Advanced Thermal Energy Storage
Heat Transfer Study with Use of
Comsol and Matlab

Petter Johansson
Abstract

The interest in storing latent energy in phase change materials has risen over the last years as the need grows for more energy efficient systems. By storing energy, free chilling and heat can be saved for later use during high load hours. Thus the gap between supply and load can be overcome. It is an efficient way to provide both cooling and heating to buildings using phase change material (PCM) as they take up much less volume compared to a corresponding water-cistern with the same amount of stored energy.

Low thermal conductivity of most of the PCMs can be compensated with advanced heat transfer design, however impact of different heat transfer mechanisms is not explicitly studied. In this work, a heat transfer study has been made on a finned cylindrical PCM heat exchanger with focus on determining the heat transfer effect of convection in non-gelled PCMs and the different ways to model such a system in a two dimensional axis-symmetric plane.

The first and simpler numerical model of the two was built using Matlab, where the convection effect was simulated using an enhanced-conduction factor based on empirical equations. The other model was built in a CFD environment and simulates the convection with more complexity and more realistic behavior.

The results show that the convection may contribute to 65% of the total heat transfer in non-gelled PCMs at a certain time and that using empirical equations for simulating convection is a fast and easy way to estimate the heat transfer, though not a recommended method for high accuracy results. The study also showed that because of the gravity-induced convection, the angle of the cylindrical finned heat exchanger affects the heat transfer and that more fins, while increasing the overall heat transfer rate, inhibits the effect of convection in a vertically positioned heat exchanger.
# Table of Contents

Abstract .................................................................................................................. 2  
Table of Contents ................................................................................................. 3  
Abbreviations ....................................................................................................... 4  
Nomenclature ........................................................................................................ 5  
Acknowledgements ............................................................................................... 6  
1 Introduction ....................................................................................................... 7  
1.1 Background .................................................................................................... 7  
1.2 Objectives ..................................................................................................... 7  
1.2.1 What is not included in this report ......................................................... 8  
1.3 Literature review ........................................................................................... 8  
1.3.1 Energy storage methods ......................................................................... 8  
1.3.2 Areas of possible PCM implementation ............................................... 8  
1.3.3 PCM classifications and characteristics ................................................. 9  
1.3.4 PCM Heat exchangers ........................................................................... 9  
1.3.5 Solving phase change problems ............................................................ 10  
2 Model description ............................................................................................ 13  
2.1 Enhanced conduction model ....................................................................... 15  
2.2 CFD model .................................................................................................. 16  
3 Results ............................................................................................................. 17  
3.1 Result comparison without convection ...................................................... 18  
3.2 Result comparison with convection ............................................................ 20  
3.3 Fin parametric study .................................................................................... 23  
3.4 Vertical and horizontal heat exchanger comparison .................................. 25  
4 Discussion and Conclusion ............................................................................ 26  
Appendix A: Simplified geometry of the heat exchanger ............................... 29  
Appendix B: Properties of Rubitherm RT9 ....................................................... 30  
Appendix C: Matlab code .................................................................................. 31  
Appendix D: Scaled up jagged zone in melting RT9 ......................................... 37  
Appendix E: CFD simulation of convection in a phase change process ........ 38  
Appendix F: Interpolated curves from Table 1 .................................................. 39  
Appendix G: Vertical and horizontal heat exchanger comparison ................. 41  
Appendix H: Temperature profiles of three different angled heat exchangers .. 43
**Abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>PCM</td>
<td>Phase Change Material</td>
</tr>
<tr>
<td>HTF</td>
<td>Heat Transfer Fluid</td>
</tr>
<tr>
<td>TES</td>
<td>Thermal Energy Storage</td>
</tr>
<tr>
<td>LHTES</td>
<td>Latent Heat Thermal Energy Storage</td>
</tr>
<tr>
<td>SHTES</td>
<td>Sensible Heat Thermal Energy Storage</td>
</tr>
</tbody>
</table>
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>[J/kg]</td>
<td>Latent heat of fusion</td>
</tr>
<tr>
<td>$\rho$</td>
<td>[kg/m$^3$]</td>
<td>Density</td>
</tr>
<tr>
<td>$k$</td>
<td>[W/m$^2$K]</td>
<td>Thermal conductivity</td>
</tr>
<tr>
<td>$k_{eq}$</td>
<td>[W/m$^2$K]</td>
<td>Equivalent thermal conductivity</td>
</tr>
<tr>
<td>$T$</td>
<td>[K]</td>
<td>Temperature</td>
</tr>
<tr>
<td>$t$</td>
<td>[sec]</td>
<td>Time</td>
</tr>
<tr>
<td>$c_p$</td>
<td>[J/kgK]</td>
<td>Specific heat</td>
</tr>
<tr>
<td>$c_{p,s}$</td>
<td>[J/kgK]</td>
<td>Specific heat in solid state</td>
</tr>
<tr>
<td>$c_{p,l}$</td>
<td>[J/kgK]</td>
<td>Specific heat in liquid state</td>
</tr>
<tr>
<td>$T_m$</td>
<td>[°C]</td>
<td>Melting temperature</td>
</tr>
<tr>
<td>$T_s$</td>
<td>[°C]</td>
<td>Solid state temperature</td>
</tr>
<tr>
<td>$T_l$</td>
<td>[°C]</td>
<td>Liquid state temperature</td>
</tr>
<tr>
<td>$v$</td>
<td>[m$^2$/s]</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$d_c$</td>
<td>[m]</td>
<td>Characteristic length</td>
</tr>
<tr>
<td>$Q$</td>
<td>[J]</td>
<td>Heat</td>
</tr>
<tr>
<td>$m$</td>
<td>[kg]</td>
<td>Mass</td>
</tr>
<tr>
<td>$\beta$</td>
<td></td>
<td>Volumetric expansion coefficient</td>
</tr>
</tbody>
</table>
Acknowledgements

This study would not have been possible without the previous work made by project supervisor Justin Chiu. More so, Justin has been a source of constant support throughout the entire the work period, supplying the tools and methods needed to deal with many of problems that arose as the project evolved and giving encouragements and guidance through more struggling periods.

I would also like to give thanks to my examiner Viktoria Martin who has guided the project in the right direction and restricted the workload to match the timeline of the project.

Finally I would like to thank the support at Comsol, especially Per Backlund and Fredrik Nääs, who introduced and supported my approach to Comsol Multiphysics and improved my knowledge in CFD modeling.
1 Introduction

1.1 Background

The interest in using renewable energy for producing electricity has risen with the potential threat of global warming due to greenhouse gas emissions and rising fuel prices. Solar and wind power are such technologies that are considered an important part of the future electricity supply. However for these technologies, there is often a mismatch between energy demand and energy production. Developing efficient technologies to store the harvested wind or solar power for use during high load is therefore considered an important part of making efficient systems with renewable energy sources. (A. Sharma, 2009)

The high-energy storage density of phase change materials (PCMs) and the small temperature swing of PCMs make them especially interesting to be used in several different fields, such as warming and cooling of buildings. However, due to the special characteristics of PCMs, case specific studies need to be conducted for every possible use of PCMs to verify that PCM implementation in a certain system is economically justifiable. (Yvan Dutil, 2011)

The researcher who wishes to do such an implementation in a system needs to look into the characteristics of the PCM, such as what melting point and temperature range is required, what container is compatible with the PCM and what would be a good heat exchanger for supplying efficient enough heat transfer so that energy can be extracted and inserted in the required time of the system. (A. Sharma, 2009)

Good knowledge of the heat transfer mechanisms in the PCM is therefore required to know what heat exchanger design is needed to meet the set requirements.

