Dynamic Behaviour of the New Årsta Bridge to Moving Trains

Simplified FE-Analysis and Verifications

Ignacio González

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Preface

This master thesis was carried out at the Department of Civil and Architectural Structural Engineering, the division of Structural Design and Bridges, at the Royal Institute of Technology in Stockholm. The thesis was conducted under supervision of Professor Raid Karoumi to whom I want to thank for valuable guidance and advice and who also was the examiner. I want also to thank PhD Candidate Johan Wiberg for the help provided and interesting discussions.

Stockholm, June 2008

Ignacio González
Abstract

Society demands today for faster, cleaner and safer means of transportation. A modern train system is one of the most interesting answers to this challenge. However, the ever increasing demand for more effective and efficient trains, leads to constant efforts to raise the speed and the axel load limits.

The dynamic response at high speed affects both the train and the railway in a very complex manner, and the vibrations induced reduce the service lives of vehicles as well as the railway infrastructure.

Bridges are especially sensitive to the dynamic effect of high speed trains. Unlike motorway bridges, the mass of the vehicles that transit railway bridges is usually comparable to that of the structure itself, so that for train speeds over 200 km/h the dynamic effect normally becomes the governing factor in structural design. Many of the dynamic characteristics of structures are not yet fully understood or are too complex to be modelled and studied in an efficient manner. This often leads to overdimensioned and expensive bridges with very high safety factors in order to ensure the reliability of the structure. In some cases, it has lead to underestimations of the dynamic effects with vast costs to society, monetary and sometimes even in human lives.

The purpose of this thesis is to study the possibility of accurately predicting the dynamic response of an existing railway bridge by implementing a simplified Finite Element (FE) Model with the aid of the program DynSolve. The bridge chosen is the New Årsta Bridge that communicates the southern part of Stockholm with the suburban areas at the south of the city. It was chosen because of the highly complex studies on its dynamic behaviour that have been carried out in recent years at the Division of Structural Engineering and Bridges, KTH. These studies were very time consuming and required greats amounts of computational power. Studying the possibility of satisfactorily assessing the bridge’s dynamic behaviour by simpler means could, in the future, save time and resources. The bridge is furthermore equipped with a very advanced dynamic measurement system. This, as well as the previous studies, allows for great amounts of data to compare the results obtained and check their reliability.

In order to assert the accuracy of the model, the different parameters governing the response are studied. Some results are compared with results obtained with other commercial FE-programs, and with the actual measured response of the bridge.

Keywords: Bridge, Train, Dynamic response, Dynamic amplification, Acceleration, Monitoring, Model updating.
Sammanfattning

Samhället kräver idag snabbare, säkrare och miljövänligare transportmedel. Ett modernt tågnät är en av de mest intressanta lösningarna till denna utmaning. Den växande efterfrågan för effektivare och konkurrenskraftigare tågsystem leder till konstanta ökningar i tillåtna hastigheter och axellaster.

Den dynamiska responsen vid höga hastigheter påverkar tåget och banan på ett mycket invecklat sätt, och vibrationerna som tillkommer minskar livslängden på fordon, banor och broar.

Broar är särskilt känsliga för de dynamiska effekterna av höghastighetståg. I motsats till motorvägsbroar, kan massan hos fordonen som kör på järnvägsbroar bli jämförlig med den av strukturen själv, så att för hastigheter över 200 km/h blir de dynamiska effekterna oftast den dimensionerande faktorn. Många av de dynamiska egenskaperna hos denna typ av strukturer är ännu idag inte helt förstådda, eller alldeles för complexa för att bli modellerade och studerade på ett effektivt sätt. Detta har lett till överdimensionerade och dyra broar med höga säkerhetsfaktorer för att kunna säkra strukturens hållbarhet. I enstaka fall har det lett till en undervärdering av de dynamiska effekterna med väldiga kostnader för samhället, pengamässigt och även i mänskliga liv.


Nyckelord: Bro, Tåg, Dynamisk respons, Dynamisk förstoring, Acceleration, Modelluppdatering, Övervakning.
## Contents

Preface ........................................................................................................................................... i

Abstract ..................................................................................................................................... iii

Sammanfattning ....................................................................................................................... v

1 Introduction ............................................................................................................................ 1

1.1 Aims of the Study ........................................................................................................... 1

1.2 Review of some Interesting Previous Work ............................................................. 2

1.2.1 Vehicle-Structure interaction ........................................................................... 3

1.2.2 System Identification ......................................................................................... 4

1.2.3 Impact forces .......................................................................................................... 6

1.2.4 Structure-Soil Interaction .................................................................................... 9

1.2.5 ERRI reports............................................................................................................ 10

2 Structural Dynamics ...................................................................................................... 23

2.1 Undamped systems .................................................................................................... 23

2.2 Damped Systems ......................................................................................................... 26

2.3 System with Multiple Degrees of Freedom .............................................................. 30

3 Signal Analysis ............................................................................................................. 37

3.1 Fourier Transform ........................................................................................................ 39

3.2 Filtering .......................................................................................................................... 45

4 Modelling ....................................................................................................................... 51

4.1 Bridge description ....................................................................................................... 51

4.1.1 The Bridge ............................................................................................................ 51

4.1.2 Construction ......................................................................................................... 52

4.1.3 Instrumentation ..................................................................................................... 54

4.2 The Bridge Model ....................................................................................................... 58

4.2.1 DynSolve............................................................................................................ 58

4.2.2 Model.................................................................................................................... 58

4.3 Convergence Studies ................................................................................................. 59

4.3.1 Number of Modes ............................................................................................... 59

4.3.2 Time step ............................................................................................................... 60

4.3.3 Number of Elements ............................................................................................ 61

4.4 Comparison ................................................................................................................... 61
5 Results ..............................................................................................................................65
  5.1 Resonance Risk ........................................................................................................65
      5.1.1 Support Stiffness ........................................................................................67
  5.2 Measurements and Updating .................................................................................68
      5.2.1 Updating ........................................................................................................73
      5.2.2 Predicted and Measured Accelerations ........................................................74

6 Conclusions and Suggestions for Further Research ..................................................81
  6.1 Conclusions ..............................................................................................................81
  6.2 Suggestions for further research ..............................................................................82

Bibliography .....................................................................................................................85

A Appendix A ...................................................................................................................89
  A.1 Predicted Mode Shapes and Frequencies ...............................................................89
1 Introduction

Dynamic effects on railway bridges can not be regarded as unimportant. For speeds over 200 km/h the dynamic effects often become the dimensioning factor when designing such structures.

In the worst case, when static calculations are not enough to accurately predict the response of a structure, the bridge has to be proved safe for all the ten HSLM trains (see chapter 1.2.5). This results in more than a hundred time-histories that have to be simulated and analysed, highly complicating the calculations needed for bridge designing.

In addition, the dynamic responses are not easy to understand. Results obtained from incorrect calculation or invalid assumptions are many times accepted without any further consideration.

Due to its complexity advanced dynamic investigations are avoided as much as possible by practical engineers. In its final report on dynamic effect on high speed railway bridges (ERRI, 1999 a), the European Rail Research Institute (ERRI) states: “In view of the potential unfamiliarity of many bridge engineers with the dynamic behaviour of structures the opportunity has been taken to provide guidance on some items beyond that normally expected in a UIC final report”. And even though a special effort was made in order to achieve a self explanatory report, some of the national railway administrations throughout Europe still found it necessary to look for further simplification to achieve a “user friendly” normative (Flesch, 2006).

Table 1.1: Some speed record for conventional Trains, modified from Fröidh and Nelldal (2006).

<table>
<thead>
<tr>
<th>Speed</th>
<th>Year</th>
<th>Country</th>
<th>Train</th>
</tr>
</thead>
<tbody>
<tr>
<td>210 km/h</td>
<td>1903</td>
<td>Germany</td>
<td>AEG</td>
</tr>
<tr>
<td>230 km/h</td>
<td>1931</td>
<td>Germany</td>
<td>Schienenzeppelin</td>
</tr>
<tr>
<td>331 km/h</td>
<td>1955</td>
<td>France</td>
<td>Aboard Train V150</td>
</tr>
<tr>
<td>380 km/h</td>
<td>1981</td>
<td>France</td>
<td>TGV</td>
</tr>
<tr>
<td>407 km/h</td>
<td>1988</td>
<td>Germany</td>
<td>ICE</td>
</tr>
<tr>
<td>515 km/h</td>
<td>1990</td>
<td>France</td>
<td>TGV-A</td>
</tr>
<tr>
<td>575 km/h</td>
<td>2007</td>
<td>France</td>
<td>TGV (modified)</td>
</tr>
</tbody>
</table>

1.1 Aims of the Study

The aim of this work is to study the possibility of accurately asserting the dynamic response of the New Årsta Bridge through a simplified 2D Finite Element Model. The main interest is the acceleration levels. For this effect a model of the New Årsta Bridge was performed and the vibrations induced by different kinds of train loads were simulated. The results from this model will be compared with those achieved with a more complex full 3-D model and with real response measured in the structure.
1.2 Review of some Interesting Previous Work

In 1851 Willis wrote his “Essay on the Effects Produced by Causing Weights to Travel Over Elastic Bars”, demonstrating for the first time that a load travelling through a beam caused larger deflections than the corresponding static loads. He also demonstrated that the effects increase with the speed of the moving load, initiating the field of structural dynamics. Willis’ demonstration was merely empirical. He observed the effects of a carriage crossing a beam at different speeds (see figure 1.1) and compared with the static effects of the carriage on the beam (Willis, 1849).

He tried to develop a mathematical theory to support his results but with only limited success. These results were lately improved by the famous mathematician G. G. Stokes giving birth to the first theoretical model of structural dynamics (Stokes, 1896). This theory disregarded though a very important effect of dynamic loads, namely vibration. A theory of vibration had just been worked out some years before by Lord Rayleigh, achieving remarkable results. Among others he introduced the fundamental concept of oscillation of a linear system about an equilibrium configuration and demonstrated the existence of eigenmodes and eigenfrequencies in discrete as well as continuous systems (Rayleigh, 1877).

Vibrations are perhaps the most important dynamic effect on railway bridges. The theory developed by Rayleigh could in theory be used to defined the differential equations governing the vibrations of any linear systems. But the amount of work required to solve reasonable complex structures was so large that rendered any attempt fruitless. The use of these concepts in bridge engineering had to wait to the introduction of modern computers.

At present date the theory of dynamics is very advance and complete, both mathematically and empirically. But there are still areas that are poorly understood. Some of interesting previous work is briefly described in the following sections.

Figure 1.1: Railroad track used by Willis to test beams from (Timoshenko, 1953).
1.2.1 Vehicle-Structure interaction

A train passing at a high speed on a bridge induces vibrations to the structure. The inevitable track irregularities are considered as the main initiator of these vibrations. This vibration in the deck of the bridge affects then the contact forces between the train and the structure and causes both train and bridge to interact dynamically. The response of the bridge is important to assert in order to ensure its structural reliability. In a similar way the behaviour of the vehicle needs to be known to guarantee the running stability and the comfort of the passengers. These two effects are more accurately described when studied together in its full interconnectedness. The problem is difficult to tackle and many factors have to be accounted for. The complexity of trains’ suspension systems, the irregularities of the track and the not completely understood bridge internal damping properties are among the aspects that complicate this kind of studies enormously (Xia and Zhang, 2005).

Xia and Zhang have done a very complete simulation of vehicle-structure interaction. Each train was modelled as a number of vehicles composed of a car body, two bogies and four wheel-sets, with spring-dashpot suspensions between the components. Each idealized vehicle had a total of 27 degrees of freedom.

The bridge response was calculated through mode superposition. Only the lowest modes were considered. These modes are the relevant ones when estimating the dynamic response and mode superposition has the advantage of needing less computational effort than direct time integration (Géradin and Rixen, 1994). The vertical and the lateral track irregularities were measured and taken into the model.

Figure 1.2: Sketch of the train model used by (Xia and Zhang, 2005).
Chapter 1. Introduction

Figure 1.3: Some of the results obtained in (Xia and Zang, 2005). Note how scattered the acceleration produced are for a given speed, even though they were induced by the same train.

To account fully for the interaction, the mass of the vehicle was considered as influencing the vibration of the bridge, so the mass matrix of the system changed with every time step.

Even with such a complex model the levels of acceleration were very difficult to estimate. The model predicted accurately only the average levels of acceleration induced by the train (see figure 1.3). The max accelerations caused by a train crossing the bridge a number of times were very scattered even when the train speed remained constant.

1.2.2 System Identification

System Identification is a very large area of investigation. An important part of today research within structural dynamic is focused on System Identification. The following section is a review of two simpler and extensively used techniques in System Identification.

FE Modelling is a very useful tool to study the dynamics of bridge structures. However, model connection to reality has to be verified. Thus, results of the simulation have to be compared with the real structure’s behaviour and the model calibrated according to measured responses. Dynamic test in real bridges can be very expensive. In order assert a reliable connection between input forces and output accelerations and deflections the forces applied to the bridge had to be carefully controlled. This almost always implies shutting the bridge to the traffic, given the traffic loads’ stochastic nature. Therefore a number of techniques have been developed to be able to find the crucial dynamic parameters, such as mode shape and damping, out of ambient excitation (traffic load, wind, earthquakes, etc). A very good description of these methods can be found in (Siringoringo and Fujino, 2007). As described in Siringoringo and Fujino, these techniques rely on initial information about the impulse response function (IRF), i.e., the response of the structure when an initial deflection (or velocity) is forced on it and no other load is applied. One of the most widely extended techniques for the obtaining the IRF is known as Random Decrement (RD) and it is based on the assumption that the response of a structure at an instant $t + t_0$ depends:
1.2. REVIEW OF SOME INTERESTING PREVIOUS WORK

1. Deterministically on the initial displacement at \( t=t_0 \)
2. Deterministically on the initial velocity \( v_0 \)
3. Non-Deterministically on the random load applied between \( t_0 \) and \( t + t_0 \)

First an adequate initial value of the response is selected. From this initial value many equally long time histories are recorded (see figure 1.4). The random part can be averaged out, remaining only the deterministic free-decay. To avoid cancelling out the deterministic part of the signal two initial conditions can be chosen: (a) constant non-zero level, giving the free-decay step response, or (b) zero level and only positive (or only negative) slope, giving the positive (negative) impulse response.

If enough free-decay responses are recorded the eigenmodes, eigenfrequencies and the damping can be identified by means of solving an eigenvalue problem. This technique is based in the fact that two free-decay signals are identical up to a phase change and an amplification factor and is called Ibrahim Time Domain Method (ITD). More details can be found in (Ibrahim and Mikulcik, 1977).

Briefly the IDT can be explained as follows. Consider the matrix equation for free-decay:

\[ X = \Phi \Lambda \]  
\( (1.1) \)

where

\[
X = \begin{bmatrix}
  x_1(t_1) & x_1(t_2) & \cdots & x_1(t_L) \\
  x_2(t_1) & x_2(t_2) & & x_2(t_L) \\
  \vdots & \ddots & \ddots & \vdots \\
  x_q(t_1) & x_q(t_2) & \cdots & x_q(t_L)
\end{bmatrix},
\]

\[
\Phi = \begin{bmatrix}
  \phi_{11} & \phi_{12} & \cdots & \phi_{12N} \\
  \phi_{21} & \phi_{22} & \cdots & \phi_{22N} \\
  \vdots & \ddots & \ddots & \vdots \\
  \phi_{q1} & \phi_{q2} & \cdots & \phi_{q2N}
\end{bmatrix},
\]

\[
\Lambda = \begin{bmatrix}
  e^{\lambda_1 t_1} & e^{\lambda_1 t_2} & \cdots & e^{\lambda_1 t_L} \\
  e^{\lambda_2 t_1} & e^{\lambda_2 t_2} & & e^{\lambda_2 t_L} \\
  \vdots & \ddots & \ddots & \vdots \\
  e^{\lambda_{2N} t_1} & e^{\lambda_{2N} t_2} & \cdots & e^{\lambda_{2N} t_L}
\end{bmatrix}.
\]

\( \Phi \) being the responses measured in \( q \) places during \( L \) time instants, \( \Phi \) the matrix of \( 2N \) eigenvectors given in \( q \) locations and \( \Lambda \) the eigenvalue matrix in which the element \((i,j)\) is exponential of the \( i \)-th eigenvalue and the \( j \)-th time step. Now, the same free-decay signal only time shifted an interval \( \Delta t \) will produce the equation:

\[ X' = \Phi' \Lambda \]  
\( (1.2) \)
Where the elements of $X'$ are related to those in $X$ by $x'_i(t_k) = x_i(t_k + \Delta t)$ and the elements in $\Phi'$ to those in $\Phi$ by $\phi'_{ij} = \phi_{ij} e^{\lambda_j \Delta t}$. After some matrix manipulations we arrive to the result:

$$ (A - I e^{\lambda_j \Delta t}) \phi_j = \bar{0} $$

(1.3)

where $A$ can be calculated by use of the pseudo-inverse method from the equation $AX = X'$. Thus the system’s eigenvectors $\phi_i$ can be obtained, for each of the eigenvalues of $A$.

