HIGH FREQUENCY OSCILLATION MODES IN A TRANSFORMER WINDING DISC

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Abstract – The high frequency resonant behavior of a winding disc is studied by measurements on a test setup, and by computer simulation of lumped-element circuit models. It is shown that in order to achieve satisfactory agreement between both, every turn must be described by several segments in the model. The reason is that the high frequency end of the spectrum is dominated by “azimuthal” resonance modes which are not present in models with lower resolution. The intermediate frequency range is characterized by “radial” resonances, present even in lower resolution models. The different physical character and properties of these modes are discussed. Our findings provide new insight into the interpretation of frequency response measurements on power transformers.

Introduction

In the field of transformer winding modeling, various approaches and tools are available. Among the most common tools are lumped-element circuits [1-3]. Usually all the turns of one or two discs are lumped together into one inductive element (“segment”) of the model, which leads to a decreased computation time but also to a reduction of the model’s upper frequency limit, typically to values around some 100 kHz.

In this paper, we study lumped-element models with much higher resolution (up to 4 segments per turn) of a single winding disc with \( n \) turns, in order to increase their upper frequency limit.

Experimental setup

The disc is chosen to have a quadratic shape so that all the self and mutual inductances can be calculated by simple analytic formulas [5]. Measurements on an experimental set-up with the same geometry (see Fig. 1 for a schematic view) have been carried out and are compared to our model calculations. The set-up consists of \( n = 10 \) turns of varnished copper wire with rectangular cross section (7 mm \( \times \) 3 mm). The inner sides of the square disc have a length of 1.2 m, and the gap between any two neighboring conductors (turns) is varying between 0.4 mm (= twice the insulation thickness) and about 1 mm because of manufacturing irregularities.

Simulation model

Three different model resolutions are studied: in our models labeled 1, 2, and 3, each turn is modeled by one, two, or four segments, respectively. This implies that there will be one, two, or four capacitances between any two neighboring turns in the winding, respectively (see Fig. 1 from right to left). Every segment consists of one resistance in series with one self inductance and there are mutual inductances between any two parallel segments of the winding. For simplicity, self inductances and resistances are shown in Fig. 1 on one segment only, and some of the mutual inductances are indicated by arrows. In case of model 1, also the connection of the voltage source for impedance measurement is
shown. All the inductance and capacitance parameters (i.e., all model parameters except for the damping resistances) are estimated from the physical winding geometry, and not fitted to measurements. The models are analyzed by solving their state space equations [2-4] in the frequency domain.

Fig. 1. The three different levels of discretization with the nodes numbered in an increasing sequence from one winding end to the other. The resolution increases from model 1 to model 3.

The circuit parameters are calculated by simple analytical formulas. We use the relation for the capacitance between two planar surfaces

\[ C = \text{permittivity} \times \frac{\text{area}}{\text{distance}}. \]

The total capacitance \( C_n \) between two turns in the disc is then given by

\[ C_n = 4 \varepsilon_i \varepsilon_0 d \frac{h + 2 \tau_i}{\tau_i}, \quad (1) \]

where \( h \) is the height of the conductor, \( \tau_i \) is twice the insulation thickness, \( \varepsilon_i \) is the relative permittivity of the insulation, \( d \) is the mean side lengths of the disc, and the addition of \( 2 \tau_i \) to \( h \) accounts for the fringing effect.

For the calculation of the self and mutual inductances, formulas from Ref. [5] are used. The self inductance \( L \) of each straight segment with the length \( l \), height \( h \) and width \( w \) in the disc is

\[ L = \frac{\mu_0}{2\pi} l \left( \ln \left( \frac{2l}{0.2235(w + h)} \right) - 1 \right). \quad (2) \]

The mutual inductance \( M \) between two segments which are perpendicular to each other is zero. That between two parallel segments of length \( l \) in Fig. 1, separated by a distance \( x \), is given by

\[ M = \frac{\mu_0}{2\pi} l \left( \ln \left( \frac{l}{x} - \sqrt{1 + \left( \frac{l}{x} \right)^2} \right) - \sqrt{1 + \left( \frac{l}{x} \right)^2 + \frac{x}{l}} \right). \quad (3) \]