1.2 Objectives

The target of this thesis work is to carry out an advanced heat transfer analysis for a phase change thermal energy storage system by looking at the heat transfer mechanisms in a finned cylindrical PCM heat exchanger. Focus will be on determining the convection induced heat transfer effect in a non-gelled PCM.

The heat exchanger used in this work has been made by project supervisor Justin Chiu and it is further described in chapter 2. Numerical models corresponding to the model used in the experimental studies is to be developed. Modification of the numerical models to match the experimental results is required as well as a parametric study to verify the correctness of the numerical model.

A literature study is also included in the work to provide the correct theory for the numerical models and to provide this report with a review of PCMs as heat storage technology.

The numerical models are developed using Matlab and the computation fluid dynamics (CFD) environment Comsol Multiphysics. Requiring good knowledge of these software is also an important part of the work.

The numerical model should be written in a manner that provides stability and high numerical accuracy, and implemented to a method that deals with the convection phenomena that occurs in non-gelled materials.
1.2.1 What is not included in this report

Economical studies of the importance of knowing and mastering the PCM thermochemical mechanisms or practically implementing the use of PCM in a system are not included in this report. Neither is the possible thermal resistance in a heat exchanger taken into account, nor the lifespan or degradation of PCM over time.

1.3 Literature review

The literature study has mainly focused on journal articles and course literature, accessible at the KTH library and through KTH given courses in the field of thermodynamics and heat transfer.

There are a substantial amount of scientific studies done on PCMs and many articles to be found on the subject. Among these, there are several reviews on PCMs and some on how to numerically model phase change. (V. Voller, 1981) (B. Zalba, 2003) (A. Sharma, 2009)

The articles from Comsol Conference user presentations that cover the area of phase change were also used, with presentations coming from both industrial companies and university researches, such as (C.W. Foong, 2010).

1.3.1 Energy storage methods

There are several ways to store energy, such as

- Mechanical energy storage
- Electrical storage
- Thermal energy storage (TES)

Each one of these technologies has its benefits and setbacks. One of the advantages of TES is that it is a comparatively easy and safe way to store energy.

TES can furthermore be divided into two groups: latent heat thermal energy storage (LHTES) and sensible heat thermal energy storage (SHTES). SHTES is storing energy by raising or lowering the temperature in a solid or liquid, while LHTES uses the phase change process in a material to store energy. The phase change can be from solid to solid, solid to liquid, liquid to gas or solid to gas and vice versa. During these phase changes, the material absorbs or releases heat, and so more energy can be stored in a smaller temperature span than compared to sensible energy storages. This also leads to smaller volumes of heat storages. (A. Sharma, 2009)

1.3.2 Areas of possible PCM implementation

There are many areas where using PCM could further increase the efficiency of a system. Examples where PCMs could be used are

- In heat exchangers for thermal energy storage.
- Built directly into the walls, roofs or floors of a building.
- Off-peak electricity storage.
- Solar water/air heating systems.

Several different approaches have been studied for building PCM directly into the structure of buildings where the PCM would act as a temperature regulator. The high heat storage density of
PCMs gives the advantage that floors or walls can be made thinner and therefore weigh less, which can facilitate certain areas when constructing buildings. (A. Sharma, 2009)

Also PCMs can be used to store off-peak electricity energy by heating a LHTES using the cheaper off-peak electricity so that the heat is stored for use during period of high-load with more expensive electricity costs. (M. Costa, 1997)

### 1.3.3 PCM classifications and characteristics

The PCMs can be divided into different groups: organic, inorganic and eutectic. Organic PCM is furthermore divided into paraffin or non-paraffins, while inorganic PCM is divided into salt-hydrates and metallics. Eutectic PCMs is the third group that is a composition of two or more components of organic and/or inorganic PCMs.

Paraffin wax has benefits such as being reliable, safe and non-corrosive. However, paraffin also has the downsides of having generally low thermal conductivity and it does not work well with plastic containers. (A. Sharma, 2009)

Depending on the type and geometry of the heat exchanger and the PCMs characteristics, if it is crystalline, gelled or non-gelled, convection may be a factor that needs to be addressed for correct heat transfer calculations. If so, calculations and simulations including natural convection require verification from experimental results because of the complex heat transfer process in phase change materials. If there are no available experimental results however, a parametric study using only conduction can still provide valuable knowledge for designing thermal energy storage. (M. Costa, 1997)

### 1.3.4 PCM Heat exchangers

When using PCM as thermal energy storage, the general low conductivity of PCMs becomes a bigger problem compared to using it as a temperature regulating material in buildings. With low conductivity in PCMs comes that the heat transfer efficiency of the heat exchangers is of greater importance.

The easiest way to increase the heat transfer is increasing the ratio between the heat transfer surface and the energy storage volume. If a finned geometry is used, increasing the number of fins would be one solution to increase this ratio. However, more fins generally decrease the volume of the PCM and raises the complexity and cost of the heat exchanger. (Yvan Dutil, 2011)

Other ways to increase the heat transfer area and volume ratio is using packed beds. With the PCM put in small capsules and surrounded by a heat transfer medium of much higher conductivity, it increases the heat transfer efficiency. This is though a technology somewhat limited by the high cost. (B. Zalba, 2003)

Other types of heat exchangers use porous or fibrous heat transfer mediums such as silica or carbon. Advantages of these heat exchanger types are the low volume fraction and high thermal conductivity of these materials. (B. Zalba, 2003)

Although there are many different types of heat exchangers, some studies show that different types and configurations of heat exchangers give the same performance if the surface and volume ratio is the same. (K. Sasaguchi, 1990)
1.3.5 Solving phase change problems

Calculating and solving phase change problems using numerical methods requires a somewhat special procedure due to the boundary layer between the solid and the liquid phases in the medium. A few methods on how to approach this problem have been presented in various studies. In 1889 Stefan presented a solution on how to deal with the phase change boundary in his work on thermal behavior in the freezing ground.

He then stated that the solid to liquid interface is a moving boundary, and he expressed this by using the energy equation over the solid-liquid interface in one dimension, as seen in eq. (1). This is today called the Stefan boundary condition or the moving boundary condition. (A. Sharma, 2009)

$$\lambda \rho \left( \frac{ds(t)}{dt} \right) = k_s \left( \frac{\delta T_s}{\delta t} \right) - k_l \left( \frac{\delta T_l}{\delta t} \right)$$  \hspace{1cm} (1)

where $\lambda$ is the latent heat of fusion, $\rho$ is the density, $s$ the energy, $t$ the time, $k$ the conductivity and $T$ the temperature.

Assuming that the heat transfer is due only to conduction, the Fourier equation can be used to describe the Stefan problem with a pair of equations, where eq. (2) is the solid and eq. (3) the liquid equation in a one dimensional plane, where the time dependent $X(t)$ is the location of the solid-liquid interface. (V. Voller, 1981)

$$\frac{d}{dx} \left( k_s \frac{dT_s}{dx} \right) = \rho c_{ps} \frac{dT_s}{dt}, 0 \leq x \leq X(t)$$  \hspace{1cm} (2)

$$\frac{d}{dx} \left( k_l \frac{dT_l}{dx} \right) = \rho c_{pl} \frac{dT_l}{dt}, X(t) \leq x$$  \hspace{1cm} (3)

where $c$ is the specific heat and $X(t)$ the time dependent location on the x-axis.

There are both approximate analytical and numerical techniques for solving phase change problems with the Stefan condition. The approximate analytical techniques have the drawback of being very complicated if solved for anything but a one-dimensional problem. The numerical techniques however, solved with finite element methods or finite difference methods, have a wider range of applicability and are therefore normally used for solving practical problems. (A. Sharma, 2009)

The numerical techniques can generally be divided in two groups: moving mesh and fixed mesh. If the problems are solved using a time dependent mesh that changes with the moving boundary, the problem can be solved with high accuracy, but it is also limited to simpler geometries and problems. Another drawback of this method is that it does not work for mushy-region problems, which occurs when using a storage material such as paraffin.

