



KTH Electrical Engineering

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Proceedings of IEEE International Conference on Communications (ICC)

5-9 June, Kyoto, Japan, 2011

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Stockholm 2011

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KTH Report: IR-EE-SB 2011:006

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Abstract—In this paper, we propose a multiple-state interference model, which takes into account the interference variability and uncertainty created by scheduling and other fast resource allocation adaptivity. This situation is modeled by a set of states, each described by a spatial covariance matrix and a probability. In order to illustrate the usefulness of such a model, we study two receive beamforming design problems, one maximizing the average data rate and one maximizing the worst-case data rate. We compare the resulting performance with what can be obtained when the state information is not available. We show theoretically and numerically that exploring the multiple-state interference structure can improve the receive beamforming design efficiency, especially for cell-edge users in an interference-limited system.

I. INTRODUCTION

In single cell multiuser Multiple Input Multiple Output (MIMO) systems, transmit and receive beamformer designs have been studied extensively, see [1], [2] and references therein. At the receiver side, both signal and interference channels are usually assumed known.

In multi-cell multiuser MIMO systems, beamformer designs should preferably take other-cell interference (OCI) into account. If instantaneous information on the OCI channels is available, many multiuser detection (MUD) technologies [3]–[5] can be applied to combat the OCI. However, unlike intra-cell interference, OCI information is not as easy to obtain at the receiver [6]. For more practical designs, one way to handle the OCI is to model it as spatially white noise [1], then the problem falls into the scope of single-cell designs. Another option is to model the OCI as spatially colored noise [7]. After estimating the long-term covariance matrix of the OCI, the receiver employs a whitening filter to suppress it. Both models rely on the OCI being relatively stationary over time, in order to facilitate parameter estimations and increase model accuracy. However, with the introduction of fast adaptive resource allocation schemes (scheduling, power control, beamforming, etc.), the OCI can change rapidly and significantly over time, and these properties are not reflected in the long-term modeling.

One observation from multi-cell systems is that mostly only a very limited number of interferers contribute significantly to the OCI, and these interferers are called dominant interferers. In [3], it is shown by simulations that the number of dominant interferers for a downlink channel is often at most two. This

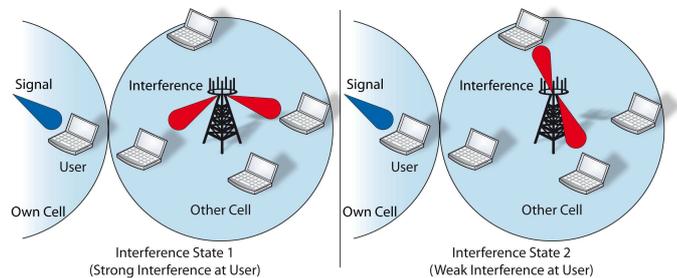


Fig. 1. Illustration of multiple-state OCI model for a cell-edge user. The two OCI states describe different behaviors of the dominant interferer (i.e., base station in the adjacent cell).

observation inspires us to relate the OCI modeling with the behavior of the dominant interferers. Instead of modeling the OCI having a single distribution, we assume a mixed distribution, where the OCI sources are in one of K states. Each state corresponds to a certain combination of the resource allocation used at the dominant interferers, as is illustrated in Fig. 1, where a cell-edge user has the adjacent cell's base station as its dominant interferer. With a certain probability, the base station serves a user in the same direction as our user, which constitutes one state of strong OCI. When the base station sends streams elsewhere, it gives rise to another state of OCI with weaker impact on our user. This is naturally modeled as a multi-state OCI model, where each state has a certain probability and is characterized by a certain spatial covariance matrix. This model itself could emanate from an underlying hidden Markov model (HMM). HMMs have been proposed earlier for interference modeling [8], [9], but to our knowledge only for pure scheduling purposes and not incorporating any spatial information. The estimation of such a model is outside the scope of the current paper, the focus here is rather to illustrate how it may be exploited in receive beamforming design and analyze the performance of the different designs.

In Section II, we present the system model and formulate two receive beamforming design problems, (P1) and (P2), given parameters of the multiple-state interference model. Section III and IV solve (P1) and (P2), respectively. In Section V, we present reference designs when state information is not available, and when assuming perfect state information

at the receiver. Section VI concludes our study by numerical illustrations.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a multi-cell multiuser downlink system, where each base station is equipped with multiple transmit antennas and each user has N receive antennas. There is no cooperation between base stations.

