Optimal Power Allocation in Multi-Hop Cognitive Radio Networks

Maksym A. Girnyk, Ming Xiao and Lars K. Rasmussen
School of Electrical Engineering and the ACCESS Linnaeus Center,
KTH Royal Institute of Technology, Stockholm, Sweden
email: {mgyr, mingx, lkra}@kth.se

***Abstract*—Optimal power allocation in a multi-hop cognitive radio network is investigated. Information transmitted from the source passes through several wireless relay nodes before reaching the destination. At each hop, the received signal is decoded, re-encoded and retransmitted to the following node. Transmissions at every hop are overheard by nearby nodes and therefore cause interference. We study optimal power allocation strategies that maximize the end-to-end throughput of the network under the constraint of strictly limited interference to external users. We show that for networks that can be modeled as a line topology the optimal solution is achieved when the capacities of every intermediate link are equal and the interference power constraint is satisfied with equality. High- and low-SNR approximations that simplify the problem of finding the optimal power allocation are presented as well. The numerical results show good performance compared to schemes with equal power allocation.**

I. Introduction

New applications and services demand higher data rates and reliability. A natural approach to achieve such requirements is to use more bandwidth. The wireless spectrum is therefore becoming an increasingly more precious commodity. However, a scan of the radio spectrum reveals the presence of so-called *spectrum holes*, where the spectrum is used inefficiently and thus valuable bandwidth resources are wasted. The concept of *cognitive radio* (CR) was proposed by Mitola [1] to increase spectrum efficiency. CR networks (CRN's) allow users to sense and learn the surrounding spectrum environment and dynamically adapt transmission strategies to increase spectral efficiency [2].

A CRN may operate simultaneously with a so-called *primary network*; a deployed network with license to operate within a certain spectrum band. A CRN is hence considered as an unlicensed *secondary network* that may use the spectrum under the condition that interference to the primary network is kept below strict limits. Thus, a main challenge in a CRN is how to allocate resources among the secondary users such that service requirements of both the primary and the secondary networks are met, with priority towards the primary system.

In this paper we focus on the problem of the resource allocation in a multi-hop CRN. Such multi-hop networks allow the same infrastructure to be reconfigured in different ways in order to deliver the information from source to destination [3]. For example, *blind zones* can appear due to shadowing from surrounding buildings or in tunnels. The presence of wireless nodes around such zones can help in relaying information in a multi-hop fashion to avoid deep fading.

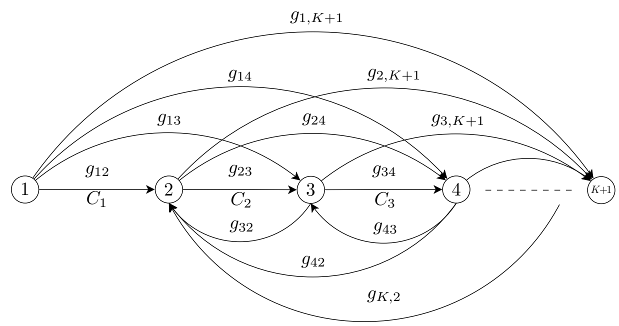

Fig. 1. Multi-hop transmission with overhearing upstream and downstream.

The relaying strategies can be roughly divided into two types: non-regenerative and regenerative. A non-regenerative relay simply amplifies and forwards the received signal to the following node, while a regenerative relay decodes, re-encodes and retransmits the received signal. Clearly, the latter has the benefit of avoiding noise propagation.

Practical systems have limited available resources, such as power and bandwidth. Moreover, the interference to a primary receiver created by the CRN is determined by the aggregated power of secondary transmitters weighted with the channel gains of the paths to the primary receiver. Here we address the issue of optimal power allocation in a multi-hop CRN such that the end-to-end data throughput is maximized under a constraint on the accumulated secondary interference (ASI) power to a set of nearest primary receivers.

Optimal power allocation for two-hop non-CR networks with a regenerative relay node was studied in [4] and [5]. In [6] and [7] bandwidth and power allocation for ergodic capacity maximization was studied for FDMA- and OFDMA-based systems respectively. Yet, interference within the network was not considered. Here we study the optimal power allocation for a CRN in which the nodes operate within the same frequency band and hence interfere with each other.