The other technique is the fixed grid method. This method can be applied to more complex problems and geometries and is usually easier to implement practically. (A. Sharma, 2009)

One of the most common and popular ways to solve a phase change problem with fixed grid is by using the enthalpy method. With this method the phase change front can be derived afterwards and is not simultaneously tracked with the calculated temperatures. So the exact position of the moving boundary must not be known to be able to calculate the heat transfer. (Nedjar, 2002)
The enthalpy method is based on the energy equation and has the temperature as the only dependent variable. Unlike the moving grid solution, the same equation is applied on the whole domain and thus the Stefan equations, as seen in eq. (2) and eq. (3), are absorbed and reduced into a single enthalpy equation as seen in eq.4. Here with the assumption of constant thermal properties. (A. Sharma, 2009)

\[
\frac{dH}{dt} = \nabla(k\nabla T)
\]  

(4)

where \( H \) is the enthalpy expressed according to the following equation

\[
H = h(T) + \rho \lambda f(T)
\]  

(5)

where \( f(T) \) is a temperature dependent function that tells the liquid or solid fraction in the medium and \( h(T) \) is defined according to eq. (6).

\[
h = \int_{T_m}^{T} \rho c_p dT
\]  

(6)

If defining a temperature \( T_m \) to the temperature of the mushy zone and assuming isothermal phase change, the function \( f(T) \) in eq. (5) can be described as

\[
f_T = 0 \text{ if } T < T_m \text{ (solid)}, \quad f = 1 \text{ if } T > T_m \text{ (liquid)}
\]  

(7)

where the region \( 0 < f < 1 \) is called the mushy region. (M. Costa, 1997)

However, for PCMs such as paraffin, the phase change transition is not isothermal, as expressed in eq. 7, and the mushy region must be defined in a different manner for these PCMs. In eq. (8-10) the temperature \( T_m \) is replaced by a temperature region between the temperature \( T_s \) where all PCM is in solid state and \( T_l \) where all PCM is in liquid state. (G. Vidalain, 2009)

\[
f_T = 0, \quad T < T_s
\]  

(8)

\[
f_T = \frac{T - T_s}{T_l - T_s}, \quad T_s < T < T_l
\]  

(9)

\[
f_T = 1, \quad T > T_l
\]  

(10)

The governing equation for heat transfer within the phase change materials is the energy equation, as seen in the following equation. (C.W. Foong, 2010)

\[
\rho c_p \frac{dT}{dt} + \rho c_p \bar{u} \nabla T = \nabla(k \nabla T)
\]  

(11)

The convective term is dependent on the movement within the medium, and to know the movement in the medium the equations of continuity, motion and energy needs to be solved simultaneously, which is a very complex and demanding procedure.
To simplify the problem, but still taking the convection term into account, a method called enhanced conduction can be used. With this method the heat transfer factor $k_{eq}$, seen in eq. (12), is defined by empirical equations and is a function of dimensionless numbers for different geometries. (Kutateladze, 1966)

The heat transfer equation seen in eq. (11) can then be simplified as seen in the following equation

$$\rho c_p \frac{dT}{dt} = \nabla (k_{eq} \nabla T) \tag{12}$$

where $k_{eq}$ is a variable depending on the liquid-solid fraction function. If $f_l = 0$ then $k_{eq} = k_c$ where $k_c$ is the conductivity of the medium, and if $f_l = 1$ then $k_{eq} = k_c \cdot e_c$ where $e_c$ is a function of the dimensionless Grashof and Prandtl numbers. Assuming that conductivity is only depending on these relations, eq. (12) can then be rewritten as

$$\rho c_p \frac{dT}{dt} = k_{eq} \Delta T \tag{13}$$

There are several empirical equations describing the natural convection for different geometric shapes. Kutateladze and Borishanskii describe the general heat transfer effect from natural convection in enclosed cylindrical spaces with the following equations. (Kutateladze, 1966)

$$10^4 < \text{Pr} \cdot \text{Gr} < 10^7, e_c = 0.062[\text{Pr} \cdot \text{Gr}]^{1/3} \tag{14}$$

$$10^7 < \text{Pr} \cdot \text{Gr} < 10^{10}, e_c = 0.22[\text{Pr} \cdot \text{Gr}]^{1/4} \tag{15}$$

where Pr and Gr are dimensionless numbers described in equation (16) & (17)

$$\text{Pr} = \frac{v \rho c_p}{k} \tag{16}$$

$$\text{Gr} = \frac{g \beta (dT)d_c^3}{v^2} \tag{17}$$

where $v$ is the kinematic viscosity, $\rho$ is the density, $k$ is the conduction, $\beta$ is the volumetric expansion coefficient, $dT$ is the temperature difference and $d_c$ is the characteristic length of the geometry.

### 1.3.5.1 Numerical methods

A conduction only heat transfer problem can easily be solved using numerical methods in an environment such as Matlab, especially if the geometry allows for a simple grid.

When solving such a problem, either an implicit or explicit method can be used, or a method that is a mix of the two, though these methods are usually seen as being implicit as well. The trapezoidal method is an example of such a method that is both explicit and implicit. The accuracy of the methods depends on the order, starting at first order accuracy. (Edsberg, 2008)
The simplest explicit method is called Euler forward and the corresponding implicit method is called Euler backwards. These two are both of first order accuracy, while the trapezoidal method is of second order accuracy.

The explicit method is based on the already known values to calculate the values of the next time step. With the explicit method, each point in the grid can be calculated separately since the values in the earlier time step are already known for all points. This sets up for solving the problem with iterations, which has the benefit of being simple to set up, but with the disadvantage of being instable for too large time steps.

The implicit method demands setting up matrices solved using Gaussian elimination. These methods often demand more computer power than explicit methods, but have the benefit of a much larger stability range. (Edsberg, 2008)

1.3.5.2 CFD set-up

There are many things to consider when simulating a phase change process in a PCM that goes from solid to liquid or vice versa in a CFD environment such as Comsol Multiphysics.

Two major phenomena that occur in the phase change transition that have to be addressed in a CFD environment are the seemingly lowered heat transfer rate, caused by the materials absorption or release of energy, and the zero velocity condition for the solid state.

To simulate the change of heat transfer in the phase change transition, the specific heat of the material can be set to rapidly increase over this temperature range so that the phase change process takes longer time.

There are several methods to simulate the change of movement conditions in a phase changing process. A moving mesh is one possibility, as discussed in an earlier chapter. But for a fixed grid the zero velocity condition in the solid phase has to be modeled in another way. Either a computation cell can be set up, stating that there is zero velocity below a certain temperature. Another method is setting a function that increases the viscosity below a certain freezing point that also results in a principally zero velocity condition. (V.R. Voller, 1987)

2 Model description

The heat exchanger consists of four round aluminum plates connected to an aluminum pipe in which the heat transfer fluid (HTF) is transported. The heat exchanger is placed inside a cylindrical container consisting of glass walls and where the bottom and the upper lids are also made of aluminum. The simplified geometry of the model is presented in Appendix A.

A set of thermocouples is set up inside the PCM filled container connected to a Keithley data logger that gives the temperature readings. The data logger is in turn connected to a PC that saves the readings.

