

# Distributed Multichannel Random Access Networks with Selfish Users

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**Abstract**—Dynamic spectrum access (DSA) schemes allow the users to share spectrum resources by taking advantage of the variations in spectrum demand over time and space. Carrying out dynamic spectrum allocation centrally, however, can be a complex task. For this reason, distributed schemes in which users can access the available channels independently may be preferable to centralized DSA schemes. Cognitive radio systems, which enable user terminals to sense their environment and form their action accordingly, are particularly well-suited for distributed systems. On the other hand, the freedom in distributed schemes gives the users the option to act selfishly, which has decisive effects on system performance. In this paper we consider a distributed multichannel wireless random access system where users selfishly access the channels in the system. We analyze the behavior of the selfish users by modeling the system as a non-cooperative game and we identify all stable operating points (Nash equilibria) of this game. We then compare the performance of this system with a number of cooperative distributed DSA schemes in terms of user utilities. Our results show that the performance of the selfish multichannel random access system can be comparable to cooperative schemes.

## I. INTRODUCTION

Dynamic spectrum access (DSA) schemes are gaining importance with the emergence of cognitive radio and “smart” radio concepts. Such sophisticated radio equipment capable of accessing different segments of spectrum along with narrow-band modulation capabilities make various forms of spectrum sharing feasible [1]. For instance, reusing temporarily unused spectrum, so-called *whitespace*, in the form of an overlay system has been proposed in literature.

Performing dynamic spectrum allocation centrally in such systems would be overwhelmingly complex and possibly NP-hard. Problems with system complexity can be avoided through distributed spectrum sharing mechanisms in which users access the medium either by following set rules or at their own discretion. These distributed mechanisms can be cooperative, where nodes try to collectively maximize the system performance, or they can be non-cooperative, where the nodes selfishly try to maximize their own benefits only. The study of cooperative random access systems dates back to the 70’s with the ALOHA systems as most popular representatives. These schemes are analytically tractable, and despite their simplicity they capture many of the characteristics of wireless random access systems.

Studies regarding random access systems where the users are selfish have so far been confined to single-channel settings

[2]–[5]. In [2], [3] the authors analyze a single channel ALOHA system with complete information using game theory and derive the Nash equilibria. Under *complete information* assumption, every user knows the channel conditions and associated transmission costs for all of the users in the system. In [4], [5] the authors consider a single channel ALOHA system with packet capture under incomplete information assumption, that is, a user has exact knowledge of only its own channel conditions and it has solely statistical knowledge about the channel conditions of the other users. They show that users’ best response is a threshold strategy where users transmit on the channel if their pathgain is above a certain level.

One extension of this concept is the ALOHA scheme with multiple channels, which better captures the fundamental properties of DSA. In [6] a Markovian analysis of such a multichannel ALOHA system was given where throughput and stability of the system is considered.

In this paper we extend the previous work on selfish random access systems to a multi-channel random access (MRA) system in which users have complete information. We first formulate the selfish MRA system as a non-cooperative game and analytically calculate the transmission strategies at all possible Nash equilibria in the system. Then we compare the performance of the selfish MRA system with three cooperative systems in terms of utilities: a cooperative scheduling system which tries to maximize the sum utility in the system, a cooperative system with fairness as its objective, and a “classic” multichannel ALOHA system.

## II. SYSTEM MODEL AND ASSUMPTIONS

In this work we compare a selfish MRA system with three cooperative MRA systems, namely a scheduling system, a maxmin-fair system and a multichannel ALOHA system. It is important to note that all of the four systems considered here are distributed systems, that is, there is no central entity in the systems which determine users’ actions. Users decide on their actions in each time slot on their own. Another property common to all systems is that the users, whether they are selfish or cooperative, are rational; they try to maximize their own utilities or the sum of utilities in the system respectively using the information available to them. These users can be communicating with a common access point or each user

can be communicating with its respective receiver, which are within the same collision domain.

In the *selfish MRA system*, users have complete information about the system, which means that every user knows the channel conditions of every other user. Because the users in this system are selfish, they act to maximize their individual utilities without any regard to the utilities of other users.

Although complete information is difficult to implement in practice, we make this assumption to obtain a bound on system performance. We analyze a similar MRA system with selfish users under the assumption that users have incomplete information about the system in [7].

In the *cooperative scheduling system* users have complete information of the channel conditions, similar to the users in the selfish system. But unlike the selfish system, the users in the cooperative scheduling system aim to maximize the sum utility of the system in a given time slot. Thus, using the information they have, each user calculates by itself the transmission schedule which will maximize the sum utility in the system for that time slot, and each user acts upon this schedule in that time slot. Since the cooperative scheduling system maximizes the sum utility, the performance of this system will indicate the upper bound of the performance that can be achieved by cooperative systems.