### 1.2.3 Impact forces

A mass-spring assembly crossing a beam at a certain speed $v$ may momentarily detach from it, only to land on it after a time interval causing an impact to occur. Such separations are usually not considered when modelling bridges and how these interact with the vehicles that transit them. Some new studies had shed some light on the issue, showing that separation is likely to occur under configurations that are common within bridge engineering, and that they can have important effects in the response of the structure (Stancioiu et al., 2007).

The conditions necessary to the separation are unproblematic. It is the re-attaching that present more serious complication. If the mass of the vehicle can be disregarded in comparison to that of the structure, the speed of the beam remains unchanged during the impact and the speed of the vehicle will experiment a discontinuity to match that of the structure.
If the mass of the moving system is considerable, then the speed of structure must also undergo a “jump” at the impact instant. Different approaches had been used to tackle this problem. Stancioiu et al. use the modal functions of the beam. The velocity of the unsprung mass in the mass-spring assembly is determined by momentum theory. Bouncing or skimming is not considered and all the impact are supposed to be plastic, i.e., directly after the impact the unsprung mass sticks to the beam. The equation governing the beam motion under impact is given by:

\[ EI \frac{\partial^4 w}{\partial x^4}(x,t) + \rho A \frac{\partial^2 w}{\partial t^2}(x,t) = -p \delta(x - vt) \delta(t - t_r) \]  

(1.4)

Where \( t_r \) is the re-attachment instant, \( p \) impulse due to impact and \( \delta(x) \) the Dirac Delta Distribution.

The equations of motion for the oscillator (see figure 1.5) are:

\[
\begin{align*}
    m_s \ddot{z}(t) &= -k(z(t) - u(t)) - c(\dot{z}(t) - \dot{u}(t)) - m_s g \\
    m_u \ddot{z}(t) &= k(z(t) - u(t)) + c(\dot{z}(t) - \dot{u}(t)) - m_u g + p \delta(t - t_r)
\end{align*}
\]

(1.5)

From these formulae the discontinuity in the modal velocity can be calculated as:

\[
\dot{q}_n(t_r^-) - \dot{q}_n(t_r^+) = -\frac{p}{\rho AL} \psi_n(vt_r) \text{ for all } n.
\]

(1.6)

where \( \psi_n \) is the modal shape, \( A \) the cross section area and \( \rho \) the material density.

By imposing the velocity of the unsprung mass and that of the beam to be equal at the impact point/instant, \( p \) can be solved, so that equation (1.6) can be rewritten as:

\[
\dot{q}_n(t_r^+) = \dot{q}_n(t_r^-) + \frac{\dot{u}(t_r^-) - \dot{w}(vt_r, t_r^-) \psi_n(vt_r)}{(1/M) + (1/m_u) / \rho AL}
\]

(1.7)
where

\[ M = \frac{\rho AL}{\sum \psi_n^2(v_i)} \]  

(1.8)

This method was proved in numerical simulations in an Euler-Bernoulli beam of length 4.5 m, bending stiffness 63,000 Nm² and linear density 20.245 kg/m. The beam was considered as undamped, but damping should not affect the validity of the results obtained. The properties of the oscillator were \( m_s = 50 \) kg, \( m_u = 20 \) kg, \( k = 10 \) kN/m and \( c = 200 \) Ns/m.

With the oscillator running a 360 km/h considering the impact interaction importantly changed the response of the structure.

The most critical parameter that influence the separation and impact were found to be the moving speed, the \( m_s \) to \( m_u \) ratio and the spring stiffness of the oscillator. And important conclusion was that, when separation is not considered there is virtually no difference between a stiff oscillator and a single unsprung moving mass. But when separation is taken into account these two models can differ significantly.

![Figure 1.6: Beam deflection caused by the moving oscillator, considering impact (continuous line) and without impact (dotted line) from (Stancioiu et al. 2007).](image-url)
1.2.4 Structure-Soil Interaction

As with every structure, an important part of the behaviour observed in bridges depends on the interaction between the bridge’s foundations and the ground. Such interactions are, for the sake of simplicity, many times obviated or regarded as unimportant. Modelling a realistic bridge-soil interaction is especially necessary when the dynamic response is to be asserted with accuracy. In a study carried on by (Alfonso, 2007), the dynamic response of a bridge under the load of a high speed train was recorded and afterwards numerically simulated. In the simulations the ground support was modelled as linear springs, with different stiffness, varying from the typical values for soft clay to bed rock foundations, and a parametrical study was performed to discover how it affected the dynamic response of the structure.

The study showed that the vertical stiffness of the supports have a very important effect in the bridge’s eigenfrequencies and that, considering fixed bearings, i.e. infinitely stiff supports, move the resonance peaks towards higher speeds. This is potentially dangerous, since model using this approximation could oversee possible resonance problems. At the same time, reducing the stiffness of the supports moves the resonance speed towards lower velocities. In addition it increases the mass participation, causing a reduction in the acceleration levels. The stiffness of the supports also affects the shape of the modes of vibration. If the stiffness of the support was set low enough, it could even cause the appearance of new rigid body modes.
Chapter 1. Introduction

The stiffness of the supports are in general difficult to estimate with precision. By measuring the dynamic deflections of a bridge models can be updated, to better reflect the properties of the structure. This allows for more accurate predictions and also for a better understanding of the factors important to considerate in future simulations.

1.2.5 ERRI reports

The method of impact factor was adopted by the International Union of Railways (UIC), as a tool for including the dynamic effects on a structure by simply enlarging the static effects. This technique has been largely adopted by many countries through Europe. The method is described in the code of practice (UIC, 1979), and it was the fruit of the study of 350 dynamic measurement and simulation carried on 37 bridges during the early 1970’s. According to it, the static effects are to be multiplied by a Dynamic Amplification Factor (DAF), to account for the dynamic effects. The DAF can be obtained as a relative simple function of the level of track maintenance, the load speed and bridge length and eigenfrequencies.

\[ DAF = 1 + \varphi = 1 + \varphi' + \lambda \varphi'' \]  

(1.9)

where \( 1 + \varphi' \) is the DAF of an ideal, roughness free track. The term \( \varphi' \) is the dynamic impact component for a particular train expressed in:

\[ \varphi = \frac{K}{1 - K + K^4} \text{ with } K = \frac{v}{2L_\phi n_0} \]  

(1.10)

with \( v \) being the velocity of the train, \( n_0 \) the first natural frequency and \( L_\phi \) the determinant length, that coincide with the span length for simply supported structures (an equivalence table is provided for other structural configurations). For equation (1.9) to be valid certain limits for its parameters have been stipulated in the UIC leaflet 776-1R.

To take into consideration the irregularities of the track the additional term \( \varphi'' \) was introduced. For high speed trains it takes the form:
1.2. REVIEW OF SOME INTERESTING PREVIOUS WORK

\[ \varphi'' = \frac{1}{100} \left[ 56e^{-\left(\frac{x}{10}\right)^2} + 50 \left( \frac{L_wn}{80} - 1 \right) e^{-\left(\frac{x}{30}\right)^2} \right] \] (1.11)

The value of the factor \( \lambda \) in equation (1.9) is tabulated in (UIC, 1979) and depend only on the level of maintenance of the track.

But the DAF method soon revealed itself insufficient, since it was based on a set of assumptions that no longer could be taken as valid. One of the main drawbacks of this method was its inadequacy to consider resonance effects (Goicolea et al., 2002).

Maximum allowable train speeds became higher and higher, and not only short experimental train circulated at speeds over 200 km/h, but also long passenger trains that had a much greater resonance potential. In addition advances made in the construction techniques and materials has led to lower structure to vehicle mass ratios, and to changes in the dissipative characteristics of the bridges towards lower damping levels.

In 1999 the UIC decided for a Specialist Sub-Committee to be set up to study the dynamic effects including resonance in railway bridges for speeds up to 350 km/h, with especial attention paid to the deck acceleration levels, a critical parameter for the stability of ballast and satisfactory wheel/rail contact. The D214 Committee was thus formed by the European Rail Research Institute (ERRI). The study resulted in the production of 9 reports; to serve as guidelines for the design of high speed railway bridges with consideration to the dynamic effects. The 9th and final report is a detailed summary of the results obtained by the committee. Some of the more important results are summarized below.

**Train Signature**

ERRI developed a method, called Train Signature, to separate the two inherent aspects of the dynamic response of the total dynamic system, namely, the characteristics of the train and the characteristics of the bridge.

The greatest advantage of separating these two aspects is that the dynamic effects of different trains at resonance and away from resonance can be compared without any reference to the characteristics of a bridge, because the Train Signature is a function of only the axel load and the axel spacing.

Using this method enables a rapid comparison of different trains to be made. If a bridge has been demonstrated adequate for certain trains at a given speed it can safely be assumed as adequate for any new train with a Train Signature of lesser magnitude than that of the already studied trains, at the same speed.

The main disadvantage of the Train Signature is that it can not be used as a defined loading for complex bridges that are not line beams bridges.

The Train Signature of a train is calculated with a method called Decomposition of Excitation at Resonance (DER), described in (ERRI, 1999 b). According to this
method the mid-span deck acceleration of simply supported bridge is the product of three terms:

- A constant
- The influence line of the bridge
- The Train Spectrum

This last term, the Train Spectrum, shows the excitation due to the train and the response of the deck to resonance, it shows basically what wave lengths the train excites, and depends only on the axel spacing, axel load, and the damping of the bridge. By assuming zero damping and disregarding the length of the bridge the Train Spectrum can be completely disassociated from the bridge characteristics, giving:

\[
G(\lambda) = \frac{L}{\zeta(L+X_{N-1})} \sqrt{\left(\sum_{k=0}^{N-1} P_k \cos\left(\frac{2\pi x_k}{\lambda}\right)\right)^2 + \left(\sum_{k=0}^{N-1} P_k \sin\left(\frac{2\pi x_k}{\lambda}\right)\right)^2} \cdot \left(1 - \exp\left(-2\pi\zeta X_{N-1} \frac{L}{\lambda}\right)\right)
\]

(1.12)

where \(\lambda\) is the ratio between the load speed and first eigenfrequency, \(P_k\) the \(k\)th load, \(x_k\) the distance between the first and the \(k\)th axel, \(\zeta\) the damping ratio of the bridge’s first mode, \(L\) the bridge length and \(X_{N-1}\) the length of the \(Nth\) sub-train.

Figure 1.9: Train Signatures for European high-speed trains from (Goicolea et al. 2002).
Since the most critical (maximum) response for a train is not necessarily obtained when the entire train has crossed the bridge, it becomes necessary to check the effect due to all of the sub-train. A sub-train is conformed by the \( n \) first axels of a train. To ensure that all the cases are taken into account, the Train Spectrum is defined as the envelope of all the possible sub-trains, including off course the whole train.

The Train Spectrum description of dynamic loading is valid at and away from resonance whenever the deflected shape of a structure can be adequately represented by a single sine term, as it is the case in simple supported bridges.

ERRI has defined a universal train signature envelope called “Eurocode envelope” that covers all the existing and envisaged train models within Europe. Furthermore, all the trains designed in the future should have a signature of a lower intensity than that covered by the prescribed envelope.

The idea behind the Eurocode envelope is that, if every bridge within Europe is designed in relation to it, then the different trains used in the European countries could safely cross the boundaries and run on every track ensuring interoperability.

To avoid the calculation of every single train with Train Signature contained within the Eurocode envelope, ERRI developed ten so-called Reference Trains or High Speed Load Model (HSLM) which, in a sense, sweeps the whole of Eurocode envelope. In this way, a structure that shows itself reliable for these ten reference trains should also be reliable for any other train with a Train Signature being under the Eurocode envelope.

The reference trains were chosen as the train configurations that best fitted the Eurocode envelope, with bogie spacing of 2, 2.5 and 3 m and with axel load that were multiples of 10 kN. There were trains that better covered the Eurocode envelope (for example with a bogie spacing of 2.3 m and axel load of 173 kN) but to keep things simple, as it should be in a regulatory document, the results were rounded to fit the criteria named above.

Table 1.2: Definition of the 10 HSLM-A trains used in middle and long-span bridges from (ERRI 2002).

<table>
<thead>
<tr>
<th>Universal Train</th>
<th>Number of intermediate coaches</th>
<th>Coach length D (m)</th>
<th>Bogie axle spacing d (m)</th>
<th>Point force P (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1</td>
<td>18</td>
<td>18</td>
<td>2.0</td>
<td>170</td>
</tr>
<tr>
<td>A-2</td>
<td>17</td>
<td>19</td>
<td>3.5</td>
<td>200</td>
</tr>
<tr>
<td>A-3</td>
<td>16</td>
<td>20</td>
<td>2.0</td>
<td>180</td>
</tr>
<tr>
<td>A-4</td>
<td>15</td>
<td>21</td>
<td>3.0</td>
<td>190</td>
</tr>
<tr>
<td>A-5</td>
<td>14</td>
<td>22</td>
<td>2.0</td>
<td>170</td>
</tr>
<tr>
<td>A-6</td>
<td>13</td>
<td>23</td>
<td>2.0</td>
<td>180</td>
</tr>
<tr>
<td>A-7</td>
<td>13</td>
<td>24</td>
<td>2.0</td>
<td>190</td>
</tr>
<tr>
<td>A-8</td>
<td>12</td>
<td>25</td>
<td>2.5</td>
<td>190</td>
</tr>
<tr>
<td>A-9</td>
<td>11</td>
<td>26</td>
<td>2.0</td>
<td>210</td>
</tr>
<tr>
<td>A-10</td>
<td>11</td>
<td>27</td>
<td>2.0</td>
<td>210</td>
</tr>
</tbody>
</table>
In (ERRI, 1999c) it is proven that the moving force loading model is conservative when compared to techniques based in vehicle-bridge interaction. Therefore the HSLM train modes include only axel loads and axel spacing disregarding the complex suspension system that characterises modern high speed trains. Since the conditions in short-span bridges differ from middle and long-span bridges two different model were developed: the HSLM-A (see figure 1.10 and table 1.2) for middle and long-span bridges and the HSLM-B for short-span bridges.

**Damping**

The study of damping ratios of different bridges carried out by ERRI (ERRI, 1999a) showed that two structures having identical form and materials can exhibit wide variations in damping, partly due to the foundation properties. Damping has proven very difficult to predict and there exist a wide variety of mathematical models to describe damping: viscous damping proportional to velocity, frictional damping proportional to displacement or proportional damping especially used in FE-modelling (see chapter 2.3). At the same time the overall dynamic behaviour is much more sensitive to the value of damping than to the mathematical model assumed.

Differences in the damping estimation were even observed for the same bridge and the same pass of train, depending on the magnitude of signal measured. In general, the ERRI advice to be extremely careful when interpreting historical data, since the amplitude of the response and the sensitivity/accuracy of the instruments used can change considerably the results obtained.

Damping estimation is full with uncertainties (non-linearity, excitation techniques, mathematical model, etc) but an overestimation of this parameter can completely change the response of a structure, especially at resonance. For this reason a lower bound should be used for design purposes. Among the conclusions reached are:

- Reducing damping to a single value is an oversimplification.
- Different lower bounds should be used, depending on whether the structure is Steel/Composite, Prestressed Concrete or Reinforced concrete/filler beam, but using finer bridge categories will not lead to better results.
There is an important correlation between span length and damping.

When estimating the damping for assessment purposes, ERRI warns about certain conditions that could increase the damping as the bridge ages, but could suddenly disappear under increased dynamic loading or track maintenance. Among such effects are earth pressure effects at the ends of the bridges, condition of ballast and possible composite action relating to fill.

Taken all this into account ERRI recommends the following lower bounds:

Table 1.3: Values of damping to be assumed for design purposes.

<table>
<thead>
<tr>
<th>Bridge Type</th>
<th>Span Length</th>
<th>( \zeta ): lower limit (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel and Composite</td>
<td>( L &lt; 20m )</td>
<td>( \zeta = 0.5 + 0.125(20 - L) )</td>
</tr>
<tr>
<td></td>
<td>( L \geq 20m )</td>
<td>( \zeta = 0.5 )</td>
</tr>
<tr>
<td>Prestressed Concrete</td>
<td>( L &lt; 20m )</td>
<td>( \zeta = 1.0 + 0.07(20 - L) )</td>
</tr>
<tr>
<td></td>
<td>( L \geq 20m )</td>
<td>( \zeta = 1.0 )</td>
</tr>
<tr>
<td>Reinforced concrete and Filler beam</td>
<td>( L &lt; 20m )</td>
<td>( \zeta = 1.5 + 0.07(20 - L) )</td>
</tr>
<tr>
<td></td>
<td>( L \geq 20m )</td>
<td>( \zeta = 1.5 )</td>
</tr>
</tbody>
</table>

Mass of Bridge

As reported in (ERRI, 1999 d), the natural frequencies of a structure tend to decrease as the mass of the structure increases, if the other parameters are kept constant. An underestimation of the mass will overestimate the resonance frequency and thus overestimate the minimum velocity required for resonance phenomenon to occur. Therefore safe upper bound estimates of bridge mass are required to ensure that safe lower bound predictions of resonant speeds are made.