As HTF, the coolant Kryo-40 is used, with a working range from -40°C to 60°C. In the experimental studies, the coolant was used in the range of -10°C to 30°C. The coolant has been cooled and warmed in a Lauda RE 1050 thermostat.
The PCM used in the experimental studies is organic paraffin from the company Rubitherm called RT9 which typically changes phase around 9°C, but in reality changes phase in a span between 5-11°C. The freezing and melting temperature range of RT9 paraffin has both different temperature ranges and different melting/freezing points. The phase change process can be described by an increase in specific heat in the phase change temperature range. The phase change induced characteristics on the RT9 specific heat have been derived by project supervisor Justin Chiu and is showed in Figure 2-1, where the hysteresis between the freezing and melting process is clearly shown. The properties of RT9 paraffin as given by Rubitherm is shown in Appendix B: Properties of Rubitherm RT9.

For model verification, a gelled salt PCM with melting temperature at 21°C is used. The reason for using this salt is that there is no convection in gelled salt, and so the functionality of the numerical heat transfer models for conduction only can be verified when comparing to experimental results with this salt PCM. Here as well Justin Chiu had derived the specific heat function of the salt prior to this project.

![Figure 2-1: The specific heat of melting and freezing RT9 paraffin](image)

The geometry of the heat exchanger suggests the usage of cylindrical coordinates for modeling the heat transfer. By assuming axis symmetry in the heat exchanger, the problem can be reduced to a two dimensional planar problem. The problem can also be reduced by just looking at the area between two fins instead of looking at the whole heat exchanger between the top and bottom lid, as seen illustrated in Figure 2-2.
2.1 Enhanced conduction model

The governing heat equation, described in eq. (13), can be rewritten in two-dimensional cylindrical coordinates according to eq. (18) to match the plane shown in Figure 2-2.

\[
\frac{dT}{dt} = \frac{k_{eq}}{\rho c_p} \left( \frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} + \frac{d^2T}{dz^2} \right)
\]  

(18)

A sketch of the two dimensional plane used for the enhanced conduction model can be seen in Figure 2-3, where boundary 1 is the outer side of the inner pipe that transports the HTF and boundary 4 is the inner side of the PCM containing cylinder. Boundary 2 and 3 are the upper and the lower fins.

The pipe and the fin temperatures determine the conditions for boundaries 1, 2 and 3. The condition for boundary 4 at the inner side of the cylinder depends on the convection rate to the ambient.
The possible contact thermal resistance at boundary 1, 2 and 3 is not considered in this work because of the difficulty in determining exact values for this, even though the fins in the experimental model were corroded during earlier experiments due to the usage of salts, which is very likely to give an increased thermal contact resistance.

The enhanced conduction model has been built using the trapezoidal method, giving good stability and sufficient accuracy. The method used to simulate the enhanced conduction came from equations (14) and (15) and are given by Kutateladze and Borishanskii for enclosed cylindrical spaces.

The code has been written in the software Matlab, which gives the benefits of having complete control over all inputs and parameters in the code. The complete trapezoidal Matlab code is attached in Appendix C: Matlab code.

### 2.2 CFD model

![Figure 2-4: Illustration of the axis-symmetric 2-d plane used for the CFD model](image)

The geometry used in the CFD environment has the same outer measurements as for the model built in Matlab, but also have defined domains for the solid aluminum pipe and fin components. In Figure 2-4 the pipe is seen next to boundary 1, and the fins are seen next to boundaries 2 and 3. Boundary 4 is the inside of the glass container and has the same convection rate to the ambient set as for the Matlab model.

The CFD model calculates the heat transfer with calculations based on the equations of continuity, motion and energy, so the movement in the material is simulated as well as the increased heat transfer due to the movement, and thus gives a very accurate and complex calculation of the heat transfer due to convection, as seen in Figure 2-5 where the movement in the melted material is illustrated by proportionally sized arrows. The color red represents the melted area in and the solid area is represented by the color white. The region between the white and red domains is the mushy-zone interface of the two phases where the phase change transition occurs.
This CFD model simulates the phase change phenomena by modifying the specific heat and using the increased-viscosity method to reach the zero velocity condition in the solid state, as described in earlier chapters.

### 3 Results

The experimental studies have been conducted using several different sets of parameters. The final set that was chosen for the model comparison studies was with the HTF at \(-10{\degree}C\) when freezing and at \(30{\degree}C\) when melting the PCM. Although several studies were made on both the freezing and melting, only one representative result from freezing and melting was chosen to be compared against the results from the numerical models. A photograph sequence of the freezing process can be seen in Figure 3-1, where it is shown that the PCM closest to the fin surface freezes first. The same was shown for the melting process, where the PCM closest to the fins melted first.

![Figure 3-1: The freezing process of paraffin RT9 in the experimental rig.](image-url)
3.1 Result comparison without convection

To verify the correctness of the two numerical models, a comparison was first made with melting and freezing of gelled salt-hydrates. Since there is no convection in gelled salt-hydrates, the function of the numerical models could be investigated without taking the models convection functions into account, but calculating with the more easily handled heat transfer through conduction only equations.

The experimental data was given by project supervisor Justin Chiu and compared with the CFD model and the enhanced conduction model made in Matlab. The comparison between the simulation and experimental results is shown in Figure 3-2 where it is seen that, though the temperature profiles varies somewhat, the final results reach almost identical values after 1h and 15 minutes. The average difference from the experimental results is of 2% for the model built in Matlab and 4% for the CFD model.

The variation in profiles may depend on several factors, but since the specific heat is given by a mathematical adapted Dirac function for the two numerical models, it is likely that this is an explanation why the profiles deviates somewhat from the experimental results.

It may also depend on the difference in geometry set up, where the geometry of the CFD model is more similar to the real model as seen in figure 2-4, while the model made in Matlab is based on a more simple square geometry as seen in figure 2-3 and has a coarser node-grid.

![Figure 3-2: Freezing process from experimental and simulation results](image)

After the two models have been verified to match empirical results to a sufficient degree, a parametric study was conducted on the CFD model to verify what size of the viscosity is needed to achieve the zero velocity condition within the solid state. The results shown in Figure 3-3 show that the simulation reaches more or less the same results when the viscosity reaches a value over $10^5 \text{ Pa s}$. All further simulations have been made using the value of $10^5 \text{ Pa s}$, to be sure that the viscosity has a high enough value.
A parametric study on the mesh size was also made in Comsol to determine the optimal mesh size to use without losing much accuracy. The results shown in Figure 3-4 indicate that the results are under 1% difference between “extremely coarse” and “finer” mesh size, but also shows that the finer the mesh size, the more accurate the results become.

For the finest meshes with very small time-steps of 1 second or below, the CFD simulations take up a few days to finish. Therefore a balance between the simulation time and the required accuracy exists. With simulation time limited to 12 hours, a time-step of 5 seconds and mesh size varying from normal to finer can be utilized for the studied case.
3.2 Result comparison with convection

When running the two models with paraffin RT9, the temperature profiles deviate more from each other as compared to when run with the salt, as seen in Figure 3-5. Here the initial value of the PCM is 29°C and the HTF is set to -10°C.

After a few simulations, it was clear that the enhanced conduction method given by Kutateladze and Borishanskii did not work for cooling, so the temperature profile for the numerical Matlab code seen in blue in Figure 3-5 is completely without convection effect and so here shows the results for heat transfer through conduction only.

The CFD temperature profile is somewhat too fast but shows similar characteristics compared to the experimental results, though the CFD profile seems more affected by the steep specific heat profile shown in Figure 2-1. There might be some thermal contact resistance lowering the heat transfer process in the experimental model, partly explaining the temperature profile deviations.