At the user's side, single stream decoding is used. The received signal from a certain data stream is given by

$$\mathbf{y} = \mathbf{h}_0 x + \mathbf{n}, \quad (1)$$

where x is the transmitted signal on this data stream, $\mathbf{h}_0 \in \mathbb{C}^{N \times 1}$ is the effective channel vector, and $\mathbf{n} \in \mathbb{C}^{N \times 1}$ is the interference plus noise vector. The estimated signal after receive beamforming is

$$\hat{x} = \mathbf{w}^H \mathbf{y} = \mathbf{w}^H \mathbf{h}_0 x + \mathbf{w}^H \mathbf{n}, \quad (2)$$

where $\mathbf{w} \in \mathbb{C}^{N \times 1}$ is the receive beamformer.

We denote the channel correlation matrix of the received signal by \mathbf{R}_0 . When \mathbf{h}_0 is known at the receiver,

$$\mathbf{R}_0 = \mathbf{h}_0 \mathbf{h}_0^H$$

is a rank 1 matrix. Otherwise,

$$\mathbf{R}_0 = \mathbb{E}\{\mathbf{h}_0 \mathbf{h}_0^H\}$$

can have rank higher than 1.

For the interference plus noise vector \mathbf{n} , we apply the multiple-state interference model, where the interference is in one of the K states. Each state is characterized by an interference plus noise covariance matrix

$$\mathbf{R}_k = \mathbb{E}\{\mathbf{n}\mathbf{n}^H | \text{state} = k\} \quad (3)$$

and a probability p_k , $k = 1, 2, \dots, K$. How to obtain and estimate such a model is out of the scope of the current paper, for now we assume that this state information is given.

Our task is to design receive beamformers \mathbf{w} based on the multiple-state interference model, according to different performance criteria. We divide our designs into three categories:

- Perfect state information design: the receiver knows exactly the current state
- State-aware design: the receiver does not know the exact state, but knows all the state parameters \mathbf{R}_k and p_k .
- State-blind design: the receiver does not know anything about the states, and treat the interference as white or colored noise.

Our focus is on state-aware designs, and the performance will be compared with perfect state information and state-blind designs.

In the following, we define mathematically two state-aware receive beamforming design problems (P1) and (P2).

Problem 1. Maximize the average data rate among all states:

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize}} \quad & c(\mathbf{w}) = \sum_{k=1}^K p_k \log \left(1 + \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}} \right), \quad (\text{P1}) \\ \text{subject to} \quad & \mathbf{w}^H \mathbf{w} \leq 1. \end{aligned}$$

Notice that the received Signal to Interference plus Noise Ratio (SINR) in state k is

$$\text{SINR}_k = \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}},$$

and the achievable data rate given SINR_k is $\log(1 + \text{SINR}_k)$. (P1) designs a receive beamformer that gives the best average data rate.

Problem 2. Maximize the worst-case data rate among all states:

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize}} \quad & \min_{k=1,2,\dots,K} \log \left(1 + \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}} \right), \quad (\text{P2}) \\ \text{subject to} \quad & \mathbf{w}^H \mathbf{w} \leq 1. \end{aligned}$$

(P2) is interesting when we want a guaranteed performance and the design maximizes the worst data rate that can be expected from different states. This problem is similar to the uplink SINR balancing problem in [10], but with the difference that the same receive beamformer is used in all rate expressions. It is also related to but not equivalent with the multicast beamforming problem [11].

In the following sections, we will study these two problems. Section III gives optimal and suboptimal solutions to (P1), and Section IV gives a solution to (P2).

III. MAXIMUM AVERAGE DATA RATE DESIGN

In this section, we will solve (P1). Besides, we propose a suboptimal design which is computationally more efficient.

A. Optimal Design

(P1) is a non-convex optimization problem, standard non-linear optimization techniques in general will find a local optimum [12]. By setting different initial points, it is more likely to end up in an global optimal point. Grid search is a global optimization method. With a sufficiently fine grid, it will find the global optimum to any specified accuracy. For this problem, one can do a grid search through all unit-norm vectors $\mathbf{w} \in \mathbb{C}^{N \times 1}$ to obtain the optimal solution. Below, we derive an alternative parameterization that reduces the number of real valued parameters as long as $K < 2N$.