On the other hand, the maximum acceleration of a structure increases as the mass decreases, so overestimation of the structure’s mass will result in an underestimation of the acceleration produced by dynamic loads. There is then a need of a safe lower bound to the bridge mass to ensure that safe estimates of peak dynamic acceleration effects are obtained.

The displacement and the dynamic load increment are unaffected by changes in the bridge distributed mass.

It is therefore undesirable to increase the structure’s mass to lower the peak acceleration, since it will on its turn also lower the critical speed of the structure.
Figure 1.11: Variation of the displacement and acceleration peaks for different bridge masses from (ERRI, 1999 d).

**Stiffness of the Bridge**

Stiffness is one of the primary parameter affecting the resonance frequencies of a structure. Unlike the other important parameters governing this factor, such as mass, span length and boundary conditions, it is very difficult to quantify accurately. For other than simply supported bridges (for example cross girders), the uncertainty of the overall stiffness of the bridge may even make it difficult to predict the boundary conditions.

ERRI studied thoroughly how to improve the assessment of the dynamic stiffness of a structure, exploring the differences between dynamic and static behaviour of materials and studying the elements of behaviour, which contribute to the stiffness of a structure.
1.2. REVIEW OF SOME INTERESTING PREVIOUS WORK

Figure 1.12: Variation of displacement and acceleration peaks for different length to first eigenfrequency ratios from (ERRI, 1999 d). A higher length to eigenfrequency ratio represents a stiffer bridge.

Increasing the stiffness of a structure is beneficial, because it raises the critical speed at which resonance effects occur. Therefore it is paramount to predict it accurately especially when resonant peaks occur just above the speed range allowed.

In these circumstances a lower bound of the stiffness should be used, and the maximum speed allowed should be calculated based on this lower bound.

Increasing the stiffness of a structure has a major effect upon costs. Thus for economic design, is necessary to be able to make accurate predictions of the stiffness of a structure.
ERRI initiated a study of the codes of practice of different counties and their approach to calculating the stiffness of concrete and steel structures. Important differences were found among the different methods. ERRI found that many issues were unsatisfactory treated in these regulations, some of them were:

- How to calculate the stiffness of cracked concrete slabs.
- How to calculate the stiffness of deck type composite bridges in hogging zones over intermediate piers.
- Identifying assumptions which are conservative when analysing a structure for strength purposes but lead to inaccuracy in calculation deflections.
- What values of Young’s Modulus should be used for concrete
- The difference in material properties for short term static deflection calculations and behaviour at frequencies corresponding to the dynamic response of a structure.
- A number of test results on bridges that illustrate varying discrepancies between predicted and measured deflections (for both bending and torsional behaviour).

The most critical parameters for the overall stiffness of a bridge are of course the materials' Young’s and shear Modula. The data studied by ERRI revealed that due to the effect of the speed of deformation expected for high speed railway bridges, the stiffness and strength will increase with 5-10%. This strengthening only affects the dynamic loads, and can not be used for own weight and dead loads. Thus the modulus of elasticity should be considered depending on the load speed and at least two different values, static and dynamic, should be considered.

Another problem regarding the elasticity modulus is that, due to safety considerations, the nominal value of this parameter is always lower than the real one. This can be disregarded when studying static effects, because it is always on the safe side to assume a structure weaker than what it really is, but in dynamic analyses it can lead to misestimating the critical frequencies and the critical speeds.

**Track Irregularities**

The expression in equation (1.11) which accounts for track irregularities was produced during the seventies with a series of assumptions that no longer could be consider valid for modern structures. Among the most important assumption made by UIC that are in need of reviewing were: The very high damping ratios assumed, between 2.5 and 17%, and the consideration of only short high speed trains.

The study performed by UIC did not consider any resonance phenomena, and did not include a bridge type that is very sensitive to track irregularities, namely very stiff short span bridges.

With all this in mind, ERRI carried a number of calculations to see if the $\phi$ factor could still be used as a reliable estimation of the track irregularities effects. The calculations included a number of bridges with span ranging between 10 and 20 m and a 5 m stiff bridge. These lengths were chosen because shorter spans tend to benefit from load distribution, which reduces the effects of track imperfections and for longer spans there is clear reduction in the effects of track irregularities.
1.2. REVIEW OF SOME INTERESTING PREVIOUS WORK

In order to make the study comparable with the one performed by UIC, the same kind of track defect were introduced in the models. The defect consists of a single sine shaped dip at mid-span. Two different dip dimensions were considered, 1mm depth with 1 m length and 3 mm depth with 6 meters length.

Since the main purpose of the study was to validate the UIC formula at resonance, only critical speeds under 350 km/h were considered and low damping ratios were assumed: 3.5% for the 5 m span and 1% for the rest.

To avoid an overestimation of the effects of the imperfections the dissipative effects of the track and ballast were accounted for.

The result of the study showed that contact forces could grow enormously due to wheel lift-off. At certain speeds the contact force will reach even 4 times the static value over very short periods of time when the wheel impacts the track. This impact forces excites high frequency modes that produce very high accelerations but almost no deflections whatsoever. The acceleration observed with track defects were in some cases up to 6 times those obtained with perfect track. These high accelerations are unrepresentative, since track maintenance usually is enough to avoid wheel lift-off. Therefore the results were filtered using a low-pass filter with a cut-off frequency of 20 Hz.

It was found that the UIC formula in equation 1.11 for dynamic deflections due to track irregularities can be used as a conservative value, even when resonance takes place. When the track maintenance is such to prevent wheel-track separation, the factor $\phi$ can also account for acceleration to a great accuracy. However, discrepancies were found at low speeds, where the calculated increment in the acceleration were higher that those estimated by UIC. At such speeds the accelerations effects are not so critical, so the values of $\phi$ calculated in accordance with UIC provide reasonable estimates for the increase in acceleration.
Bridge/Train Interaction

In order to determine the significance of bridge/train interaction effects a number of calculations were performed by ERRI. The calculations considered the full train primary and secondary suspension characteristics and associated axle, bogie and vehicle masses and rotary inertia. Two different finite element programs were used, as well as two different generic train types (ICE 2 and Eurostart).

The results showed important differences between full interaction models and moving forces models only in short bridges near resonance.

The full interaction models produced lower displacement and accelerations than the travelling force model and a slight shift of the resonance frequency was observed. This can be readily explained by considering the axle forces. The vibrations in the deck increase rapidly over time when the bridge is near resonance. The vibrations in the deck plus those of the wheel set cause the axle forces to vary as the train passes on the bridge. Energy is transfer in this way from the beam into the vehicle suspension.

For simply supported spans it is possible to derive particular solutions with travelling point load excitation. Such solutions are not possible when interaction is considered, not even for the simplest case of interaction, namely travelling mass model. Thus, general partial equations describing the system have to be solved numerically.

Nonetheless an analytical solution can be found for systems at resonance, provided that some simplifications are assumed:

- Only first mode of a simply supported line beam taken into account (torsional effects are disregarded)
- The train is modelled as a series of travelling sprung masses comprising a mass connected to the beam by a spring and damper, each mass subject to its own static load
- The speed of the train corresponds to the resonant loading. All transient effects had been damped away, and a steady state has been reached.

By means of calculating the unknown Fourier coefficients for the first mode of vibration of the beam a particular solution of the forced vibration equation can be obtained. This solution takes into account the static axle load and the prescribed displacement due to the vibration of the beam, describing it as a Fourier series.

A more general method is needed to analyse more complex geometric configurations as well as non resonant and transient responses under bridge/train interaction.

The dynamic response of a bridge can be significantly affected by dynamic interaction with train vehicles. This interaction is very expensive to simulate, in term of computational costs. In search for economical means to take into consideration this factor, ERRI studied the possibility of adding a certain amount of damping to the bridge. The added damping could correspond to the damping effect of the train suspension and mass.
By comparing the travelling force simulation (with added damping) and the full interaction models ERRI could deduce an empirical formula to represent the extra damping caused by bridge/train interaction. Good agreement in both displacement and accelerations was observed between the travelling force and interaction models for almost all the speeds, spans and train types studied. A safe lower bound for the added damping can be expressed in:

\[
\Delta \zeta = \frac{a_1 L + a_2 L^2}{1 + b_1 L + b_2 L^2 + b_3 L^3}
\]

(1.13)

This formula accounts for several physical conditions. The added damping \( \Delta \zeta \) tends to zero whenever the span length \( L \) tends to zero or infinity, and has a maximum between \( L = 10 \text{ m} \) to \( L = 20 \text{ m} \). The explanation to these characteristics lies in the fact that the added damping is supposed to represent the energy transferred from the structure to the vehicle. If the spans considered are too short the energy transferred will be small, and if they are too long the motion of the primary suspension reverses while the train is still on the bridge, re-transferring the energy to it.

The difference is more evident in the intermediate range of span lengths, where enough energy is transferred to the suspension, but it cannot be re-transferred to the structure, because the train as already left the bridge when reversal occurs.
2 Structural Dynamics

For vibration analysis, many structures can be idealized as systems having one degree of freedom. The most intuitive single degree of freedom system is a mass attached to a linear spring (see figure 2.1 a). Every linear system can be reduced to this configuration, at least as a first approximation. The meaning of “mass”, “displacement” and “spring rigidity” can vary when reinterpreting a single degree of freedom (SDOF) system into our sprung mass paradigm. If the system is a disk fixed by a shaft, the rotation angle of the disk will be interpreted as the “displacement”, its rotational inertia as the “mass” and the torsional stiffness of the shaft as the “spring rigidity” (see figure 2.1 b). Thus many different problems involving vibration can be understood by studying the sprung mass system. Of special interest are the cases when a harmonic force is applied and when the system is provided with some dissipation mechanism, or damper. Many systematic studies on linear structural dynamics can be found in the literature. The exposition made in this chapter is taken from (Weaver et al., 1990).

2.1 Undamped systems

The equation governing the motion of an undamped (conservative) one degree of freedom sprung mass system (see figure 2.1) is:

\[ m\ddot{u} + ku = 0 \]  \hspace{1cm} (2.1)

Figure 2.1: Undamped Single Degree of Freedom (SDOF) System (a), and other systems that can be idealized as undamped SDOF systems (b).
where $m$ is the mass, $k$ the spring stiffness and $u$ the displacement.

This equation has a very simple solution in its original coordinate system. It suffices to replace $u=A \sin(\omega t + \theta)$ where $\omega^2 = k/m$ and find an appropriate value for $A$ and $\theta$ depending on the initial configuration of the system. Now, when an external force $F$ is applied to the system, the equation (2.1) turns into:

$$m\ddot{u} + ku = F(t) \quad (2.2)$$

which is difficult to solve analytically for other than a harmonic external force of the form: $F(t) = F_\theta \sin(\Omega t + \phi)$. It may seem artificial to require such law-abiding kind of force in real physical problems, but they are very common. Further, even though many load cases in the real world are non-harmonic, they are very often periodic. Thanks to a very powerful mathematical tool called Fourier analysis (first studied by Joseph Fourier) we are allowed to decompose any periodic function into a sum of harmonic function, allowing for a simple solution of the problem (see chapter 3).

Dividing the harmonic forced form of equation (2.2) by $m$ and introducing the terms $\omega^2 = k/m$ and $q = F_\theta / m$ we now get (the phase constant $\beta$ is set to 0 without loss of generality)

$$\ddot{u} + \omega^2 u = q \cdot \sin(\Omega t) \quad (2.3)$$

The homogenous solution of equation (2.3) is $u_h = A \sin(\omega t + \theta)$ and the particular solution is:

$$u_p = \frac{q \cdot \sin(\Omega t)}{\omega^2 - \Omega^2} \quad (2.4)$$

The homogenous solution can be considered representing the free vibration, while the particular solution represents the forced vibration. Ignoring the free vibrations the solution can be written as

$$u_p = \frac{F_\theta}{k \cdot \sin(\Omega t)} \left( \frac{1}{1 - \Omega^2 / \omega^2} \right) \quad (2.5)$$

Where the term $F_\theta / k \sin(\Omega t)$ represents the deflection caused by $F(t)$ if it were acting statically, and $1/(1 - \Omega^2 / \omega^2)$ is the magnifying factor that accounts for the dynamic action. The absolute value of the later quantity is usually called magnification factor and is a function of only the ratio between $\Omega$ and $\omega$ (see figure 2.2).

Note that when the load frequency $\Omega$ equals the system’s natural frequency $\omega$ the magnifying factor grows to infinity. It means that if the periodic force acts on the vibrating system with its natural frequency the amplitude of vibration increases indefinitely, provided that there is no dissipation of energy. This phenomenon is known as resonance.
2.1. UNDAMPED SYSTEMS

Figure 2.2: Magnification Factor as a function of the loading frequency ratio for an undamped SDOF system.

Resonance should not be interpreted as if the system assumed immediately a steady state with infinitely amplitude. Its physical meaning is rather that no steady state solution exists at resonance (without dissipation), and that the amplitude of the vibration increases with every cycle.

The general solution (homogeneous plus particular solutions) of equation (2.3) can be written as follows:

\[ u = u_0 \cos \omega t + \frac{\dot{u}_0}{\omega} \sin \omega t + \frac{q}{\omega^2 - \Omega^2} \left( \sin \Omega t - \frac{\Omega}{\omega} \sin \omega t \right) \] (2.6)

If the initial conditions are set to \( u_0 = \dot{u}_0 = 0 \) the equation (2.6) can be simplify to

\[ u = \frac{q}{\omega^2 - \Omega^2} \left( \sin \Omega t - \frac{\Omega}{\omega} \sin \omega t \right) \] (2.7)

Introducing the notation \( \omega - \Omega = 2 \varepsilon \), and with the help of trigonometric identities this solution can be rewritten as:

\[ u = -\frac{q}{2\omega} \left( \frac{1}{\varepsilon} \sin \varepsilon t \cos (\omega - \varepsilon) t - \frac{1}{\omega - \varepsilon} \cos \varepsilon t \sin (\omega - \varepsilon) t \right) \] (2.8)

Evaluating this solution in the limit we obtain:

\[ \lim_{\varepsilon \to 0} u = -\frac{q}{2\omega} (\omega t \cos \omega t - \sin \omega t) \] (2.9)
It can clearly be seen that the amplitude, at least of the first of the terms, increases indefinitely with time.

### 2.2 Damped Systems

In the previous section the free vibration amplitude of the system was found to be constant in time, however experience shows otherwise. A system set in motion will eventually come to rest if no disturbing forces are applied to it. Our study also led to unlimited increasing amplitudes at resonance. But we know that because of damping there always exists some finite amplitude of steady-state response, even at resonance.

Damping is usually assumed to be viscous, i.e. proportional to the speed of vibration. The equation of motion of a damped one degree of freedom sprung mass system is:

\[ m\ddot{u} + c\dot{u} + ku = F(t) \]  \hspace{1cm} (2.10)

If we assume a sinusoidal disturbing force of the form \( F(t) = F_0 \cos(\Omega t) \) and introduce the notation:

\[
\omega^* = \frac{k}{m}, \quad 2n = \frac{c}{m} \quad \text{and} \quad q = \frac{F_0}{m}
\]  \hspace{1cm} (2.11)

into equation (2.10) we obtain:

![Damped SDOF system](image.png)

**Figure 2.3:** Damped SDOF system.
\[ \ddot{u} + 2n\dot{u} + \omega^2 u = q \cos(\Omega t) \quad (2.12) \]

that has a particular solution of the form:

\[ u_p = M \cos(\Omega t) + N \sin(\Omega t) \quad (2.13) \]

Replacing equation (2.13) into (2.10) and taking into consideration that it has to be fulfilled for all time \( t \), we get:

\[
M = \frac{q(\omega^2 - \Omega^2)}{(\omega^2 - \Omega^2)^2 + 4n^2\Omega^2}
\]

\[
N = \frac{q(2n\Omega)}{(\omega^2 - \Omega^2)^2 + 4n^2\Omega^2}
\]

In the equivalent phase-angle form, equation (2.13) will be of the form:

\[ u_p = A \cos(\Omega t - \theta) \quad (2.15) \]

where

\[
A = \sqrt{M^2 + N^2} = \frac{q / \omega^2}{\sqrt{1 - \Omega^2 / \omega^2} + 4n^2\Omega^2 / \omega^2}
\]

and

\[
\theta = \tan^{-1} \left( \frac{N}{M} \right) = \tan^{-1} \left( \frac{2n\Omega / \omega^2}{1 - \Omega^2 / \omega^2} \right)
\]

Now, introducing the damping ratio \( \gamma \) as the ratio between the damping of the oscillator and its critical damping:

\[ \gamma = \frac{n}{\omega} = \frac{c}{c_{cr}} \quad (2.18) \]

We may substitute (2.15) into (2.12) using the values of \( \omega \) and \( q \) as defined before in order to get:

\[ u = \frac{F_u}{k} \beta \cos(\Omega t - \theta) \quad (2.19) \]

With \( \beta \) being

\[ \beta = \frac{1}{\sqrt{1 - \Omega^2 / \omega^2}^2 + (4\gamma \Omega / \omega)^2} \quad (2.20) \]
Chapter 2. Structural Dynamics

This means that the total steady-state forced response amplitude can be written as the static response $F_0/k$ times a magnification factor $\beta$ that is a function of the ratio between the load frequency and the eigenfrequency and of the damping ratio.