![Figure 3-5: Results comparison of freezing paraffin RT9](image)

For a melting process of the paraffin, seen in Figure 3-6 with initial values set to -2°C and the HTF fluid set to 30°C, the enhanced conduction method worked to a better extent than for the freezing process. However the enhanced conduction simulation does not show the same characteristics as the experimental study, with a jagged temperature curve after the actual phase change temperature range around 9°C. An up-scaled figure of this jagged that appears after the phase change can be seen in Appendix D.

The convection currents that appear in the melted phase and passes by the middle node are the cause of this jaggedness, explaining why this effect is not shown in the enhanced-conduction simulation while it rather seems enhanced in the CFD simulation results.

It should also be noted that the initial temperature is set to be uniform in all the domains for the numerical models simulated in Figure 3-5 and Figure 3-6, where the set initial temperature is taken from the experimental results at the middle node at $t=0$. But it is unlikely that the experimental model had this middle node temperature as a uniform temperature for the whole model at $t=0$. 

-20-
The phase change process in the CFD model is shown in Appendix E and further shows how the convection affects the melting process, with the warmer PCM flowing up, causing more melt in the higher regions.

The simulated release of warm PCM from the bottom fin can be seen in Appendix E. This release of warm PCM in currents is a recurring process in the CFD melting paraffin model, and it explains the oscillation in temperature profile. In Figure 3-7 the same conditions are set as in Figure 3-6, but here the relative magnitude of the movement within the PCM is also shown.
The extra effect that comes out from the convection effect is shown in Figure 3-8 when comparing two CFD simulations of melting paraffin RT9 from -2°C with HTF at 30°C. One CFD simulation is made with normal heat transfer process, including convection, and the other with conduction only.

The power curves in Figure 3-8 is calculated using the following equation

\[ Q = m \cdot c_p \cdot dT \]

where the bulk temperature difference \( dT \) is represented by taking the temperature value from the middle point in the geometry, shown as a red dot in Figure 2-3.

The power difference reaches a maximum after about 45 minutes as seen in Figure 3-9 while the temperature and power reaches the same value after 2.5h.

![Figure 3-8: Conduction plus convection and conduction only heat transfer comparison in melting paraffin RT9](image)

![Figure 3-9: Difference in heat storage of conductive plus convective and conductive only heat transfer in melting paraffin RT9](image)
3.3 Fin parametric study

In Figure 3-10 the phase change results are shown for a melting process set at -2°C with HTF 30°C with a changing number of fins ranging from four to seven.

![Figure 3-10: Showing the increasing heat transfer due to increased number of fins](image)

The same four sets of various heat exchanger types are simulated with and without convection shown in Figure 3-11. The plots in Figure 3-11 show that the extra effect that comes from convection is very small for the heat exchanger with seven fins while it has an important effect for the heat exchanger design with only four fins.
Figure 3-11: The increased effect of heat transfer due to increased number of fins

Figure 3-11 also shows that the conduction only model is strongly affected by the increased numbers of fins, while the model where the convection effect is included is much more mildly affected, as seen in Table 1 and illustrated with figures in Appendix F.

Table 1: The increased temperature at a certain time compared to the 4-fin model

<table>
<thead>
<tr>
<th></th>
<th>5 fins</th>
<th>6 fins</th>
<th>7 fins</th>
</tr>
</thead>
<tbody>
<tr>
<td>After 50 min</td>
<td>22%</td>
<td>33%</td>
<td>35%</td>
</tr>
<tr>
<td>After 70 min</td>
<td>8%</td>
<td>11%</td>
<td>12%</td>
</tr>
<tr>
<td>After 50 min conduction only</td>
<td>107%</td>
<td>137%</td>
<td>154%</td>
</tr>
<tr>
<td>After 70 min conduction only</td>
<td>39%</td>
<td>48%</td>
<td>51%</td>
</tr>
</tbody>
</table>
3.4 **Vertical and horizontal heat exchanger comparison**

Since the convection effect comes from gravity, a melting simulation with the same design of heat exchanger has been run with three different angles for the heat as seen in Figure 3-12 and Appendix G.

![Figure 3-12: Melting process in paraffin RT9 for vertical, horizontal faced up and horizontal faced down heat exchangers](image)

The difference in results from the three models is presented Appendix H, where it can be seen that all three different sets of angles give different results. To give a better view of the overall heat transfer process, the bulk temperature has been taken as an average from four radially spread out point in the middle space instead of just one central point.

The results for the average values from the two horizontal simulations are presented in Figure 3-13 where it is compared to the values from the vertical heat exchangers. The results show that changing the angle of the heat exchanger to a horizontal position gives an overall extra power of roughly 20% after 50 minutes.

![Figure 3-13: Power comparison of horizontal and vertical heat exchangers](image)
4 Discussion and Conclusion

The enhanced conduction model has shown both benefits and weaknesses. Using an empirically based relationship for describing the enhanced heat transfer due to convection is both a simple and fast way to calculate the increased heat transfer and it can be used to estimate the increased heat transfer effect due to convection. However, this method is unable to show the characteristics of the phase change process, such as how the phase change is affected by the gravitation and how the rising temperature somewhat levels out right after a certain point has melted due to the convection.

Using a CFD environment to model the phase change has proven much more useful for the types of simulations that have been made in this work where the results from the CFD simulations have shown similar characteristics as the experimental study and have followed the temperature profiles of the experimental results to a better degree. A drawback is the loss of complete control over the model and the long simulation times.

The jagged area of the temperature profiles, showed in Appendix D, is caused by the temperature being evened out after the phase change process and occurs because the values are taken from just one specific point in the domain. It is also shown that once the temperature is evened out within the domain, the temperature starts to rise again.

During the freezing process, the temperature clearly levels out during the phase change, due to the increase in specific heat. But in the melting process, this leveling out of the temperature over the phase change range can almost not be noted at all. This may be partly because the phase change process for melting ranges over a larger temperature region and is a smoother process compared to freezing.

Figure 3-9 shows the practical difference of the extra heat transfer that occurs from the convection effect, where at one point the extra heat transfer effect due to convection reaches 65%. Depending on the purpose and requirements of using a certain PCM heat exchanger, the convection effect must therefore be taken into account if an optimum heat transfer rate is sought. Convection may be neglected under the conditions that the PCM should take on close to the same temperature as the HTF since the two power curves come together after a certain amount of time when reaching temperatures close to that of the HTF.

However, it must also be noted that the bulk temperature for the PCM when melting or freezing is taken from just one point in between the two fins. So even though it is a large temperature difference at a certain time between the conduction plus convection model and the convection only model, it must be remembered that this value is taken from one point only and may therefore be misleading for representing the average bulk temperature.

The effect of the convection has though been proven not to be negligible, and it is therefore interesting to see the effects when increasing the number of fins. With more fins, the heat transfer surface is increased and should therefore increase the heat transfer rate. But the enclosures are also smaller, and so inhibiting the convection effect, which is clear to see in Appendix F where there is a big change in the heat transfer between the heat exchanger using four fins and the heat exchanger using six fins. Between the heat exchangers with six and seven fins, the heat transfer difference is not very big. The conduction only simulations show a larger difference than the total heat transfer difference with convection included. This is because the design with seven fins inhibits the
convection effect more than that of the design with six fins. So for this example, using six fins instead of seven would probably be the better choice since the extra heat transfer for the seven fin model is close to negligible due to the inhibited convection.

When using the same design on a cylindrically finned heat exchanger but putting it in different angles, it is also shown that the position of the heat exchanger affects the overall heat transfer effect, as shown in Figure 3-13. This is because the vertically placed fins inhibit the convections less as there is larger space for gravity assisted convective mechanism. However, it should be considered that the assumption that an axis-symmetric 2-d model can represent the heat exchanger might be faulty when changing the angle of the heat exchanger from being completely vertical.

