\end{align*}
\]

6.1.5 Loads and Load Cases

Two different types of loads are used in the design of flexible culverts; dead load from the soil and live loads. The live loads are calculated as concentrated and distributed loads. As previously discussed, the MathCAD routine handles the loads slightly differently than the handbook.

6.1.5.1 Concentrated loads

To specify the concentrated loads in a load case, you make one vector where you enter the magnitude of each concentrated load, with dimensions of course, and another vector for specifying the area that is loaded with the corresponding concentrated load. It is defined by its center point and its width in the x- and y direction. The origin of the coordinate system can be chosen at will, but it must be the same for the entire load case. It is possible to use different coordinate systems for different load cases if you are only interested in the equivalent traffic.
load, but it is not recommended if you are interested in the location of the maximum vertical
stresses. The position will be correct in the coordinate system for the load case, but there is a
high risk of confusing the different coordinate systems as a user.

The routine also provides standard load cases, taken from the Swedish bridge codes. The
origins of the coordinate systems in those load cases are positioned at the center of the
concentrated load at the bottom left corner of the group of concentrated loads in that specific
load case. In other words, all the concentrated loads have positive coordinates for their center
points. See appendix C and D for details about the predefined loads.

\[
\begin{pmatrix}
P_1 \\ P_2
\end{pmatrix}
\begin{pmatrix}
x_1 & y_1 & x_1.\text{width} & y_1.\text{width} \\
x_2 & y_2 & x_2.\text{width} & y_2.\text{width}
\end{pmatrix}
\]

It is also possible to choose one of the predefined load cases from BRO 04,

\[
\begin{pmatrix}
P_{\text{ekv.1}} & P_{\text{ekv.2}} & P_{\text{ekv.3}} & P_{\text{fatigue}} & P_{\text{pos fatigue}}
\end{pmatrix}
\]

Figure 6:6 Method for describing the loads.

Once the load cases have been defined, they should be put in vectors used by the functions in
the routine. Those vectors are named \( P \) and \( \text{load}_{xy} \). You also have to specify which of the load
cases that are used as fatigue load cases. This is done in the vector called \( \text{fatigue} \). In this
example, the predefined load cases provided by the routine are used.

\[
\begin{pmatrix}
P_{\text{fatigue}} \\ P_{\text{ekv.1}} \\ P_{\text{ekv.2}} \\ P_{\text{ekv.4}}
\end{pmatrix}
\begin{pmatrix}
\text{pos fatigue} \\ \text{pos ekv.1} \\ \text{pos ekv.2} \\ \text{pos ekv.4}
\end{pmatrix}
\]

\[
\text{fatigue} := (0)
\]

Figure 6:7 How to arrange the load cases.
(Vägverket) also require checking the type loads described in VV Publ 1998:78. For some heights of cover, one of those is actually the worst load case. When they are not the worst load case, the classification loads A and B need to be calculated. The type loads are also predefined in the routine. In this example they are first collected in their own vectors to make adjustments to them before they are added to the load case vectors.

\[
\begin{align*}
\mathbf{p}_{\text{class}} & \leftarrow \text{Klass}\mathbf{Last}_a \\
& \quad \text{Klass}\mathbf{Last}_b \\
& \quad \text{Klass}\mathbf{Last}_c \\
& \quad \text{Klass}\mathbf{Last}_d \\
& \quad \text{Klass}\mathbf{Last}_e \\
& \quad \text{Klass}\mathbf{Last}_f \\
& \quad \text{Klass}\mathbf{Last}_g \\
& \quad \text{Klass}\mathbf{Last}_h \\
& \quad \text{Klass}\mathbf{Last}_i \\
& \quad \text{Klass}\mathbf{Last}_j \\
& \quad \text{Klass}\mathbf{Last}_k \\
& \quad \text{Klass}\mathbf{Last}_l \\
& \quad \text{Klass}\mathbf{Last}_{II} \\
& \quad \text{Klass}\mathbf{Last}_{IIA}
\end{align*}
\]

\[
\begin{align*}
\mathbf{q}_{\text{class}} & \leftarrow (0)\text{kPa} \\
& \quad (0)\text{kPa} \\
& \quad (0)\text{kPa} \\
& \quad (0)\text{kPa} \\
& \quad (0)\text{kPa} \\
& \quad (0)\text{kPa} \\
& \quad (0)\text{kPa} \\
& \quad (0)\text{kPa} \\
& \quad (0)\text{kPa} \\
& \quad (0)\text{kPa} \\
& \quad (0)\text{kPa}
\end{align*}
\]

\[
\begin{align*}
\mathbf{p}_{\text{pos}} & \leftarrow (1\ 2\ 3\ 4)m \\
& \quad (1\ 2\ 3\ 4)m \\
& \quad (1\ 2\ 3\ 4)m \\
& \quad (1\ 2\ 3\ 4)m
\end{align*}
\]

\[
\begin{align*}
\mathbf{q}_{\text{pos}} & \leftarrow \text{Klass}\mathbf{pos}_g \\
& \quad \text{Klass}\mathbf{pos}_h \\
& \quad \text{Klass}\mathbf{pos}_i \\
& \quad \text{Klass}\mathbf{pos}_j \\
& \quad \text{Klass}\mathbf{pos}_k \\
& \quad \text{Klass}\mathbf{pos}_l \\
& \quad \text{Klass}\mathbf{pos}_{II} \\
& \quad (1\ 2\ 3\ 4)m
\end{align*}
\]

\[
\begin{align*}
\mathbf{q}_{\text{pos}} & \leftarrow \text{Klass}\mathbf{pos}_g \\
& \quad \text{Klass}\mathbf{pos}_h \\
& \quad \text{Klass}\mathbf{pos}_i \\
& \quad \text{Klass}\mathbf{pos}_j \\
& \quad \text{Klass}\mathbf{pos}_k \\
& \quad \text{Klass}\mathbf{pos}_l \\
& \quad \text{Klass}\mathbf{pos}_{II} \\
& \quad (1\ 2\ 3\ 4)m
\end{align*}
\]

Figure 6:8 The type loads from VV Publ. 1998:78 are prepared to be added to the load cases.

Because the dynamic effect is not included in the type loads, it is applied before they are put in the same vector as the other loads. It is calculated as prescribed in section 2.3.2.2.2 in VV Publ 1998:78.

The partial coefficients in Bro 2004 and VV Publ. 1998:78 are slightly different. For the backfill material they differ in the serviceability and ultimate limit states. The routine uses the coefficients from Bro 2004 for all load cases, which is on the safe side. For the traffic load, the minimum value for the partial coefficient is the same in both codes, 0.7, but the maximum value is 1.3 in VV Publ. 1998:78 and 1.5 in Bro 2004. This is handled in the routine by multiplying the loads from VV Publ. 1998:78 with 1.3/1.5 and using the partial coefficients from Bro 2004. The minimum value for the traffic load partial coefficient is not needed, since only one variable load is used in each load case.

The type load cases are also fatigue load cases. To compensate for the decrease in magnitude for the type loads in the fatigue limit state, the fatigue load from Bro 2004 is reduced with the same factor and the partial coefficient for the fatigue limit state is increased correspondingly.
Dynamic effect for the classification loads

\[
P_{\text{class}} := \text{for } i \in 0..\text{last}(P_{\text{class}})\\
P_i \leftarrow P_{\text{class}}_i \cdot \min\left(\frac{740}{20 + 2 \frac{D}{m}}, 1.35\right) \text{ if } h_c \leq 0.5\text{ m}\\
P_i \leftarrow P_{\text{class}}_i \left[ \min\left(\frac{740}{20 + 2 \frac{D}{m}}, 0.35\right) \left(1 - \frac{h_c - 0.5\text{ m}}{2.5\text{ m}}\right) + 1 \right] \text{ if } h_c > 0.5\text{ m} \land h_c \leq 3\text{ m}\\
P_i \leftarrow P_{\text{class}}_i \text{ otherwise}
\]

Compensation for the different partial coefficients

\[
\text{for } i \in 0..\text{last}(P_{\text{class}})\\
P_0 := P_0 \cdot 1.3 \land \frac{\phi_{t.f}}{\phi_{t.f}} := \frac{\phi_{t.f}}{\phi_{t.f}} \cdot 1.5
\]

\[
\begin{align*}
& P_{\text{class}}_i \leftarrow P_{\text{class}}_i \cdot 1.3 \\
& q_{\text{class}}_i \leftarrow q_{\text{class}}_i \cdot 1.3 \\
& \text{max}(\phi_{t.f})
\end{align*}
\]

Figure 6:9 The dynamic effect is added to the classification loads and a compensation for different partial coefficients is made.

Instead of adding all the type loads as new load cases for the fatigue limit state, the type loads used in the serviceability and ultimate limit state are used for the fatigue limit state as well. This means that there are two load fields in the fatigue limit state instead of one as prescribed by VV Publ. 1998:78. The effect of this is limited and on the safe side.

\[
\text{fatigueadd} := \text{first } \leftarrow \text{rows}(P)\\
\text{last } \leftarrow \text{rows}(P_{\text{class}}) + \text{first}\\
\text{for } i \in 0..\text{last} - \text{first} - 1\\
\text{fatigueadd}_i \leftarrow \text{first} + i\\
P := \text{stack}(P, P_{\text{class}})\\
\text{load}_{xy} := \text{stack}(\text{load}_{xy}, \text{pos}_{\text{class}})
\]

Figure 6:10 The vector listing the fatigue load cases is updated and the type loads are added to the load case vectors.
6.1.5.2 Distributed Loads

In the handbook the distributed loads are assumed to be evenly distributed over the span of the pipe. Hence it only has a single value denoted by \( q \). Because the MathCAD routine already has the functions for performing the calculations with surface pressures instead of point loads, it provides the user with the possibility to handle the distributed loads in the same manner. This way, you can enter different pressures for different lanes of traffic, as prescribed for the equivalent load cases in the codes from (Vägverket). There is a specific vector for this kind of distributed loads in the routine called \( q_i \). The subscript \( i \) stands for infinite, because this load has an infinite length.

There is also the possibility to enter other distributed loads that might be imposed on the structure. The vector \( q_b \) is intended for these loads. Of course you can also handle the distributed loads as they are handled in the handbook. Then you just enter a single value in the vector \( q_{\text{handbook}} \) for that load case. This is what is done in the example in this report.

Distributed loads to be calculated according to Boussinesq

If there aren't any loads to be calculated this way, just put the value for \( q \) to 0 for that load case, and put arbitrary values in the \( \text{pos} \) matrix. Every load case has to have at least one row in the matrices. The load cases should be ordered as in \( P \).

For load \( n \) in a load case, specify \( q_{n'}, y_{n'}, \text{start}_{n'}, \text{length}_{n'}, \text{width}_{n'} \) \( \left( \begin{array}{c} q_1 \\ q_2 \end{array} \right) \left( \begin{array}{c} y_1 \text{start}_1 \text{length}_1 \text{width}_1 \\ y_2 \text{start}_2 \text{length}_2 \text{width}_2 \end{array} \right) \)

\[ q_b := \left( \begin{array}{c} (0)\text{kPa} \\ (0)\text{kPa} \\ (0)\text{kPa} \\ (0)\text{kPa} \end{array} \right) \quad \text{pos} := \left( \begin{array}{c} (1 2 3 4)\text{m} \\ (2 4 9 1 1)\text{m} \\ (4 3 2 3)\text{m} \\ (1 2 3 4)\text{m} \\ (1 2 2 4)\text{m} \end{array} \right) \]

\[ q_b := \text{stack}(q_b, q_{\text{class}}) \quad \text{pos} := \text{stack}(\text{pos}, \text{pos}_q) \]

**Figure 6:11** Distributed loads in the routine.
Distributed loads with infinite length to be calculated according to Boussinesq

If there aren't any loads to be calculated this way, just put the value for q to 0 for that load case, and put arbitrary values in the pos matrix. Every load case has to have at least one row in the matrices. The load cases should be ordered as in P.

For load n in a load case, specify \( q_n \), \( y_n \), \( \text{start}_n \), \( \text{length}_n \), \( \text{width}_n \)

\[
\begin{bmatrix}
q_1 \\
q_2 \\
\vdots \\
q_i \\
\end{bmatrix}
\begin{bmatrix}
y_1 \\
\text{start}_1 \\
\text{length}_1 \\
\text{width}_1 \\
\end{bmatrix}
\begin{bmatrix}
y_2 \\
\text{start}_2 \\
\text{length}_2 \\
\text{width}_2 \\
\end{bmatrix}
\]

\[
q_i := \begin{bmatrix}
(0) \text{kPa} \\
(0) \text{kPa} \\
(0) \text{kPa} \\
(0) \text{kPa}
\end{bmatrix}
\]

\[
pos_i := \begin{bmatrix}
(1 2 3 4) \text{m} \\
1 \times 10^{-3} 2 \times 10^{-3} 3 \\
4 \times 10^{-3} 2 \times 10^{-3} 3 \\
(1 2 3 4) \text{m} \\
(1 2 3 4) \text{m}
\end{bmatrix}
\]

\[
q_{i,\text{class}} := \text{for} \ i \in 0.. \text{last}(P_{\text{class}}) \quad \text{pos}_{i,\text{class}} := \text{for} \ i \in 0.. \text{last}(P_{\text{class}})
\]

\[
q_{i,\text{class}} := (0) \text{kPa} \\
pos_{i,\text{class}} := (1 2 3 4) \text{m}
\]

\[
q_i := \text{stack}(q_{i,\text{class}})
\]

\[
pos := \text{stack}(\text{pos}_{i,\text{class}}, \text{pos}_i)
\]

\[\text{Figure 6:12 Distributed loads with infinite length.}\]

Distributed load for use according to the handbook

Set the value for q for each load case. If all distributed loads are calculated according to Boussinesq, this value should be zero for that load case. If you use this value instead, \( q_i \) should be zero for that load case.

\[
q_{\text{handbook}} := \begin{bmatrix}
0 \text{kPa} \\
4 \text{kPa} \\
4 \text{kPa} \\
0 \text{kPa}
\end{bmatrix}
\]

\[
q_{\text{h.class}} := \text{for} \ i \in 0.. \text{last}(P_{\text{class}})
\]

\[
q_{\text{h.class}} := 5 \text{kPa}
\]

\[\text{Figure 6:13 The vector for distributed loads according to the handbook.}\]

6.1.5.3 Dynamic Effect

For the loads in the bridge codes from (Vägverket) the dynamic effect is included. This is not the case for the type loads in VV Publ 1998:78 or the loads from railway traffic, prescribed by (Banverket). Therefore it is necessary to specify whether or not the dynamic effect is included in the load.