The homogeneous solution to (2.12) is found similarly by supposing a solution of the form $u=Ce^{rt}$, as it is usual for homogeneous linear differential equations with constant coefficients. Introducing this solution into equation (2.12) leads us to the second degree equation in $r$

$$r^2 + 2nr + \omega^2 = 0 \text{ from which } r = -n \pm \sqrt{n^2 - \omega^2} \quad (2.21)$$

Now, whenever $n < \omega$, the most typical case in bridge structures, we obtain two complex roots:

$$r_1 = -n + i\omega_d \text{ and } r_2 = -n - i\omega_d \text{ with } \omega_d^2 = \omega^2 - n^2 \quad (2.22)$$

Substituting these roots gives us two solutions. Any linear combination of these solutions will also be a solution, so in order to get rid of the imaginary term we write:

$$u_1 = \frac{C_1}{2}(e^{r_1 t} + e^{r_2 t}) = C_1 e^{-\omega_d t} \cos(\omega_d t)$$

$$u_2 = \frac{C_2}{2i}(e^{r_1 t} - e^{r_2 t}) = C_2 e^{-\omega_d t} \sin(\omega_d t) \quad (2.23)$$
So that the general solution for the free vibration case acquires the form

\[ u = e^{-nt} \left( C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t) \right) \]  \hspace{1cm} (2.24)

with \( C_1 \) and \( C_2 \) constants that remain to be determined depending on the initial conditions. The factor \( e^{-nt} \) decreases with time and the vibrations originally generated will be damped out eventually. Notice that the angular frequency of damped vibration \( \omega_d \) is somewhat lower than \( \omega \) the ratio between them being:

\[ \frac{\omega_d}{\omega} = \sqrt{1 - \frac{n^2}{\omega^2}} \] \hspace{1cm} (2.25)

For bridge engineering purposes, the quote \( n/\omega \) practically never surpasses a limit of about 0.1 so that the damped frequency of vibration can be considered equal with the undamped one.

**Logarithmic decay**

The free vibration solution (2.24) can be rewritten as:

\[ u = A e^{-nt} \cos(\omega_d t - \alpha) \] \hspace{1cm} (2.26)

![Figure 2.5: Damped free vibration, with successive points of extreme displacement m1, m2 and m3 marked.](image-url)
with

\[ A = \sqrt{C_1^2 + C_2^2} \quad \text{and} \quad \alpha = \tan^{-1}\left( \frac{C_2}{C_1} \right) \] (2.27)

The velocity can then be obtained by means of derivating with respect to time:

\[ \dot{u} = -Ae^{-nt} \omega_d \sin(\omega_d t - \alpha) - Ae^{-nt} \cos(\omega_d t - \alpha) \] (2.28)

Setting the velocity equal to zero, we get:

\[ \tan(\omega_d t - \alpha) = -\frac{n}{\omega_d} \] (2.29)

Thus points of extreme displacement are separated by equal time intervals of length

\[ t = \frac{\pi}{\omega_d} = \tau_d/2. \]

The ratio between two successive maximal amplitudes is:

\[ \frac{u_{mi}}{u_{m(i+1)}} = \frac{Ae^{-nt_d}}{Ae^{-n(t_d + \tau_d)}} = e^{n\tau_d} = e^\delta \] (2.30)

The quantity \( \delta = n\tau_d \) is called the logarithmic decrement, and can be used to estimate the damping \( n \) in the system, in accordance with

\[ \delta = \ln \frac{u_{mi}}{u_{m(i+1)}} = n\tau_d = \frac{2\pi n}{\omega_d} \approx \frac{2\pi n}{\omega} \] (2.31)

It is only necessary to determine experimentally the ratio of two successive amplitudes of vibration. However, greater accuracy is obtained is the ratio of two amplitudes \( j \) cycles apart is used, in which case the logarithmic decay is calculated as:

\[ \delta = \frac{1}{j} \ln \frac{u_{mi}}{u_{m(i+j)}} \] (2.32)

### 2.3 System with Multiple Degrees of Freedom

The set of equations governing the free vibrations of an \( n \) degrees of freedom system, with no external forces take the general form:
2.3. System with Multiple Degrees of Freedom

\[ \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & \cdots & M_{1n} \\ M_{21} & M_{22} & M_{23} & M_{24} & \cdots & M_{2n} \\ M_{31} & M_{32} & M_{33} & M_{34} & \cdots & M_{3n} \\ M_{41} & M_{42} & M_{43} & M_{44} & \cdots & M_{4n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ M_{n1} & M_{n2} & M_{n3} & M_{n4} & \cdots & M_{nn} \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \\ \ddot{u}_4 \\ \vdots \\ \ddot{u}_n \end{bmatrix} + \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & \cdots & S_{1n} \\ S_{21} & S_{22} & S_{23} & S_{24} & \cdots & S_{2n} \\ S_{31} & S_{32} & S_{33} & S_{34} & \cdots & S_{3n} \\ S_{41} & S_{42} & S_{43} & S_{44} & \cdots & S_{4n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ S_{n1} & S_{n2} & S_{n3} & S_{n4} & \cdots & S_{nn} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_n \end{bmatrix} = 0 \]  

(2.33)

Or, in its shorter form

\[ \mathbf{M}\ddot{\mathbf{D}} + \mathbf{S}\mathbf{D} = 0 \]  

(2.34)

As before, a harmonic solution can be assumed:

\[ \mathbf{D}_i = \Phi_{Mi}\sin(\omega_i t + \phi_i) \]  

(2.35)

For some \( \omega_i \) and \( \phi_i \). The symbol \( \mathbf{D}_i \) denotes a column matrix, or vector, of the displacement for the \( i \)th mode, and \( \Phi_{Mi} \) denotes the corresponding mode shape. Replacing (2.35) into (2.34) we get the set of algebraic equations:

\[ \mathbf{H}_i\Phi_{Mi} = 0 \text{ where } \mathbf{H}_i = \mathbf{S} - \omega_i^2\mathbf{M} \]  

(2.36)

To obtain a nontrivial solution the determinant of \( \mathbf{H}_i \) must be zero, giving an \( n \)th degree polynomial in \( \omega_i^2 \). If \( \mathbf{M} \) is positive-definite and \( \mathbf{S} \) is either positive-definite or positive-semidefinite then all the eigenvalues \( \omega_i^2 \) are nonnegative.

To obtain the eigenvectors \( \Phi_{Mi} \) we can take advantage of the fact that:

\[ \mathbf{H}_i^\dagger\mathbf{H}_i = |\mathbf{H}_i|\mathbf{I} = 0 \]  

(2.37)

Where \( \mathbf{H}_i^\dagger \) denotes the adjoint matrix (defined as the transpose of the cofactor matrix). Equation (2.37) indicates that the vectors \( \Phi_{Mi} \) are proportional to any nonzero column in \( \mathbf{H}_i^\dagger \). Since the eigenvector can be arbitrarily scaled it may be taken to be either equal to such a column or normalized as desired.

Consider now the modes \( i \) and \( j \). According to equation (2.36) we have:

\[ \mathbf{S}\Phi_{Mi} = \omega_i^2\mathbf{M}\Phi_{Mi} \]

\[ \mathbf{S}\Phi_{Mj} = \omega_j^2\mathbf{M}\Phi_{Mj} \]  

(2.38) a, b

Since both \( \mathbf{S} \) and \( \mathbf{M} \) are symmetric, we can premultiply the first expression with \( \Phi_{Mj}^T \) and postmultiply the transpose of the second by \( \Phi_{Mi} \) getting.
\[
\Phi^\top_M S \Phi_{Mi} = \omega_i^2 \Phi^\top_M M \Phi_{Mi} \tag{2.39} \text{a, b}
\]

The left-hand sides of these expressions are equal, and so must the right-hand sides be. Thus:

\[
(\omega_i^2 - \omega_j^2) \Phi^\top_{Mj} M \Phi_{Mi} = 0 \tag{2.40}
\]

On the other hand if we divide (2.39)a by \(\omega_i^2\) and (2.39)b by \(\omega_j^2\) and then subtract both expressions we get

\[
\left( \frac{1}{\omega_i^2} - \frac{1}{\omega_j^2} \right) \Phi^\top_{Mj} S \Phi_{Mi} = 0 \tag{2.41}
\]

To satisfy (2.40) and (2.41) when the eigenvalues are distinct \((\omega_i^2 \neq \omega_j^2)\), the following relation must follow:

\[
\Phi^\top_{Mj} S \Phi_{Mi} = \Phi^\top_{Mi} S \Phi_{Mj} = 0 \tag{2.42}
\]

and

\[
\Phi^\top_{Mj} M \Phi_{Mi} = \Phi^\top_{Mi} M \Phi_{Mj} = 0 \tag{2.43}
\]

These expressions represent the orthogonality relationships among the principal modes of vibration. We see that the eigenvectors are orthogonal with respect to \(M\) and also with respect to \(S\).

For the case when \(i=j\), equations (2.42) and (2.43) yield:

\[
\Phi^\top_{Mi} M \Phi_{Mi} = M_{Pi} \tag{2.44}
\]

And

\[
\Phi^\top_{Mi} S \Phi_{Mi} = S_{Pi} \tag{2.45}
\]

Where \(M_{Pi}\) and \(S_{Pi}\) are constants depending on how the eigenvector \(\Phi_{Mi}\) is normalized.

Placing all the vectors as row in a modal matrix of the form

\[
\Phi_M = [\Phi_{M1} \quad \Phi_{M2} \quad \ldots \quad \Phi_{Mn}] \tag{2.46}
\]

we can summarize the previous equations into:

\[
\Phi^\top_M M \Phi_M = M_{P} \tag{2.47}
\]
and

\[ \Phi_{M}^{T}S\Phi_{M} = S_{p} \]  

(2.48)

\( M_{p} \) and \( S_{p} \) are diagonal arrays referred to as principal mass matrix and principal stiffness matrix respectively. In order to take advantage of the diagonal character of this coefficient matrix the equation (2.34) of free vibration for a multidegree system can be premultiplied by \( \Phi_{M}^{T} \) and the identity \( I = \Phi_{M} \Phi_{M}^{-1} \) matrix can be inserted after \( M \) and \( S \) obtaining

\[ \Phi_{M}^{T}M\Phi_{M}\Phi_{M}^{T}\mathbf{D} + \Phi_{M}^{T}S\Phi_{M}\Phi_{M}^{-1}\mathbf{D} = 0 \]  

(2.49)

which can be rewritten as

\[ M_{p}\mathbf{D}_{p} + S_{p}\mathbf{D}_{p} = 0 \]  

(2.50)

The new displacement and acceleration vectors are defined in accordance to

\[ \mathbf{D}_{p} = \Phi_{M}^{T}\mathbf{D} \]  

(2.51)

and

\[ \mathbf{\ddot{D}}_{p} = \Phi_{M}^{T}\mathbf{\ddot{D}} \]  

(2.52)

The generalized displacements \( \mathbf{D}_{p} \) are called the principal coordinates, for which the equation of motion have neither inertial nor elasticity coupling. This results in \( n \) distinct and independent single degree of freedom systems, instead of the original coupled \( n \)-degrees system. This allows the system to be solved by the techniques used in the previous section about SDOF systems, merely by a change in the coordinate system. Of course, the coordinate system can readily be changed back as to obtain the original displacements.

**Normal-Mode response to applied forces**

Consider the case of a multiple-degree system subjected to applied forces in the original coordinate system. The equation of motion for such a system will be:

\[ \mathbf{M}\ddot{\mathbf{D}} + \mathbf{S}\mathbf{D} = \mathbf{Q} \]  

(2.53)

Where \( \mathbf{Q} \) represents the vector of time-varying applied forces:

\[ \mathbf{Q} = \begin{bmatrix} Q_{1} \\ Q_{2} \\ \vdots \\ Q_{n} \end{bmatrix} = \begin{bmatrix} F_{1}(t) \\ F_{2}(t) \\ \vdots \\ F_{n}(t) \end{bmatrix} \]  

(2.54)
Premultiplying (2.53) by $\Phi_M^T$ and inserting the identity $I = \Phi_M \Phi_M^{-1}$ matrix after $M$ and $S$ as in the previous section we obtain:

$$\Phi_M^T M \Phi_M \ddot{\Phi}_p + \Phi_M^T S \Phi_M \dot{D}_p = \Phi_M^T Q$$

(2.55)

Introducing the symbol $Q_p$ to denote the vector of applied forces in the principal coordinates, defines as:

$$Q_p = \Phi_M^T Q$$

(2.56)

We can rewrite (2.55) as:

$$M_p \ddot{D}_p + S_p \dot{D}_p = Q_p$$

(2.57)

with $M_p$ and $S_p$ as in (2.47) and (2.48). Thus the system turns into $n$ different uncoupled SDOF systems and can be solved with the techniques for forced SDOF explained previously in this chapter.

**Damping in multiple-degree systems**

As mentioned before, damping is of greatest importance when resonance occurs. In multiple-degree systems resonance can take place when the frequency of loading is close to any of the eigenfrequencies of the system. The importance of the damping is difficult to assert a priori. It is related to the duration of the excitation, as well as to its frequencies and many other factors. It should, therefore, always be considered until its effects are shown not to be significant.

If viscous damping is considered the equation of motion take the form:

$$M \ddot{D} + C \dot{D} + S D = Q$$

(2.58)

In which the damping matrix $C$ has the general form:

$$C = \begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & \cdots & C_{1n} \\
C_{21} & C_{22} & C_{23} & C_{24} & \cdots & C_{2n} \\
C_{31} & C_{32} & C_{33} & C_{34} & \cdots & C_{3n} \\
C_{41} & C_{42} & C_{43} & C_{44} & \cdots & C_{4n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
C_{n1} & C_{n2} & C_{n3} & C_{n4} & \cdots & C_{nn}
\end{bmatrix}$$

(2.59)

For the principal modes of vibration to exist, it is necessary that the transformation that decouples the equation of motion also diagonalizes the damping matrix $C$. This does not occur in reality in systems with considerable damping. The eigenvalues of this kind of systems have significant imaginary components that cause complex phase relationships in the natural modes that do exist.
In lightly damped systems this need not to be considered. The nature of damping is so poorly understood that attempts to determine the coefficients in the damping matrix are bounded to fail. A simpler approach consists of experimentally obtaining (or assuming) the damping ratio $\gamma_i$ for each of the natural modes, and neglect the inter-modal damping influence.

A practical way of applying this approach is to assume $C$ to be a linear combination of $M$ and $S$. Since the transformation to principal coordinates diagonalizes both $M$ and $S$, it will also diagonalize any linear combination of these two matrices, achieving the uncoupling of the system. A damping matrix of the form:

$$C = aM + bS$$  \hspace{1cm} (2.60)

Will result in the set of equations, after decoupling:

$$u_{Ni} + (aM_{Pi} + bS_{Pi})u_{Ni} + \omega_i^2 u_{Ni} = q_{Ni} \text{ for } i=1,2,3...n$$  \hspace{1cm} (2.61)

Since $M_{Pi}$ depends on the normalization of the $i$th eigenvector it can always be set to 1. If this is done, the value of $S_{Pi}$ will be $\omega_i^2$ in accordance to (2.39 a and b), (2.47) and (2.48). In this way we have:

$$u_{Ni} + (a + b\omega_i^2)u_{Ni} + \omega_i^2 u_{Ni} = q_{Ni} \text{ for } i=1,2,3...n$$  \hspace{1cm} (2.62)

Thus each normal mode will have a damping ratio:

$$\gamma_i = \frac{a + b\omega_i^2}{2\omega_i} = \frac{a}{2\omega_i} + \frac{b\omega_i}{2} \text{ for } i=1,2,3...n$$  \hspace{1cm} (2.63)

Meaning that an increase of the constant $a$ will result in a considerable increase of the damping ratio only for the low-frequency eigenmodes, while an increase in the constant $b$ will lead to a raise in the damping ratios of the high-frequency modes.
3 Signal Analysis

In bridge engineering the signals interesting to measure are typically the deflections, accelerations, applied forces or any other relevant physical magnitude of a structural element. Thus, we could be interested, for example, in keeping a record of the deflections caused at some point of a beam by a train crossing a bridge at a certain speed. Mathematically, this “record” or signal is treated as a function, often with the time as domain. For any given time instant there is one and only one value for the deflection in a defined point.