The conclusions can be drawn that the usage of empirical equations to simulate the enhanced heat transfer should always be verified before usage. The empirical equation always depends on the medium and the geometry and it is therefore difficult to find standardized empirical equations for the enhanced heat transfer exactly matching that of a specific heat exchanger. If only coarse estimations are needed, it is however a fast and simple method to use.

It is also clear that convection gives an important heat transfer effect in non-gelled PCMs and must always be taken into account in calculations for optimizing the use of a heat exchanger together with non-gelled PCMs.

It is furthermore important to master the design of the heat exchanger. If a cylindrically finned heat exchanger is to be used in a system, it is necessary to do a thorough pre-study to verify that there are not too few fins resulting in not enough heat transfer surface, and neither that too many fins are used, decreasing the maximum possible PCM volume and inhibiting the convection effect while probably increasing the production cost of the heat exchanger.

It is also important to look at the installation possibility, if the heat exchanger should be vertically or horizontally positioned. This study has shown benefits in using the cylindrically finned heat exchangers in a horizontal position, but there might be other drawbacks that are not included in this report.

There is much remaining work in this field of study. It would be interesting to further look at other heat exchanger designs, such as using a graphite fibered matrix as heat transfer medium with the PCM. Also empirical heat transfer equations could be derived to better fit this kind of geometry and parameters.

An economic study on the importance of the convection effect and how this projects model could be applied to an industrial process would be a continuation of this line of work.
**Bibliography**


Appendix A: Simplified geometry of the heat exchanger
Appendix B: Properties of Rubitherm RT9

Melting area: 6-11°C (typically 9°C)
Congealing area: 22-19 °C (typical being 22°C)
Heat storage capacity (in temp range 15-30°C): 195 kJ/kg
Density solid (at 15°C): 0.86 kg/l
Density liquid (at 25°C): 0.76 kg/l
Volume expansion (in phase change range): 10.47%
Heat conductivity: 0.2 W/m*K
Kin. Viscosity (at 50°C): 21.33 mm²/s
Flash point (PCM): 134°C
Chemically inert with respect to most materials
WGK class-1.¹

¹ German water hazard class ranging from 0-3

Data collected from Rubitherm.de/eng (Rubitherm, 2011)
Appendix C: Matlab code

```matlab
%% Trapezoidal code, by Petter Johansson
% The following is set for the melting process
clc; clear; close all;
dt=1; %time step
r=0.036; %radius
z=0.03; %height (fin spacing)
pb=0.5; % temperature probe position to the radius
nr=12; %number of nodes radial wise
nz=10; %number of nodes height wise
dr=r/nr;
dz=z/nz;
nr1=nr+1;
Tp=11; %T pipe
Ti=-2.3; %T initial pcm
Td=20; %desired temperature
Ta=24; %T air
Tm=9; %melting temperature

%variables
lambda=0.2; %W/m K
lambda_c=0.2;
rho=860; %Kg/m^3
rho_s=860;
rho_l=760;
Cp=195e3; %J/Kg
cps=10e3; %J/Kg K
hm=60e3; %140e3; %J/Kg

h=5; %heat transfer coefficient at the fin border considering it as fixed
v=21.33e-6; % [m^2/s] kinematic viscosity
d_c=r+2; % [m]
B=0.000764; %thermal expansion of paraffin oil according to The Engineering toolbox
g=9.82; % gravity [m/s^2]

%(aluminum 6082 ALss 4212):
lambdafin=180;
rhofin=2700;
cpfin=894;

%% Del 1
% Initierar vektorer, matriser och variabler:
T=[Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ta]; % en 13 variabler lång vektor
T=[T T T T T T T T T T T T T T]; % ..som upprepas 10 ggr, till 130 lång
T_old=[Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ta]; % Samma f`r T_old som f`r T
T_old=[T T T T T T T T T T T T T T];
Tfin=[Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ti Ta]';
Afin=[]; A_i=[]; A_e=[];
```
a_v=[]; b_v=[]; c_v=[]; % Används för att bygga Afin

r_v=dr:dr+dr; % Skapar vektor för radien som, 13 lång
r_v=[r_v r_v r_v r_v r_v r_v r_v r_v r_v r_v]; % Gör vektorn 130 lång

Tfinmon=[]; Tmon=[]; cpmon=[];

no=0;
A_i=diag(ones(130,1));
A_e=diag(ones(130,1));

% Importerar data från ett excel-ark med empiriska studievärden

cp_heating1=[xlsread('RT9','RT9sheet','m7:m94')];
deg_heating1=[xlsread('RT9','RT9sheet','j7:j94')];

% Imp. av temp-värden fr empiriska studier för s–k 5 med kylmedel
pcm_fin_mid=[xlsread('paraffin_kryo40_2','warming','c6:c451')];
pipe_temp=[xlsread('paraffin_kryo40_2','warming','k6:k451')];
fin_mid=[xlsread('paraffin_kryo40_2','warming','q6:q451')];
time=[xlsread('paraffin_kryo40_2','warming','b6:b451')];

% curve=fit(time,pcm_fin_mid,'poly9');

cp_heating1(24)=cp_heating1(25);
cp_heating1(28)=cp_heating1(29);
cp_heating1(42)=cp_heating1(41);
cp_heating1(45)=cp_heating1(44);
cp_heating1(37)=cp_heating1(36);
cp_heating1(38)=cp_heating1(41)+2e3;
cp_heating1(39)=cp_heating1(41)+3e3;
cp_heating1(40)=cp_heating1(41)+2e3;
cp_heating1(40)=cp_heating1(41)+2e3;

mp_mf=fit(deg_heating1,smooth(cp_heating1),'smoothingspline');

cp_cop=[]; x_c=[];
for qq=1:0.2:17;
    if qq<2;
        cp_cop=[cp_cop 10000];
    else
        cp_cop=[cp_cop cp_mf(qq)];
    end
    x_c=[x_c qq];
end

cp_cop(1)=cp_cop(3);
cp_cop(2)=cp_cop(3);
x2=44000;

pipe_spline=spline(time(1:x2/10),pipe_temp(1:x2/10),1:x2);
pipe_spline=smooth(pipe_spline);

%% Del 2
% Beräknar fläns-temperaturen

tic

% Definiera värmekonstanter för Afin:
alphafin=lambdafin/(cpfin*rhofin);
\[ \text{Fo}_\text{fin} = \alpha_f \text{fin} \cdot \text{dt}/\text{dr}^2; \]
\[ \text{Bi}_\text{fin} = h \cdot \text{dr}/\lambda; \]

% Initierar och nollställer vektorerna f"r flänsarna:
\[ a_v_{\text{fin}} = []; b_v_{\text{fin}} = []; c_v_{\text{fin}} = []; \]

% Bygger mitten på vektorerna f"r A_fin:
\[ \text{for } i = 2:nr-1; \]
\[ \quad a_v_{\text{fin}} = [a_v_{\text{fin}} \text{Fo}_\text{fin} \cdot (\text{dr}/(2 \times r_{\text{v}(i)}) - 1)]; \]
\[ \quad b_v_{\text{fin}} = [b_v_{\text{fin}} 1 + \text{Fo}_\text{fin} \cdot (2)]; \%
\]
\[ \quad c_v_{\text{fin}} = [c_v_{\text{fin}} - \text{Fo}_\text{fin} \cdot (1 + \text{dr}/(2 \times r_{\text{v}(i)}))); \]
\[ \text{end} \]

% Modifierar vektorerna:
\[ a_v_{\text{fin}} = [a_v_{\text{fin}} 0]; b_v_{\text{fin}} = [1 b_v_{\text{fin}} 1]; c_v_{\text{fin}} = [0 c_v_{\text{fin}}]; \]