Consider the following problem:

$$\begin{aligned} \underset{\mathbf{w}}{\text{maximize}} \quad & \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \left(\sum_{k=1}^K \alpha_k \mathbf{R}_k \right) \mathbf{w}} \quad (2) \\ \text{subject to} \quad & \mathbf{w}^H \mathbf{w} \leq 1. \end{aligned}$$

Theorem 1. *There exists an $\boldsymbol{\alpha} = [\alpha_1, \dots, \alpha_K]^T \in \mathbb{R}^{K \times 1}$, with $\sum_k \alpha_k = 1$ and $\alpha_k \geq 0$, $\forall k$, such that (2) has the same solution as (P1).*

Proof: Suppose that \mathbf{w}_{opt} is the solution to (P1) and set

$$\gamma_k = \frac{\mathbf{w}_{\text{opt}}^H \mathbf{R}_0 \mathbf{w}_{\text{opt}}}{\mathbf{w}_{\text{opt}}^H \mathbf{R}_k \mathbf{w}_{\text{opt}}}.$$

Then, \mathbf{w}_{opt} is also a solution to the feasibility problem

$$\begin{aligned} \text{find} \quad & \mathbf{w} \\ \text{subject to} \quad & \mathbf{w}^H \mathbf{R}_0 \mathbf{w} \geq \gamma_k \mathbf{w}^H \mathbf{R}_k \mathbf{w} \quad \forall k. \end{aligned}$$

Using the Lagrange multipliers β_1, \dots, β_K , the Lagrangian of this feasibility problem is

$$\mathcal{L} = \sum_k \beta_k (\gamma_k \mathbf{w}^H \mathbf{R}_k \mathbf{w} - \mathbf{w}^H \mathbf{R}_0 \mathbf{w}).$$

Differentiating with respect to \mathbf{w} and equating to zero yields

$$\left(\sum_k \beta_k \mathbf{R}_0 \right) \mathbf{w} = \left(\sum_k \beta_k \gamma_k \mathbf{R}_k \right) \mathbf{w}.$$

This necessary condition is a generalized eigenvalue problem. To prove that \mathbf{w}_{opt} is also a solution to (2), assume for the purpose of contradiction that for λ_{max} strictly larger than one, there exists a (normalized) eigenvector \mathbf{w}_{max} such that

$$\begin{aligned} \left(\sum_k \beta_k \mathbf{R}_0 \right) \mathbf{w}_{\text{max}} &= \lambda_{\text{max}} \left(\sum_k \beta_k \gamma_k \mathbf{R}_k \right) \mathbf{w}_{\text{max}}. \\ &= \left(\sum_k \beta_k \tilde{\gamma}_k \mathbf{R}_k \right) \mathbf{w}_{\text{max}}. \end{aligned} \quad (3)$$

where $\tilde{\gamma}_k = \lambda_{\text{max}} \gamma_k > \gamma_k$. Then, we can use this \mathbf{w}_{max} to increase the utility in (P1): $\sum_k p_k \log(1 + \tilde{\gamma}_k) > \sum_k p_k \log(1 + \gamma_k)$. This contradiction means that \mathbf{w}_{opt} is also the solution to the generalized Rayleigh quotient

$$\max_{\mathbf{w}; \|\mathbf{w}\|=1} \frac{(\sum_k \beta_k) \mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H (\sum_k \beta_k \gamma_k \mathbf{R}_k) \mathbf{w}}. \quad (4)$$

Finally, by setting $\alpha_k = \beta_k \gamma_k / (\sum_l \beta_l \gamma_l)$ and ignoring the scaling factor in the numerator of (4), we achieve the problem in (2) and that \mathbf{w}_{opt} is a solution to (2). ■

The theorem establishes a relationship between (P1) and the generalized Rayleigh quotient in (2) for a proper α . If this parameter is known, then the solution of (P1) is easily computed by solving a generalized Rayleigh quotient problem.

To conclude, we have two strategies searching for the optimal solution to (P1):

1. Grid search over \mathbf{w} , pick the one that maximizes $c(\mathbf{w})$ in (P1).
2. Grid search over α , calculate \mathbf{w} from (2), pick the one that maximizes $c(\mathbf{w})$.