When the predefined type loads in the routine are used together with loads where the dynamic factor is included, the dynamic effect should be applied to the type loads before they are put in the same vector as the other loads. This way all the loads will have the dynamic effect included.
Dynamic effect \( \text{inc} := 0 \)

Is the dynamic effect included in the live load? If so, inc should be put equal to 0. Otherwise inc should be put equal to 1.

Figure 6:14 Specify whether or not the dynamic effect is included in the live load.

6.1.6 Options for the Numerical Method

There are a few options for the numerical method used to find the maximum vertical pressure. \( x_{\text{divisions}} \) and \( y_{\text{divisions}} \) simply specify how dense the original grid of calculation points should be. For more details about the numerical method, see appendix A. As explained in part 5.2 of this thesis, the Boussinesq formulae used by the routine can lead to negative values for the vertical pressure. The variable \( \text{neg} \) specifies if the routine should use those negative values or put them equal to zero.

Specify the number of calculation points to find \( p_{\text{traffic}} \)

\[
\begin{align*}
x_{\text{divisions}} & := 15 \\
y_{\text{divisions}} & := 10 \\
\text{Number of original calculation points:} & = (x_{\text{divisions}} + 1)(y_{\text{divisions}} + 1) = 176
\end{align*}
\]

Disregard negative values of \( \sigma_v \)? \( \text{neg} := 0 \)

If \( \text{neg} = 0 \), negative values for the pressure \( \sigma_v \) are put equal to zero. If \( \text{neg} = 1 \) the negative values for the pressure will be used in the calculations.

Figure 6:15 Input for the numerical method used to calculate the vertical stresses in the soil.

6.1.7 Soil Parameters

The handbook provides two methods for calculating the design tangent modulus for the structural backfill. For the simplified method (Method A) you only need to specify the relative degree of compaction in the units Standard Proctor, \( \text{RP}^{\text{std}} \). For the more precise method (Method B) you need to have more data to describe the soil.

Define the degree of compaction, the standard Proctor value \( \text{RP} \):

\( \text{RP} := 97 \)

Select method, set \( \text{Meth} = 1 \) for the simplified method (Method A) or \( \text{Meth} = 2 \) for the more precise method (Method B)

\( \text{Meth} := 2 \)

Figure 6:16 Basic input for the soil.

The input needed for Method B is the optimal density for the cover material, \( \rho_{\text{opt}} \), which is used together with the degree of compaction to calculate the density of the material above the culvert, \( \rho_{\text{cv}} \), as written in appendix 2 in the handbook. You also need to specify the particle size distribution and the uncertainty partial coefficient \( \gamma_m \) for the soil. If different material than what is used above the culvert is used to the side of the culvert, \( \rho_1 \) is used to define its
density, and for the material below the culvert, $\rho_2$ is used. In this example, the same material is used all around the culvert, so $\rho_1$ and $\rho_2$ are put equal to $\rho_{cv}$.

Define the optimal density $\rho_{opt}$, the density $\rho_1$, the average density $\rho_2$, the particle size distribution $d_{10}, d_{50}, d_{60}$ and $\gamma_{m, soil}$.

\[
\begin{align*}
\rho_{opt} &:= 19.4 \text{ kN/m}^3 \\
d_{10} &:= 0.4 \text{mm} \\
d_{50} &:= 0.9 \text{mm} \\
d_{60} &:= 1.32 \text{mm} \\
\gamma_{m, soil} &:= 1.3
\end{align*}
\]

$\rho_{cv} := \frac{RP}{100} \rho_{opt}$  \hspace{1cm} $\rho_{cv} = 18.8 \text{ kN/m}^3$

$\rho_1 := \rho_{cv}$  \hspace{1cm} $\rho_2 := \rho_{cv}$

Figure 6:17 Soil input for the more precise method.

### 6.2 Calculations

The calculations are structured the way they are presented in *the handbook* to make them easy to follow. The order of some calculations has been changed relative to *the handbook* when the result from one calculation is needed for the other. In order to make the routine efficient, most of the equations from *the handbook* are made as functions, which can be called multiple times with different send variables.

#### 6.2.1 Soil Tangent Modulus

As previously mentioned, there are two methods for calculating the tangent modulus of the soil. The choice of method is done in the input section of the routine. The calculations are made in functions containing the formulas from *the handbook*. Before you can calculate the tangent modulus with Method B, you need to calculate the arching coefficient for the soil. It is done in the function called arch(). It uses equations (4.d) through (4.g) and (b2.f) in *the handbook*. The function used for calculating the tangent modulus is called soil(). It uses equations (b2.a) through (b2.i) in *the handbook*.

$\rho_{cv} := \frac{RP}{100} \rho_{opt}$  \hspace{1cm} $\rho_{cv} = 18.8 \text{ kN/m}^3$

$\rho_1 := \rho_{cv}$  \hspace{1cm} $\rho_2 := \rho_{cv}$

Figure 6:18 Calculating the tangent modulus for the soil.
\[
\text{arch}(\text{RP}, d, h_c, D, \gamma_n, \gamma_m) :=
\begin{align*}
C_u & \leftarrow \frac{d_2}{d_0} \\
\varphi_{cv,k} & \leftarrow \left(26 + 10 \frac{\text{RP} - 75}{25} + 0.4 C_u + 1.6 \log \left(\frac{d_1}{\text{mm}}\right)\right) \text{deg} \\
\varphi_{cv,d} & \leftarrow \arctan \left(\frac{\tan(\varphi_{cv,k})}{\gamma_n/\gamma_m}\right) \\
S_v & \leftarrow \frac{0.8 \tan(\varphi_{cv,d})}{\left(1 + \tan(\varphi_{cv,d})^2 + 0.45 \tan(\varphi_{cv,d})\right)^2} \\
\kappa & \leftarrow 2S_v \frac{h_c}{D} \\
S_{ar} & \leftarrow \frac{1 - e^{-\kappa}}{\kappa} \\
\text{return } S_{ar}
\end{align*}
\]

**Figure 6.19** Function for calculating the arching coefficient, \( S_{ar} \).
\[
\text{soil}(\text{Meth}, \text{RP}, h_c, H, \gamma_n, \gamma_m, d, \rho_{\text{opt}}, \rho_{\text{cv}}, \rho_2, S_{\text{ar}}) :=
\]

\[
\begin{align*}
\text{if Meth} = 1
& \quad 1 \\
E_{s,d} & \leftarrow \frac{1.2}{\gamma_n} \cdot 1.17^{\text{RP}-95} \left(1.25 \ln \left(\frac{h_c}{m} + \frac{H}{2m}\right) + 5.6\right) \text{MPa} \\
\text{if Meth} = 2
& \\
\rho_s & \leftarrow 26 \text{ kN/m}^3 \\
e & \leftarrow \frac{\rho_s}{\rho_{\text{cv}}} - 1 \\
C_u & \leftarrow \frac{d_2}{d_0} \\
m & \leftarrow 282 C_u - 0.77 e - 2.83 \\
\beta & \leftarrow 0.29 \log \left(\frac{d_1}{0.01 \text{mm}}\right) - 0.065 \log(C_u) \\
\phi_k & \leftarrow 26 + 10 \frac{\text{RP} - 75}{25} + 0.4 C_u + 1.6 \log \left(\frac{d_1}{\text{mm}}\right) \text{deg} \\
\phi_d & \leftarrow \arctan \left(\frac{\tan(\phi_k)}{\gamma_m \gamma_n}\right) \\
k_v & \leftarrow \frac{3 - 2 \sin(\phi_k)}{2 - \sin(\phi_k)} \\
E_{s,d} & \leftarrow 0.42 m \cdot 100 \text{kPa} \cdot k_v \cdot \left[\frac{\left(1 - \sin(\phi_k)\right) \rho_2 S_{\text{ar}} \left(h_c + \frac{H}{2}\right)}{100 \text{kPa}}\right]^{1-\beta} \\
\text{if Meth} \neq 1 \land \text{Meth} \neq 2
& \quad 0 \\
E_{s,d} & \leftarrow 0 \\
\text{return } E_{s,d}
\end{align*}
\]

**Figure 6:20** Function for calculating the tangent modulus.
6.2.2 Culvert Sectional Properties and Stiffness Parameter

To calculate the sectional properties for the corrugated plates equation (b1.a) in the handbook is used to calculate the tangent length, \( m_t \), and the angle \( \alpha \), both defined as in figures B1.2 through B1.5 in the handbook. In the routine, the Find() function in MathCAD is used to solve the equation system. It requires guess values for the unknowns, which are set using the equations in table B1.1 in the handbook for the 200x55 profile. The function culvProp() used to calculate the sectional properties uses equations (b1.b) through (b1.g) in the handbook.

\[
\begin{align*}
\text{Culvert profile} \\
\text{Calculations} \\
\text{Given} \\
h_{\text{corr}} &= 2R\left(1 - \cos(\alpha_{\text{guess}})\right) + m_t\cdot\sin(\alpha_{\text{guess}}) \\
c_{\text{val}} &= 4R\cdot\sin(\alpha_{\text{guess}}) + 2m_t\cdot\cos(\alpha_{\text{guess}}) \\
\begin{bmatrix} m_{\text{emp}} \\ \alpha \end{bmatrix} &= \text{Find}\left(\begin{bmatrix} m_t \\ \alpha \end{bmatrix}, \alpha_{\text{guess}}\right) \\
m_t &= m_{\text{emp}} \\
2R\left(1 - \cos(\alpha)\right) + m_t\cdot\sin(\alpha) - h_{\text{corr}} &= 1 \times 10^{-9} \text{ m} \\
4R\cdot\sin(\alpha) + 2m_t\cdot\cos(\alpha) - c_{\text{val}} &= -1.3 \times 10^{-9} \text{ m} \\
\text{dim} &= \left(R \ t \ c_{\text{val}} \ h_{\text{corr}} \ m_t\right) \\
\text{props} &= \text{culvProp}(\text{dim}, \alpha) \\
A_s &= \text{props}_{0,0} \text{ m} \\
I_s &= \text{props}_{0,1} \text{ m}^3 \\
W_s &= \text{props}_{0,2} \text{ m}^2 \\
Z_s &= \text{props}_{0,3} \text{ m}^2
\end{align*}
\]

\[
\text{Calculations} \\
A_s = 6.1 \text{ mm}^2 \\
I_s = 2246.2 \text{ mm}^4 \\
W_s = 74.9 \text{ mm}^3 \\
Z_s = 106.2 \text{ mm}^3 \\
\frac{Z_s}{W_s} = 1.4
\]

Figure 6:21 Calculating sectional properties for the steel profile.
When the sectional properties for the corrugation and the tangent modulus of the soil are known it is possible to calculate the stiffness parameter, $\lambda_f$, using equation (4.3) in the handbook.

$$\lambda_f := \frac{\mathcal{E}_s \mathcal{D}^3}{E I_s}$$

$\lambda_f = 8645.2$
6.2.3 Reduction of the Effective Depth of Cover

Because of the fact that the culvert is deformed during backfilling it might be necessary to reduce the effective height of cover. The routine uses the function cRise() to calculate the crown rise according to equation (b3.b) in the handbook. When the crown rise is known it is simply subtracted from the original height of cover according to equation (4.a) in the handbook.

**Crown rise**

<table>
<thead>
<tr>
<th>Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_{\text{crown}} := \text{cRise}(h_c, D, H, \lambda_f, \rho_1, E_s, \text{profile}) )</td>
</tr>
<tr>
<td>( h_{c, \text{red}} := h_c - \delta_{\text{crown}} )</td>
</tr>
</tbody>
</table>

\[ \begin{align*}
\delta_{\text{crown}} & = 41.19 \text{mm} \\
0.015 D & = 90.75 \text{mm} \\
h_{c, \text{red}} & = 0.96 \text{m}
\end{align*} \]

**Figure 6:24** Calculating the reduced height of cover.

\[ \begin{align*}
\text{cRise}(h_c, D, H, \lambda_f, \rho_1, E_s, \text{profile}) := \left( f_h \leftarrow 0.013 \left( \frac{H}{D} \right)^{2.8} \cdot \lambda_f \right) \right.
\]

\[ \delta_{\text{crown}} \leftarrow \rho_1 \frac{D^2}{E_s} f_h \]

\[ \delta_{\text{crown}} \leftarrow 0 \text{m if profile = "B"} \lor \text{profile = "G"} \]

\[ \delta_{\text{crown}} \leftarrow \frac{\delta_{\text{crown}}}{4} \text{ if profile = "F"} \]

return \( \delta_{\text{crown}} \)

**Figure 6:25** Function for calculating the crown rise.

6.2.4 Dynamic Factor

There are two different cases when it comes to the dynamic factor. If the dynamic effect is included in the load a reduction factor is used when the height of cover is larger than 2 m. If the dynamic effect is not included in the load, the dynamic amplification factor is first calculated, then reduced if the height of cover is larger than 1.2 m.