A signal recorded this way, the parameter measured for any given point, is called a time-history, because it describes what happens with the parameter under study as time flows. Thus the parameter we are interested in is represented as a function of the time variable.

This is the most natural and intuitive way of representing and recording a signal, but it has its drawbacks, since much of the useful information is present but hidden, as we will see later.

Theoretically such functions carry an infinite amount of information. Between any two time instants in the time line there is an infinite number of points for which the value

![Figure 3.1: An acceleration time-history recorded at the New Årsta Bridge, and a zoom-in. The signal was recorded with a sampling frequency of 600 Hz.](image)
of the deflection must be stored. Our capacity to store information, thought enhanced by the arrival of computers, is certainly limited. To overcome this problem the analog signal, containing information from infinite points with infinite precision has to be turned into a discrete signal and then into a digital signal that can be handled by a computer.

Discrete and digital signals have important technical differences, though they are used as synonyms for many engineers. Discrete signals are defined for a finite (or at most countable infinite) domain. Typically this domain will be a set of time instants in which the interested parameter is measured and recorded. If we start recording the (discrete) time-history of the deflection at a beam’s mid-span from time \( t_0 = 0 \), we will then store the values of this parameter at time instants \( t_i \) for \( i = 0, 1, 2, 3 \ldots \) a positive integer.

Unlike analog signals, which have a continuous domain (a time line or its mathematical equivalent the set of the real numbers), discrete signals take values on the set of the integers. Therefore, between two data points (two time instants with the respective value of the measured parameter) there is only a finite number of data point to be recorded and stored.

If we choose to record the deflection in a point on a beam, say each hundredth of a second, starting from \( t=0 \) then we will have information about the variable we chose at times \( t=0.01, 0.02, 0.03 \ldots \) but we will know nothing about what happened in between those measurements.

In a second step the discrete values have to be transformed to binary numbers so that they can be stored in a computer. This transformation also implies a lost in accuracy.

These two reductions, from continuum time to discrete time, and from the actual physical value to binary are called discretization and digitalization respectively.

The whole process of converting an analog signal into a digital one is called sampling. The samples are usually taken at regular spaced time intervals, \( i.e., \) the sampling frequency is constant. Sampling frequency is critical for the usefulness of a discrete signal. If too low a frequency is chosen relevant information could be lost and if the sampling frequency is too high it will only result in larger computational and storage costs. The optimal sampling frequency is very system dependant; in general it should be set to many times the highest relevant frequency to be measured. This is because an analog low-pass filter is usually used before discretising the signal. The propose of the

![Figure 3.2: The dots represent a discrete signal, sampled with 4 Hz. As seen in the figure, it could correspond to a signal with frequency 1.5 Hz as well as 2.5 Hz.](image-url)
3.1. FOURIER TRANSFORM

low pass filter is to prevent a phenomenon known as aliasing, thus it is usually referred to as anti-alias filter.

When a signal is converted from analog to digital, there is always a loss of information of the frequency of the original analog signal. In this way it is impossible to completely determinate the frequency of the original analog signal. All the frequencies that are higher than half of the sampling frequency are “mirrored” into a frequency lower than half of the sampling frequency. In figure 3.2 two signals are showed, one with a frequency of 1.5 Hz and another with 2.5 Hz. If the signals are sampled with a sampling-frequency of 4 Hz (red dots in figure 3.2), both digital time-histories will be identical. If information is only available from the digital signal, there is no way to tell if the original frequency was 1.5 or 2.5 Hz. The frequency of 2.5 Hz has been aliased into 1.5 Hz.

Half the sampling frequency is a very important parameter governing aliasing. This frequency is sometimes called the Nyquist frequency after Harry Nyquist (1887-1976), one of the pioneers of the signal analysis.

In general a frequency $f_i$ higher than the Nyquist Frequency will alias into a frequency $f$ lower than the Nyquist Frequency if:

$$f + f_i = f_s \cdot k \quad \text{or} \quad f - f_i = f_s \cdot k$$

for $k$ an integer. (3.1)

where $f_s$ is the sampling frequency.

3.1 Fourier Transform

To better understand the characteristics of a signal we introduce the frequency-domain representation as an alternative to the much more intuitive time-domain representation or time-history.

From the designers point of view it is very interesting to know with what frequencies the bridge vibrates, since it can give an idea of the eigenfrequencies of the structure, of the loading frequencies and of possible resonance problems. By directly studying a time-history, like the one in figure 3.1, is difficult to imagine how it could even have something like a frequency, since it is not nearly periodic, but there are mathematical tools to tell us the frequencies of almost any signal, periodic or not.

A very important result in signal analysis is that any waveform signal that exists in the real world can be decomposed into a sum of sine signals of the form:

$$F_{A,\omega,\theta} = A \sin(\omega t + \theta)$$

(3.2)

So that the signal we are interested in analyzing is represented as the sum of many simple sine-shaped functions, with their own amplitude $A$, frequency $\omega$ and phase $\theta$. By comparing then the amplitudes associated to the different frequencies it can be discovered which frequencies make the most important contributions to the total signal.
In figure 3.3 we see a signal in time-domain and its decomposition in the frequency axis. This signal is the sum of 3 different sine-shaped signals, all with different amplitudes, frequencies and phases. The frequency-domain representation is equivalent to a graph of the amplitude against the frequency, instead of the amplitude against the time, as in time-domain representation. Say, for example, that the frequencies that compose the signal described in figure 3.3 are $\omega_1$, $\omega_2$ and $\omega_3$, and that they have respective amplitudes $A_1$, $A_2$ and $A_3$. If this were the case, then the frequency-domain representation will be a function that is zero constantly for all the frequencies, except at $\omega_1$, $\omega_2$ and $\omega_3$, where it will jump to the values of $A_1$, $A_2$ and $A_3$ respectively.

So loosely speaking the spectrum of a signal is a measure of how much it vibrates at different frequencies, also called the frequency content of the signal.

The Discrete Fourier Transform (DFT) has been known to mathematicians for a very long time, but before the appearance of the telegraph and the telephone in the nineteenth century it had no practical applications, so it was an issue that only concerned abstract mathematicians. These two inventions marked the beginning of signal generation and signal processing and made the DFT a very important and useful tool in this new field of engineering.

The DFT of a vector $y = (y_1, y_2, y_3, y_4, \ldots y_n)$ of $n$ samples will then be:

$$Y_t = \sum_{k=1}^{n} y_k e^{\frac{2\pi i (t-1)(k-1)}{n}} \text{ with } t = 1, 2, 3 \ldots n$$

$$\text{ (3.3) }$$

Figure 3.3: A signal composed of 3 combined sine-shaped signals and it decomposition into the frequency domain. Note the phase difference among the 3 components.
By establishing the notation:

\[ W_n = e^{-2\pi i/n} \]  \hspace{1cm} (3.4)

We can conveniently rewrite equation (3.3) in matrix form:

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
\vdots \\
Y_n
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
W_n^1 & W_n^2 & W_n^{n-1} \\
W_n^2 & W_n^4 & W_n^{2(n-1)} \\
\vdots & \vdots & \ddots & \vdots \\
W_n^{n-1} & W_n^{2(n-1)} & \cdots & W_n^{(n-1)^2}
\end{bmatrix} \begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
\vdots \\
y_n
\end{bmatrix}
\]  \hspace{1cm} (3.5)

This matrix, called the Fourier Matrix, has the complex elements:

\[ F_{nk} = W_n^{(k-1)(r-1)} \]  \hspace{1cm} (3.6)

It is a simple exercise to prove that pre- or postmultiplying the Fourier Matrix by the matrix whose elements are:

\[ F'_{nk} = W_n^{-(k-1)(r-1)} \]  \hspace{1cm} (3.7)

will result in \( nI \), where \( I \) is the \( n \times n \) identity matrix.

So the inverse DFT will be defined by:

\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
\vdots \\
y_n
\end{bmatrix} = \frac{1}{n} \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
W_n^{-1} & W_n^{-2} & W_n^{-(n-1)} \\
W_n^2 & W_n^4 & W_n^{2(n-1)} \\
\vdots & \vdots & \ddots & \vdots \\
W_n^{-(n-1)} & W_n^{-(2(n-1))} & \cdots & W_n^{-(n-1)^2}
\end{bmatrix} \begin{bmatrix}
Y_1 \\
Y_2 \\
Y_3 \\
\vdots \\
Y_n
\end{bmatrix}
\]  \hspace{1cm} (3.8)

or in its compact form:

\[ y_t = \frac{1}{n} \sum_{k=1}^{n} Y_k W_n^{-(k-1)(r-1)} \quad \text{with} \quad t = 1, 2, 3 \ldots n \]  \hspace{1cm} (3.9)

We can equivalently write, recalling the definition of \( W_n \) in equation (3.4):

\[ y_t = \frac{1}{n} \sum_{k=1}^{n} Y_k e^{2\pi i(k-1)(r-1)/n} \quad \text{with} \quad t = 1, 2, 3 \ldots n \]  \hspace{1cm} (3.10)

It is important to notice that the vector \( y \) typically represent the values of a variable measured at different time instants, so the variable \( t \) actually represents the (discrete)
time. Observe also that the exponent in the right-hand side if equation (3.10) is pure imaginary representing the oscillating nature of the different terms of the sum.

If the vector \( \mathbf{y} \) was real-valued from the beginning then the imaginary part of the right-hand side should, and in fact does, vanish for every \( t = 1, 2, 3... n \). So if we write \( Y_k = (a_k - i b_k) \) and discard the imaginary part, we can transform equation (3.10) into:

\[
y_t = \frac{1}{n} \sum_{k=1}^{n} a_k \cos \left( \frac{2\pi(k-1)(t-1)}{n} \right) + b_k \sin \left( \frac{2\pi(k-1)(t-1)}{n} \right)
\]

(3.11)

or in the phase form:

\[
y_t = \frac{1}{n} \sum_{k=1}^{n} c_k \sin \left( \omega_k (t-1) + \theta_k \right)
\]

(3.12)

with the constant:

\[
c_k = \sqrt{a_k^2 + b_k^2}, \quad \theta_k = \arctan \frac{a_k}{b_k} \text{ and } \omega_k = \frac{2\pi(k-1)}{n}
\]

(3.13)

arriving to our starting point, namely that the DFT transform a discrete signal to a sum of sinus-shaped signals, all with their different amplitudes, frequencies and phases.

As with every discretization of a phenomenon that is intrinsically continuous, the DFT has its drawbacks, many inherited of the discretization of the signal itself.

The domain of the variable \( k \) spans from 1 to \( n \), so that the lowest frequency \( \omega_k \) considered is always 0 (the constant or non-oscillatory term of the signal) and the highest is \( (n-1)/n \) times the sampling frequency. Given that the number of samples is usually large it is a good approximation of the sampling frequency.

The DFT will decompose the signal into as many frequency components as data points registered in the signal, the lowest term having zero frequency and the largest the sampling frequency. The number of samples taken will necessarily be the duration of the signal times the sampling frequency. So that the frequency resolution can be expressed as:

\[
\Delta f = \frac{f_s}{n} = \frac{f_s}{\tau} = \frac{1}{\tau}
\]

(3.14)

It follows then that the frequency resolution \( \Delta f \) will be equal to the inverse of the length of the total measured time \( \tau \). The length of the signal to be analyzed must therefore be chosen carefully so that a sufficient frequency resolution is achieved.

It was said above that there is no way of distinguishing between two aliased frequencies, but the DFT consider frequencies from zero up to the sampling frequency so that the higher frequencies should alias. How does the DFT algorithm discriminate between two aliased frequencies? It does not. It assigns half of the amplitude to the
frequency below the Nyquist Frequency and the other half to the one above. An example will help to illustrate this phenomenon: Suppose a perfect sinusoidal signal with a frequency of 100 Hz and amplitude of 1 is sampled with 600 Hz. It will be impossible to tell, after digitalizing, if the frequency recorded had a frequency of 100 or 500 Hz. Under these conditions the DFT will show amplitudes of 0.5 for both 100 and 500 Hz, so that the sum of both amplitudes corresponds to the original amplitude. Since this is truth for all aliased frequencies, it follows that the half of the spectrum that is beyond the Nyquist Frequency is only a mirror image of the first half of it.

In fact the situation is more complicated than that. Frequencies higher that the sampling frequency will also alias as shown in figure (3.4). Aliasing can be visualized as a folding of the spectrum in those frequencies that are integer multiples of the Nyquist Frequency, resulting in a spectrum that “packs” together information of all the frequencies in the frequency interval from 0 to $f_s/2$.

If no vibration occurs at frequencies higher than the Nyquist Frequency then the original signal can be completely reconstructed from the discrete signal (or from its spectrum). This result is known as the Sampling Theorem. It is a very strong and somewhat anti-intuitive assertion whose mathematical details and their demonstration are beyond the scope of this study, but it is a very useful tool.

Theoretically a signal could be perfectly reconstructed from its discrete version. In the real world this is impossible; the signals recorded are not only discrete but digital as well, which make them only an approximation. Other practical problems arise: noise is always present in measurements and filtering can greatly reduce the content of higher frequencies but never completely remove it, while it also produces a frequency dependant phase shift that distort the signal somewhat.

If an analog signal is filtered by a low-pass (anti-alias) filter so that the frequency content is zero beyond the Nyquist Frequency, then no aliasing will take place. This allows the engineer to overcome the difficulties named in the above, namely that the original frequency of a digitalized signal can not be completely determined. The analysis becomes thus much simpler, and more information about the vibrations can be harvested from the frequency spectrum.

The spectrum is not only useful to determine the frequencies at which a system vibrates, allowing identification of eigenmodes and loading frequency. It can also be used to estimate the amount of damping an eigenmode has.

As showed above, the FT of a perfect sinusoidal signal will be a single sharp peak at the signal’s frequency. If a damped signal is analyzed with the FT the peak will tend to “spread out” to neighboring frequencies, reducing also its height.

The relation between the peak height and width gives a very good approximation of the damping of the eigenmode under study for lightly damped structures.
Chapter 3. Signal Analysis

Figure 3.4: An analog signal’s spectrum and the spectrum of the same signal after discretization. All the frequencies higher than half the sample frequency will be “folded” so that they appear to be lower frequencies in the discrete spectrum.

Let \( \omega_a \) and \( \omega_b \) be the angular frequencies at which the response amplitude is \( 1/\sqrt{2} \) times the peak amplitude, at the left and right of the maximum response and \( f_a \) and \( f_b \) the corresponding frequencies. If the structure is only lightly damped then the damping ratio \( \zeta \) can be calculated as:

\[
\zeta \approx \frac{\omega_b - \omega_a}{2\omega_{\text{max}}} \approx \frac{\omega_b - \omega_a}{\omega_b + \omega_a} = \frac{f_b - f_a}{f_b + f_a} \tag{3.15}
\]

This formula is originally derived from the Magnification Factor (see chapter 2.2) that measures the steady-state response of SDOF loaded with a harmonic applied force. It can be generalized to more general load patterns, provided the damping is low. Together with Logarithmic Decrement is one of the most used tools to experimentally determine the damping of different modes. One of its main drawbacks is that when several peaks are close to each other so that they overlap it becomes very difficult to distinguish between the amplitude contributions of the different peaks (see figure 3.6). Methods to overcome this difficulty had been developed during the years. The constant chosen to determinate \( \omega_a \) and \( \omega_b \), \( 1/\sqrt{2} \) in the half-power band width method, can actually be set to higher values, with slight modifications in equation (3.15); minimising thus the effects of near by peaks. These methods often need a higher frequency resolution, though, and they tend to overestimate the damping, which is potentially dangerous (Yin, 2008).
3.2. Filtering

A recorded signal contains a great amount of information. Not all this information is useful for the engineer, so it may be helpful to have a set of signal processing function that could enhance the relevant information and remove the information that lacks physical meaning or that is not of interest for the problem in study. The most typical operations are noise reduction and unwanted frequency content removal. The circuiting designed to achieve this, or the algorithm implemented in the case of a digital signal, is called a filter.

Many types of filter exist, as many types of operations can be performed in signals. Given the actual omnipresence of computers and their speed and capacity, digital filtering is the most commonly used nowadays. They have many advantages, over analog filters: while analog filters are a piece of hardware, digital filters are only algorithms that can be performed by a computer. They can achieve very sharper frequency discriminations than practical analog filters and they also introduce less noise into the signal compared with analog filters (Smith, 1997).

Many of their disadvantages are related to the drawbacks of digitalizing in general, such as lost of precision and sample rate limitations.
Figure 3.6: Two superposed frequency peaks (above) and their separated graph (below). If the damping is estimated with the half power band width method from the superposed graph, the values are overestimated with a factor 1.25 for the peak at 6 Hz.