One strategy can be computationally preferable to the other mainly depending on dimensions of \mathbf{w} and α .

B. Suboptimal Design - State search method

The grid search method can lead us to a solution with any specified accuracy, but the computational complexity makes it impractical. In order to reduce the complexity of the design, we propose a suboptimal solution.

From the *proof* for **Theorem 1**, we can see that the optimal value of α is a function of optimal \mathbf{w} . Here, we relax the problem by ignoring the dependence between α_{opt} and

\mathbf{w}_{opt} . This leads to the following problem:

$$\begin{aligned} &\underset{\mathbf{w}, \alpha}{\text{maximize}} \quad \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \left(\sum_{k=1}^K \alpha_k \mathbf{R}_k \right) \mathbf{w}} \\ &\text{subject to} \quad \sum_{k=1}^K \alpha_k = 1, \alpha_k \geq 0, \forall k \\ &\quad \quad \quad \mathbf{w}^H \mathbf{w} \leq 1. \end{aligned} \quad (5)$$

Notice that now \mathbf{w} and α can be optimized separately. For each value of \mathbf{w} , the optimal α is given by

$$\alpha_k = \begin{cases} 1, & \text{if } \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}} \geq \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_l \mathbf{w}}, \forall l \neq k \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

Hence, (5) reduces to

$$\begin{aligned} &\underset{\mathbf{w}}{\text{maximize}} \quad \max_{k=1, \dots, K} \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}} \\ &\text{subject to} \quad \mathbf{w}^H \mathbf{w} \leq 1. \end{aligned} \quad (7)$$

To better understand how (7) is obtained from (5), notice that according to Jensen's inequality,

$$\max_{\alpha} \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \left(\sum_{k=1}^K \alpha_k \mathbf{R}_k \right) \mathbf{w}} \leq \max_{\alpha} \sum_{k=1}^K \alpha_k \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}}. \quad (8)$$

Besides,

$$\max_{\alpha} \sum_{k=1}^K \alpha_k \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}} \leq \max_{k=1, \dots, K} \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}}. \quad (9)$$

Equalities hold for both (8) and (9) if and only if α is given by (6).

In problem (7), we first find the best beamformer for each state, and then select the state that achieves the highest SINR. The solution is the beamformer associated with the selected state. Since we have to search over all different states, we call it *state search solution*. The complexity of problem (7) is equal to K generalized Rayleigh quotient problems.

IV. MAXIMUM WORST-CASE DATA RATE DESIGN

Now, we turn our attention to (P2), which provides the best worst-case performance, in contrast to (P1) which maximizes the average performance.

Problem (P2) may be equivalently reformulated as

$$\begin{aligned} &\underset{\mathbf{w}, \eta}{\text{minimize}} \quad \eta \\ &\text{subject to} \quad \mathbf{w}^H \mathbf{R}_0 \mathbf{w} = 1 \\ &\quad \quad \quad \mathbf{w}^H \mathbf{R}_k \mathbf{w} \leq \eta, \quad k = 1, 2, \dots, K. \end{aligned} \quad (10)$$

To understand this, observe that through different states, maximizing minimum data rate is equivalent to minimizing the maximum interference given a certain received signal power. A similar reformulation was used in [13] in the context of spectrum-sharing.

In general, (10) is a non-convex problem, but it can be solved efficiently using a semidefinite relaxation [14]. Introducing the matrix $\mathbf{W} = \mathbf{w} \mathbf{w}^H$ and note that $\mathbf{w}^H \mathbf{R}_k \mathbf{w} =$

$\text{tr}(\mathbf{R}\mathbf{w}\mathbf{w}^H) = \text{tr}(\mathbf{R}\mathbf{W})$, problem (10) can be reformulated as

$$\begin{aligned} & \underset{\mathbf{W}, \eta}{\text{minimize}} \quad \eta \\ & \text{subject to} \quad \text{tr}(\mathbf{R}_0 \mathbf{W}) = 1, \\ & \quad \text{tr}(\mathbf{R}_k \mathbf{W}) \leq \eta, \quad k = 1, 2, \dots, K \\ & \quad \mathbf{W} \succeq 0 \\ & \quad \text{rank}[\mathbf{W}] = 1. \end{aligned} \quad (11)$$

Disregarding the rank constraint, we achieve a relaxed problem that is convex and thus can be efficiently solved using CVX, a package for specifying and solving convex programs [15]. In the special case when \mathbf{R}_0 is rank one, Lemma 1 in [16] proves that the solution to the relaxed problem always satisfies $\text{rank}[\mathbf{W}] = 1$. In general, the solution may have larger rank, although our simulation results show that this is seldom the case. If $\text{rank}[\mathbf{W}] > 1$, then an approximately optimal \mathbf{w} can be calculated using randomization techniques, see [14].