<table>
<thead>
<tr>
<th>Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_d := \text{dyn}(D, h_c, h_{c, \text{red}}, \text{inc}) )</td>
</tr>
</tbody>
</table>

**Figure 6:26** Calculating the dynamic factor.
The function dyn() handles both cases in the routine. If the dynamic effect is included in the load (specified by the variable inc), it uses the sub function redFac() to calculate the reduction factor according to equation (3.a) in the handbook. If the dynamic effect is not included, it uses the sub function dynFac() to calculate the dynamic amplification factor and, if needed, the reduction of it according to equation (b6.a) in the handbook.

\[
dynFac(D, h_c) := \begin{align*}
  df & \leftarrow 1 + \frac{4m}{8m + 2D} \\
  \text{if } h_c & > 1.2m \\
  \Delta df & \leftarrow 0.1 \left( \frac{h_c}{m} - 1.2 \right) \\
  df & \leftarrow \max(1.0, df - \Delta df) \\
  \text{return } df
\end{align*}
\]

\[
\text{redFac}(h_{c,\text{red}}) := \begin{align*}
  r_d & \leftarrow 1.0 \text{ if } h_{c,\text{red}} < 2m \\
  r_d & \leftarrow 1.10 - 0.05 \frac{h_{c,\text{red}}}{m} \text{ if } h_{c,\text{red}} \geq 2m \land h_{c,\text{red}} < 6m \\
  r_d & \leftarrow 0.8 \text{ otherwise} \\
  \text{return } r_d
\end{align*}
\]

\[
dyn(D, h_c, h_{c,\text{red}}, \text{inc}) := \begin{align*}
  \text{if } \text{inc} & = 1 \\
  \text{dynFac}(D, h_c) & \\
  \text{redFac}(h_{c,\text{red}}) & \text{otherwise}
\end{align*}
\]

Figure 6:27 Functions for calculating the dynamic factor.

6.2.5 Axial Forces

The axial forces that arise in the culvert are first calculated with a characteristic value. Then the design value is calculated using the partial coefficient method. There are different coefficients for different load types so two cases are calculated, the permanent load consisting of the load from the surrounding soil, and the variable load consisting of traffic loads.

6.2.5.1 Load from the Surrounding Soil

During the process of backfilling the crown of the culvert rises. The maximum rise occurs when the backfill reaches the level of the crown. This results in a negative moment in the crown and this stage could be the dimensioning capacity of the culvert. Therefore the normal force at this stage is calculated. In the routine it is given the name \( N_{s,\text{surr}} \). When the structure is complete, the final normal force in the culvert caused by the surrounding soil is needed for design checks. It is simply the sum of the normal force from the surrounding soil, and the soil above the crown.
The routine uses the function \( N_s.f() \) to calculate the two parts of the total normal force. It uses equation (4.c) in the handbook and returns the two parts of the equation before they are added in order to get the needed value for \( N_s.surr \). As indicated in the handbook, the equation is a little different for horizontal ellipses and box culverts. This is because the geometrical differences in the culverts make the volume of the surrounding soil different.

\[
N_s.surr = 68.8 \frac{kN}{m} \quad N_s.cover = 68.4 \frac{kN}{m} \quad N_s = 137.2 \frac{kN}{m}
\]

\textbf{Figure 6:28 Calculating normal forces from the soil.}

6.2.5.2 \textit{Live Loads}

As previously discussed, a numerical method is used to find the point where the live loads give the highest vertical stresses in the soil. The principles of the numerical method used in the routine is presented in appendix A. Graphs showing the equivalent line load, \( p_{\text{traffic}} \), as a function of the effective height of cover, \( h_c \), for each of the load cases in this example are presented in appendix C and D.

The results from the numerical method are shown as a 3-D plot in the routine. It is the plot of the stress at point \((x, y)\) at depth \( h_{c,\text{red}} \) for the worst load case, and the coordinate system is the one associated with that load case. Another plot shows the positions of the concentrated loads in that load case.
All the calculated stresses for all load cases are available in the routine, but the only stresses needed for the design are the maximum stresses for the worst load case and for the worst fatigue load case. They are then used to calculate two values for the equivalent line load according to equation (4.k) in the handbook. It is also necessary to know the maximum stresses for type load a and the worst of the other type loads described in VV Publ. 1998:78. This is needed for the classification of the bridge.

![Diagram of vertical pressure and point loads](image)

**Figure 6:30** Display of the vertical stresses in the routine.

The equivalent line load is calculated according to equation (4.k) in the handbook and used to calculate the normal forces in the culvert arising from traffic loads. If distributed loads have been calculated with the boussinesq method, they are included in the equivalent line load. This is not the case for the distributed load calculated with the method in the handbook. Besides equivalent line load for the worst load case in the different limit states, you also need to know the equivalent line load in the fatigue limit state for type load a and the worst of the other type loads from VV Publ. 1998:78.
Figure 6:31 Calculating the equivalent line loads and the normal forces from traffic.

To calculate normal forces from traffic loads the function \( N_t() \) in the routine uses equations (4.1') through (4.1'''). If all distributed loads are calculated with the boussinesq method, the value for the variable \( q \), used for the handbook method is zero. As mentioned earlier, the effect of the distributed loads is then included in \( p_{traffic} \) instead.

\[
\begin{align*}
p_{traffic} & := \frac{\pi h_c}{2} \cdot \text{max}(\sigma_v) & \quad \text{\( p_{traffic} = 134.4 \text{ kN/m} \)} \\
p_{traffic,\text{fatigue}} & := \frac{\pi h_c}{2} \cdot \text{max}(\sigma_{v,\text{fatigue}}) & \quad \text{\( p_{traffic,\text{fatigue}} = 66.6 \text{ kN/m} \)} \\
p_{traffic,\text{fatigue},A} & := \frac{\pi h_c}{2} \cdot \sigma_{v,A} & \quad \text{\( p_{traffic,\text{fatigue},A} = 66.6 \text{ kN/m} \)} \\
p_{traffic,\text{fatigue},B} & := \frac{\pi h_c}{2} \cdot \sigma_{v,B} & \quad \text{\( p_{traffic,\text{fatigue},B} = 56.5 \text{ kN/m} \)} \\
N_t & := N_t(h_{c,\text{red}},D,q,p_{traffic}) & \quad \text{\( N_t = 146.5 \text{ kN/m} \)} \\
N_{t,\text{fatigue}} & := N_{t,\text{f}}(h_{c,\text{red}},D,0,p_{traffic,\text{fatigue}}) & \quad \text{\( N_{t,\text{fatigue}} = 66.6 \text{ kN/m} \)} \\
N_{t,\text{fatigue},A} & := N_{t,\text{f}}(h_{c,\text{red}},D,0,p_{traffic,\text{fatigue},A}) & \quad \text{\( N_{t,\text{fatigue},A} = 66.6 \text{ kN/m} \)} \\
N_{t,\text{fatigue},B} & := N_{t,\text{f}}(h_{c,\text{red}},D,0,p_{traffic,\text{fatigue},B}) & \quad \text{\( N_{t,\text{fatigue},B} = 56.5 \text{ kN/m} \)}
\end{align*}
\]

Figure 6:32 Function for calculating normal forces from the distributed and concentrated surface loads.
6.2.5.3 Design Axial Forces

Depending on the load, the maximum and minimum moments in the culvert can have either positive or negative sign. This means that the design stresses can arise with the low coefficient for the soil as well as with the high coefficient. Both cases need to be checked. The routine calculates all possible combinations of high and low values for the coefficients and then uses the maximum and minimum values of the calculated combinations. Then, the dimensioning values for the normal forces are calculated according to equations (4.m) through (4.o) in the handbook.

\[
\begin{align*}
\varphi_{f,s,\text{mod}} & := \begin{pmatrix}
\varphi_{f,s,0} \\
\varphi_{f,s,1} \\
\varphi_{f,s,1} \\
\varphi_{f,s,0}
\end{pmatrix} & \quad & \varphi_{f,t,\text{mod}} & := \begin{pmatrix}
\varphi_{f,t,0} \\
\varphi_{f,t,1} \\
\varphi_{f,t,0} \\
\varphi_{f,t,1}
\end{pmatrix} \\
\varphi_{f,s,\text{u,mod}} & := \begin{pmatrix}
\varphi_{f,s,u,0} \\
\varphi_{f,s,u,1} \\
\varphi_{f,s,u,1} \\
\varphi_{f,s,u,0}
\end{pmatrix} & \quad & \varphi_{f,t,\text{u,mod}} & := \begin{pmatrix}
\varphi_{f,t,u,0} \\
\varphi_{f,t,u,1} \\
\varphi_{f,t,u,0} \\
\varphi_{f,t,u,1}
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
N_{d,s} & := \max(\varphi_{f,s,\text{mod}} \cdot N_s + \varphi_{f,t,\text{mod}} \cdot N_t) \\
& \quad \min(\varphi_{f,s,\text{mod}} \cdot N_s + \varphi_{f,t,\text{mod}} \cdot N_t) \\
N_{d,u} & := \max(\varphi_{f,s,\text{u,mod}} \cdot N_s + \varphi_{f,t,\text{u,mod}} \cdot N_t) \\
\Delta N_{d,f} & := \varphi_{f,t,\text{f,\text{fatigue}}} \cdot N_t \\
\Delta N_{d,f,A} & := \varphi_{f,t,\text{f,\text{fatigue,A}}} \cdot N_t \\
\Delta N_{d,f,B} & := \varphi_{f,t,\text{f,\text{fatigue,B}}} \cdot N_t
\end{align*}
\]

\[
\begin{align*}
N_{d,s} & = \frac{297.4}{123.5} \text{ kN/m} \\
N_{d,u} & = 356.9 \text{ kN/m} \\
\Delta N_{d,f} & = 76.8 \text{ kN/m} \\
\Delta N_{d,f,A} & = 76.8 \text{ kN/m} \\
\Delta N_{d,f,B} & = 65.2 \text{ kN/m}
\end{align*}
\]

Figure 6:33 Calculating design axial forces.

6.2.6 Bending Moments

6.2.6.1 Characteristic Bending Moments

As with the normal force, it is necessary to know the bending moment from the surrounding soil as well as the moment from the soil for the complete structure. The moment from the live
load is applied together with the moment from the complete backfill. The characteristic values are calculated for the moment from the soil and from the live loads.

**Bending moments**

\[
\begin{align*}
R_{\text{send}} & := \begin{pmatrix} R_t \\ R_s \end{pmatrix} \\
\begin{pmatrix} M_{s,\text{surr}} \\ M_{s,\text{cover}} \\ M_t \\ f_s \end{pmatrix} & := M_h \left( h_{\text{c.red}}, H, D, \lambda_f, \rho_1, \rho_{cv}, S_{\text{ar}}, R_{\text{send}}, p_{\text{traffic}} \right)
\end{align*}
\]

\[
\begin{pmatrix} f_1 \\ f_{2,\text{surr}} \\ f_{2,\text{cover}} \\ f_3 \\ f_4 \\ f_{-4} \\ f_{-4}-4 \end{pmatrix} := \frac{f_s}{N}
\]

\[
\begin{align*}
\begin{pmatrix} M_{s,\text{surr}.\text{fatigue}} \\ M_{s,\text{cover}.\text{fatigue}} \\ M_{t,\text{fatigue}} \\ f_{s,\text{fatigue}} \end{pmatrix} & := M_h \left( h_{\text{c.red}}, H, D, \lambda_f, \rho_1, \rho_{cv}, S_{\text{ar}}, R_{\text{send}}, p_{\text{traffic}.\text{fatigue}} \right) \\
\begin{pmatrix} M_{s,\text{surr}.\text{fatigue}.A} \\ M_{s,\text{cover}.\text{fatigue}.A} \\ M_{t,\text{fatigue}.A} \\ f_{s,\text{fatigue}.A} \end{pmatrix} & := M_h \left( h_{\text{c.red}}, H, D, \lambda_f, \rho_1, \rho_{cv}, S_{\text{ar}}, R_{\text{send}}, p_{\text{traffic}.\text{fatigue}.A} \right) \\
\begin{pmatrix} M_{s,\text{surr}.\text{fatigue}.B} \\ M_{s,\text{cover}.\text{fatigue}.B} \\ M_{t,\text{fatigue}.B} \\ f_{s,\text{fatigue}.B} \end{pmatrix} & := M_h \left( h_{\text{c.red}}, H, D, \lambda_f, \rho_1, \rho_{cv}, S_{\text{ar}}, R_{\text{send}}, p_{\text{traffic}.\text{fatigue}.B} \right)
\end{align*}
\]

**Calculations**

\[
\begin{align*}
f_1 & = 1 \\
f_{2,\text{surr}} & = 9 \times 10^{-4} \\
f_{2,\text{cover}} & = 3.2 \times 10^{-3} \\
f_3 & = 2 \\
f_{-4} & = 0.14 \\
f_{-4} & = 0.05 \\
f_{-4} & = 2.42 \\
f_{-4} & = 1
\end{align*}
\]

\[
\begin{align*}
M_{s,\text{surr}} & = -7.49 \text{kN}m \\
M_{s,\text{cover}} & = 2.03 \text{kN}m \\
M_t & = 13.79 \text{kN}m \\
M_{t,\text{fatigue}} & = 6.61 \text{kN}m \\
M_{t,\text{fatigue}.A} & = 6.61 \text{kN}m \\
M_{t,\text{fatigue}.B} & = 5.61 \text{kN}m
\end{align*}
\]

*Figure 6:34 Calculating bending moments.*
The function \( M_f() \) in the routine calculates the three needed moments according to equations (4.q) through (4.y) in the handbook. It also returns the values for the eight help parameters to make it easy to check the calculations. In the example, the help parameters for the serviceability and ultimate limit states are shown.