% Bygger A_fin utifrån vektorerna:
\[ \text{Afin} = \text{diag}(a_v_{\text{fin}}, -1) \times \text{diag}(b_v_{\text{fin}}, 0) \times \text{diag}(c_v_{\text{fin}}, 1); \]
\[ \text{Afin}(13, 13) = 1; \text{Afin}(nr1 * 2, nr1 * 2) = 1; \]

%% Del 3 - Beräkningar f"r PCM

\[ \text{for } qq = 1:x2; \]
\[ \quad \text{lambda} = \text{lambda}_c; \]
\[ \quad \text{no} = \text{no} + 1; \]
\[ \quad \text{Tfin}(nr1 + 1: nr1 + 2) = \text{T}(nr1 + 1: nr1 + 2); \]
\[ \text{for } k = 1: nr1 * nz; \]
\[ \quad \text{if } T(k) > = 4; \]
\[ \quad \quad \text{cp}(k) = \text{cp}_m(T(k)); \]
\[ \quad \text{else} \]
\[ \quad \quad \text{cp}(k) = 9000; \]
\[ \quad \text{end} \]
\[ \quad \text{if } T(k) > 13.2; \]
\[ \quad \quad \text{cp}(k) = 4000; \]
\[ \quad \text{end} \]
\[ \quad \text{if } T(k) >= 6; \]
\[ \quad \quad \text{rho} = \text{rho}_l; \]
\[ \quad \text{else} \]
\[ \quad \quad \text{rho} = \text{rho}_s; \]
\[ \quad \text{end} \]
\[ \quad \text{alpha}(k) = \text{lambda}/(\text{rho} \times \text{cp}(k)); \]
\[ \text{end} \]

% Räknar ut enhanced thermal conduction:
\[ \text{if } T(k) >= 5; \]
\[ \quad \text{Pr} = \text{v} \times \text{rho} \times \text{cp}(k)/\text{lambda}; \]
\[ \quad \text{Gr} = g \times B \times r^3 \times v^2 \times abs(T(nr1+nz/2+nr1/2+0.5)-T(nr1*nz/2)); \]
\[ \text{if } \text{Pr} \times \text{Gr} > 10^7; \]
\[ \quad f_c = 0.22 \times (\text{Pr} \times \text{Gr})^{(1/4)}; \]
\[ \text{else} \]
\[ \quad f_c = 0.062 \times (\text{Pr} \times \text{Gr})^{(1/3)}; \]
\[ \text{end} \]
\[ \text{if } \text{Pr} \times \text{Gr} < 10^3; \]
\[ \quad f_c = 1; \]
\[ \text{end} \]
\[ \text{if } T(k) <= 11; \]
\[ \quad \text{distr} = (T(k) - 5)/(11 - 5); \]
\[ \quad f_c = f_c \times \text{distr}; \]

-33-
\[ \rho = 760 + (1 - \text{distr}) \times 100; \]

\[ \text{if } f_c < 1; \]
\[ f_c = 1; \]
\[ \text{end} \]
\[ \lambda = \lambda_c \times f_c; \]
\[ \text{else} \]
\[ \lambda = \lambda_c; \]
\[ \text{end} \]
\[ \text{if } T(k) > 14; \]
\[ \text{if } T(k) > 14; \]
\[ \text{cp}(k) = 4000; \]
\[ \text{end} \]
\[ \alpha(k) = \lambda / (\rho \times \text{cp}(k)); \]
\[ \text{end} \]
\[ \text{for } i = 1: n_r; \]
\[ \text{for } i = 1: n_r; \]
\[ \text{Foz} = \lambda_c / (\rho \times \text{cp}(k)) \times \text{dt} / \text{dz}^2; \]
\[ \text{Afin}(i, n_r + i) = (-2 \times \text{Foz}); \]
\[ \text{Afin}(i, i) = 1 + \text{Fo}_\text{fin}(2) + 2 \times \text{Foz}; \]
\[ \text{Afin}(n_r + i, n_r + i) = 1; \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{Afin}(1, 14) = 0; \]
\[ \text{Afin}(1, 1) = 1; \]
\[ \text{Afin}(13, 13) = 1; \]
\[ \text{Afin}(12, 12) = 1; \]
\[ \text{Afin}(n_r, n_r + n_r) = 0; \]
\[ \text{Afin}(n_r, n_r + 2) = 0; \]
\[ \% \text{Sätter yttre randvillkoret för } T_\text{fin} \]
\[ T_\text{fin}(n_r) = \lambda_c / (\lambda_c + \text{dr} \times h) \times (T_\text{fin}(n_r - 1)) + \text{dr} \times h / (\lambda_c + \text{dr} \times h) \times T_a; \]
\[ \% \text{ Sätter temperaturen till ret efter empiriska studier} \]
\[ T_\text{fin}(1) = \text{pipe}_\text{temp}(1 + \text{round(no/10)}); \]
\[ \text{end} \]
\[ \% \text{Beräknar flänsstemperaturen} \]
\[ T_\text{fin} = \text{Afin} \setminus T_\text{fin}; \]
\[ \% \text{Sätter flänsstemperaturen på stora } T \text{-vektorn} \]
\[ T(1:10) = T_\text{fin}(1:10); \]
\[ T(11:13) = T((11 + n_r):(13 + n_r)); \]
\[ T(118:127) = T_\text{fin}(1:10); \]
\[ T(128:130) = T((128 + n_r):(130 - n_r)); \]
\[ \% \text{Följande bygger implicita } A \text{-matrisen } A_i \]
\[ \text{for } k = 1 + n_r; \]
\[ \text{for } k = 1 + n_r; \]
\[ A_i(k + 1, k) = -\alpha(k + 1) \times \text{dt} / (2 \times \text{dr}^2); \]
\[ A_i(k + 1, k + 1) = 1 + \alpha(k + 1) \times \text{dt} / (2 \times \text{dr}^2 + 1 / (\text{r}_v(k) \times \text{dr}) + 2 / \text{dz}^2); \]
\[ A_i(k + 1, k + 2) = -\alpha(k + 1) \times \text{dt} / (2 \times \text{dr}) \times (1 / \text{r}_v(k) + 1 / \text{dr}); \]
\[ \% \text{Följande if slinga sköter Foz värdena i matrisen} \]
\[ \text{if } k > n_r \& k < n_r \times n_z - n_r - 1; \]
\[ A_i(k + 1, k + 1 - n_r) = -\alpha(k + 1) \times \text{dt} / (2 \times \text{dz}^2); \]
\[ A_i(k + 1, k + 1 + n_r) = -\alpha(k + 1) \times \text{dt} / (2 \times \text{dz}^2); \]
\[ \text{end} \]
\[ \% \text{Följande if slinga ger villkor för randvärden} \]
\[ \text{if } \]
\[ \text{k} = n_r \| \text{k} = n_r + 2 \| \text{k} = n_r + 3 \| \text{k} = n_r + 4 \| \text{k} = n_r + 5 \| \text{k} = n_r + 6 \| \text{k} = n_r + 7 \| \text{k} = n_r + 8 \| \]
\[ \text{k} = n_r + 9 \| \text{k} = n_r + 10; \]

\[ \]
\begin{align*}
A_i(k,k-1) &= 0; \\
A_i(k,k) &= 1; \\
A_i(k,k+1) &= 0; \\
A_i(k+1,k-1) &= 0; \\
A_i(k+1,k) &= 1; \\
A_i(k+1,k+1) &= 0; \\
\text{if } k > nr1 & \land k < nr1*nz-nr1; \\
A_i(k,k-nr1) &= 0; \\
A_i(k,k+nr1) &= 0; \\
A_i(k+1,k-nr1) &= 0; \\
A_i(k+1,k+nr1) &= 0; \\
\end{align*}

\begin{align*}
F_r &= \alpha(k-1)*dt/((dr/2+dz/2)^2); \\
Bi &= h*dr/(\lambda); \\
\end{align*}