The third system we consider in our comparison, the *maxmin-fair system*, is also a cooperative system like the scheduling system, and the users of the maxmin-fair system too have complete information about all the channel conditions. Unlike the scheduling system, the users in the maxmin-fair system calculate their actions in order to increase the utility of the worst-off user as much as possible. We consider this system to see the effect of fairness in system performance.

The final distributed MRA system in our comparison is the *multichannel ALOHA system*. The users in this system are also cooperative, and using the number of available channels and the number of users in the system they calculate the transmission probability which will maximize throughput of the system. In contrast to all three systems mentioned so far, users in the multichannel ALOHA system do not take into account the channel conditions in the system when they choose their actions but transmit with a fixed probability. The reason to consider the multichannel ALOHA system is to observe the performance degradation in distributed systems when the users do not use or do not have information about the channel conditions.

To simplify the analysis of the multichannel random access systems we assume time is divided into slots and when two or more users transmit on the same channel all of the colliding packets are lost. We also assume that users have full buffers, so they always have a packet to transmit.

The metric we use to compare the performance of the MRA systems we have mentioned is *user utility*. In general terms, the utility associated with an action is the gain or loss that the user experiences by following that action. We base our definition of utility on the model proposed in [4], which defines utility as a function of both the throughput that the user obtains and the energy that it spends in the

transmission. When a user  $n$  transmits a packet on channel  $k$  and it is successfully received, the user enjoys a benefit from this transmission. Obviously, the user obtains no benefit if the transmission fails. Though every time the user transmits a packet, it spends some energy irrespective of the outcome of the transmission, therefore an energy cost  $e_{nk}$  is associated with every transmission attempt. We normalize the benefit of successful transmission to the maximum transmission cost that the user can afford. Let  $T_{nk}(\mathbf{S})$  denote the probability that user  $n$  successfully transmits a packet on channel  $k$ , where  $\mathbf{S}$  is a vector that denotes the actions of all the users in the system. Then we can express the expected utility that user  $n$  obtains from transmitting on channel  $k$  as the following:

$$\begin{aligned} U_{nk}(\mathbf{S}) &= T_{nk}(\mathbf{S})(1 - e_{nk}) + (1 - T_{nk}(\mathbf{S}))(-e_{nk}) \\ &= T_{nk}(\mathbf{S}) - e_{nk} \end{aligned} \quad (1)$$

In this model we assume that the  $e_{nk}$  are a function of the user's transmission power and they vary in each timeslot according to the pathgain of the user. In comparison, the transmission costs in [4] are the same for all users and do not change from one time slot to another.

In this work we focus on average sum utilities and average user utilities when comparing the performance of the four MRA systems we mentioned. Sum utility reflects the efficiency in utilization of resources, therefore it indicates the overall performance of the system. On the other hand, user utility indicates the utility that an individual user obtains from the system, so it is a measure of the system performance from the user's point of view.

In a given time slot, the selfish MRA system may be at one of many possible operating points (i.e. set of actions taken by the users). For this reason we use distributions of the sum utilities and user utilities when comparing the performance of the four MRA systems in question.

It is important to note that  $e_{nk}$  associated with the transmission of a packet represents the perceived expense of this transmission by the user. This cost can be a function of the propagation loss associated with the channel conditions, remaining battery capacity of the terminal, or a similar measurement. Therefore, this quantity depends on the user preferences and there may be different ways to model this cost. In this work, we define the normalized transmission cost  $e_{nk}$  in a given slot as the ratio of the transmit power employed by the user in that slot ( $P_t$ ) to the maximum transmit power that the user can afford ( $P_{\max}$ ), which we assume to be common for all users. Since capture cannot occur, users transmit their packets at a power level which is just enough to satisfy the SNR requirement at the receiver to minimize their transmission costs:

$$P_r = P_0 = \frac{P_t}{r^\alpha} \cdot S \cdot R \cdot c \Rightarrow P_t = \frac{P_0 \cdot r^\alpha}{S \cdot R \cdot c} \quad (2)$$

where  $r$  is the distance of the user to the receiver,  $\alpha$  is the pathloss exponent, the shadow fading component  $S$  is a lognormally distributed random variable with unit mean and standard deviation  $\sigma$ , the fast-fading component  $R$  is an exponentially distributed random variable with unit mean,

and  $c$  is a scaling factor. The required received power for successful reception in the absence of interference ( $P_0$ ) can also be expressed as the average received power of a user who is located at the cell border, who is transmitting at maximum power:

$$P_0 = \frac{P_{\max}}{r_0^\alpha} \cdot c \Rightarrow P_{\max} = \frac{P_0 \cdot r_0^\alpha}{c} \quad (3)$$

where  $r_0$  is the nominal cell radius and  $S = R = 1$  because we consider average received power in this case. Using (2) and (3) we can write the transmission cost as:

$$e_{nk} = \frac{P_t}{P_{\max}} = \frac{D}{S \cdot R} \quad (4)$$

where  $D = (r/r_0)^\alpha$  is the distance dependent component of the transmission cost, and to approximate an urban environment we assumed  $\alpha = 3$  and  $\sigma = 4$ .