\[
M_f(h_c, H, D, \lambda_f, \rho_1, \rho_{cv}, S_{ar}, R, p_{traffic}, q) := \\
\text{check} \leftarrow \frac{H}{D} \\
f_1 \leftarrow 0.67 + 0.87(\text{check} - 0.2) \text{ if check} > 0.2 \land \text{check} \leq 0.35 \\
f_1' \leftarrow 0.8 + 1.33(\text{check} - 0.35) \text{ if check} > 0.35 \land \text{check} \leq 0.5 \\
f_1'' \leftarrow 2 \text{check} \text{ if check} > 0.5 \land \text{check} \leq 0.6 \\
f_{2,surr} \leftarrow 0.0046 - 0.0010 \log(\lambda_f) \text{ if } \lambda_f \leq 5000 \\
f_{2,surr} \leftarrow 0.0009 \text{ if } \lambda_f > 5000 \\
f_3 \leftarrow 6.67 \text{check} - 1.33 \\
f_{2,cover} \leftarrow 0.018 - 0.004 \log(\lambda_f) \text{ if } \lambda_f \leq 5000 \\
f_{2,cover} \leftarrow 0.0032 \text{ otherwise} \\
M_{s,surr} \leftarrow \rho_1 D^3 (-f_1) f_3 f_{2,surr} \\
M_{s,cover} \leftarrow D^3 S_{ar} \rho_{cv} \frac{h_c}{D} \left( \frac{R_0}{R_1} \right)^{0.75} \cdot f_1 \cdot f_{2,cover} \\
f_{4} \leftarrow 0.65 \left( 1 - 0.2 \log(\lambda_f) \right) \\
f_{4}' \leftarrow 0.120 \left( 1 - 0.15 \log(\lambda_f) \right) \text{ if } \lambda_f \leq 100000 \\
f_{4}'' \leftarrow 0.030 \text{ otherwise} \\
f_{4}''' \leftarrow \left( \frac{h_c}{D} + 0.15 \right)^{-0.75} \\
f_{4}'''' \leftarrow \left( \frac{R_0}{R_1} \right)^{0.25} \\
M_{t} \leftarrow \min(f_{4} \cdot f_{4}'''', 1.0) \cdot f_{4}'' \cdot f_{4}'''' \cdot D \cdot p_{traffic} + S_{ar} \left( \frac{R_0}{R_1} \right)^{0.75} \cdot f_1 \cdot f_{2,cover} \cdot q \cdot D^2 \\
f \leftarrow \left( f_1 \cdot f_{2,surr} \cdot f_{2,cover} \cdot f_3 \cdot f_{4} \cdot f_{4}' \cdot f_{4}'' \cdot f_{4}''' \cdot f_{4}'''' \right) \\
\text{return } \begin{bmatrix} M_{s,surr} \\ M_{s,cover} \\ M_{t} \\ f \cdot N \end{bmatrix}
\]

**Figure 6.35** Function for calculating bending moments.
6.2.6.2 Design Bending Moments

Because different material can be used for the backfill surrounding the culvert and for the backfill above the culvert, the two soil volumes can independently have either the high or the low load coefficient. In the routine, all possible combinations of high and low coefficients for the two soil volumes are calculated, and the maximum and minimum values are used. Equations (4.w') through (4.ae) in the handbook are used to determine the design values for the moments. (4.ae) corresponds to equation (4.ä) in the Swedish version of the handbook.

Design bending moments

Calculations

\[ M_{td,s} := \max(\phi\gamma_{t,s}) M_t \]
\[ M_{sd,s} := \max\left(\phi\gamma_{s,s,1} M_{s,surr} + \phi\gamma_{s,s,2} M_{s,cover}\right) / \min\left(\phi\gamma_{s,s,1} M_{s,surr} + \phi\gamma_{s,s,2} M_{s,cover}\right) \]
\[ M_{td,u} := \max(\phi\gamma_{t,u}) M_t \]

Calculations

\[ M_{sd,s} = \begin{pmatrix} -4.5 \\ -6.4 \end{pmatrix} \text{kNm/m} \]
\[ M_{td,s} = \begin{pmatrix} 13.8 \\ -6.9 \end{pmatrix} \text{kNm/m} \]
\[ M_{ds,s} := M_{sd,s} + M_{td,s} \]
\[ M_{td,u} = \begin{pmatrix} -5.7 \end{pmatrix} \text{kNm/m} \]
\[ M_{sd,u} = \begin{pmatrix} 20.7 \end{pmatrix} \text{kNm/m} \]
\[ M_{du,u} := M_{sd,u} + M_{td,u} \]

\[ \Delta M_{d,f} := \phi\gamma_{t,f} M_{t,\text{fatigue}}^{1.5} \]
\[ \Delta M_{d,f,A} := \phi\gamma_{t,f} M_{t,\text{fatigue,A}}^{1.5} \]
\[ \Delta M_{d,f,B} := \phi\gamma_{t,f} M_{t,\text{fatigue,B}}^{1.5} \]

Figure 6:36 Calculating design bending moments.
6.3 Design Checks

The method of partial coefficients is used for verifying the capacity of the culvert. There are a number of required checks listed in the handbook. One of the necessary checks is about the settlement of the soil volume around the culvert. This check should be made according to geotechnical specifications and is not covered in the handbook, hence not in the scope of this thesis. This also applies to the check against earth slide. The check of the concrete footings for arches is not in the scope of this thesis since it is not covered in the handbook and it is not any different for flexible culverts than for other bridges. As long as the radii’s of the culvert are chosen according to section 1.2.3 in the handbook, the check for the radial soil pressure against the lower corner plates is not needed. As written in the handbook, the check for safety against instability is not necessary when the checks against the building of a plastic hinge for all sections with a constant radius are performed.

The numbering of the checks in the MathCAD routine follows the numbering in the handbook. Therefore some numbers are not used in the routine.

6.3.1 Yielding in the Serviceability State

Yielding in the serviceability state needs to be checked for the construction stage as well as for the finished bridge. It is done using Navier’s equation. The check is made for the upper part of the walls in the pipe according to equation (5.a) in the handbook.

![Figure 6:37](image-url) Checking safety against yielding in the serviceability state.
6.3.2 Building of a Plastic Hinge

The check is made for the upper part of the culvert, with the use of equation (5.b) together with equations (5.c) and (b1.h) in the handbook. Two checks are prescribed in the handbook, one with maximum normal force and no moment, and one with both normal force and moment. However, when consulting with Lars Pettersson during the work with this thesis, the question of whether or not the check with zero moment is necessary for BoxCulverts have been discussed. Perhaps in a future edition of the handbook the check for the building of a plastic hinge for BoxCulverts will be different.

3) Check against the building of a plastic hinge

Calculations

\[ \eta := \frac{Z_s}{W_s}, \quad M_u := Z_s \cdot f_{yd}, \quad M_{u_cr} := \min \left[ 1, 1.429 - 0.156 \ln \left( \frac{m_t}{t} \right), \sqrt{\frac{f_{yk}}{227 \text{MPa}}} \right] \cdot M_u \]

\[ N_{cr,1} := \secO\left( h_{c, \text{red}}, R_t, F_{s,d}, E_{s}, f_{yd}, A_s, 1 \right) \]

\[ N_{cr,2} := \secO\left( h_{c, \text{red}}, R_t, F_{s,d}, E_{s}, f_{yd}, A_s, 0 \right) \]

\[ \alpha_{c,1} := \max \left( 0.8, \eta \cdot \frac{N_{cr,1}}{A_s \cdot f_{yd}} \right), \quad \alpha_{c,2} := \max \left( 0.8, \eta \cdot \frac{N_{cr,2}}{A_s \cdot f_{yd}} \right) \]

Calculations

\[ N_{cr,1} = 1.1 \times 10^3 \text{ kN/m}, \quad N_{cr,2} = 652.2 \text{ kN/m}, \quad M_{u_cr} = 30.4 \text{ kNm/m} \]

\[ \left( \frac{N_{d,u}}{N_{cr,1}} \right)^\alpha_{c,1} + \max \left( \frac{M_{d,u}}{M_{u_cr}} \right) = 0.8 \]

\[ \text{check} \left[ \left( \frac{N_{d,u}}{N_{cr,1}} \right)^\alpha_{c,1} + \max \left( \frac{M_{d,u}}{M_{u_cr}} \right) \right] = "OK!" \]

\[ \left( \frac{N_{d,u}}{N_{cr,2}} \right)^{\alpha_{c,2}} = 0.6 \]

\[ \text{check} \left[ \left( \frac{N_{d,u}}{N_{cr,2}} \right)^{\alpha_{c,2}} \right] = "OK!" \]

For some types of metal culverts \( N_u \) and \( M_u \) should be divided by 1.15, see the handbook.

Figure 6.38 Checking safety against a plastic hinge.

In order to check for the building of a plastic hinge, the buckling load according to second order theory, \( N_{cr} \), is needed. Because the parameter \( \zeta \) in the equations is calculated differently for the two checks, two values of \( N_{cr} \) need to be calculated. In the routine it is done by using the function \( \secO() \) two times with different send variables. It uses equations (b5.a) through (b5.h) in the handbook. The function also handles a third case, which is used when checking the capacity in the lower part of the pipe.
When a BoxCulvert is used it is necessary to check for the building of a plastic hinge in the corner section. The dimensioning bending moment is reduced according to equation (4.2) in the handbook. The expression for the critical buckling moment is different because of the cross-corrugation of the corner plates. It is calculated as 60% of the moment capacity. As stated in appendix 1 in the handbook, this is valid if the plate thickness is 5 mm or more. Of course it is necessary to calculate new values for the critical buckling normal forces since the corner section has a smaller radius than the top section.
Check of the corner section for BoxCulverts

\[ N_{cr.1.c} := \sec \left( h_{c,red}, R_s, E_s, d, E_l, f_{yd}, A_s, 1 \right) \]
\[ N_{cr.2.c} := \sec \left( h_{c,red}, R_s, E_s, d, E_l, f_{yd}, A_s, 0 \right) \]
\[ \alpha_{c.1.c} := \max \left( 0.8, \eta^2 \frac{N_{cr.1.c}}{A_s f_{yd}} \right) \quad \alpha_{c.2.c} := \max \left( 0.8, \eta^2 \frac{N_{cr.2.c}}{A_s f_{yd}} \right) \]
\[ M_{ucr.c} := 0.6 M_u \quad M_{ucr.c} = 18.2 \text{kN} \]
\[ M_{d.u.c} := M_{sd.u} \frac{2}{3} + M_{ld.u} \frac{1}{3} \quad M_{d.u.c} = \begin{pmatrix} 3.8 \\ 3.1 \end{pmatrix} \text{kN} \]
\[ \left( \frac{N_{d.u}}{N_{cr.1.c}} \right)^{\alpha_{c.1.c}} + \frac{\max \left( M_{d.u.c} \right)}{M_{ucr.c}} = 0.5 \]
\[ \text{check} \left[ \left( \frac{N_{d.u}}{N_{cr.1.c}} \right)^{\alpha_{c.1.c}} + \frac{\max \left( M_{d.u.c} \right)}{M_{ucr.c}} \right], 1.0 \right] = "OK!" \]
\[ \left( \frac{N_{d.u}}{N_{cr.2.c}} \right)^{\alpha_{c.2.c}} = 0.6 \]
\[ \text{check} \left[ \left( \frac{N_{d.u}}{N_{cr.2.c}} \right)^{\alpha_{c.2.c}} \right], 1.0 \right] = "OK!" \]

**Figure 6:40** Checking the corner section for BoxCulverts.
6.3.3 Capacity in the Lower Part of the Pipe

When a culvert with different radii’s is used, all sections with a constant radius need to be checked. Since the section with the top radius, $R_t$, has already been checked, the radii’s in the lower part of the culvert are now checked. Here the capacity for normal force is the only thing needed to be checked, specified by equation (5.d) in the handbook.

### 4) Check for sufficient capacity in the lower part of the pipe

#### Calculations

- $N_d := \max\left(\left|\frac{N_{d.s}}{N_{d.u}}\right|\right)$
- $N_{cr} := \text{every}_R\left(h_c, R_{\text{check}}, E_{s,d}, E_{I_s}, f_{yd}, A_s, 2\right)$
- $\alpha_c := \text{for } i \in 0..\text{last}\left(N_{cr}\right)$
  \[ \alpha_i := \max\left(0.8, \eta^2 \frac{N_{cr_i}}{A_s f_{yd}}\right) \]

#### Calculations

\[
\begin{align*}
N_d &= 356.9 \text{ kN/m} \\
N_{cr} &= \begin{pmatrix} 1.5 \times 10^3 \text{ kN/m} \\ 1.6 \times 10^3 \text{ kN/m} \end{pmatrix} \\
\left(\frac{N_d}{N_{cr}}\right)^{\alpha_c} &= \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix} \\
\text{check} \left(\frac{N_d}{N_{cr}}\right)^{\alpha_c}, 1.0 &= \begin{pmatrix} \text{"OK!"} \\ \text{"OK!"} \end{pmatrix}
\end{align*}
\]

**Figure 6:41** Checking the capacity in the lower part of the pipe.

In order to efficiently calculate $N_{cr}$ for multiple sections, the function everyR() is used in the routine. It in turn uses the function secO() for all the different radii’s that are to be checked. Because the parameters $\gamma_j$ and $\epsilon$ are calculated differently than before, the third case covered by secO() is used.

\[
\text{every}_R\left(h_c, R_{\text{check}}, E_{s,d}, E_{I_s}, f_{yd}, A_s, \epsilon_{\text{case}}\right) := \text{for } i \in 0..\text{last}(R)
\]

\[
N_{cr_i} \leftarrow \text{secO}\left(h_c, R_i, E_{s,d}, E_{I_s}, f_{yd}, A_s, \epsilon_{\text{case}}\right)
\]

**Figure 6:42** Function for calculating the effects of second order theory for the lower part of the pipe.
6.3.4 Capacity of the Bolted Connections

In the handbook the specifics for flexible metal culverts when it comes to checking the bolted connections are described. For the rest of the design checks you are referred to the Swedish code BSK 99.

\[
\frac{f_{bud}}{1.2\gamma_n} = f_{ud} = \max\left\{\frac{f_{uk}}{1.2.1.1\gamma_n},f_{yd}\right\}
\]

**Figure 6:43** Calculating the dimensional values for the stress capacity.

6.3.4.1 Ultimate Limit State

The required checks in the ultimate limit state are for the shear capacity and the capacity for combined tension and shear. Equations 6:431 through 6:433 in BSK 99 are used. The distance from the centre of the hole to the free edge is called \( a \) in the routine instead of \( e_1 \) as in BSK 99.