The easiest way of understanding a filter is to visualize its Transfer Function, i.e., the ratio between the input and the output signals' spectra for the different frequencies. A low-pass filter, for example, is a filter that allows all the frequencies lower that a certain given value (cut frequency) while rejecting all higher frequencies. The Transfer Function of a perfect low-pass filter will then be a step function with value 1 from 0 to the cut frequency $\omega_c$, and with value zero for frequencies higher than $\omega_c$. This kind of filtering is impossible to achieve in real life situation. Instead of a “discrete jump” what we get when we implement a low-pass filter is a continuous (if very steep) transition that tends rapidly to zero.

The most common digital filters express the output (filtered) signal as a linear combination of the $n+1$ last input values and the $m$ previous output values. This kind of filters is called linear filters. Because they only depend on past values of the input signal they are said to be causal (causality means that they can be calculated while the signal is sampled, since they do not rely of future values of the signal).
3.2. FILTERING

Figure 3.7: Pass, transition, and stop-band for three of the most used linear digital causal low-pass filters. Note how higher sharpness is achieved by allowing distortions (ripples) to occur.

The most common digital filters express the output (filtered) signal as a linear combination of the \( n+1 \) last input values and the \( m \) previous output values. This kind of filters is called linear filters. Because they only depend on past values of the input signal they are said to be causal (causality means that they can be calculated while the signal is sampled, since they do not rely of future values of the signal).

For designing such a filter the first step is to write the desired transfer function as a coefficient between two polynomials, the nominator polynomial of degree \( n \) and the denominator polynomial of degree \( m \). By mean of the FFT this can be converted to an equivalent expression that gives the “current” output value as a linear function of the “current” input, the \( n \) past input values and the \( m \) past output values. Thus to multiply the spectrum of a signal \( x \) by Transfer Function of the form:

\[
H(\omega) = \frac{b_0 w^n + b_1 w^{n-1} + \cdots + b_{m-1} w^1 + b_m}{w^n + a_1 w^{n-1} + \cdots + a_{n-1} w^1 + a_n}
\]

would be equivalent to obtain the spectrum of the signal \( y \) defined as:

\[
y_i = b_0 x_i + b_1 x_{i-1} + \cdots + b_{m-1} x_{i-m} - a_1 y_{i-1} - a_2 y_{i-2} - \cdots - a_n y_{i-n}
\]
Filters not only affect the amplitude of the different frequency contents but also change their phase. Designing the right filter is often a compromise between the sharpness with which the filters cut off unwanted frequencies and the fidelity with which it represent the wanted frequency content (in both amplitude and phase). Optimal filters have been design for the different needs that arise in signal analysis. Butterworth filters are designed for maximal flatness at both $\omega=0$ and $\omega=\infty$ (see figure (3,7)) while relaxing one of these two condition will result in the so called Chebyshev filters, that are flat either in the pass-band or in the stop-band but have a sharper cut-off. The elliptic filters are design for maximal cut-off steepness, while allowing for ripples on both the stop and the pass-band. There are also filter that minimise the phase distortion of the signal, Bessel filters, and a battery of different filter with thoroughly studied characteristics that give powerful tools for signal analysing.

**Signal Smoothing**

Another very useful family of filter is those designed not so much to remove unwanted frequencies as to smooth out noise from noisy signals. The most simple of such filters in

![Figure 3.8: Top panel: A noise free signal together with a 50% noise contaminated signal. Second panel: Smoothed signal using moving average with window-width 10 (light) and 20 (dark) data points. Third panel: Smoothed signal using median filter with window-width 10 (light) and 20 (dark) data points. Bottom panel: Smoothed signal using Savitzky-Golay filter. An 8\textsuperscript{th} degree polynomial is used to fit 121 data points (light) and A 4\textsuperscript{th} degree polynomial is used to fit 61 data points. Notice the superior performance of Savitzky-Golay filter in preserving the amplitude.](image)
the moving average filter in which replace each data point with the average of the data point itself and a number of data points to the left and to the right of the “actual” data point. Given the random characteristics of noise it is natural to expect that averaging a number of samples will tend to reduce its presence. This kind of filters has the disadvantage of also reducing the amplitude of high frequency content, since the work as a low-pass filter. In despite of its simplicity, moving average filter is the best possible linear filter for reducing random white noise while keeping step response as sharp as possible (Smith, 1997). This smoothing procedure can be further varied by assigning weighting factors to the data points to be averaged. Another noise reduction filter is the median filter. Instead of taking the average or mean value of a number of samples, the median is used as output value. This kind of filters smoothes the signal while preserving eventual edges. They also include no float point operations what makes them very fast and suitable for real-time applications.

A third very useful low-pass filter that is specially well-equipped to smooth noisy signals is the Savitzky-Golay filter. This filter uses the less-square method to fit with a polynomial of a given degree a number of samples. Typically it fits $2n+1$ data points ($n$ points to the left, $n$ to the right and the “actual” input value) by a polynomial $p$ of degree $m \ll 2n+1$. Thus the filtered value $(y'_0,t_0)$ will be $(p(t_0),t_0)$. This method requires a large number of polynomials fitting, since for each sample a polynomial of degree $m$ has to be found. This involves matrix inversion, which is very time-consuming. Fortunately since the process of least-squares fitting involves only a linear matrix inversion, the coefficients of a fitted polynomial are themselves linear in the values of the data. That means that we can do all the fitting in advance, for fictitious data consisting of all zeros except for a single 1, and then do the fits on the real data just by taking linear combinations (Cambridge University, 1992).
4 Modelling

4.1 Bridge description

The bridge under study is The New Årsta Bridge, a railway and pedestrian bridge connecting Stockholm with the suburbs located at the south of the city, over the Årsta bay. The bridge is a very complex and highly optimized pre-stressed concrete structure that achieves a very high degree of slenderness. It was planned and built to increase the railway traffic capacity connecting Stockholm to the south. Up to 2005 all the traffic went on the old Årsta Bridge built in 1929, which could no longer cope with the new traffic needs of the city.

The design is the result of an international competition in which 24 different projects participated. The winning proposal, submitted by Foster and Partner together with the design company Ove Aarup A/S resulted in a doubled-curved underside structure, very elegant and aesthetically simple. But the aesthetics of the structure had its backside. The resulting complex curved and slender structure had as consequence an increased construction cost. However, the design presented other technical advantages mainly in maintenance and noise prevention. These advantages rendered it as the winning proposal in despite of its high cost around SEK 1 500 million.

4.1.1 The Bridge

The bridge is an 833 m long and 19.5 m wide pre-stresses concrete structure. It has 11 spans; the nine central ones have the same shape (though some are slightly curved in the horizontal plane) and are each 78 m long. The northernmost and southernmost spans are slightly different shaped and are 48 and 65 m long, respectively. The bridge allows for a 40 m wide navigation channel with 26 m vertical clearance. The superstructure at the piers is 5.2 m high and at the mid span it is only 3.5 m high.

The two railway tracks are embedded with 1.2 m parapet clad with vibration-absorbent wool to reduce the noise produced by train passages. At the west side of the bridge there is a 3.5 m wide pedestrian way, and a similar corridor at the east side is used as maintenance and service road. At each of the pier sections there is a 2 m thick transversal wall that works as stiffener.

In order to be able to achieve the maximum possible slenderness the designers opted for a ballastless track configuration that reduced considerably the structure’s weight. The track was instead mounted directly on the concrete upper slab.

The concrete quality used for the construction was K60 defined in the Swedish Standard BBK94 (Boverket, 1994) and the properties of the concrete were tested thoroughly, both at the plant and at the construction site. The bridge was supposed to be of a “Falun” shade of red; very typical for Sweden and it was decided to use coloured concrete instead of painted due to the lower maintenance costs, even though it raised somehow the construction costs.
Reinforcement steel was needed in huge amounts. Not considering the 50 pretensioning cables the superstructure has 220 kg/m$^3$ of reinforcement, almost twice as much as in regular reinforced concrete bridge structures.

Besides the piers P9 and P10, which are poled, all the foundations rest on the bedrock.

### 4.1.2 Construction

The bridge has a slight curve in the horizontal direction in the 4 northernmost spans. For this reason conventional scaffolding had to be used. The resulting provisional structure was almost 200 m long and more than 20 m high. Such an enormous amount of scaffolding as needed for the project could not be found in Sweden, not even in Europe for that matter, so most of it was shipped from abroad.

For the southern part the different sections of the bridge were moulded in a launched formwork mounted in a tower wagon. This formwork was used and reused in all the straight middle spans. Given the complex geometry of the bridge deck the formwork needed to be large enough as to allow for a whole span to be cast before launching it to the next span. The resulting structure was enormous, weighting about 1 800 tons and, when full extended, reaching 180 m long. With the reinforcement in place and all the concrete cast each support has to bear around 4 000 tons.
4.1. BRIDGE DESCRIPTION

Figure 4.2: Scaffolding needed for the construction of the curved northern spans.

Figure 4.3: Detail of the launching platform used. The plastic pipes used for the tendons are clearly visible.
The formwork consisted of 2 longitudinally symmetrical halves. When the concrete was cast in place the formwork was removed by separating the halves sideways and then launched to the next span.

The schema for casting, pre-tensioning and launching was very complex but very critical for the success of the project. Also in order to assert the arising stresses and strains it was important to monitor when each construction stage took place.

4.1.3 Instrumentation

For monitoring the bridge during the construction and in the operation stage it was instrumented with a number of sensors: strain transducers, thermocouples, fibre optic strain sensors, accelerometers and a linear variable differential transformer (LVDT). Instrumentation is a very reliable method for asserting the behaviour of the bridge and it adequacy to the theoretical one. It has been successfully used in the past to predict the need for additional reinforcement work and other kinds of maintenance processes (Ko and Ni, 2005).

From the point of view of a dynamic analysis, the interesting instruments were mostly located in 4 different cross-sections throughout the bridge, 3 of which were on the navigable channel between piers P8 and P9. These 3 sections were labelled A, B and C; A being the section at pier 9; B the quarter section between P9 and P8 (nearer P9) and C representing the mid-span section. The fourth instrumented section, labelled D, was located equidistant from piers P7 and P8.

In order to identify the different sensors they were given a nomenclature that will be used throughout this study. Thus the notation BK4 means that it is a strain transducer located in cross-section B, mounted by KTH in the 4th position in that section. For the accelerometer the same notation was used but a two letters suffix was added to differentiate them from the strain transducers and to indicate its orientation. CK3 AV, for instance, should be read Vertical Accelerometer mounted in the 3th position (by KTH) in cross-section C.

Modern accelerometers are based on the piezoelectric effect, a well studied property of some crystals and ceramics to generate electric potential in response to applied mechanical stress. Simplified an accelerometer consist in a mass attached by some elastic material or spring to the surface to be measured. As the surface accelerates the spring will deform to transmit the acceleration to the attached mass. This deformation can be measured in different manners. Old fashioned seismographs just have a pencil fixed to the mass, that “draws” the deformation in a rolling sheet of paper, but modern accelerometers measure the electric potential difference in the piezoelectric material the spring is made of.

The accelerometers used are the Si-Flex\textsuperscript{TM} SF1500S accelerometers, mounted in protective cylinders designed by Claes Kullberg at KTH. These accelerometers can measure full scale signal from DC (i.e. 0 Hz or static signal) to 1500 Hz, and for small signals, such as the present in bridge vibrations, it can register vibrations up to 5000 Hz. More information about the bridge instrumentation can be found in (Enckell and Wiberg, 2005)
Figure 4.4: Instrumentation placed at the quarter point section between P8 and P9 (closest to P9). Strains are measured in the longitudinal, transversal and vertical directions. Acceleration is only measured in the vertical direction in the east edge beam. From (Wiberg, 2006)
Figure 4.5: Instrumentation placed at the mid-span point section between P8 and P9. Strains are measured in the longitudinal, transversal and vertical directions. Acceleration is measured in the vertical direction in the east and west edge beam and in the middle point. Transversal acceleration is also measured in the middle point. From (Wiberg, 2006)
Figure 4.6: Instrumentation placed at the mid-span point section between P7 and P8. Only acceleration in the vertical direction is measured in the east and west edge beam. From (Wiberg, 2006)
4.2 The Bridge Model

4.2.1 DynSolve

DynSolve, the computer program used in this study, has been developed by Raid Karoumi at The Royal Institute of Technology (KTH), Sweden. The program was originally developed with the intention of studying the moving load problem of cable supported bridges. The bridge is modelled in 2D using the finite element method. Different types of elements are available including cable, beam and truss elements. Classical bridge damping is considered. In addition, the user can include discrete damping devices as Tuned Mass Dampers (TMD) allowing for a study of the effect of such devices. The vehicles are modelled as either moving constant point forces, moving masses, or as simple sprung mass systems. Rail surface roughness and defects can be considered. The bridge-train dynamic interaction is taken into account using an iterative procedure. The user can choose between two methods for evaluating the dynamic response, i.e. the mode superposition technique for linear dynamic analysis and the direct integration methods for nonlinear dynamic analysis. Further details are given in (Karoumi, 1998). DynSolve is written based for the commercial software MATLAB® (The Math Works, 1998)

4.2.2 Model

The bridge was modelled as a 2 dimensional structure composed of modified Euler-Bernoulli beam elements. The material was considered homogeneous, so the pre-stressing cables and other reinforcement were not accounted for in the model. However the over all E-modulus of the material was increased as to resemble that of the “real” structure. The model will be updated afterwards to match the response measured in the structure so the input parameters adopted in first instance are not of great importance. Damping ratio of 1% was used for all the eigenmodes in the simulations in agreement with (Banverket, 2004). This parameter was not updated.

The deck of the bridge is made up of a single 815 m long beam with varying cross-sectional properties. The values of the area and of the moment of inertia of each

![Figure 4.7: Area and Moment of Inertia along the deck of the New Årsta Bridge. The transversal walls at each pier are clearly visible.](image-url)
4.3. Convergence Studies

4.3.1 Number of Modes

The number of modes used for the simulations was chosen with regard to the maximal vertical acceleration in the bridge deck. A number of runs were simulated using the HSLM-A1 train as reference and increasing the number of included modes by one, from 5 to 20. Beyond the 13th eigenmode of vibration no significant contribution was made to the overall levels of vertical acceleration (see figure 4.9), so 13 modes were included in the subsequent simulations. These simulations were performed with a time step of 0.007 s, more than enough to include at least two data points per cycle for the highest eigenmode considered, the eigenfrequency of the 20th mode being 7.9 Hz.
Figure 4.9: Max/min acceleration for every point along the bridge deck induced by the HSLM-A1 train at two speeds and considering 13 and 20 modes of vibration.

4.3.2 Time step

Rather than indicating the length of the time step to be used in the simulations, it was chosen to use as an input the number of increments which a train passage on the bridge was to be divided into. This method had the advantage of better controlling the size of the output files, as well as adapting the length of the time step to the speed of the crossing train. Since a vehicle crossing at higher speeds will take a shorter time to travel through the bridge, dividing the time required for it to cross by a fixed number will result in shorter time step and in higher accuracy with increasing speed.

It is usually recommended to have a time step shorter than half the period of the highest mode of vibration considered, so that the time-histories obtained contain sufficient information (see chapter 3). In this study, the highest vibration mode considered had a frequency of 4.76 Hz, thus a time step shorter than \((4.76 \text{ Hz})^2/2 = 0.11 \text{ s}\) is required. The HSLM trains used for the simulations never exceed 400 m in length, so the total distance covered by the train from the instant when the first axel load enters the bridge to the moment when the last axel load leaves is never surpasses 1215 m (the length of the train plus the length of the bridge). Given the range speed studied, between 150 to 300 km/h, this distance is covered in less than 30 s but more than 14 s.
The simulation were then divided into 3000 increments, so that the time step used was between 0.01 \( s \) and 0.005 \( s \) depending on the speed of the train, allowing for at least 20 increments per period. A comparison was also performed between simulations carried out with 3000 and 8000 increments, and no important differences were found in the max vertical deck acceleration at the range of speeds studied.

In the simulations used to compare the result obtained here with other studies, a somewhat different model was used. The vehicle speed used for the comparison was only 70 km/h, so using only 3000 time increments resulted in a very long time step that showed to cause convergence problems, so 8000 time iterations were considered instead. It also made the different studies more comparable since Wiberg used 8400 time iteration in his study (Wiberg, 2008).

### 4.3.3 Number of Elements

The 20 first eigenfrequencies of the structure converged already with 3 meter long beam elements or, equivalently, with 270 elements for the main girder. The crossing of a train of type HSLM-A1 was simulated to see how the division of the main girder into elements affected the accelerations and displacements calculated. Five different models were prepared, with 4, 3, 2, 1.5 and 1 meter elements. No significant differences were found between the last four models, so an element length of 3 meters was chosen for checking the resonance risk where many simulations had to be performed in an efficient way. For the other simulations used to compare the obtained results with measurements and results obtained by other studies a 2 meters element length was used. In such a manner each load remained in the same element for at least 7 time increments.