For non-CR networks transmission strategies were proposed in [8], where all the nodes in the network take part in

The authors would like to thank the European Union for providing partial funding of this work through the EU FP7 project INFSO-ICT-248303 QUASAR. The work has further been supported in parts by the ARC Grant DP0986089, and the VR grant 621-2009-4666.

transmitting the information from the source to the destination using transmissions overheard from downstream nodes. Here we consider a CRN with the tandem topology shown in Fig. 1, where the source signal goes through multiple hops to the destination. However, desired transmissions are only between neighboring nodes. Nodes are full-duplex, transmit simultaneously, and as shown in Fig. 1 a node receives a desired signal as well as upstream/downstream intra-network interference through overhearing. We show that in a CR multi-hop tandem network with arbitrary intra-network interference, the end-to-end throughput is maximized when all individual link capacities are equal and the power is allocated such that the AS interference constraint is met with equality. Further, we show that high- and low-SNR approximations can greatly simplify the optimal power allocation problem while still providing a reasonable precision of the solution.

The remainder of the paper is organized as follows. In the next section we describe our system model, while we show the optimal power allocation solution and discuss approximations in Section III. In Section IV we apply our results to a simple network topology, and demonstrate simulation results in Section V. Finally, concluding remarks are made in Section VI.

II. System Model

Consider a K-hop CRN with a tandem topology as illustrated in Fig. 1. We assume that the relaying nodes operate in full-duplex mode. Namely, nodes can receive and transmit information simultaneously. Transmission from node i to node $i+1$ is said to be *overheard* if the signal from node i reaches node $j, (j \neq i+1)$. In certain scenarios, e.g., cooperative communications, overhearing can be helpful leading to increased diversity. However, since we assume that node $i+1$ knows only its incoming channel from node i, it treats its overheard signals as interference. Overhearing interference can come from the nodes downstream referred to as *forward overhearing interference* (FOI). Interference from upstream nodes is called *backward overhearing interference* (BOI). If both FOI and BOI are present we refer to this case as *general overhearing interference* (GOI), as illustrated in Fig. 1. In the case of BOI, we assume that the channel gains to downstream nodes are smaller (in absolute value) than those to upstream nodes. For instance, this can be achieved by using directional antenna transmissions with main lobes directed upstream [9].

The links of the primary and CR networks are additive white Gaussian noise (AWGN) channels subject to slow fading such that the channel gains are random but constant during a transmission suite from source to destination. The transmit power of CRN node i is denoted as P_i and the noise variance is σ^2. The channel gain $g_{i,j}$ between nodes i and j in the CRN absorbs path-loss and Rayleigh fading effects. The channel gain $\tilde{g}_{i,j}$ between node i in the CRN and node j in the primary network is defined similarly. We also assume that all channel gains are known at the source before the transmission starts, which can be obtained with an initial training phase. In this way, the source can determine the optimal power allocation and forward the allocation information upstream as overhead with the message. The remaining nodes can also get information of the channel gains of their incoming links during the training phase to implement the regenerative relaying. Thus, according to the network model, the capacity of link i is

$$C_i = \log_2\left(1+\frac{g_{i,i+1}P_i}{\sigma^2+\sum_{j=1}^{L_f} g_{i-j,i+1}P_{i-j}+\sum_{j=1}^{L_b} g_{i+1+j,i+1}P_{i+1+j}}\right), \quad (1)$$

where L_f and L_b are corresponding maximum numbers of hops that create FOI and BOI.

Suppose, the primary network consists of M receivers operating simultaneously with the CRN within the same area. In order to limit the ASI to the primary network the CRN has a constraint

$$\sum_{i=1}^{K}\sum_{j=1}^{M}\tilde{g}_{i,j}P_i \leq \gamma. \quad (2)$$

We denote $\varphi_i \triangleq \sum_{j=1}^{M}\tilde{g}_{i,j}$ and rewrite the ASI constraint in the following form

$$\sum_{i=1}^{K}\varphi_i P_i \leq \gamma. \quad (3)$$

III. Optimal Power Allocation

From [10] we know that the end-to-end throughput of the network with a single source and a single sink is determined by the minimum capacity of the channels in the network. Since overhearing links cannot be helpful for information transmission, then in the case of the tandem topology the capacity is determined by the weakest link, i.e. $C = \min\{C_1, ..., C_K\}$. Therefore, the end-to-end throughput maximization problem for a given channel realization becomes

$$\begin{aligned} \max_{P_i} \quad & \min\{C_1, ..., C_K\} \\ \text{s.t.} \quad & \sum_{i=1}^{K}\varphi_i P_i \leq \gamma \\ & P_i \geq 0, \ \forall i. \end{aligned} \quad (4)$$