% Ställer randvärden f för implicita A matrisen
\begin{align*}
A_i(k-1,k-2) &= -2*F_r; \\
A_i(k-1,k-1) &= (1+2*F_r*(2+Bi)); \\
A_i(k-1,k) &= -2*Bi*F_r; \\
\text{if } k > nr1 & \land k < nr1*nz; \\
A_i(k-1,k-nr1-1) &= -F_r; \\
A_i(k-1,k+n1-1) &= -F_r; \\
\end{align*}

\begin{align*}
\text{end}
\end{align*}

% Följande bygger explicita A_e-matrisen
\begin{align*}
A_e(k+1,k) &= \alpha(k+1)*dt/(2*dr^2); \\
A_e(k+1,k+1) &= 1 - \alpha(k+1)*dt/2*(2/dr^2+1/(r_v(k+1)*dr)+2/dz^2); \\
A_e(k+1,k+2) &= \alpha(k+1)*dt/(2*dr)*(1/dr+1/r_v(k+1)); \\
\end{align*}

\begin{align*}
\text{if } k > nr1 & \land k < nr1*nz-nr1-1; \\
A_e(k+1,k+1-nr1) &= \alpha(k+1)*dt/(2*dz^2); \\
A_e(k+1,k+1+nr1) &= \alpha(k+1)*dt/(2*dz^2); \\
\end{align*}

\begin{align*}
\text{end}
\end{align*}

\begin{align*}
\text{if } k == nr1 & \lor k == nr1*2 & \lor k == nr1*3 & \lor k == nr1*4 & \lor k == nr1*5 & \lor k == nr1*6 & \lor k == nr1*7 & \lor k == nr1*8 & \lor k == nr1*9 & \lor k == nr1*10; \\
A_e(k,k-1) &= 0; \\
A_e(k,k) &= 1; \\
A_e(k,k+1) &= 0; \\
A_e(k+1,k-1) &= 0; \\
A_e(k+1,k) &= 1; \\
A_e(k+1,k+1) &= 0; \\
\text{if } k > nr1 & \land k < nr1*nz; \\
A_e(k,k-nr1) &= 0; \\
A_e(k,k+nr1) &= 0; \\
A_e(k-1,k-1-nr1) &= 0; \\
A_e(k-1,k-1+nr1) &= 0; \\
\end{align*}

\begin{align*}
\text{end}
\end{align*}

\begin{align*}
\text{if } k > nr1 & \land k < nr1*nz-nr1; \\
A_e(k+1,k-1-nr1) &= 0; \\
A_e(k+1,k+1-nr1) &= 0; \\
\end{align*}

\begin{align*}
\text{end}
\end{align*}

\begin{align*}
A_e(k+1,k+1-nr1) &= \lambda/(dr*h+\lambda); \\
A_e(k+1,k+1+nr1) &= 0; \\
A_e(k+1,k) &= dr*h/(dr*h+\lambda); \\
\end{align*}

\begin{align*}
\text{end}
\end{align*}

\begin{align*}
\text{if } k > nr1 & \land k < nr1*nz-nr1; \\
A_e(k,k-1) &= 0; \\
A_e(k,k+1) &= 0; \\
A_e(k,k+2) &= \lambda/(dr*h+\lambda); \\
A_e(k+1,k) &= 0; \\
A_e(k+1,k+1) &= 0; \\
A_e(k+1,k+2) &= \lambda/(dr*h+\lambda); \\
\end{align*}

\begin{align*}
\text{end}
\end{align*}

% Denna rad lser T med trapezoidal-metoden:
\begin{align*}
T &= A_i\backslash(A_e*T);
\end{align*}
% Läser och lagar temp vid mittnoden för T och cp
Tmon=[Tmon T((nr+1)*nz/2+(nr+1)/2+0.5)];
cpmon=[cpmon cp((nr+1)*nz/2+(nr+1)/2+0.5)];

% Bevakar flänsstempaturen vid mittnoden:
Tfinmon=[Tfinmon Tfin(7)];

for q1=1:nz;
    for q2=1:(nr+1);
        T_matrix(q1,q2)=T((nr+1)*(q1-1)+q2);
        cp_matrix(q1,q2)=cp((nr+1)*(q1-1)+q2);
    end
end
T_matrix;
toc

%% PLOTS
figure
plot(Tmon,'green')
hold on
plot(Tfinmon)
title('Temperature Profile - Fully implicit method.')
xlabel('Time step')
ylabel('Temperature')
hold on
plot(time(1:x2/10),pcm_fin_mid(1:x2/10),'r');
hold on
plot(time(1:x2/10),fin_mid(1:x2/10),'m');
hold on
plot(time(1:x2/10),pipe_temp(1:x2/10),'x');
plot(pipe_spline,'b');
legend('PCM','fin','pcm_fin_mid','fin_mid','pipe_temp')
figure
plot(Tmon,cpmon)
title('cp-profile for melting RT9')
xlabel('Temperature')
hold on
plot(deg_freezing2,cp_freezing2,'m');
hold on
plot(deg_heating1,cp_heating1,'r');
disp('The time in seconds to reach the set degree');disp(no*dt);
disp('The difference in percent against empirical results:');
disp((Tmon(1)-Tmon(x2))/(pcm_fin_mid(1)-pcm_fin_mid(x2/10))*100-100);
disp('The integrated Cp between 2 and 17degC (should be 195kJ/kg):')
disp(9000*2+sum(cp_mf(4:13))+4000*4);
Appendix D: Scaled up jagged zone in melting RT9

The following figure shows the zone where the jaggedness occurs right after the phase change process has occurred in the middle node, as seen in Figure 3-5, where the jaggedness appears due to convection.
Appendix E: CFD simulation of convection in a phase change process

The convection effect in the melted PCM is shown with proportionally sized arrows in the following figure. The white part of the figure is the solid domain of the PCM while the blue is the mushy zone where the material is undergoing the phase change process. A warm current rising from the bottom fin can also be seen in the mid-bottom of the figure.
Appendix F: Interpolated curves from Table 1

The following figures show the increased temperature in the middle node between two fins and the pipe and inner cylinder surface of a horizontally positioned cylindrical finned heat exchanger after 30 minutes with HTF of 30°C and initial condition of -2°C. The x-axis shows the increased number of fins and the y-axis the increased heat transfer rate.

After 50 min, simulation run with convection effect:

![Graph showing increased temperature and heat transfer rate after 50 min with convection effect.]

After 70 min, simulation run with convection effect

![Graph showing increased temperature and heat transfer rate after 70 min with convection effect.]

After 50 min, simulation run with conduction only conditions:

![Graph showing increased temperature and heat transfer rate after 50 min with conduction only conditions.]

-39-
After 70 min, simulation run with conduction only conditions:
Appendix G: Vertical and horizontal heat exchanger comparison

The simulations was run with an initial temperature of -2°C and with HTF of 30°C.

Results after 30 min for the vertically positioned finned cylindrical heat exchanger:

Results after 30 min for the horizontally positioned finned cylindrical heat exchanger:
Results after 40 min for the vertically positioned finned cylindrical heat exchanger:

Results after 40 min for the horizontally positioned finned cylindrical heat exchanger:
Appendix H: Temperature profiles of three different angled heat exchangers

The following figure presents the heat transfer difference for three cylindrical heat exchangers placed in different angles. The average temperature representing the bulk temperature has been taken from four radially spread out points.