**Shear capacity:**

\[
d_s := d \quad f_{ud} = 323.7\text{MPa} \quad f_{bud} = 606.1\text{MPa}
\]

\[
F_{Rvd} := 0.6 A_s b f_{bud} \quad F_{Rvd} = 89.1\text{kN}
\]

\[
F_{Rbd} := 1.2 \left(\frac{a}{d} - 0.5\right) d_s f_{ud} \quad F_{Rbd} = 128.2\text{kN}
\]

\[
N_{d,u} = 356.9\frac{\text{kN}}{\text{m}} \quad \min\left(n\cdot F_{Rvd}, n\cdot F_{Rbd}\right) = 1.5 \times 10^3\frac{\text{kN}}{\text{m}}
\]

\[
\text{check}\left[\left(N_{d,u}\right), \min\left(n\cdot F_{Rvd}, n\cdot F_{Rbd}\right)\right] = "OK!"
\]

**Combined tension and shear:**

\[
F_{St} := \frac{\max\left(2 \left|\frac{M_{d,u}}{a\cdot n}\right|\right)}{a\cdot n} \quad F_{SV} := \frac{N_{d,u}}{n} \quad F_{St} = 24.7\text{kN} \quad F_{SV} = 21\text{kN}
\]

\[
F_{Rtd} := \phi_t A_s b f_{bud} \quad F_{Rtd} = 89.1\text{kN}
\]

\[
\left(\frac{F_{St}}{F_{Rtd}}\right)^2 + \left(\frac{F_{SV}}{F_{Rvd}}\right)^2 = 0.1
\]

\[
\text{check}\left[\left(\frac{F_{St}}{F_{Rtd}}\right)^2 + \left(\frac{F_{SV}}{F_{Rvd}}\right)^2\right]_{1.00} = "OK!"
\]

**Figure 6:44** Checking safety for shear and tension in the ultimate limit state.
6.3.4.2 *Fatigue Limit State*

In the fatigue limit state, checks are made for pure tension, pure shear and for combined tension and shear. To calculate the characteristic fatigue stress capacity, \( f_{rk} \), equations 6:523a and 6:523c are used together with table 6:523 and figure 6:523 in BSK 99. The checks are done according to equations 6:512a through 6:512c.

<table>
<thead>
<tr>
<th>Fatigue limit state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculations</td>
</tr>
</tbody>
</table>
| \( \phi_m := \begin{cases} 
1.0 & \text{if } f_{uk} < 410\text{MPa} \\
1.10 & \text{if } f_{uk} < 450\text{MPa} \land f_{uk} \geq 410\text{MPa} \\
1.15 & \text{if } f_{uk} < 490\text{MPa} \land f_{uk} \geq 450\text{MPa} \\
1.20 & \text{if } f_{uk} < 600\text{MPa} \land f_{uk} \geq 490\text{MPa} \\
1.25 & \text{if } f_{uk} \geq 600\text{MPa} 
\end{cases} \) |
| \( \phi_{\text{dim}} := \max \left[ \frac{25\text{mm}}{t} \right]^{0.0763} \cdot 1.0 \) |
| \( f_{rk} := \phi_m \cdot \phi_{\text{dim}} \cdot \frac{2 \cdot 10^6}{n_t} \cdot \text{MPa if } n_t < 5 \cdot 10^6 \) |
| \( \phi_m \cdot \phi_{\text{dim}} \cdot 0.885C \cdot \frac{2 \cdot 10^6}{n_t} \cdot \text{MPa if } n_t < 10^8 \land n_t \geq 5 \cdot 10^6 \) |
| \( \phi_m \cdot \phi_{\text{dim}} \cdot 0.885C \cdot \frac{2 \cdot 10^6}{10^8} \cdot \text{MPa otherwise} \) |
| \( f_{rd} := \frac{f_{rk}}{1.1 \cdot \gamma_n} \) |
| \( f_{rvd} := 0.6f_{rd} \) |

<table>
<thead>
<tr>
<th>Calculations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C = 45 )</td>
</tr>
<tr>
<td>( n_t = 4 \times 10^5 )</td>
</tr>
<tr>
<td>( \phi_m = 1.2 )</td>
</tr>
<tr>
<td>( \phi_{\text{dim}} = 1.1 )</td>
</tr>
<tr>
<td>( a = 76\text{mm} )</td>
</tr>
<tr>
<td>( n = 17 \frac{1}{\text{m}} )</td>
</tr>
<tr>
<td>( \sigma_{rd} := \frac{2}{a} \left</td>
</tr>
<tr>
<td>( \tau_{rd} := \frac{\Delta N_{d.f}}{n \cdot A_{s,b}} )</td>
</tr>
</tbody>
</table>

**Figure 6:45** Calculating needed stress capacities and the design stresses in the fatigue limit state.
### Tension:

\[
\sigma_{rd} = 72.3 \text{ MPa} \quad f_{rd} = 82.7 \text{ MPa}
\]

Check \([\sigma_{rd}, f_{rd}] = "OK!"

### Shear:

\[
\tau_{rd} = 18.4 \text{ MPa} \quad f_{rvd} = 49.6 \text{ MPa}
\]

Check \([\tau_{rd}, f_{rvd}] = "OK!"

### Combined shear and tension:

\[
\sqrt{\left(\frac{\sigma_{rd}}{f_{rd}}\right)^2 + \left(\frac{\tau_{rd}}{f_{rvd}}\right)^2} = 1
\]

Check \[\sqrt{\left(\frac{\sigma_{rd}}{f_{rd}}\right)^2 + \left(\frac{\tau_{rd}}{f_{rvd}}\right)^2}, 1.10 = "OK!"

**Figure 6:46** Checking safety for shear and tension in the fatigue limit state.

### 6.3.5 Stiffness for Installation and Handling

As described in *the handbook*, this check is not an absolute requirement, more a recommendation. The final choice of stiffness for the backfilling stages should be done with calculations for those stages, where the case \(h_c = 0\) is especially important. The recommended values in *the handbook* for this parameter is \(\gamma_m < 0.13\) for circular sections and \(\gamma_m < 0.20\) for arch-formed and low-rise sections. This is according to equation (5.g).

7) Check to ensure that the construction has adequate stiffness for installation and handling etc.

\[
\eta_m := \frac{D^2}{E \cdot I_s} \quad \eta_m = 0.1 \text{ m/kN}
\]

**Figure 6:47** Checking the stiffness for installation and handling etc.
6.4 Classification Loads A and B

(Vägverket) require classification values to be calculated for all bridges. They are simply theoretical values of axle and boogie loads in the type loads in VV Publ. 1998:78, called A and B. This is done in order to know what loads can be accepted for real vehicles that could pass over the bridge.

Since the worst stresses for the type loads are already calculated, you only need to scale the values of A and B respectively to make the stress from the worst type loads as big as the stress from the worst load case altogether. Another check has to be done for the fatigue limit state. The smallest of the values in these two checks will be the classification loads for the bridge.

Find the classification values A and B:

\[
\begin{align*}
\sigma_{v,A} &= 42.4 \text{kPa} \\
\sigma_{v,B} &= 36 \text{kPa} \\
\max(\sigma_v) &= 85.5 \text{kPa} \\
\Delta N_{d.f.A} &= 76.8 \text{kN/m} \\
\Delta N_{d.f.B} &= 65.2 \text{kN/m} \\
\Delta M_{d.f.A} &= 11.4 \text{kNm/m} \\
\Delta M_{d.f.B} &= 9.7 \text{kNm/m} \\
\end{align*}
\]

\[
\begin{align*}
\text{lcases}_A &= \text{"Klass Last a"} \\
\text{lcases}_B &= \text{"Klass Last k"} \\
A_{\sigma} &= \frac{\Delta N_{d.f.A}}{\sigma_{v,A}} \\
B_{\sigma} &= \frac{\Delta N_{d.f.B}}{\sigma_{v,B}} \\
A_N &= \frac{\Delta N_{d.f.A}}{\Delta N_{d.f.B}} \\
B_N &= \frac{\Delta N_{d.f.A}}{\Delta N_{d.f.B}} \\
A_M &= \frac{\Delta M_{d.f.A}}{\Delta M_{d.f.B}} \\
B_M &= \frac{\Delta M_{d.f.A}}{\Delta M_{d.f.B}} \\
A_{\sigma} &= 242.1 \text{kN} \\
A_N &= 120 \text{kN} \\
A_M &= 120 \text{kN} \\
B_{\sigma} &= 427.9 \text{kN} \\
B_N &= 212.1 \text{kN} \\
B_M &= 212.1 \text{kN} \\
A_{\text{val}} &= \min(A_{\sigma}, A_N, A_M) \\
B_{\text{val}} &= \min(B_{\sigma}, B_N, B_M) \\
A &= 120 \text{kN} \\
B &= 180 \text{kN} \\
A_{\text{val}} &= 120 \text{kN} \\
B_{\text{val}} &= 212.1 \text{kN} \\
\end{align*}
\]

Figure 6:48 Calculating the classification loads, A and B.
PART 3

Study of Failure Loads
7 Ultimate Loads

The design method in the handbook is not made to calculate deflections, thrusts or moments in the culvert; it is made to provide a safety against failure. Therefore, when comparing calculated values to field test measurements, the most important values are the failure loads. This has been done by finding the theoretical load that makes the check against the building of a plastic hinge fail. After consulting Lars Pettersson, it was decided that for Box Culverts, used in the Järpås field tests, the design check with the moment equal to zero could be disregarded.

Besides comparing calculations to field tests it is interesting to study the impact of some of the parameters used in the design method. This was done by comparing failure loads for culverts where one or two parameters were changed. The Enköping 97 field test was used as the original culvert.

In this report the Enköping field tests are named Enköping followed by the relative degree of compaction of the specific field test. The Järpås field tests are named Järpås followed by the span of the specific field test.

The setup for the load was the same for all the field tests. It was a single axis with two contact surfaces to simulate tires. The area of a contact surface was 0.2 x 0.6 m². The distance between the centre points of the contact surfaces was 2.0 m in the direction of the long side of the rectangular surfaces. The load was increased until failure occurred. The failure load is the axle load, which is distributed equally to both contact surfaces. The following figures are from the Järpås field tests and show the loading of the structures.

![Figure 7:1 The way the load was applied in the field tests.](image)
The calculations were made with all the partial coefficients set to 1 in order to find the built in safety of the design method when comparing the calculations to the field tests. The partial coefficients were also set to 1 for the parameter study. All calculations were made for the effective height of cover; i.e. $h_{c,red}$ was put equal to $h_c$.

The only design check that was used was the check against the building of a plastic hinge at the top of the culvert. The axle load was increased until one of the two checks failed, either the interaction of normal force and moment, or the normal force without moment. For the BoxCulverts in the Järpås field tests the load was increased until the interaction of normal force and moment check failed. The check of normal force without moment was disregarded for BoxCulverts.
7.1 Comparison to Field Tests

7.1.1 Data for the Calculations

The following data have been used to perform the calculations for the Enköping field tests. Notice that the plate thickness is different for the Enköping 100 culvert. Data for the bolts are not used in the check against the building of a plastic hinge, hence not of interest. The name of the profile “E” corresponds to the figures in the handbook, i.e. it is a low-rise culvert. The high value for the elastic yield stress limit is because of cold forming of the steel plates.

<table>
<thead>
<tr>
<th>Type of profile</th>
<th>Enköping 100</th>
<th>Enköping 97</th>
<th>Enköping 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of the profile</td>
<td>H</td>
<td>3.02</td>
<td>3.02</td>
</tr>
<tr>
<td>Theoretical span width</td>
<td>D</td>
<td>6.05</td>
<td>6.05</td>
</tr>
<tr>
<td>Top radius</td>
<td>R_t</td>
<td>3.052</td>
<td>3.052</td>
</tr>
<tr>
<td>Side radius</td>
<td>R_s, R_t, R_t</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Corner radius</td>
<td>R_c</td>
<td>1.308</td>
<td>1.308</td>
</tr>
<tr>
<td>Depth of cover</td>
<td>h_c</td>
<td>0.75</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**Steel plates**

<table>
<thead>
<tr>
<th>Thickness</th>
<th>Enköping 100</th>
<th>Enköping 97</th>
<th>Enköping 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>2.95</td>
<td>3.25</td>
<td>3.25</td>
</tr>
<tr>
<td>Corrugation height</td>
<td>t_corr</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>Wave-length</td>
<td>c_val</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Radii of curvature</td>
<td>r</td>
<td>53</td>
<td>53</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>E_s</td>
<td>203</td>
<td>203</td>
</tr>
<tr>
<td>Elastisk yield stress limit</td>
<td>f_yk</td>
<td>350</td>
<td>350</td>
</tr>
</tbody>
</table>

**Soil**

<table>
<thead>
<tr>
<th>Relative degree of compaction</th>
<th>Enköping 100</th>
<th>Enköping 97</th>
<th>Enköping 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>RP</td>
<td>100</td>
<td>97</td>
<td>90</td>
</tr>
<tr>
<td>Optimal density (Modified Proctor)</td>
<td>/_opt</td>
<td>19.4</td>
<td>19.4</td>
</tr>
<tr>
<td>Particle size distribution d_{10}</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Particle size distribution d_{50}</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Particle size distribution d_{60}</td>
<td>1.32</td>
<td>1.32</td>
<td>1.32</td>
</tr>
<tr>
<td>Unit weight of soil 1</td>
<td>/_1</td>
<td>/_cv</td>
<td>/_cv</td>
</tr>
<tr>
<td>Unit weight of soil 2</td>
<td>/_1</td>
<td>/_cv</td>
<td>/_cv</td>
</tr>
</tbody>
</table>

Table 7.1 Data for the Enköping field tests.
The two culverts in the Järpås field tests were BoxCulverts. According to the test report (Bayoglu Flener, 2006), the relative degree of compaction of the soil was 88% Standard Proctor. The particle size was 0-90 mm and the maximum density was measured to 2,14 g/cm³ Standard Proctor. Because the particle size distribution was not specified, the calculations have been performed for the three types of soil listed in table B2.2 in the handbook. For comparison the calculations have also been performed for 97% relative degree of compaction of the soil, which is a normal value and what the table in the handbook is made for. To be able to make comparisons between the two tests, the calculations have been performed for the two plate thicknesses, 4 mm and 7 mm, for both structures.