V. STATE-BLIND DESIGNS AND PERFECT STATE INFORMATION DESIGN

In the above, we have solved the two state-aware design problems (P1) and (P2), based on the multiple-state interference model. In this section, we will study two reference cases: State-blind designs and Perfect state information design. The design criterion is to maximize received SINR, which is equivalent to maximizing achievable data rate.

A. State-blind designs

When the receiver has no information about the state parameters p_k and \mathbf{R}_k , the interference can only be treated as noise. This results in a Maximum Ratio Combining (MRC) design

$$\mathbf{w}_{\text{MRC}} = \arg \max_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_0 \mathbf{w}. \quad (12)$$

When treating the interference as colored noise, the receiver measures the long-term interference plus noise covariance matrix

$$\mathbf{R}_I = \mathbb{E}\{\mathbf{R}_k\} = \sum_{k=1}^K p_k \mathbf{R}_k. \quad (13)$$

Given \mathbf{R}_I , the beamforming design problem is

$$\begin{aligned} & \underset{\mathbf{w}}{\text{maximize}} \quad u(\mathbf{w}) = \log \left(1 + \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_I \mathbf{w}} \right) \\ & \text{subject to} \quad \mathbf{w}^H \mathbf{w} \leq 1. \end{aligned} \quad (14)$$

Notice that (14) can be reduced to a generalized Rayleigh quotient problem

$$\begin{aligned} & \underset{\mathbf{w}}{\text{maximize}} \quad \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_I \mathbf{w}} \\ & \text{subject to} \quad \mathbf{w}^H \mathbf{w} \leq 1. \end{aligned} \quad (15)$$

Based on the multiple-state interference model, $u(\mathbf{w})$ is a lower bound on $c(\mathbf{w})$ in (P1). since by replacing \mathbf{R}_I with p_k and \mathbf{R}_k in (13), and using Jensen's inequality,

$$u(\mathbf{w}) \leq \log \left(1 + \sum_{k=1}^K p_k \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}} \right) \leq c(\mathbf{w}). \quad (16)$$

B. Perfect state information design

Here, we assume that the receiver can detect the interference state and design the receive beamformer based on this information. This results in the problem

$$\begin{aligned} & \underset{\mathbf{w}}{\text{maximize}} \quad v(\mathbf{w}) = \log \left(1 + \frac{\mathbf{w}^H \mathbf{R}_0 \mathbf{w}}{\mathbf{w}^H \mathbf{R}_k \mathbf{w}} \right) \\ & \text{subject to} \quad \mathbf{w}^H \mathbf{w} \leq 1 \\ & \quad \text{state} = k, \end{aligned} \quad (17)$$

which is a generalized Rayleigh quotient problem.

VI. NUMERICAL RESULTS

In this section, based on the multiple-state interference model, we compare the performance of the following receive beamformer designs through Monte Carlo simulations:

- 1) *Perfect state information* design from (17).
- 2) *Max states average* design from (P1).
- 3) *State search* design from (7).
- 4) *Best worst-case performance* design from (P2).
- 5) *Maximum ratio combining (MRC)* design from (12)
- 6) *Colored noise* design from (15).

Among these, 1) is a perfect state information design. 2)-4) are state-aware designs. 5) and 6) are state-blind designs. Different designs are compared according to two performance metrics: **average data rate**, which is the utility function in (P1); and **worst-case data rate**, which is the utility function in (P2). We want to find out:

- When the state-aware designs work well?
- How close the performance of the suboptimal *State search* design is to the optimal *Max states average* design?