### 4.4 Comparison

The response of a single point load moving at a constant speed was simulated and the results compared with those obtained by Wiberg based on the commercial FEM program SOLVIA®. Two different sets of conditions were used (see figure 4.10). In the first one a full 3D model was implemented, allowing for vertical, horizontal and torsional effects. The point studied was not located at the centre of the cross-section but under one of the tracks so that the signal simulated accounts also for the torsional bending. In the second simulation only vertical deformation was allowed to occur, blocking all the transversal and torsional components of the eigenmodes in order to make the results more comparable with those obtained in this study and to be able to estimate the overall importance of the three-dimensional deformations.

The two 2-dimesional models lead to very similar results with accelerations that, to a great extent were identical (see figure 4.10). More important, from the designer’s point of view, the results showed very similar levels of acceleration. This speaks for the reliability of the model, since the two calculations were carried from to different models, considering different eigenmodes and with a different discretization of the structure.

The comparison between these two almost coinciding results and the result from the full 3-dimesional analysis reveals that the effects of torsional deformation are too
important to be disregarded. Even though the peak acceleration is much the same for all 3 plots the RMS of the acceleration obtained with the 3D model almost double those from the 2D model through out the measured time. Many of the eigenmodes found in the 3D analysis had important torsional and lateral components. When these were locked, the eigenmodes disappeared or changed considerably in shape and eigenfrequency.

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It can be noticed that the acceleration peak is always reached at the start of the simulation, when the train is entering the bridge relatively far away from the studied point, and before any important resonance effects could have taken place. This is highly unintuitive, and was suspected to be due to discretizations problems.

---

**Figure 4.10:** Upper plot: Acceleration predicted by Wiberg, using a full 3D model (Wiberg, 2008). Middle plot: Acceleration predicted by Wiberg, with torsional deformation constrained. Lower plot: Acceleration predicted by the author with a 2D model.
When comparing the simulated results with measurements done in the structure, special attention was given to this phenomenon to see if it was present also in them (see chapter 5.2.2). This acceleration peak did not coincide with large deflections so a first guess was to study if it was caused by the high frequency modes included in the analysis.

It was found, though, that this phenomenon persisted even when the 13th mode, the highest considered in this study, was not taken into consideration. Further, it did not really depend on one single mode. The early acceleration peak was already visible when considering only the 5 first modes, and it increased steadily with the number of modes considered.

The infinitely stiff roller bearings that were used to model piers P9 and P10 and the south abutment SL could be also suspected as one of the sources for such anomalies, therefore a new model was considered in which the infinitely stiff supports were replaced by a linear spring and different stiffness constants were tried. The model with linear elastic supports not only failed to reduce the early high acceleration levels but it even increased their magnitude for almost all the stiffness constants tried, between infinitely stiff and \(3 \times 10^9\) N/m.

This phenomenon was not visible, at least not directly, in the measured signal, where the acceleration levels steadily grow to reach a maximum when the load train is crossing the measured point. Its presence in both models and its persistence through different modification introduced in the models was a source of concern.
5 Results

In this chapter results are presented from mode superposition analysis. The analysis was performed with the FE-program DynSolve using a 2D bridge model with 405 beam elements and including the first 13 modes of vibration. Details about the bridge model and the program can be found in chapter 4. All the train passages were simulated with the train entering the bridge from the southern end, since the actual structure is crossed normally only by trains travelling in that direction.

5.1 Resonance Risk

The HSLM-A10 train caused the largest vertical accelerations in the bridge deck (see figure 5.2). For virtually every point, there was a speed at which the acceleration caused by the HSLM-A10 train surpassed the accelerations caused by all other trains at any of the speeds checked during the simulations. The only exception to this was the HSLM-A1 train crossing at 300 km/h, the highest speed simulated (see chapter 1.2.5 for a description of the HSLM train model). This was the only train for which the acceleration exceeded 0.3 m/s². This acceleration value was reached only in the middle of the northern span, and can be considered as a very local phenomenon since the other spans had significantly lesser maximal acceleration values. In general the largest and better differentiated resonance peaks are observed at the northern span. By studying the shape of the eigenmodes, it can be concluded that the 13th mode is very critical to the excitation caused by the passage of the train. The eigenfrequency of this mode is 4.76 Hz and it has the particularity of being a highly localized mode, where only the two northernmost spans are deformed considerably.

From studying the signal produced by the HSLM-A10 at resonance speed (232 km/h, see figure 5.1) it is observed that the highest levels of excitation occurred when the first axels of the train reach the northern span, located between NL and P1. Before that the highest acceleration measured are all below $5 \cdot 10^{-2}$ m/s² and the frequencies of vibration are relatively low. This could be expected since the places where the normalized mode shape vector has the largest values are also the places where a given input force can cause the largest response.

The distance between two consecutive bogies in the main body of the HSLM-A10 train (i.e. the non-engine wagons) is 27 m. At the resonance speed that distance is covered in 0.42 s, or equivalently, the loading frequency is 2.38 Hz. That is exactly half of the resonance frequency for the 13th eigenmode, meaning that two complete vibrations cycles occur during the time it takes for two successive bogies to load the bridge. Thus the different axel loads build up on the previous axel effects, causing the bridge to resonate.

In the case of the HSLM-A1 a similar reasoning leads to a critical speed of 308 km/h. Whereas for the HSLM-A10 the resonant loading occurred every two cycles, in the case of the HSLM-A1, the bridge is loaded at the critical frequency with no unloaded cycles in between two loads. This reduces the dissipative effects of the damping and explains why the HSLM-A1 train causes considerably larger acceleration, even though the load per axel is significantly lower compared with that of the HSLM-A10 train.
Chapter 5. Results

Figure 5.1: Max vertical deck acceleration caused by the HSLM-A10 at different speeds. Note that a higher resolution has been used for the critical speed between 230 and 240 km/h.

There are reasons to suspect that these results could differ from reality. The deformations of the 13\textsuperscript{th} eigenmode are focalized in a few meters (see Appendix A). This means that an input force in those places could potentially cause very large deflections. This would cause the bridge to also deform in other directions than vertical. It is true that the bridge is much stiffer for horizontal bending than for vertical bending and that for small deformation caused by traffic loads it can be considered completely stiff in the horizontal direction, but the effect of lateral deformation and torsion can not be neglected for very large deflections. As a matter of fact, a similar mode was found in the study carried out by Wiberg, but it had an important horizontal component. Actually, the horizontal component was the main one and the frequency associated with this mode was much lower (only 3.56 Hz) since the bridge could be deformed horizontally and rotate freely making it much less stiff (Wiberg, 2006).

According to the results obtained in this study, the New Årsta Bridge can be considered safe from any dangerous resonance phenomenon. The only resonance observed was always connected to the 13\textsuperscript{th} mode of vibration and there are reasons to consider these problems more of the model than of the real structure. Even if the resonance is to be considered, it occurs at velocities far beyond the ones allowed for the bridge and does not cause accelerations above the limits of what could be considered safe (Banverket, 1997).
5.1. RESONANCE RISK

Figure 5.2: Max vertical deck acceleration caused by all the HSLM-A trains at different speeds. All the visible peaks are caused by resonance of the 13\textsuperscript{th} eigenmode. The different resonance speeds are due to the different wagon lengths for each HSLM-A trains.

5.1.1 Support Stiffness

In a test loading carried out in December 2007 on which a report is being prepared by Wiberg at KTH, the bridge was statically loaded with two engine wagons standing on top of the pier P9, which was one of the two piers founded on piles. The goal of the measurement was to assert the equivalent spring coefficient of the support. The measurement was repeated a number of times in order to achieve a more reliable result by means of averaging out possible errors. The displacement was 0.5 mm for a static load of 1530 kN, revealing a support stiffness of around 3\times10\textsuperscript{9} N/m, more than one order of magnitude less than the obtained by supposing elastic piers on a completely rigid foundation.

The transversal area of the pier is 13.9 m\textsuperscript{2}, and the elasticity modulus assumed was 55\times10\textsuperscript{9} Pa so the resulting theoretical support stiffness is 3.6\times10\textsuperscript{10} N/m considering that the piers have an average height of about 20 m. Different calculations were performed for different support stiffness, ranging from idealised stiff support to linear spring support with a spring constant of 3\times10\textsuperscript{9} N/m. The expected support stiffness in the vertical direction for the other supports is much higher, since these piers rest directly on the bed rock. Therefore a value of 3\times10\textsuperscript{9} N/m as found during the measurement is a more than safe lower bound to the real support stiffness of the structure.
Chapter 5. Results

Figure 5.3: The ten lowest eigenfrequencies against vertical support stiffness. Practically no difference exists between $1.8 \cdot 10^{10}$ N/m and infinitely stiff support.

No important differences were found between the results from the stiff support model and the models with a support stiffness of $1.8 \cdot 10^{10}$ N/m for the 15 lowest modes of vibration. The mode shapes remained mainly the same and the natural frequencies changed as little as 1.8% at the most (see figure 5.3).

After this stiffness limit the highest modes of vibration start showing important differences in both shape and eigenfrequency. It is a very important change, because in reducing the critical loading rate it also reduces the speeds at which resonance is triggered. With the stiff model, for example, a very important factor, at least in the simulations, was the frequency of the 13th mode of vibration 4.76 Hz. It was when the moving load caused this mode to resonate that the highest deck accelerations were reached. But the frequency of this eigenmode is so high that it can only be loaded at half the critical rate, allowing for two complete cycles to occur before reloading, if one is to keep velocities under 300 km/h. Now, with the less stiff model this mode of vibration still exist but at a much lower frequency of only 3.9 Hz. This means that the HSLM-A trains with short bogie distance could achieve the critical load rate at speeds under 300 km/h, the highest speed investigated in this study.

5.2 Measurements and Updating

In order to validate the model developed in this study, the displacements and accelerations obtained from the simulations were to be compared with those measured in the structure itself. To obtain reliable simulation results it is important to identify
the natural frequencies of the structure. Critical parameters for the estimation of the natural frequencies such as structural damping and Young’s Modula and density of the materials are difficult to estimate. Therefore it is recommended to compare the results obtained from a first model with those measured in the structure and then update the model modifying the values of these parameters to match the ones obtained from the measurements.

As a preliminary study the acceleration induced in the bridge by a train of type X-2000 crossing the bridge at 120 km/h were measured and analyzed in the frequency domain. The whole of the signal was taken into account and not only the free vibration to avoid windowing. In general good agreement is reached between the modes calculated and what can be deduced for the measured signals. No information is available from the measurements about the longitudinal vibrations, since accelerometers were only placed in the plane orthogonal to the bridge length (see chapter 4.1.3). There is a similar lack of information about the transversal vibrations in the model, since it was simplified to a 2D model. Therefore only for the vertical displacements a comparison between model and measurements could be carried out.

The first clear visible top occurs at a frequency of 1.14 Hz. It is very clearly visible in all the vertical accelerometers (see figure 5.4). The transverse accelerometer, on the other hand, does not detect any vibration in that frequency. The magnitude of the peak is somewhat lesser in the accelerometer placed at section B.

![Spectra of the acceleration induced in the New Arsta Bridge](image)

**Figure 5.4:** Spectra of the acceleration induced in the New Arsta Bridge by a train of type X-2000 crossing at 120 km/h. The positions of sections B, C and D is described in chapter 4.1.3.
The first theoretical mode of vibration found in the study corresponds with this description to a very great extent (see table 5.1). Studying the mode shape one could expect comparable acceleration levels for all the accelerometer, except the one placed in B, given that it is placed in the quarter-span instead of in the mid-span like the others.

The eigenfrequency found in the model was 1.31 Hz. This differs from the value of 1.15 Hz found in the measurements by 13%. This difference can be due to erroneous assumptions in the material properties such as E-modulus, damping ratio or density.

A clear peak can also be seen for the accelerometer placed in section D at 1.64 Hz. None of the other instruments registered identifiable peaks in that frequency. This could indicate the excitation of a mode whose shape is considerably larger in section D than in the other instrumented sections. This is the case of the 5th and 9th mode founded through the 2D model. Their natural frequencies are 1.90 and 3.20 Hz respectively. Note that the frequency of the measured peak is exactly 13% lower than the calculated frequency of the 5th mode. Updating the material properties to match the first natural frequency will automatically result in a perfect accordance for this 5th mode too.

A third clearly visible peak for all measured sections was found at 2.39 Hz. In this case section D once more registers the larger acceleration, although comparable levels are registered in section C, and a lower but still recognizable peak is visible at B. In section C, the accelerations were measured in three different points: in the west edge beam (at the pedestrian passageway), in the middle of the section and in the east edge beam. The acceleration measured at the edge beams present no significant difference, but the accelerations measured in the middle of the section are notably lower. This could speak for a combined bending/torsional mode of vibration.

The 8th mode calculated fulfills the condition of having larger deformations at section D and its frequency was found to be 2.86 Hz. The torsional component could not be registered by the model in this study since it was limited to a 2D model. However, (Wiberg, 2006) finds in his 3D model a mode of identical shape and of natural frequency 2.85 Hz with an important torsional component. The measured frequency is 17% lower than the calculated one.

The 4th peak in magnitude is found at 2.10 Hz. Unlike the other is it only clearly visible for the sensors placed in C. The 7th mode found in the model has also much larger deformations in section C compared to the other instrumented sections, and its natural frequency lays in 2.52 Hz. The measured frequency is 17% lower than the calculated one.

Beside the peaks named here there are 3 more peaks at 1.27, 1.34 and 1.39 Hz, but they are too close together to carry a detailed analysis. The first one appears to have important torsional and vertical components. The acceleration ratio between section C and D are close to unit for the two first. The third one reveals relatively high accelerations in section D.

Lesser peaks coincide to a great extent with the 2nd, 4th and 6th modes in both shape and frequency (considering the overall shift in frequencies of around 15%). So in
general the predicted modes of vibration had a reflection in the measured signals (the 3\textsuperscript{rd} mode predicted by the model was not considered in the comparison, being a longitudinal one).

In this particular measurement there were some peaks that could not be accounted for from the model. However, agreement is found the 5 most important peaks between the model and the measurements for frequencies up to 4.7 Hz (the frequency corresponding highest mode considered).

The signals analyzed in order to find the natural frequencies of the structure (and to a certain extent the natural modes of vibration) present all a common problem: both the frequencies and the relative height of the peaks varies. The variation in the frequencies are very small and can be attributed to the different length of the time-histories measured (that result in different frequency-spectrum resolution). However some peaks present in many of the signals analyzed were completely absent in other signals. This difference is natural considering that different train configurations and different speeds excite different modes of vibration. If only a few numbers of signals is analyzed there is a risk of overseeing important information, or of judging as corresponding to natural modes of the structure, peaks that only are due to the cyclic loading process. Therefore it was decided to study large amounts of time-histories induced by different train configurations and at different speeds. The fact that sensors are placed at different cross-sections allows for an easy coarse estimation of the speed of the crossing trains by simply measuring the time the train takes to pass through the different

Table 5.1: Values of predicted (before updating) and measured eigenfrequencies and the corresponding modal shapes at sections B, C and D. See Appendix A
Figure 5.5: Stresses induced by two different trains on the New Årsta Bridge (below) and the frequency content of the signals measured by the accelerometers in the structure (above). In both cases peaks correspond roughly to the same frequencies, but their relative height varies strongly. For the location of the different accelerometers see chapter 4.1.3.

instrumented sections. The register from the instrument BK5 (see figure 4.4) can be used also to obtain an idea of the train configuration, since each axel load can be clearly differentiated even for the maximum speed of 120 km/h. Thus, one can ensure that a wide variation in train configurations and velocities is considered in order to discriminate the train-dependant acceleration peaks from the ones of the structure itself.

An average signal obtained from 200 train passages was analyzed in this manner. The whole signal was analyzed, both the forced vibration and the free vibration. The results were clear and sharp peaks obtained at the frequencies where it could be expected, mainly between 1 and 3 Hz.

The largest accelerations were though found at 57 Hz. It was a troubling fact since it is very far from the bridge frequencies that usually are excited by trains and because many codes of practice regard the higher modes of vibration as unimportant. How could they be unimportant if the acceleration peaks they registered were more than four times larger than the ones for the first eigenmodes? Wiberg, estimates the eigenfrequency of the concrete slab on which the track is fixed to around 60 Hz, offering this fact as a possible explanation (Wiberg, 2006). Wiberg’s calculation approximates the track slab to a simply-supported, pinned beam; in reality there is some rotational stiffness at the supports (see figures 4.4-4.6), so that an eigenfrequency higher than 60 Hz could be expected. However, a certain amount of cracking has been observed in the track slab. This could explain the relatively low eigenfrequency observed in the measurements.
5.2. MEASUREMENTS AND UPDATING

Figure 5.6: High frequency acceleration for the different accelerometers installed on the bridge. The amplitude is normalized with respect to the highest amplitude in the range 0 to 5 Hz. The signal that dominates at 57 Hz is the one measured directly on the track slab. For the location of the different accelerometers see chapter 4.1.3.