Table 7:2 Data for the Järpås field tests.

<table>
<thead>
<tr>
<th></th>
<th>Järpås 8</th>
<th>Järpås 14</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of profile</strong></td>
<td>profile</td>
<td>G</td>
</tr>
<tr>
<td><strong>Height of the profile</strong></td>
<td>H</td>
<td>2.44</td>
</tr>
<tr>
<td><strong>Theoretical span width</strong></td>
<td>D</td>
<td>8.087</td>
</tr>
<tr>
<td><strong>Top radius</strong></td>
<td>R_t</td>
<td>8.892</td>
</tr>
<tr>
<td><strong>Side radius</strong></td>
<td>R_s</td>
<td>1.088</td>
</tr>
<tr>
<td><strong>Depth of cover</strong></td>
<td>h_c</td>
<td>0.45</td>
</tr>
<tr>
<td><strong>Steel plates</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Thickness</strong></td>
<td>t</td>
<td>4</td>
</tr>
<tr>
<td><strong>Corrugation height</strong></td>
<td>h_corr</td>
<td>140</td>
</tr>
<tr>
<td><strong>Wave-length</strong></td>
<td>c_val</td>
<td>381</td>
</tr>
<tr>
<td><strong>Radii of curvature</strong></td>
<td>r</td>
<td>76.2</td>
</tr>
<tr>
<td><strong>Young's modulus</strong></td>
<td>E_s</td>
<td>205.7</td>
</tr>
<tr>
<td><strong>Elastisk yield stress limit</strong></td>
<td>f_yk</td>
<td>352</td>
</tr>
<tr>
<td><strong>Soil</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Unit weight of soil 1</strong></td>
<td>ρ_1</td>
<td>ρ_cv</td>
</tr>
<tr>
<td><strong>Unit weight of soil 2</strong></td>
<td>ρ_2</td>
<td>ρ_cv</td>
</tr>
</tbody>
</table>

Table 7:2 Data for the Järpås field tests.
7.1.2 Results

The results for the three Enköping field tests are shown in the following table. They were all calculated to fail by the interaction of normal force and moment.

<table>
<thead>
<tr>
<th>Field test</th>
<th>Calculated</th>
<th>Measured</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enköping 100</td>
<td>590</td>
<td>676</td>
<td>87.3%</td>
</tr>
<tr>
<td>Enköping 97</td>
<td>467</td>
<td>524</td>
<td>89.1%</td>
</tr>
<tr>
<td>Enköping 90</td>
<td>261</td>
<td>372</td>
<td>70.2%</td>
</tr>
</tbody>
</table>

Table 7:3 Calculated and measured failure loads (kN) for the Enköping field tests.

The measured failure load for Järpås 14 m was 340 kN. It wasn’t as clear what the failure load was in the Järpås 8 m field test. It could be approximated around 400 kN, but the load could be increased to 479 kN with increasing deformations. The following table lists the calculated failure loads for the Järpås field tests. Because these culverts were BoxCulverts they were all calculated to fail by the interaction of normal force and moment.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Järpås 8 m</th>
<th>Järpås 14 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprängsten</td>
<td>114</td>
<td>55</td>
</tr>
<tr>
<td>Förstärkningslager</td>
<td>141</td>
<td>85</td>
</tr>
<tr>
<td>Bärlager</td>
<td>138</td>
<td>82</td>
</tr>
<tr>
<td>Sprängsten</td>
<td>203</td>
<td>146</td>
</tr>
<tr>
<td>Förstärkningslager</td>
<td>293</td>
<td>279</td>
</tr>
<tr>
<td>Bärlager</td>
<td>324</td>
<td>367</td>
</tr>
<tr>
<td>Sprängsten</td>
<td>180</td>
<td>106</td>
</tr>
<tr>
<td>Förstärkningslager</td>
<td>218</td>
<td>148</td>
</tr>
<tr>
<td>Bärlager</td>
<td>215</td>
<td>146</td>
</tr>
<tr>
<td>Sprängsten</td>
<td>292</td>
<td>227</td>
</tr>
<tr>
<td>Förstärkningslager</td>
<td>398</td>
<td>367</td>
</tr>
<tr>
<td>Bärlager</td>
<td>446</td>
<td>433</td>
</tr>
</tbody>
</table>

Table 7:4 Calculated failure loads for the Järpås field tests.

7.2 Parameter Study

A parameter study for the culvert in the Enköping field test has been performed. The calculation data from the Enköping 97 field test were used, except for the parameters studied in each case.

7.2.1 Height of cover

The effect of the effective height of cover was studied for three different degrees of compaction of the soil. A comparison was also made for steel of lower quality. All calculated points in the graph below failed for the interaction of normal force and moment except for the smallest height of cover for RP = 97 and the two smallest heights of cover for RP = 90, which failed for the normal force without moment.
7.2.2 Degree of Compaction

The effect of the relative degree of compaction of the soil was studied for two different heights of cover. All the calculated points in the graph below failed for the interaction of normal force and moment.

Figure 7:4 The calculated failure load at different heights of cover for three different degrees of compaction of the soil.

Figure 7:5 The calculated failure load at different degrees of compaction.
7.2.3 Plate Thickness

The effect of the plate thickness on the failure load was studied for two different heights of cover. All the calculated points in the graph below failed for the interaction of normal force and moment except for the four smallest plate thicknesses for $h_c = 0.5$ m, which failed for the normal force without moment.

![Graph showing the calculated failure load for different plate thicknesses.]

**Figure 7:6** The calculated failure load for different plate thicknesses.

7.2.4 Span Width

To study the effect of the span width on the failure load a few parameters for the culvert profile were changed as the span width was changed. The original values from the Enköping 97 culvert for the height of the profile, top radius, side radius, bottom radius and corner radius were all multiplied with the same factor (the new theoretical span width divided by the original theoretical span width). This was done for two different plate thicknesses as well as for SuperCor plates. The height of cover was changed to 0.5 m and the backfill material was changed to “bärlagermaterial” according to table B2.2 in *the handbook*.

For low span widths the calculated failure occurred for the interaction of normal force and moment and for high span widths it was the check for normal force without moment that failed.
7.2.5 Backfill Material

The calculated failure load for the soil in Enköping 97 and for the three types of soil listed in table B2.2 in the handbook are listed in the table below. They were all calculated to fail in the check for normal force without moment. The height of cover was changed to 0.5 m.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Failure Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field Test Enköping</td>
<td>432</td>
</tr>
<tr>
<td>Krossad Sprängsten</td>
<td>438</td>
</tr>
<tr>
<td>Förstärkningslager</td>
<td>675</td>
</tr>
<tr>
<td>Bärlagermaterial</td>
<td>761</td>
</tr>
</tbody>
</table>

Table 7:5 Calculated failure loads (kN) for different backfill material.

Figure 7:7 The calculated failure load at different span widths.
7.3 Conclusions

The MathCAD routine developed is very a powerful tool when performing calculations according to the handbook. Without it, it would have been an enormous work to perform all the calculations needed for the parameter studies. It definitely shortens the design process and makes design optimizations possible.

The results show that both for preliminary design and for detailed design, a parameter study should be made to find the optimal solution for the structure at hand. Automated calculations as the routine developed in this master’s thesis prove to be powerful tools in such studies.

Parameter studies show that increasing the height of cover gives an almost linear effect on the failure load for the heights of cover studied.

The effect of better compaction of the backfill material is great and it increases with larger heights of cover. However, increasing the relative degree of compaction is less effective when the degree of compaction is already high to begin with.

The effect of the steel quality is small for low heights of cover and increasing with higher heights of cover. This is natural since increasing the height of cover increases the total load on the structure.

Increasing the plate thickness is not very effective and should not be the first choice for the designer to increase the failure load.

Using plates with larger corrugations do not always lead to a higher failure load. For some span widths it is more effective with plates with smaller corrugations. However, large corrugations increase the largest possible span widths greatly.

Choosing backfill material with better properties has a great impact on the failure load. It can be a good way to improve a construction when needed.

The Järpås calculations show that the effect of increasing the plate thickness is less when the soil stiffness is great compared to the plate stiffness. Increasing the soil stiffness has a greater impact on the failure load when the plate stiffness is lower. The optimal construction should be a culvert with low plate stiffness and high soil stiffness.

The measured and calculated values for the failure load correspond well for the Enköping field tests. The calculated values are closer to measured for high relative compaction of the soil and always on the safe side. The importance of the compaction of the soil is clear.

The calculated values for the Järpås field tests are not directly comparable to the measured values because of the uncertainty about the particle size distribution and the relative degree of compaction of the soil. When comparing the different calculations to each other an important note is that for highly compacted soil of good quality, the culvert with a 14 m theoretical span width can have a higher failure load than the culvert with an 8 m span width. It is also clear that improving the compaction of the soil is more effective for raising the failure load than increasing the plate thickness.
References

American Iron and Steel Institute, AISI (1971) *Handbook of Steel Drainage and Highway Construction Products, 2nd Edition*, CSPI, Corrugated Steel Pipe Institute


Järpås II – SKA KOMPLETTERAS INNAN TRYCKNING!


http://contech-cpi.com/corporate/history/255

http://encyclopedia.thefreedictionary.com

http://www.thefreedictionary.com/


Appendixes

Appendix A - Principles for the Numerical Method in the MathCAD Routine

The Boussinesq formulae work very well for calculating the vertical stresses at a certain depth at a given point. To calculate the equivalent line load you need to know the worst vertical stresses from the given loads. The problem is that you don’t know the location of the point where the worst stresses appear. Therefore the MathCAD routine uses a numerical method to find this point. It consists of a number of functions but only one of the functions needs to be called from the main routine. It is called allPoints().

Given the coordinates of every load in the load case, the area between the lowest and highest values for the coordinates of the load area corners is determined. Initial calculation points are created by using the number of divisions for x and y values specified by the user in the variables xdivisions and ydivisions. Extra points are added at the center of each load.

The vertical pressure at depth \( h_{c,red} \) is calculated for all of these initial points. The ten percent of the points with the highest vertical pressure are then used to form new calculation points. Around each of those initial points a new grid is made with a spacing that is four times smaller than the spacing for the previous grid.

When the pressures are calculated for the new points, a comparison is made between the maximum pressure from the new points and the maximum pressure from the previous calculations. This procedure is repeated until the difference between two rounds of calculations is less than five percent.

The pressures at all the calculated points are returned to the main routine along with the coordinates of those points. Information is also returned about the number of times new denser grids have been created.

![Figure A:1 Calculation points in the numerical method.](image-url)
Appendix B - Graphs of Equivalent Line Loads

The following graphs have been calculated with the numerical method described in appendix A. The input have been $x_{\text{divisions}} = 15$, $y_{\text{divisions}} = 10$ and $\text{neg} = 0$. The height of cover is the effective height of cover, i.e. the actual height of cover after the backfilling is complete.

**Figure B:1** Equivalent line load (kN/m) for equivalent load type 1 from BRO 2004 as a function of the height of cover (m). As calculated by the routine and as given in the handbook.

**Figure B:2** Equivalent line load (kN/m) for equivalent load type 2 from BRO 2004 as a function of the height of cover (m). As calculated by the routine and as given in the handbook.
Figure B:3 Equivalent line load (kN/m) for equivalent load type 4 from BRO 2004 as a function of the height of cover (m). As calculated by the routine and as given in the handbook.

Figure B:4 Equivalent line load (kN/m) for the fatigue load from BRO 2004 as a function of the height of cover (m). As calculated by the routine.
Figure B:5 Equivalent line load (kN/m) for type load a from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.

Figure B:6 Equivalent line load (kN/m) for type load b from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.
Figure B.7  Equivalent line load (kN/m) for type load "c" from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.

Figure B.8  Equivalent line load (kN/m) for type load "d" from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.
Figure B:9 Equivalent line load (kN/m) for type load e from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.

Figure B:10 Equivalent line load (kN/m) for type load f from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.
Figure B:11 Equivalent line load (kN/m) for type load g from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.

Figure B:12 Equivalent line load (kN/m) for type load h from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.
Figure B:13 Equivalent line load (kN/m) for type load i from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.

Figure B:14 Equivalent line load (kN/m) for type load j from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.
Figure B:15 Equivalent line load (kN/m) for type load k from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.

Figure B:16 Equivalent line load (kN/m) for type load I from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.
Figure B:17 Equivalent line load (kN/m) for type load II from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.