The resistance \( R \) of each segment is assumed to be of the form

\[ R = \alpha l \left( \frac{1}{\sigma wh} + \frac{1}{2(w + h)} \sqrt{\frac{\mu_0 \pi f}{\sigma}} \right), \quad (4) \]
where $\sigma$ is the conductivity of the conductor and $f$ is the frequency. The first term is the DC resistance and the second term accounts for the skin effect at higher frequencies. Since proximity losses are not included in our model, a numerical factor $\alpha > 1$ has been introduced and adjusted so that a realistic level of resonance damping is obtained in Figs. 2 and 3.

**Observations and physical interpretation**

We study the new phenomena that emerge with increasing model resolution. The magnitudes of the calculated impedances $Z(f)=U(f)/I(f)$ (see Fig. 1) are compared to each other in Fig. 2. The impedance magnitude of model 3 is compared to the measured one in Fig. 3. It can be seen that several resonances occur: the first, pronounced impedance maximum is the fundamental resonance of the winding, due to the total inductance and series capacitance of the whole disc. As we will argue below, the three following resonances can be interpreted as “radial” resonance modes, and the two after that, which form a pronounced impedance minimum above 10 MHz and do not appear in the lowest-resolution model 1, as “azimuthal” resonance modes. Our measurements (Fig. 3) show that these high-frequency azimuthal modes, which model 1 is unable to produce and which model 2 produces partially, are no model artifacts but real physical phenomena.

In our measurements the radial modes are shifted toward somewhat higher frequencies compared to the simulations, but they are fully recognizable. Such a shift is expected to occur due to the proximity effect which has not been taken into account in the model calculations reported here. Next, the physical meaning of the radial and azimuthal resonances will be explained and discussed.

**Radial resonance modes**

We call those resonances “radial” whose node voltages vary rapidly in the radial direction, but slowly in the azimuthal direction. By radial and azimuthal directions, we mean the $\rho$ and $\phi$ directions in polar coordinates, respectively (see Fig. 9).

In Figures 4–6, instantaneous node voltages for a sinusoidal excitation voltage $U$ are depicted for different resonance frequencies, each at two different instants of time during an oscillation period, obtained from simulations of our model 3. A linear voltage profile along the whole winding (which is the low-frequency limiting behavior) is subtracted, so that the values at both end nodes of the winding are equal to zero. The green line shows the geometry of the winding disc and the location of the nodes, and the thin horizontal red line shows the zero level of the voltage as a reference. Black lines connect voltage levels in radial direction, and vertical blue lines indicate the correspondence between voltage levels and nodes. The voltage distribution in the disc for the first, second and third radial resonance of
Fig. 2 is depicted in Figs. 4, 5, and 6, respectively. \( f_k \) \((k = 1, 2, \text{ or } 3)\) denotes the frequency for that particular resonance, and \( 1/f_k \) is the corresponding period time. The radial resonances appear as standing voltage waves which can approximately be described by the formula

\[
V_k (\rho, \varphi, t) \approx \cos(2\pi f_k t) \left[ A_k \sin \left( k2\pi \frac{\rho - \rho_0}{\rho_1 - \rho_0} \right) + B_k \left( \rho - \frac{\rho_0 + \rho_1}{2} \right) \right]
\]  

(5)

for \( \rho_0 < \rho < \rho_1 \), where \( \rho_0 \) and \( \rho_1 \) are the inner and outer “radii” of the disc, respectively (see Fig. 9). The resonance voltage amplitudes \( A_k \) and \( B_k \) are damping dependent. Note that the approximate expression (5) is independent of \( \varphi \). It can be seen in Figs. 4–6 that the approximation (5) is best for low resonance order \( k \). The amplitude \( B_k \) is close to zero for \( k = 1 \) and increases with increasing resonance order \( k \).
**Azimuthal resonance modes**

For “azimuthal” resonances, just like the “radial” ones, the node voltages vary rapidly in the \( \rho \) direction, but the difference is that there are also significant node-voltage variations in the \( \phi \) direction. This pattern can be seen in Figs. 7 and 8 which depict the instantaneous node voltages for the two dominant azimuthal resonances, appearing in Figs. 2 and 3 as pronounced minima close to \( f_{az} \), at two different instants of time. Again, model 3 has been employed and a linear voltage profile has been subtracted.