The track slab eigenfrequency hypothesis can only explain one peak, while the high acceleration levels are though present in a very wide range of frequencies, mainly between 40 and 70 Hz. This high acceleration levels do not induce any important deflection, but they could be dangerous in ballasted bridges, for example.

5.2.1 Updating

The eigenfrequencies of the structure are clearly overestimated by the initial model. Among the parameters controlling the eigenfrequencies, the geometrical ones (moment of inertia and cross-section area) are the most likely to be accurate, while the material parameters (elasticity modulus and density) were only roughly estimated. Consequently the logical updating step will be to change the material parameters until they fit the measurements. Assuming a linear model implies that the eigenfrequencies are proportional to the square root of the ratio between the elasticity modulus and the density. But the discrepancy between measured and calculated frequencies seems to grow as higher frequencies are considered. Thus, only updating the material parameters of the model will not be enough to match the relevant modes. As showed before (see figure 5.3 and chapter 5.1.2), variations in the stiffness of the support can greatly affect the high frequency modes while leaving the first eigenmodes almost untouched. Therefore a combination of material parameter and support stiffness updating was used to enhance the reliability of the results.
Chapter 5. Results

Figure 5.7: By combining the spectra obtained from a number of measurements the eigenfrequencies can be easily identified. The dots correspond to the eigenfrequencies predicted by the model after updating.

A reduction of the elasticity modulus to 48 GPa combined with an increase in the density to 2710 kg/m³ resulted in a perfect match for the six first eigenfrequencies. It corresponds to a reduction of 13 % in the originally estimated elasticity modulus and an increase of 13 % in the density. The changes in the density can be justify by elements in the bridge that increase the mass of the structure while not increasing the overall stiffness as the tracks, safety equipment and the train itself. The changes in the elasticity modulus, on the other hand, are more difficult to justify, since, of safety consideration, concrete delivered to the construction site has usually higher elasticity modulus than nominal.

While the first eigenfrequencies of the modified model match those measured, there is a growing difference in the higher modes of vibration: 2% in the sixth mode and 8% in the tenth.

With the total support stiffness set to $5 \times 10^9$ N/m all the 10 first frequencies are within 1% of the measured values. This value was used for all the support disregarding the fact that the foundations of some of the pier are of a different kind (see chapter 4.1.1). Recall that a linear stiffness of $3 \times 10^9$ N/m was measured at one of the piers, and that it was considered as a lower bound to all the other piers.

5.2.2 Predicted and Measured Accelerations

A comparison between the measured accelerations and the calculated ones was carried out using the updated model. The load train chosen was a single Rc6 locomotive crossing the bridge, at 70, 90 and 120 km/h respectively. This load was chosen because measurements at controlled speeds had been made at the bridge using the Rc6 locomotive, so that both the magnitude of load and the speed could be known with precision.
5.2. MEASUREMENTS AND UPDATING

Figure 5.8: Sketch of the Swedish Rc6 locomotive used for the dynamic tests. The distances between axles are given in millimetres.

Figure 5.9: Measured and modelled acceleration time-history at section C (both filtered at 2 Hz) for a single Rc6 locomotive crossing the bridge. The signals are much the same when the locomotive is near the measured point. They differ somewhat as the load train moves away, but the acceleration levels are still accurately predicted.

To obtain comparable results the accelerations’ time-history were filtered with different cut-off frequencies. It was expected that a cut-off frequency of 5 Hz will give the better results, since the highest mode considered in the simulations had an eigenfrequency of about 5.0 Hz.

The best similitude between measured and predicted values was, however, found at a much lower cut-off frequency, namely, 2 Hz. Filtered at 2 Hz the levels of acceleration predicted are practically equal through out the measurement time (see figure 5.9). Interestingly, the high acceleration at beginning of the signal (see chapter 4.5) was also found in the measurement, corroborating the result obtained by the model. Beside the instants when the locomotive is directly on the measured point, the highest
acceleration levels are registered during the first 2 second, when the load train is more than a hundred meters away from the measured section. The acceleration during the first seconds reaches up to 85 % of the maximum peak acceleration, when the load train passes through the measured section. Both the model and the measurement register this phenomenon.

In the frequency domain the similitude between measured and predicted accelerations is very satisfactory under 2 Hz. The frequency peaks are a very local phenomena and the frequency resolution used for the comparison is relatively low so the accordance shown in figure 5.10 is indeed very good, and better than the results obtained with the 3D model (see figure 5.10). The shortcomings of the model become patent for higher frequencies, but the 3D model used for comparison does not offer much more accurate results, so that the use of the simplified 2D appears as good trade off between accuracy and simplicity.

The measurements clearly show excitation of the modes with eigenfrequencies between 2 and 3 Hz (2.2, 2.4 and 3.1 Hz) and no important frequency content beyond that until around 20 Hz. The model however, failed to predict the excitation of the modes of frequency higher than 2 Hz, even though they were considered in the calculations.

Figure 5.10: Measured and simulated acceleration spectra. The current 2D model accurately predicts acceleration levels up to a frequency of 2 Hz, where above the eigenmodes becomes mainly torsional modes. The results predicted by the 3D model by Wiberg are plotted for comparison.
A possible explanation to this phenomenon can be found in whether the modes excited are bending or torsional in their nature.

A clear predominance of torsional modes in the range 3 to 10 Hz can be observed in the recorded accelerations: the central placed accelerometer hardly registers any acceleration in these frequencies while all the accelerometers in the edge beams register clear peaks at roughly the same frequencies.

In order to estimate the amount of bending and torsion present in each of the modes the time histories of the accelerometers CK1AV and CK2AV were studied in combination. These two accelerometers are placed in the same section in the west and in the east edge beam respectively (see figure 4.5). If the bridge cross-section is considered as a rigid body, then its movement can be separated into a translational component and a rotational component by:

\[
a_r = \frac{a_2 - a_1}{2} \tag{5.1}
\]

and

\[
a_t = \frac{a_2 + a_1}{2} \tag{5.2}
\]

where \(a_r\) is the acceleration registered by the instrument CK1AV, \(a_2\) is the acceleration registered by the instrument CK2AV, \(a_1\) is the translational acceleration component, \(a_t\) the rotational acceleration component and \(B\) the breadth of the cross-section.

The original signal can be recomposed from their components by

\[
a_2 = a_r + a_t \frac{B}{2} \quad \text{and} \quad a_1 = a_r - a_t \frac{B}{2} \tag{5.3}
\]

If the accelerations are separated in this manner it can be easily appreciated from the different signals recorded that the modes with eigenfrequencies from 1 to 2 Hz are predominantly of bending nature, while the ones between 2 and 3 Hz present both bending and torsional deformations and the ones higher than 3 Hz in the frequency spectrum are mainly torsional modes. This could be an explanation of why the model implemented failed to predict accurately the frequency content for frequencies higher than 2 Hz. A 2D model can not be expected to represent torsional effects.

Different train were simulated at different speed and the result was always the same. The simulations predicted with great accuracy the frequency content for frequencies lower than 2 Hz, where the natural modes of vibration are almost pure bending modes, but completely fail to give reliable result for higher frequencies. The model not always underestimated the vibration of the higher modes; it sometimes returned values much higher than the measured ones.
Figure 5.11: Torsional and bending acceleration spectra for a train of type X-2000 crossing the structure at around 120 km/h. Note how the bending component predominates in the low-frequency modes, to almost disappear in modes higher that 3 Hz.

It was mentioned before that very high acceleration levels occur at high frequencies (see chapter 5.2). The three measurements under controlled speeds reveal that these accelerations are probably due not only to eigenmodes of the structure itself but also to regularly spaced structural elements that fix the track. The first high frequency peaks appears at 19.5, 24.5 and 33.7 Hz for the speeds of 70, 90 and 120 km/h, respectively. A second peak appears for the two lower velocities at 33.3 and 43.2 Hz, suggesting that another peak should also appear for 120 km/h at around 59 Hz. Such a peak is registered but had a much lower magnitude, probably reduced by the anti-alias filter. This linear dependency in the velocity speaks against a resonance of an eigenmode; since the load train is very short (only 4 loads) to cause serious resonance problems, it travels at relatively low speeds and if the peaks were to be due to eigenmode excitation they should be registered at the same frequencies independently of the speed.

This kind of interpretation becomes much more difficult when analysing the acceleration produced by longer trains crossing the bridge. In this case the frequency content shows a very chaotic behaviour at very high frequencies and almost constant levels of acceleration from 30 Hz onwards. Literally hundreds of peaks pack together and very little is found in common between any two signals analysed. In fact when a number of signals are summed together these peaks tend to disappear, indicating that they are not bounded to a specific frequency.
Figure 5.12: Upper plot: the normalized spectra obtained from an Rc6 locomotive crossing the bridge at three different speeds (70, 90 and 120 km/h). Note the relation between speed and the frequency at which the high-frequency peaks appear in the upper plot, indicating that they are due to the equally distanced track fastening devices. Lower plot: the low frequency accelerations induced by the Rc6 locomotive at 70, 90 and 120 km/h. The frequency range shown corresponds with the frequencies considered in the theoretical analysis. Note that the low frequency peaks are common for all speeds.
Only two peaks are observable averaging 150 signals, one at 45 Hz and another at 57 Hz. The peak at 45 Hz is more or less of the same magnitude for all the accelerometers while the one at 57 Hz, though present in all accelerometers, is overwhelmingly higher for the accelerometer mounted directly in the track slab. This could be interpreted as being the eigenfrequency of the track slab (see figure 5.6), but this interpretation present some problems since the frequency content in that frequency is very low for some of the recorded signals, specially for short trains. This peak is the only one present in all of the three measurements under controlled speeds with the Rc6 locomotive.
6 Conclusions and Suggestions for Further Research

6.1 Conclusions

In the present work a simplified 2D model of a complex multi-span pre-stressed concrete railway bridge, the New Årsta Bridge, was developed. The purpose of the study was to assess the ability of the proposed model to predict the dynamic characteristics of a complex structure. The results obtained from the simplified model are compared with results from a more complex model presented in (Wiberg, 2006), in order to obtain an idea of the simplifications impact on the results. Measurements of the bridge’s accelerations and displacements were used first for updating the model in order to more accurately represent the behaviour of the real structure and then to compare and validate the results obtained by the implemented 2D model. From the study and analysis undertaken the following conclusions can be drawn:

- A 2D model is good enough to predict the bridge dynamic characteristics as eigenfrequencies and two-dimensional bending eigenmodes, even for complex multi-span unsymmetrical structures.
- A 2D model can predict the response accurately regarded that the structure’s eigenmodes are mainly of bending nature, with no important torsional component as is the case in simpler single-track bridges.
- Taking into consideration only the lower eigenmodes for simulation will accurately predict the vibrations, but greatly underestimate the accelerations induced by random crossing trains.
- As expected 2D model can not accurately predict the excitation of eigenmodes with an important torsional component. Not even the bending component of such modes is predicted satisfactorily by a 2D model.
- The point load model gives adequate results for unballasted, long-span bridges; within the limitation proper of a 2D model.
- Updating the model greatly improves the accuracy of the predicted eigenfrequencies as shown in the following table (3rd mode was not measured being a longitudinal mode):

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Before Updating:</th>
<th>After Updating:</th>
<th>Measured:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.31 Hz</td>
<td>1.15 Hz</td>
<td>1.14 Hz</td>
</tr>
<tr>
<td>2</td>
<td>1.43 Hz</td>
<td>1.26 Hz</td>
<td>1.27 Hz</td>
</tr>
<tr>
<td>3</td>
<td>1.56 Hz</td>
<td>1.38 Hz</td>
<td>Not measured</td>
</tr>
<tr>
<td>4</td>
<td>1.63 Hz</td>
<td>1.43 Hz</td>
<td>1.44 Hz</td>
</tr>
<tr>
<td>5</td>
<td>1.9 Hz</td>
<td>1.64 Hz</td>
<td>1.65 Hz</td>
</tr>
<tr>
<td>6</td>
<td>2.15</td>
<td>1.88 Hz</td>
<td>1.90 Hz</td>
</tr>
</tbody>
</table>
• The vertical stiffness of the supports is an important factor and must be considered when updating the model, especially for the high-frequency eigenmodes.
• Even thought the model implemented has its shortcomings, its accuracy is to a great extent comparable to that of the 3D model used for comparison purposes, making it a very good trade off between accuracy and complexity.
• High accelerations are measured in the frequencies from 20 Hz to 70 Hz. These are caused by the regular spaced track fastening devices and are not linked to any eigenmode in particular. A modal analysis will not suffice if these kinds of effects are to be accounted for. However, these high frequency vibrations do not cause additional strains or displacements in the structure.
• The speed dependant vibrations mentioned above could be used in the future to estimate the speed of the crossing trains.
• The New Årsta Bridge’s eigenmodes with eigenfrequency under 2 Hz are mainly bending modes. Modes between 2 and 3 Hz have both bending and torsional components, while modes higher that 3 Hz are mostly torsional modes.
• A vertical support stiffness of more than $1.8 \cdot 10^{10}$ N/m gives the same results as an infinitely stiff support model, for the 15 first modes of vibration.
• A vertical stiffness of about $3 \cdot 10^9$ N/m was obtained from the measurements carried out at one of the piers founded in piles. The stiffness obtained by assuming elastic piers on stiff foundations is $3.6 \cdot 10^{10}$ N/m.
• The New Årsta Bridge is safe from resonance risks, since the speed required to reach resonant loading frequency are far beyond the maximal allowed speed.
• The highest vertical deck acceleration obtain in the simulations is $0.35 \text{ m/s}^2$ for the HSLM-A1 train crossing at 300 km/h. The second highest acceleration is about $0.25 \text{ m/s}^2$ induced by the HSLM-A10 trains at about 225 km/h. These accelerations are much below the design code levels of $5 \text{ m/s}^2$ for unballasted tracks.

6.2 Suggestions for further research

The most natural next step for this study would be to expand the possibilities DynSolve has to offer to 3D models. The program has shown to give reliable results for 2D, so it would be interesting to see if similar results can be achieved for 3D models including complex 3-dimentional bending/torsional deformations.

The model of point load was used, as for long span bridges the train-structure interaction does not alter the results importantly. The complete interaction model still has interest as a way of asserting the accelerations in the vehicle as well as the in bridge deck, and for studying the effects of the train mass in the dynamic behaviour of the bridge. In order to take the travelling mass of the train into account a direct integration method will have to be used as opposed to the mode superposition method applied in this study. Investigating the difference between these two approaches could also give in interesting results and contribute to our understanding of the dynamic problem. Bridge-train interaction is especially important for short span bridges, so it should be considered if DynSolve is to be used for such bridges. Another factor worth studying in the future that was not considered in this work is the load distribution through the track/sleepers and possible ballast.
Of course increasing the degrees of freedom by considering a 3D structure or by introducing complex vehicle configurations will implicate enormous increases in the computational costs, that were already enough to give a 4-cored Intel processor with 4 Gb RAM some problems.

Other study that could have been introduced in the current work is estimation of the damping (at least for the low frequency modes) by means of the logarithmic decrement or the half-power bandwidth method.

The results obtained indicated too that an important component of the acceleration was due to the regular spaced track fastening devices. Designing a way to introduce this effect into the model could improve the accuracy of the calculations. Since these kinds of vibrations are not related to any eigenmode, they could be introduced considering the contact force as time dependant with a frequency depending on the speed of the train and the distance of the track fastening devises.

It was observed during the study that the highest acceleration occurred at 57 Hz. A possible explanation was that it was the eigenfrequency of the track slab, but theoretically this should be above 60 Hz. Cracking was suggested as a probable reason for the track slab being less stiff, and therefore having a lower eigenfrequency. It means that the eigenfrequency should change with time. Studying the possibilities instrumentation offers to monitor indirectly the amount of cracking by registering changes in the eigenfrequencies of different structural elements could also be a line to follow in the future.

Finally, it was particularly difficult to estimating the speeds and the loads crossing the bridge. In order to validate models using random train passages, studies should be carried on the possibility of asserting the axel load and the train speed from the signals recorded on the bridge.
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Appendix A

Predicted Mode Shapes and Frequencies

In this appendix the shapes of the 14 lowest eigenmodes predicted by the updated 2D model are illustrated together with their respective eigenfrequencies. The variation in the shapes between the original model and the updated one were very slight. The piers are omitted in the figures and only the deformation of the deck is shown. The frequency associated with each eigenmode corresponds to the updated natural frequency. Modes 3 and 12 are longitudinal modes.

1st mode. Frequency: 1.15 Hz

2nd mode. Frequency: 1.26 Hz

3rd mode (longitudinal). Frequency: 1.38 Hz

4th mode. Frequency: 1.43 Hz

5th mode. Frequency: 1.64 Hz

6th mode. Frequency: 1.88 Hz

7th mode. Frequency: 2.13 Hz

8th mode. Frequency: 2.39 Hz