Figure B:18 Equivalent line load (kN/m) for type load II A from VV Publ. 1998:78 as a function of the height of cover (m). As calculated by the routine without the dynamic effect.
Appendix C - Predefined Equivalent Loads in the Routine

Loads according to the Swedish BRO 04

Ekvivalentlast 1

\[
\begin{pmatrix}
250 \\
250 \\
250 \\
250 \\
250 \\
250 \\
170 \\
170 \\
170 \\
170 \\
170 \\
170
\end{pmatrix}
\]

\[P_{ekv.1} := 0.5 \text{kN} \cdot \begin{pmatrix}
0 & 0 & 0.2 & 0.6 \\
0 & 2 & 0.2 & 0.6 \\
1.5 & 0 & 0.2 & 0.6 \\
1.5 & 2 & 0.2 & 0.6 \\
7.5 & 0 & 0.2 & 0.6 \\
7.5 & 2 & 0.2 & 0.6 \\
0 & 3 & 0.2 & 0.6 \\
0 & 5 & 0.2 & 0.6 \\
1.5 & 3 & 0.2 & 0.6 \\
1.5 & 5 & 0.2 & 0.6 \\
7.5 & 3 & 0.2 & 0.6 \\
7.5 & 5 & 0.2 & 0.6
\end{pmatrix} \ m\]

Ekvivalentlast 2

\[
\begin{pmatrix}
310 \\
310 \\
210 \\
210
\end{pmatrix}
\]

\[P_{ekv.2} := 0.5 \text{kN} \cdot \begin{pmatrix}
0 & 0 & 0.2 & 0.6 \\
0 & 2 & 0.2 & 0.6 \\
0 & 3 & 0.2 & 0.6 \\
0 & 5 & 0.2 & 0.6
\end{pmatrix} \ m\]

Ekvivalentlast 4

\[
\begin{pmatrix}
325 \\
325 \\
325 \\
325 \\
325 \\
325
\end{pmatrix}
\]

\[P_{ekv.4} := 0.5 \text{kN} \cdot \begin{pmatrix}
0 & 0 & 0.2 & 0.6 \\
0 & 2 & 0.2 & 0.6 \\
1.5 & 0 & 0.2 & 0.6 \\
1.5 & 2 & 0.2 & 0.6 \\
7.5 & 0 & 0.2 & 0.6 \\
7.5 & 2 & 0.2 & 0.6
\end{pmatrix} \ m\]
**Utmattningslast - fatigue limit load**

\[
\begin{bmatrix}
150 \\
150 \\
150 \\
150 \\
180 \\
180 \\
180 \\
180
\end{bmatrix}
\cdot 0.5\text{kN} =
\begin{bmatrix}
0 & 0 & 0.2 & 0.6 \\
0 & 2 & 0.2 & 0.6 \\
1.5 & 0 & 0.2 & 0.6 \\
1.5 & 2 & 0.2 & 0.6 \\
7.5 & 0 & 0.2 & 0.6 \\
7.5 & 2 & 0.2 & 0.6 \\
9.5 & 0 & 0.2 & 0.6 \\
9.5 & 2 & 0.2 & 0.6
\end{bmatrix}
\]
Appendix D - Predefined Type Loads in the Routine

\[
\begin{align*}
\text{cc} & := 1.7\text{m} & \text{tl} & := 0.3\text{m} & \text{ll} & := 0.2\text{m} & \text{utbr} & := 3\text{m} \\
q_{kl} & := \frac{5\text{kN}}{\text{utbr}} & A & := 120\text{kN} & B & := 180\text{kN} & B_2 & := 160\text{kN} \\
\text{KlassLast}_a & := \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} A & \text{utba} & := \begin{pmatrix} 0 \text{m} & 0 \text{m} & \text{ll} & \text{tl} \\ 0 \text{m} & \text{cc} & \text{ll} & \text{tl} \end{pmatrix} \\
\text{KlassLast}_a & := \text{stack}\left(\text{KlassLast}_a, 0.8\text{KlassLast}_a\right) \\
\text{utba} & := \text{stack}\left(\text{utba}, \text{augment}\left(\text{utba}^{(0)}, \text{utba}^{(1)} + 3\text{m}, \text{utba}^{(2)}, \text{utba}^{(3)}\right)\right) \\
\text{KlassLast}_b & := \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \frac{0.8B}{2} & \text{utbb} & := \begin{pmatrix} 0 \text{m} & 0 \text{m} & \text{ll} & \text{tl} \\ 0 \text{m} & \text{cc} & \text{ll} & \text{tl} \end{pmatrix} \\
\text{KlassLast}_b & := \text{stack}\left(\text{KlassLast}_b, 0.8\text{KlassLast}_b\right) \\
\text{utbb} & := \text{stack}\left(\text{utbb}, \text{augment}\left(\text{utbb}^{(0)}, \text{utbb}^{(1)} + 3\text{m}, \text{utbb}^{(2)}, \text{utbb}^{(3)}\right)\right) \\
\text{KlassLast}_c & := \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \frac{B}{2} & \text{utbc} & := \begin{pmatrix} 0 \text{m} & 0 \text{m} & \text{ll} & \text{tl} \\ 0 \text{m} & \text{cc} & \text{ll} & \text{tl} \\ 1.3\text{m} & 0 \text{m} & \text{ll} & \text{tl} \\ 1.3\text{m} & \text{cc} & \text{ll} & \text{tl} \end{pmatrix} \\
\text{KlassLast}_c & := \text{stack}\left(\text{KlassLast}_c, 0.8\text{KlassLast}_c\right) \\
\text{utbc} & := \text{stack}\left(\text{utbc}, \text{augment}\left(\text{utbc}^{(0)}, \text{utbc}^{(1)} + 3\text{m}, \text{utbc}^{(2)}, \text{utbc}^{(3)}\right)\right) \\
\text{KlassLast}_d & := \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} \frac{1.10B}{2} & \text{utbd} & := \begin{pmatrix} 0 \text{m} & 0 \text{m} & \text{ll} & \text{tl} \\ 0 \text{m} & \text{cc} & \text{ll} & \text{tl} \\ 1.8\text{m} & 0 \text{m} & \text{ll} & \text{tl} \\ 1.8\text{m} & \text{cc} & \text{ll} & \text{tl} \end{pmatrix} \\
\text{KlassLast}_d & := \text{stack}\left(\text{KlassLast}_d, 0.8\text{KlassLast}_d\right) \\
\text{utbd} & := \text{stack}\left(\text{utbd}, \text{augment}\left(\text{utbd}^{(0)}, \text{utbd}^{(1)} + 3\text{m}, \text{utbd}^{(2)}, \text{utbd}^{(3)}\right)\right)
\end{align*}
\]
\[
\begin{pmatrix}
0.5 \\
0.5 \\
0.5 \\
0.5 \\
\end{pmatrix}
\] 
\[
\begin{pmatrix}
1.17B \\
3 \\
\end{pmatrix}
\]
\[
\begin{pmatrix}
0m \\
0m \\
1m \\
1m \\
\end{pmatrix}
\]
\[
\begin{pmatrix}
0m cc \\
1m cc \\
2m cc \\
\end{pmatrix}
\]
\[\text{utb}_e := \text{stack}\left(\text{KlassLast}_e, 0.8\text{KlassLast}_e\right)\]
\[\text{KlassLast}_e := \begin{pmatrix}
0.5 \\
0.5 \\
1.17B \\
3 \\
\end{pmatrix}
\]
\[\begin{pmatrix}
0m \\
0m \\
1m \\
1m \\
\end{pmatrix}
\]
\[\begin{pmatrix}
0m cc \\
1m cc \\
2m cc \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0.5 \\
0.5 \\
\end{pmatrix}
\] 
\[
\begin{pmatrix}
0.5 \\
\end{pmatrix}
\] 
\[
\begin{pmatrix}
1.32B \\
3 \\
\end{pmatrix}
\]
\[
\begin{pmatrix}
0m \\
0m \\
1.3m \\
1.3m \\
\end{pmatrix}
\]
\[
\begin{pmatrix}
0m cc \\
1m cc \\
2.6m cc \\
\end{pmatrix}
\]
\[\text{utb}_f := \text{stack}\left(\text{KlassLast}_f, 0.8\text{KlassLast}_f\right)\]
\[\text{KlassLast}_f := \begin{pmatrix}
0.5 \\
0.5 \\
1.32B \\
3 \\
\end{pmatrix}
\]
\[\begin{pmatrix}
0m \\
0m \\
1.3m \\
1.3m \\
\end{pmatrix}
\]
\[\begin{pmatrix}
0m cc \\
1m cc \\
2.6m cc \\
\end{pmatrix}
\]
\[\text{utb}_f := \text{stack}\left(\text{KlassLast}_f, 0.8\text{KlassLast}_f\right)\]
\[\text{KlassLast}_f := \begin{pmatrix}
0.5 \\
0.5 \\
0.5 \\
\end{pmatrix}
\]
\[\begin{pmatrix}
0m \\
0m \\
0m \\
\end{pmatrix}
\]
\[\begin{pmatrix}
0m cc \\
0m cc \\
0m cc \\
\end{pmatrix}
\]
\[
\begin{align*}
\text{KlassLast}_g & := \begin{pmatrix} 0.44 \\ 0.44 \\ 1.32 \\ 1.32 \\ 1.32 \\ 1.32 \\ 1.32 \\ 1.32 \\ 1.32 \end{pmatrix} \\
\text{utbg}_g & := \begin{pmatrix} 0m \\ 0m \\ 0m \\ 0m \\ 0m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \\ 5m \end{pmatrix} \\
\text{KlassLast}_g & := \text{stack}(\text{KlassLast}_g, 0.8\text{KlassLast}_g) \\
\text{utbg}_g & := \text{stack}(\text{utbg}_g, \text{augment}(\text{utbg}_g, 3m, \text{utbg}_g, \text{utbg}_g)) \\
\text{qKlass}_g & := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\
\text{Klasspos}_g & := \begin{pmatrix} \text{utbr} - 0.5 \times (\text{utbr} - \text{cc}) & -7.5m & 5m & \text{utbr} \\ \text{utbr} - 0.5 \times (\text{utbr} - \text{cc}) & 7.5m & 5m & \text{utbr} \\ \text{utbr} - 0.5 \times (\text{utbr} - \text{cc}) & 22.5m & 5m & \text{utbr} \end{pmatrix} \\
\text{qKlass}_g & := \text{stack}(\text{qKlass}_g, \text{qKlass}_g) \\
\text{Klasspos}_g & := \text{stack}(\text{Klasspos}_g, \text{augment}(\text{Klasspos}_g, 3m, \text{Klasspos}_g, \text{Klasspos}_g))
\end{align*}
\]
\[
\begin{pmatrix}
0.55 \\
0.55 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
0.5
\end{pmatrix} \cdot 0.5 \cdot B := utbh
\]

\[
KlassLast_h := \text{stack} (KlassLast_h, 0.8 \cdot \text{KlassLast}_h)
\]

\[
\begin{pmatrix}
0m \\
0m \\
3m \\
3m \\
4.3m \\
4.3m \\
9.3m \\
9.3m \\
10.6m \\
10.6m \\
11.9m \\
11.9m
\end{pmatrix}
\]

\[
\text{utb}_h := \text{stack} \left( \text{utb}_h, \text{augment} \left( \text{utb}_{h\{1\}}, \text{utb}_{h\{2\}} + 3m, \text{utb}_{h\{3\}} \right) \right)
\]

\[
q_{Klass_h} := \begin{pmatrix}
1 \\
1 \\
q_{k1}
\end{pmatrix}
\]

\[
Klasspos_h := \begin{pmatrix}
\text{utbr} \cdot 0.5 - 0.5(\text{utbr} - \text{cc}) - 7.5m \\
\text{utbr} \cdot 0.5 - 0.5(\text{utbr} - \text{cc}) + 14.4m
\end{pmatrix}
\]

\[
q_{Klass_h} := \text{stack}(q_{Klass_h}, q_{Klass_h})
\]

\[
\text{Klasspos}_h := \text{stack} \left( \text{Klasspos}_h, \text{augment} \left( \text{Klasspos}_{h\{1\}}, \text{Klasspos}_{h\{2\}} + 3m, \text{Klasspos}_{h\{3\}} \right) \right)
\]
\[
\begin{align*}
\text{KlassLast}_i & := \begin{pmatrix}
0.44 \\
0.44 \\
1.10 \\
2 \\
1.10 \\
2 \\
1.10 \\
2 \\
1.10 \\
2 \\
1.10 \\
2 \\
1.10 \\
2 \\
1.10 \\
2 \\
0.66 \\
2 \\
0.66 \\
2 \\
0.66 \\
2 \\
0.66 \\
2 \\
0.66 \\
2 \\
0.66 \\
2 \\
0.66 \\
2 \\
0.66 \\
2 \\
0.66 \\
2 \\
\end{pmatrix} \\
\text{utb}_i & := \begin{pmatrix}
0m & 0m & ll & tl \\
0m & cc & ll & tl \\
3.8m & 0m & ll & tl \\
3.8m & cc & ll & tl \\
5.6m & 0m & ll & tl \\
5.6m & cc & ll & tl \\
9.6m & 0m & ll & tl \\
9.6m & cc & ll & tl \\
11.4m & 0m & ll & tl \\
11.4m & cc & ll & tl \\
15m & 0m & ll & tl \\
15m & cc & ll & tl \\
16m & 0m & ll & tl \\
16m & cc & ll & tl \\
\end{pmatrix}
\end{align*}
\]

\[
\text{KlassLast}_i := \text{stack}\left(\text{KlassLast}_i, 0.8 \text{KlassLast}_i\right)
\]

\[
\text{utb}_i := \text{stack}\left(\text{utb}_i, \text{augment}\left(\text{utb}_i^{(0)}, \text{utb}_i^{(1)} + 3m, \text{utb}_i^{(2)}, \text{utb}_i^{(3)}\right)\right)
\]