Unfortunately, since a function of type $f(x) = \log_2(1+\frac{a}{b+cx})$ with a, b and c constants is not concave, the problem above is not a convex optimization problem. However, for this problem we have the following result.

Theorem 1. For a multi-hop tandem CR network with a constraint on the ASI the end-to-end throughput is maximized if and only if the capacities of all channels of the network are equal, i.e.,

$$C_1 = ... = C_K \quad (5)$$

and the ASI constraint is met with equality, i.e.,

$$\sum_{i=1}^{K}\varphi_i P_i = \gamma. \quad (6)$$

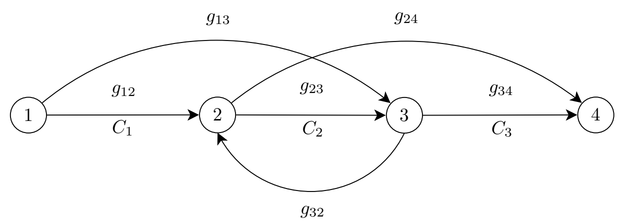

Fig. 2. 3-hop transmission with overhearing upstream and downstream.

Proof: See appendix. ■

Theorem 1 suggests that any violation of either (5) or (6) will immediately lead to sub-optimal solution. Therefore, in order to find the power allocation that achieves the maximum throughput we have to solve a system of $K-1$ non-linear equations (5) subject to the ASI constraint (6). For a large number of hops in the network it is difficult to find a closed-form solution; however, the problem can be solved numerically. Furthermore, in order to simplify the solution we can use a high-SNR approximation based on the fact that at high SNR network performance is GOI-limited and the influence of noise vanishes. Thus, if $\min_{i,j}\{g_{i,j}P_i\} \gg \sigma^2$ the latter can be neglected and the capacities become

$$C_i = \log_2\left(1+\frac{g_{i,i+1}P_i}{\sum_{j=1}^{L_f} g_{i-j,i+1}P_{i-j}+\sum_{j=1}^{L_b} g_{i+1+j,i+1}P_{i+1+j}}\right). \quad (7)$$

According to Theorem 1, from (7) we get $K-1$ equalities with K unknowns, while one more equation is formed by the ASI constraint (3). Therefore, we can compute the optimal P_i^*. In general, this system is easier to solve than (1).

Similarly, at low SNR we can use the fact that the network performance is noise-limited and hence, we can neglect the GOI. The capacities become

$$C_i = \log_2\left(1+\frac{g_{i,i+1}P_i}{\sigma^2}\right). \quad (8)$$

From (8) we get a system of $K-1$ equalities, subject to the ASI constraint (3) and thus, we can compute P_i^*.

IV. Examples

For illustration, we now give a few examples as follows.

A. 3-Hop Network with General Overhearing

Consider a 3-hop network with possible one hop GOI which contains both FOI and BOI components of the interference as shown in Fig. 2. We assume that two-hop FOI is weak and can be disregarded, i.e., $L_f = 1$ and $L_b = 1$.

For this setting the link capacities are given by

$$C_1 = \log_2\left(1+\frac{g_{12}P_1}{\sigma^2+g_{32}P_3}\right), \quad (9a)$$
$$C_2 = \log_2\left(1+\frac{g_{23}P_2}{\sigma^2+g_{13}P_1}\right), \quad (9b)$$
$$C_3 = \log_2\left(1+\frac{g_{34}P_3}{\sigma^2+g_{24}P_2}\right). \quad (9c)$$

From Theorem 1 we know that in order to obtain optimal P_1^*, P_2^* and P_3^* we have to solve the system of equations above, together with (3). This can be done numerically.