For simplicity, we generate $\mathbf{R}_0 = \mathbf{h}_0 \mathbf{h}_0^H$, where $\mathbf{h}_0 \sim \mathcal{CN}(0, \sigma_0^2 \mathbf{I})$. $\mathbf{R}_k = \mathbf{H}_k \mathbf{H}_k^H + \sigma_n^2 \mathbf{I}$, where $\mathbf{H}_k \in \mathbb{C}^{N \times N}$, $\text{vec}(\mathbf{H}_k) \sim \mathcal{CN}(0, \sigma_k^2 \mathbf{I})$, σ_k^2 is the interference power in state k and σ_n^2 is the noise power. The plotted results are averaged over 1000 realizations. We study the case with $N = 4$ receive antennas and $K = 4$ states with equal probability of occurrence, i.e., $p_1 = p_2 = p_3 = p_4 = 1/4$. In the figures, the results are plotted as a function of the pre-processing SINR, defined as

$$\text{SINR} = \sum_{k=1}^K p_k \frac{\sigma_0^2}{\sigma_k^2 + \sigma_n^2}. \quad (18)$$

We set $\text{SNR} = \frac{\sigma_0^2}{\sigma_n^2} = 20\text{dB}$. By adjusting the interference power σ_k^2 , we get different SINR values. Our focus is low to medium SINR region.

Two multiple-state interference models are considered. In **model 1**, the different states are modeled with the same interference power, i.e., $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$. This means that the different states only model different spatial properties of the interference represented by \mathbf{R}_k . In **model 2**, different states are in addition modeled with different interference powers. This is a more realistic model for cell-edge users since the interference power they experience varies considerably subject

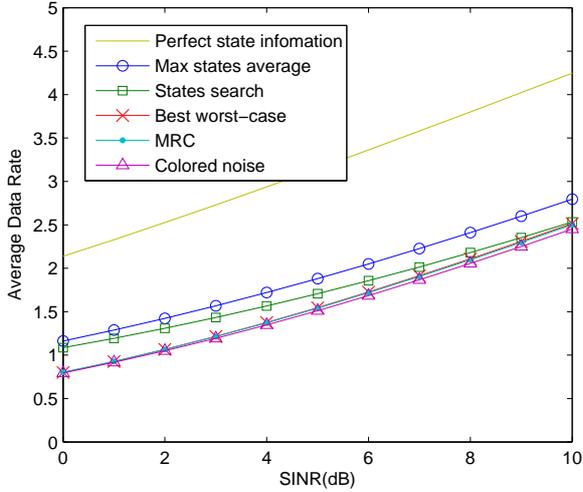


Fig. 2. Average data rate achieved by different beamformer designs based on model 1, where different states are modeled with *same* interference power.

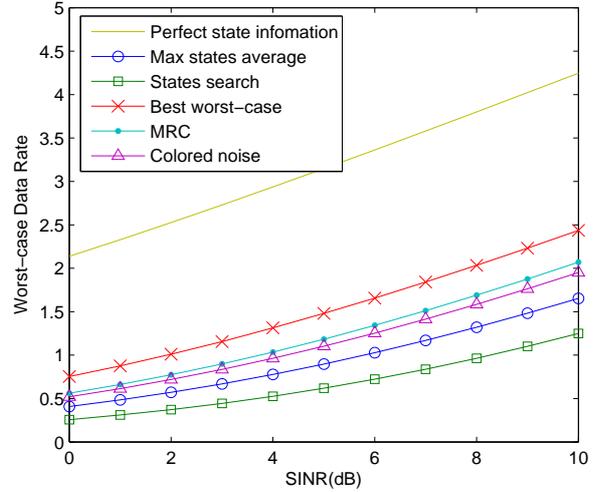


Fig. 4. Worst-case data rate achieved by different beamformer designs based on model 1, where different states are modeled with *same* interference power.

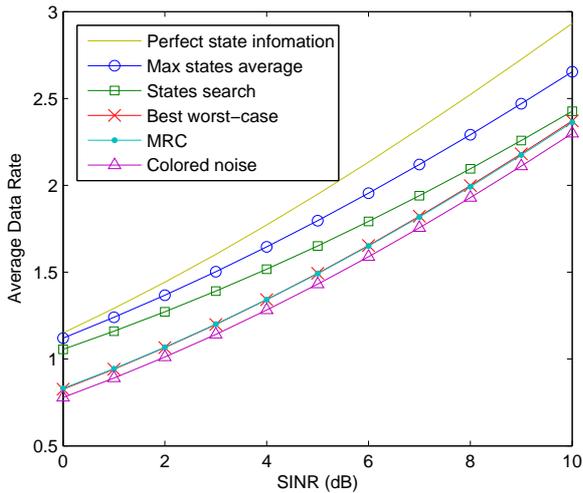


Fig. 3. Average data rate achieved by different beamformer designs based on model 2, where different states are modeled with *different* interference powers.