\[
\text{qKlass}_i := \begin{pmatrix} 1 \\ 1 \end{pmatrix} q_{kl} \\
\text{Klasspos}_i := \begin{pmatrix} \text{utbr}_i \cdot 0.5 - 0.5(\text{utbr}_i - cc) - 7.5m & 5m & \text{utbr}_i \\
\text{utbr}_i \cdot 0.5 - 0.5(\text{utbr}_i - cc) & 18.5m & 5m & \text{utbr}_i \end{pmatrix}
\]

\[
\text{qKlass}_i := \text{stack}\left(\text{qKlass}_i, \text{qKlass}_i\right)
\]

\[
\text{Klasspos}_i := \text{stack}\left(\text{Klasspos}_i, \text{augment}\left(\text{Klasspos}_i^{(0)}, \text{Klasspos}_i^{(1)} + 3m, \text{Klasspos}_i^{(2)}, \text{Klasspos}_i^{(3)}\right)\right)
\]
\[
KlassLast_j := \text{stack}\left(\begin{array}{cccc}
0.44 & 0.44 & 0.44 & 0.44 \\
0.44 & 0.44 & 0.44 & 0.44 \\
1.32 & 3 & 3 & 3 \\
1.32 & 3 & 3 & 3 \\
1.32 & 3 & 3 & 3 \\
1.32 & 3 & 3 & 3 \\
\end{array}\right)\cdot 0.5B
\]
\[
\text{utb}_1 := \left(\begin{array}{c}
0m \\
0m \\
3m \\
3m \\
3m \\
3m \\
5m \\
5m
\end{array}\right)
\]
\[
\text{utb}_2 := \text{augment}\left(\text{utb}_1^{(0)} + 10m + \max\left(\text{utb}_1^{(0)}\right), \text{submatrix}(\text{utb}_1, 0, \text{rows}(\text{utb}_1) - 1, 1, \text{cols}(\text{utb}_1) - 1)\right)
\]
\[
\text{utb}_3 := \text{augment}\left(\text{utb}_1^{(0)} + 50m + \max\left(\text{utb}_2^{(0)}\right), \text{submatrix}(\text{utb}_1, 0, \text{rows}(\text{utb}_1) - 1, 1, \text{cols}(\text{utb}_1) - 1)\right)
\]
\[
\text{utb}_4 := \text{augment}\left(\text{utb}_1^{(0)} + 10m + \max\left(\text{utb}_3^{(0)}\right), \text{submatrix}(\text{utb}_1, 0, \text{rows}(\text{utb}_1) - 1, 1, \text{cols}(\text{utb}_1) - 1)\right)
\]
\[
\text{utb}_j := \text{stack}(\text{utb}_1, \text{utb}_2, \text{utb}_3, \text{utb}_4)
\]
\[
KlassLast_j := \text{stack}(\text{KlassLast}_j, 0.8 \text{KlassLast}_j)
\]
\[
\text{utb}_j := \text{stack}(\text{utb}_j, \text{augment}(\text{utb}_j^{(0)}, \text{utb}_j^{(1)} + 3m, \text{utb}_j^{(2)}, \text{utb}_j^{(3)}))
\]
\[
\text{utb}_j \cdot \text{utb}_j^{(0)} \cdot \text{utb}_j^{(1)} \cdot \text{utb}_j^{(2)} \cdot \text{utb}_j^{(3)}
\]
\[
qKlass_j := \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}
\]
\[
Klasspos_j := \begin{pmatrix} \text{utbr} - 0.5 - 0.5(\text{utbr} - \text{cc}) - 7.5m \\ \text{utbr} - 0.5 - 0.5(\text{utbr} - \text{cc}) \\ \text{utbr} - 0.5 - 0.5(\text{utbr} - \text{cc}) \\ \text{utbr} - 0.5 - 0.5(\text{utbr} - \text{cc}) \end{pmatrix}
\]
\[
qKlass_j := \text{stack}(qKlass_j, qKlass_j)
\]
\[
Klasspos_j := \text{stack}(\text{Klasspos}_j, \text{augment}(\text{Klasspos}_j^{(0)}, \text{Klasspos}_j^{(1)} + 3m, \text{Klasspos}_j^{(2)}, \text{Klasspos}_j^{(3)}))
\]
\[
\begin{pmatrix}
0.55 & 0.55 \\
0.55 & 0.55 \\
1/2 & 1/2 \\
1/2 & 1/2 \\
1/2 & 1/2 \\
1/2 & 1/2 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
0m & 0m & ll & tl \\
0m & cc & ll & tl \\
3m & 0m & ll & tl \\
3m & cc & ll & tl \\
4.3m & 0m & ll & tl \\
4.3m & cc & ll & tl \\
9.3m & 0m & ll & tl \\
9.3m & cc & ll & tl \\
10.6m & 0m & ll & tl \\
10.6m & cc & ll & tl \\
11.9m & 0m & ll & tl \\
11.9m & cc & ll & tl \\
\end{pmatrix}
\]

\[\text{KlassLast}_k := \text{stack}
\begin{pmatrix}
0.55 & 0.55 \\
0.55 & 0.55 \\
1/2 & 1/2 \\
1/2 & 1/2 \\
1/2 & 1/2 \\
1/2 & 1/2 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
1.32 & 1.32 \\
3/3 & 3/3 \\
\end{pmatrix}, -0.5B \]

\[\text{utb}_{k1} := \begin{pmatrix}
0m & 0m & ll & tl \\
0m & cc & ll & tl \\
3m & 0m & ll & tl \\
3m & cc & ll & tl \\
4.3m & 0m & ll & tl \\
4.3m & cc & ll & tl \\
9.3m & 0m & ll & tl \\
9.3m & cc & ll & tl \\
10.6m & 0m & ll & tl \\
10.6m & cc & ll & tl \\
11.9m & 0m & ll & tl \\
11.9m & cc & ll & tl \\
\end{pmatrix}
\]

\[\text{utb}_{k} := \text{stack}(\text{utb}_{k1}, \text{utb}_{k2})
\]

\[\text{KlassLast}_k := \text{stack}(\text{KlassLast}_k, 0.8\text{KlassLast}_k)
\]

\[\text{utb}_{k} := \text{stack}(\text{utb}_{k}, \text{augment}(\text{utb}_{k}^{(0)}, \text{utb}_{k}^{(1)} + 3m, \text{utb}_{k}^{(2)}, \text{utb}_{k}^{(3)}))
\]

\[\text{qKlass}_{k} := \begin{pmatrix}
1 \\
1 \\
1 \\
\end{pmatrix}
\]

\[\text{Klasspos}_{k} := \begin{pmatrix}
\text{utbr} - 0.5 - 0.5(\text{utbr} - cc) & -7.5m & 5m & \text{utbr} \\
\text{utbr} - 0.5 - 0.5(\text{utbr} - cc) & 14.4m & 5m & \text{utbr} \\
\text{utbr} - 0.5 - 0.5(\text{utbr} - cc) & 76.3m & 5m & \text{utbr}
\end{pmatrix}
\]

\[\text{qKlass}_{k} := \text{stack}(\text{qKlass}_{k}, \text{qKlass}_{k})
\]

\[\text{Klasspos}_{k} := \text{stack}(\text{Klasspos}_{k}, \text{augment}(\text{Klasspos}_{k}^{(0)}, \text{Klasspos}_{k}^{(1)} + 3m, \text{Klasspos}_{k}^{(2)}, \text{Klasspos}_{k}^{(3)}))
\]
KlassLast\(_1\) := stack 
\[
\begin{bmatrix}
0.44 & 0.44 \\
1.1 & 1.1 \\
2 & 2 \\
1.1 & 1.1 \\
2 & 2 \\
1.1 & 1.1 \\
2 & 2 \\
0.66 & 0.66 \\
2 & 2 \\
0.66 & 0.66 \\
2 & 2 \\
0.66 & 0.66 \\
2 & 2
\end{bmatrix}
\]
-0.5B
utb\(_{11}\) :=
\[
\begin{bmatrix}
0m & 0m & ll & tl \\
0m & cc & ll & tl \\
3.8m & 0m & ll & tl \\
3.8m & cc & ll & tl \\
5.6m & 0m & ll & tl \\
5.6m & cc & ll & tl \\
9.6m & 0m & ll & tl \\
9.6m & cc & ll & tl \\
11.4m & 0m & ll & tl \\
11.4m & cc & ll & tl \\
15m & 0m & ll & tl \\
15m & cc & ll & tl \\
16m & 0m & ll & tl \\
16m & cc & ll & tl
\end{bmatrix}
\]

utb\(_{12}\) := augment\((utb\(_{11}\) \text{ (0)}) + 50m + \max(utb\(_{11}\) \text{ (0)})\), submatrix\((utb\(_{11}\), 0, rows(utb\(_{11}\)) - 1, 1, cols(utb\(_{11}\)) - 1)\)

utb\(_1\) := stack\((utb\(_{11}\), utb\(_{12}\))\)

KlassLast\(_1\) := stack\((KlassLast\(_1\), 0.8KlassLast\(_1\))\)

KlassLast\(_1\) :=
\[
\begin{bmatrix}
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1
\end{bmatrix}
\]
Klasspos\(_1\) :=
\[
\begin{bmatrix}
\text{utbr} \cdot 0.5 - 0.5(\text{utbr} - \text{cc}) & -7.5m & 5m & \text{utbr} \\
\text{utbr} \cdot 0.5 - 0.5(\text{utbr} - \text{cc}) & 18.5m & 5m & \text{utbr} \\
\text{utbr} \cdot 0.5 - 0.5(\text{utbr} - \text{cc}) & 84.5m & 5m & \text{utbr}
\end{bmatrix}
\]

qKlass\(_1\) := stack\((qKlass\(_1\), qKlass\(_1\))\)

Klasspos\(_1\) := stack\((Klasspos\(_1\), augment(Klasspos\(_1\) \text{ (0)}, Klasspos\(_1\) \text{ (1)} + 3m, Klasspos\(_1\) \text{ (2)}, Klasspos\(_1\) \text{ (3)}))\)
KlassLast \( \Pi \) := stack \( \text{KlassLast}_{\Pi}, 0.8 \text{KlassLast}_{\Pi} \)  

\[
\begin{pmatrix}
0.5 \\
0.5 \\
\frac{1}{2} \\
\frac{1}{2} \\
\frac{1}{2} \\
\frac{1}{2} \\
\frac{1}{2} \\
\frac{1}{2}
\end{pmatrix}
\]

\( 0.5B_{2} \)

utb_{\Pi} := stack \( \text{utb}_{\Pi}, \text{augment} \left( \text{utb}_{\Pi}^{(\downarrow)}, \text{utb}_{\Pi}^{(\downarrow)} + 3m, \text{utb}_{\Pi}^{(\uparrow)}, \text{utb}_{\Pi}^{(\uparrow)} \right) \)  

\[
\begin{pmatrix}
0m & 0m & \text{tl} \\
0m & \text{cc} & \text{tl} \\
4.8m & 0m & \text{tl} \\
4.8m & \text{cc} & \text{tl} \\
6m & 0m & \text{tl} \\
6m & \text{cc} & \text{tl} \\
10m & 0m & \text{tl} \\
10m & \text{cc} & \text{tl} \\
14.8m & 0m & \text{tl} \\
14.8m & \text{cc} & \text{tl} \\
16m & 0m & \text{tl} \\
16m & \text{cc} & \text{tl}
\end{pmatrix}
\]

qKlass_{\Pi} := \begin{pmatrix} 1 \\ 1 \end{pmatrix} q_{kl}  

Klasspos \( \Pi \) := \begin{pmatrix}
\text{utb} - 0.5 - 0.5(\text{utbr} - \text{cc}) & -7.5m & 5m & \text{utbr} \\
\text{utb} - 0.5 - 0.5(\text{utbr} - \text{cc}) & 18.5m & 5m & \text{utbr}
\end{pmatrix}  

Klasspos_{\Pi} := stack \( \text{Klasspos}_{\Pi}, \text{augment} \left( \text{Klasspos}_{\Pi}^{(\downarrow)}, \text{Klasspos}_{\Pi}^{(\downarrow)} + 3m, \text{Klasspos}_{\Pi}^{(\uparrow)}, \text{Klasspos}_{\Pi}^{(\uparrow)} \right) \)  

\[
\begin{pmatrix}
1 \\
1 \\
1 \\
1 \\
0.5B_{2} \\
1 \\
1 \\
1
\end{pmatrix}
\]

KlassLast \( \Pi A \) := stack \( \text{KlassLast}_{\Pi A}, 0.8 \text{KlassLast}_{\Pi A} \)  

\[
\begin{pmatrix}
1 \\
1 \\
1 \\
1 \\
1
\end{pmatrix}
\]

\( 0.5B_{2} \)

utb_{\Pi A} := stack \( \text{utb}_{\Pi A}, \text{augment} \left( \text{utb}_{\Pi A}^{(\downarrow)}, \text{utb}_{\Pi A}^{(\downarrow)} + 3m, \text{utb}_{\Pi A}^{(\uparrow)}, \text{utb}_{\Pi A}^{(\uparrow)} \right) \)  

\[
\begin{pmatrix}
0m & 0m & \text{tl} \\
0m & \text{cc} & \text{tl} \\
1.2m & 0m & \text{tl} \\
1.2m & \text{cc} & \text{tl} \\
5.2m & 0m & \text{tl} \\
5.2m & \text{cc} & \text{tl} \\
6.4m & 0m & \text{tl} \\
6.4m & \text{cc} & \text{tl}
\end{pmatrix}
\]

KlassLast_{\Pi A} := stack \( \text{KlassLast}_{\Pi A}, 0.8 \text{KlassLast}_{\Pi A} \)  

utb_{\Pi A} := stack \( \text{utb}_{\Pi A}, \text{augment} \left( \text{utb}_{\Pi A}^{(\downarrow)}, \text{utb}_{\Pi A}^{(\downarrow)} + 3m, \text{utb}_{\Pi A}^{(\uparrow)}, \text{utb}_{\Pi A}^{(\uparrow)} \right) \)