B. High-SNR Regime

When the transmission power is large, the network performance becomes interference-limited and therefore, if $\min_{i,j}\{g_{i,j}P_i\} \gg \sigma^2$ the latter is negligible and the channel capacity becomes

$$C_1 = \log_2\left(1+\frac{g_{12}P_1}{g_{32}P_3}\right), \quad (10a)$$
$$C_2 = \log_2\left(1+\frac{g_{23}P_2}{g_{13}P_1}\right), \quad (10b)$$
$$C_3 = \log_2\left(1+\frac{g_{34}P_3}{g_{24}P_2}\right). \quad (10c)$$

Let $r=\frac{g_{12}}{g_{32}}$, $s=\frac{g_{23}}{g_{13}}$ and $q=\frac{g_{34}}{g_{24}}$. Thus, we can find that

$$\frac{P_1}{P_3} = \sqrt[3]{\frac{sq}{r^2}}, \quad \frac{P_2}{P_1} = \sqrt[3]{\frac{rq}{s^2}}, \quad \sum_{i=1}^{3}\varphi_i P_i = \gamma. \quad (11)$$

Solving the system of equations (11) provides the optimal power allocation at high SNR

$$P_1^* = \frac{\gamma\sqrt[3]{\frac{sq}{r^2}}}{\varphi_1\sqrt[3]{\frac{sq}{r^2}}+\varphi_2\sqrt[3]{\frac{q^2}{rs}}+\varphi_3}, \quad (12a)$$
$$P_2^* = \frac{\gamma\sqrt[3]{\frac{q^2}{rs}}}{\varphi_1\sqrt[3]{\frac{sq}{r^2}}+\varphi_2\sqrt[3]{\frac{q^2}{rs}}+\varphi_3}, \quad (12b)$$
$$P_3^* = \frac{\gamma}{\varphi_1\sqrt[3]{\frac{sq}{r^2}}+\varphi_2\sqrt[3]{\frac{q^2}{rs}}+\varphi_3}. \quad (12c)$$

C. Low-SNR Regime

When SNR is low, the network performance is noise-limited and hence, we can neglect the interference from overhearing. The capacity expressions are simplified to

$$C_1 = \log_2\left(1+\frac{g_{12}P_1}{\sigma^2}\right), \quad (13a)$$
$$C_2 = \log_2\left(1+\frac{g_{23}P_2}{\sigma^2}\right), \quad (13b)$$
$$C_3 = \log_2\left(1+\frac{g_{34}P_3}{\sigma^2}\right). \quad (13c)$$

Equating capacities in (13) we obtain a system of equations

$$\frac{P_1}{P_2}=\frac{g_{23}}{g_{12}}, \quad \frac{P_2}{P_3}=\frac{g_{34}}{g_{23}}, \quad \sum_{i=1}^{3}\varphi_i P_i = \gamma. \quad (14)$$

Solving (14) we get the optimal transmit powers at low SNR

$$P_1^* = \frac{\gamma g_{23}g_{34}}{\varphi_1 g_{23}g_{34}+\varphi_2 g_{34}g_{12}+\varphi_3 g_{12}g_{23}}, \quad (15a)$$
$$P_2^* = \frac{\gamma g_{34}g_{12}}{\varphi_1 g_{23}g_{34}+\varphi_2 g_{34}g_{12}+\varphi_3 g_{12}g_{23}}, \quad (15b)$$
$$P_3^* = \frac{\gamma g_{12}g_{23}}{\varphi_1 g_{23}g_{34}+\varphi_2 g_{34}g_{12}+\varphi_3 g_{12}g_{23}}. \quad (15c)$$

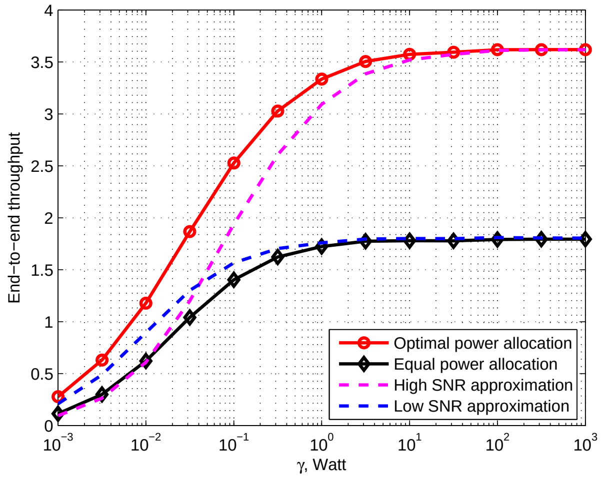

Fig. 3. End-to-end throughput for different power allocation strategies.