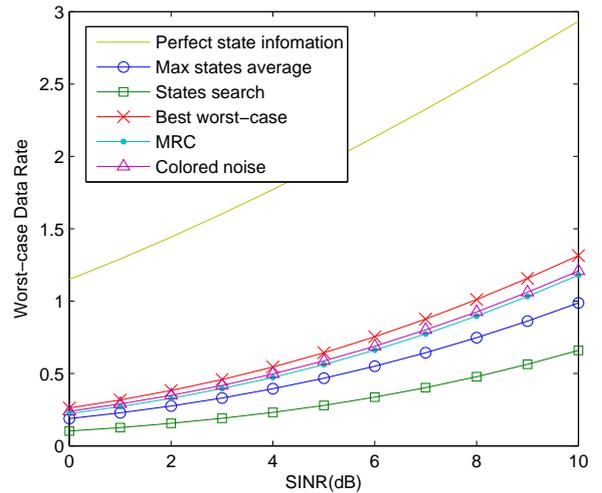


Fig. 5. Worst-case data rate achieved by different beamformer designs based on model 2, where different states are modeled with *different* interference powers.

to the behaviors of dominant interferers. In the simulations, we study an example of model 2 by setting $\sigma_1^2 = \sigma_2^2 = 0.1\sigma_3^2 = 0.1\sigma_4^2$.

Fig. 2 and Fig. 3 show the average data rate achieved by different receive beamformer designs, based on model 1 and model 2, respectively. As predicted in (16), state-aware design aiming for maximizing average data rate performs better than state-blind design.

Two observations can be made in the low SINR region: Firstly, *state search* design, which is a suboptimal solution to *max states average* design, can give near optimal performance. Secondly, in model 2, state-aware designs perform almost as well as perfect state information design. One explanation is: when the overall conditions are bad, the only significant data

is obtained in the most beneficiary interference state, so the remaining states can be ignored, as is done in the *state search* design. Especially in model 2, some states are so much worse than the others that it does not hurt to ignore them. This is why the state-aware designs can approach the performance of perfect state information design.

Another interesting observation is that the *colored noise* design performs slightly worse than the *MRC* design, especially under model 2. Thus, when the interference has large variations, modeling the average spatial interference color is not necessarily an advantage.

Fig. 4 and Fig. 5 show the worst-case data rate achieved by different receive beamformer designs, based on model 1 and model 2, respectively. *Best worst-case performance* design

from (P2) maximizes the worst-case data rate and works particularly well in model 1, where there is small variations in interference power. Besides, we see that the *max states average* design and especially the *state search* design does not provide any guarantees on the worst case performance, although they achieve good average rates.

To summary, in the low SINR region where the system is interference-limited, when state parameters are available, we propose the *state search* design which achieves almost the same performance as the *max states average* design, but is computationally preferable. When Quality of Service is the main concern, we propose the *best worst-case performance* design which gives the best guaranteed data rate. If there is no state information available at the receiver side, we propose the *MRC* design, since knowing only the long-term spatial property of rapidly changing interference is not necessarily helpful.

VII. CONCLUSION

When fast adaptive resource allocation schemes are used in a multi-cell system, the other-cell interference has a rapidly changing behavior. This property is ignored if the interference is only modeled as stationary noise. A multiple-state interference model is proposed herein to describe such interference by modeling multiple interference states with different probabilities of occurrence. Based on this model, we have studied two receive beamforming design problems. (P1) designs a beamformer maximizing the average data rate. (P2) designs a beamformer maximizing the worst-case data rate. We compare the performance of these two state-aware designs with state-blind designs where state parameters are not available. We show that by exploring the multiple-state structure of the interference, beamformers can better achieve their design purposes. The performance gain from knowing the state information is considerable, especially for the cell-edge users and in an interference-limited system.

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