In contrast to the radial resonances which are spread out in frequency, the azimuthal resonances are “clustered” (at least when viewed on a logarithmic frequency scale) around a characteristic frequency \( f_{az} \) slightly above 10 MHz. They cannot be described by a simple formula like that for the radial resonances (5), but their common characteristics is approximated by the expression

\[
V_{az}(\rho,\phi,t) \approx A_{az} \cos(2\pi f_{az} t)(1 - \cos(\phi)) \left( \rho - \frac{\rho_0 + \rho_1}{2} \right).
\]  

(6)

This fundamental behavior is indicated in Figs. 7 and 8 by dotted lines. Individual azimuthal resonance modes differ from it by additionally superposed short-wavelength modulations. The mode in Fig. 7 resembles more closely to the fundamental expression (6) than the one in Fig. 8.

![Fig. 7. Voltage profile of the first azimuthal resonance, at times \( t = 0.25/f_{az} \) (left) and \( t = 0.7/f_{az} \) (right).](image1)

![Fig. 8. Voltage profile of the second azimuthal resonance, at times \( t = 0.4/f_{az} \) (left) and \( t = 0.65/f_{az} \) (right).](image2)

In Fig. 10, measurements are shown for a winding consisting of two, three and four discs (with the same dimensions as the single disc measured in Fig. 3), respectively, connected together in a continuous way. It can be seen that the azimuthal resonances occur around the same frequency \( f_{az} \) somewhere between 10 and 20 MHz, no matter how many discs are connected together in the winding. This supports the picture that azimuthal resonances are internal oscillations in every individual disc, and are roughly independent of the other discs. Furthermore, measurements and simulations show that they are very sensitive to small changes in the winding geometry (e.g. mechanical winding deformations). For
instance, in our measurements on several winding disks of identical design but with small manufacturing differences (not shown here), the precise location and relative strength of the two dominating azimuthal resonances varied noticeably, each disk thus having its individual “finger print”.

$$\phi = 3\pi/2$$

$$\phi = \pi/2$$

$$\phi = 0$$

$$\phi = \pi$$

$$\rho_1$$

$$\rho_0$$

$$\rho_6$$

$$\rho_{4n+1}$$

$$\rho_{4n}$$

$$\phi = \pi/2$$

$$\phi = \pi$$

$$\phi_0$$

$$\phi_{4n}$$

$$\phi_{4n+1}$$

$$\phi_{4n-1}$$

$$\phi_{4n-2}$$

$$f_{az}$$

$$\rho = 0$$

$$\rho = \rho_1$$

$$\rho = \rho_6$$

$$\rho = \rho_{4n+1}$$

$$\rho = \rho_{4n}$$

$$\rho = \rho_{4n-1}$$

$$\rho = \rho_{4n-2}$$

**Conclusions**

We have defined two classes of internal resonance modes of a single transformer winding disc, the “radial” modes at lower frequencies and the “azimuthal” modes at higher frequencies (above ca. 10 MHz in our case), and studied them by measurements and model simulations.

The radial modes are characterized by a rather constant voltage amplitude within every turn, whereas the azimuthal modes describe electrical oscillations of different parts of the same turn against each other and therefore can only be seen in models with more than one segment per turn.

When several discs are connected together to a disc winding, the number and distribution of the radial modes varies strongly with the number of discs, which shows that they are “global” modes of the whole winding consisting of interacting discs. In contrast, the azimuthal modes always cluster around the same frequency $f_{az}$, leading to a very pronounced impedance minimum, which suggests that they are “local” modes, depending only on the geometry of the individual discs. As such, they are expected to be useful for probing the integrity of individual discs in frequency response (FRA) measurements on transformers. Moreover, measurements and simulations show that they are highly sensitive to small changes in the winding geometry. The application of these mode signatures to winding fault detection should therefore be further explored.

**References**