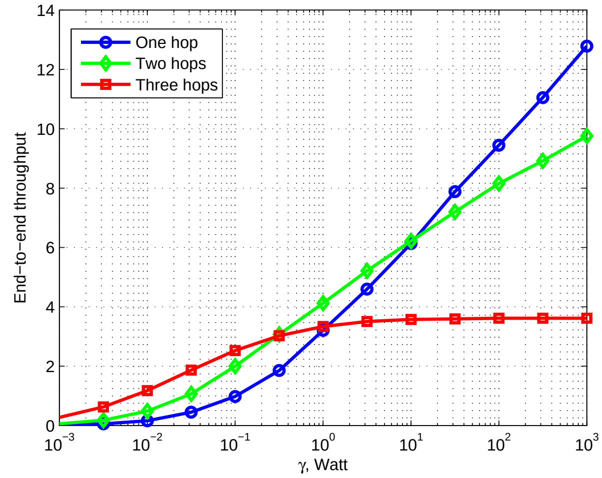

Fig. 4. End-to-end throughput for different number of hops.

V. Numerical Results

In this section we present numerical results for the 3-hop network with GOI illustrated in Fig. 2. The received power is subject to path-loss with path-loss exponent $\alpha = 4$, and channel gains are modeled as i.i.d. complex Gaussian entries, $\mathcal{CN}(0, 1)$. The distance between the transmitter and the receiver is normalized to 1. The primary network is assumed to have similar line topology with $M = 4$ receivers. The distance between the primary network and the CRN is similarly set to 1. The noise variance is chosen to be $\sigma^2 = 1$ mW. Backward channel gain is modeled as $g_{32} = Gg_{23}$, where antenna directivity attenuation G is set to 0.2.

We note that the tolerable ASI threshold γ is linearly related to the transmission power in (3). Since we consider the interference-limited networks, we use γ as x-label for plots. For instance, Fig. 3 shows the performance of the optimal power allocation found by solving (9) as well as high- and low-SNR approximation solutions (12) and (15), respectively in comparison with equal power allocation. The approximations are much easier to solve. Low-SNR approximation is suitable for γ values less than 10 mW, while the High-SNR approximation tends to the optimal solution for large γ values. We note that the low-SNR and equal power solutions are sub-optimal in the high-SNR region due to neglecting GOI which becomes significant as the SNR increases.

Fig. 4 illustrates the system performance for different number of hops $K = 1, 2, 3$ when the distance between the source and the destination is fixed to $d = 3$, similarly to the previous case. We can observe that for small γ values 3-hop transmission performs better than the other strategies. This is because it splits the link into three regenerative channels with smaller pathloss components due to shorter distances between the transmitted and the receiver at each hop. However, having more hops, the system has more interfering nodes and therefore, it drops faster to the interference-limited mode. This explains the behavior of curves for large γ values. Direct transmission from the source to the destination in this case is not affected by any interference and thus, outperforms transmission strategies with multiple hops.

VI. Conclusions

In this paper we have studied multi-hop cognitive radio networks with arbitrary overhearing interference among nodes at each hop. When the accumulated interference from the secondary network to the primary network is limited, the CRN transmit powers can be optimally allocated among hops in order to maximize the end-to-end throughput. We have shown that maximum end-to-end throughput of a tandem network is achieved when all intermediate link capacities are equated and then maximized, so that the maximum interference constraint is satisfied with equality. We have also presented high- and low-SNR approximation solutions that outperform equal power allocation and are good approximations in corresponding SNR regions.

Appendix A
Proof of Theorem 1

First, we rewrite problem (4) in epigraph form

$$\begin{aligned} \min \quad & -t \\ \text{s.t.} \quad & h_1 = t - C_1 \le 0 \\ & \vdots \\ & h_K = t - C_K \le 0 \\ & h_{K+1} = \sum_{i=1}^{K} \varphi_i P_i - \gamma \le 0. \end{aligned} \tag{16}$$

We now show that for the optimization problem (16) strong duality holds and therefore, primal and dual optimal variables must satisfy the KKT conditions [11]. To do so, we show that (16) satisfies the *linear independent constraint qualification* (LICQ) conditions [12] and hence, the duality gap of this problem is zero.

Constraints $h_j,\ j = 1, ..., K+1$ of problem (16) are said to satisfy the LICQ conditions if matrix

$$\mathbf{D}h = [\nabla_{P^*} h_1, ..., \nabla_{P^*} h_{K+1}] \tag{17}$$

has full column rank, i.e., the gradients of the active constrains are linearly independent at the optimal point $(P_1^*, ..., P_K^*)$.

The matrix of gradients has the following form

$$\mathbf{D}h = \begin{bmatrix} -A_{11} & B_{12} & \cdots & B_{1,L_b} & 0 & 0 & \cdots & 0 & \varphi_1 \\ 0 & -A_{22} & B_{23} & \cdots & B_{2,1+L_b} & 0 & \cdots & 0 & \varphi_2 \\ B_{31} & 0 & -A_{33} & B_{34} & \cdots & B_{3,2+L_b} & \cdots & 0 & \varphi_3 \\ \vdots & & & & & & \ddots & & \vdots \\ 0 & \cdots & 0 & B_{K,K-1-L_f} & \cdots & B_{K,K-2} & 0 & -A_{K,K} & \varphi_K \\ 1 & 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 0 \end{bmatrix}, \quad (18)$$

where

$$A_{i,i} = \frac{g_{i,i+1}}{(\sigma^2 + \sum_{\substack{k=j-L_f \\ k \neq i+1}}^{j+L_b} g_{k,i+1} P_k) \ln 2}, \quad (19a)$$

$$B_{i,j} = \frac{P_j g_{i,j+1} g_{j,j+1}}{(\sigma^2 + \sum_{\substack{k=j-L_f \\ k \neq j+1}}^{j+L_b} g_{k,j+1} P_k)(\sigma^2 + \sum_{\substack{k=j-L_f \\ k \neq j \\ k \neq j+1}}^{j+L_b} g_{k,j+1} P_k) \ln 2}. \quad (19b)$$

Since $g_{i,j}, \forall i, j$ are positive, non-zero with high probability and i.i.d., σ^2 and $P_i, \forall i$ are strictly positive, then $A_{i,j}$ and $B_{i,j}$ are also independent and strictly positive with high probability. Therefore, for a set of optimal powers $(P_1^*, ..., P_K^*)$ matrix $\mathbf{D}h$ in (18) has full rank with high probability and hence, the LICQ conditions are satisfied. Thus, the KKT conditions are necessary conditions for a solution to be optimal.

We next show that the KKT conditions for (16) have a unique feasible solution only when all link capacities are equal and then maximized, such that ASI constraint (3) is met with equality.

The Lagrangian for (16) can be written as

$$L = -t + \sum_{i=1}^{K} \lambda_i (t - C_i) + \omega (\sum_{i=1}^{K} \varphi_i P_i - \gamma). \quad (20)$$

From (20) we can write down the KKT conditions for a solution to be optimal

$$\frac{\partial L}{\partial P_i} = 0, \ i = 1, ..., K, \quad (21a)$$

$$\frac{\partial L}{\partial t} = 0, \quad (21b)$$

$$h_j \leq 0, \ j = 1, ..., K+1, \quad (22)$$

$$\lambda_j \geq 0, \ \omega \geq 0, \quad (23)$$

$$\lambda_j h_j = 0, \ j = 1, ..., K+1 \quad (24a)$$

$$\omega h_{K+1} = 0. \quad (24b)$$

Taking partial derivatives from (20) we get the optimality conditions (21) of form

$$\frac{\partial L}{\partial \beta_i} = \omega - A_{i,i} \lambda_i + \sum_{j=i-L_f-1}^{i+L_b} B_{i,j} \lambda_j = 0, \quad (25a)$$

$$\frac{\partial L}{\partial t} = -1 + \sum_{i=1}^{K} \lambda_i = 0, \quad (25b)$$

where $A_{i,i}$ and $B_{i,j}$ are the same as in (19).

We have only one negative component in each of the K equations (25a) and hence, all λ_i's have to be strictly positive in order to satisfy $\frac{\partial L}{\partial \beta_i} = 0 \ \forall i$. Otherwise, any $\lambda_i = 0$ will immediately lead to $\lambda_j = 0, \ \forall j \neq i$ and hence, (25b) will not be satisfied.

Finally, if $\omega = 0$, we will get a linear system of equations

$$\mathbf{M}_{K+1,K+1}(\mathbf{D}h) \lambda = \mathbf{0} \quad (26)$$

where $\lambda = [\lambda_1, ..., \lambda_K]^T$, $\mathbf{M}_{K+1,K+1}(\mathbf{D}h)$ is the minor to the entry $(K+1, K+1)$ of matrix $\mathbf{D}h$ in (18). Since $\mathbf{D}h$ has full column rank, the system (26) has a single trivial solution $\lambda = \mathbf{0}$ which does not satisfy (25b) and hence is not feasible.

All this leads us to the conclusion that the KKT conditions have a unique optimal solution corresponding to $C_1 = ... = C_K$ and $\sum_{i=1}^{K} \varphi_i P_i = \gamma$. Thus, the end-to-end throughput of a tandem network with the ASI constraint (3) is maximized if and only if all the link capacities are equal and constraint (3) is satisfied with equality.

Acknowledgement

The authors thank Hamed Farhadi for useful comments.

References

[1] J. Mitola III and G. Maguire Jr., "Cognitive radio: Making software radios more personal," *IEEE Personal Commun.*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[2] I. Akyildiz, W.-Y. Lee, M. Vuran, and S. Mohanty, "A survey on spectrum management in cognitive radio networks," *IEEE Commun. Magazine*, vol. 46, no. 4, pp. 40–48, Apr. 2008.

[3] H.-P. Shiang and M. van der Schaar, "Distributed resource management in multihop cognitive radio networks for delay-sensitive transmission," *IEEE Trans. Vehicular Tech.*, vol. 58, no.2, pp. 941–953, Feb. 2009.

[4] M. Hasna and M.-S. Alouini, "Optimal power allocation for relayed transmissions over Rayleigh fading channels," *IEEE Vehicular Tech. Conf.*, pp. 2461–2465, Jeju, Korea, Apr. 2003.

[5] Z. Qi, Z. Jingmei, S. Chunju, W. Ying, Z. Ping, and H. Rong, "Power allocation for regenerative relay channel with Rayleigh fading," *IEEE Vehicular Tech. Conf.*, pp. 1167–1171, Milan, Italy, May 2004.

[6] M. Dohler, A. Gkelias, and H. Aghvami, "Resource allocation for FDMA-based regenerative multihop links," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1989–1993, Nov. 2004.

[7] S. Jing, Z. Zhao-Yang, Q. Pei-Liang, and Y. Guan-Ding, "Subcarrier and power allocation for OFDMA-based regenerative multi-hop links," *Int. Conf. on Wireless Commun., Networking and Mob. Comput.*, pp. 207–210, Maui, Hawaii, Sep. 2005.

[8] L. Ong and M. Motani, "Myopic coding in multi-terminal networks," *IEEE Trans. Inf. Theory*, vol. 54, no. 7, pp. 3295–3314, Jul. 2008.

[9] Y. Wang and J. Garcia-Luna-Aceves, "Spatial reuse and collision avoidance in ad hoc networks with directional antennas," *IEEE Global Conf. Telecommun.*, Taipei, Taiwan, Nov. 2002.

[10] P. Elias, A. Feinstein, and C. Shannon, "A note on the maximum flow through a network," *IRE Trans. Inf. Theory*, vol. 2, no. 4, pp. 117–119, Dec. 1956.

[11] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, Mar. 2004.

[12] R. Henrion, "On constraint qualifications," *Journal of Opt. Th. and App.*, vol. 72, pp. 187–197, 1992.