We assume that the distance dependent component  $D$  of pathloss and shadow fading  $S$  are independent of the carrier frequency, so in a given realization a user experiences the same  $D$  and  $S$  across the  $K$  channels in the system. Nevertheless  $R$  will be different because we assume that the  $K$  channels in the system, whether they come from adjacent bands or separate bands, will display different propagation characteristics due to fast-fading. We also assume that channel coherence time is longer than the slot duration so that fast-fading component is essentially constant within a slot. Shadow fading components of each user are also uncorrelated in our assumptions and likewise fast fading components are uncorrelated for each user and channel.

### III. GAME THEORETIC ANALYSIS OF SELFISH RANDOM ACCESS

In this section we define the multichannel random access game (MRAG) with complete information and characterize all of the Nash equilibria of the game. For the purpose of the following analysis, having complete information essentially means that a user knows all  $e_{nk}$  in the system.

Let  $n \in \mathcal{N}$  denote a user in the set of all users in the system with  $|\mathcal{N}| = N$  and similarly let  $k \in \mathcal{K}$  denote a channel in the set of all channels in the system with  $|\mathcal{K}| = K$ . The strategic form of the multichannel random access game is then the following:

- **Players:** The players of the MRAG are the users in  $\mathcal{N}$ .
- **Strategies:** In MRAG, user  $n$  transmits a packet on channel  $k$  with probability  $p_{nk}$ . The strategies of the  $N$  users are their transmission probabilities  $p_{nk}$  on channels in  $\mathcal{K}$ . The strategy employed by user  $n$  can be denoted in vector form as  $\mathbf{S}_n = (p_{n1}, p_{n2}, \dots, p_{nK}, \dots, p_{nK})$  where  $0 \leq p_{nk} \leq 1$ . The probability that user  $n$  chooses to wait in a slot is denoted by  $p_{n0} = 1 - \sum_{k=1}^K p_{nk}$  because the probabilities of all strategies of a user should add up to 1. We can represent the set of strategies that all the users employ (i.e. the strategy profile) as  $\mathbf{S} = (\mathbf{S}_1, \mathbf{S}_2, \dots, \mathbf{S}_n, \dots, \mathbf{S}_N)^\top$ .
- **Utilities:** When user  $n$  transmits a packet on channel  $k$ , it incurs a normalized transmission cost of  $e_{nk}$ . If the

transmission is successful, it also gains a normalized utility of 1. The probability that user  $n$  successfully transmits a packet on channel  $k$  is the the probability that no other user transmits on channel  $k$  in the same slot, which is given by

$$T_{nk}(\mathbf{S}) = \prod_{\substack{i=1 \\ i \neq n}}^N (1 - p_{ik}) \quad (5)$$

Then, a user's possible actions and its respective utilities are the following:

Actions	Utilities
Transmission	$T_{nk}(\mathbf{S}) - e_{nk}$
Wait	0

The overall utility that user  $n$  obtains from its strategy  $\mathbf{S}_n$  can be calculated as:

$$U_n(\mathbf{S}) = \sum_{k=1}^K \overbrace{p_{nk} (T_{nk}(\mathbf{S}) - e_{nk})}^{U_{nk}} \quad (6)$$

$$= \sum_{k=1}^K p_{nk} \left( \prod_{\substack{i=1 \\ i \neq n}}^K (1 - p_{ik}) - e_{nk} \right) \quad (7)$$

Note that in the user utility definition we could have imposed a cost for the action of waiting, like  $-w$ , however this would not alter the solution of the game, as mentioned in [8] and shown in [4].

We use the following notation throughout the analysis:

- $\mathcal{N}_m$  is the set of users which monopolize  $|\mathcal{N}_m|$  channels in the system. A *monopolizing user* is a user which transmits with probability 1 on a channel.  $|\mathcal{N}_m| = N_m \leq K$ .
- $\mathcal{K}_m$  is the set of monopolized channels, that is, the set of channels where monopolizing users transmit.  $|\mathcal{K}_m| = K_m = N_m \leq K$ .
- $\mathcal{K}_f = \mathcal{K} \setminus \mathcal{K}_m$  is the set of channels which are not monopolized by users in  $\mathcal{N}_m$ .
- $\mathcal{M}$  is the set of users whose strategies are probability mixes of the actions (i.e. transmit or wait). A user who employs a mixed strategy chooses both of its actions with positive probability, that is, for  $n \in \mathcal{M}$  and  $k \in \mathcal{K}$  we have  $0 < p_{n0} < 1$  and  $0 < \sum_{k=1}^N p_{nk} < 1$ .
- $\mathcal{X}_n$  is the set of channels where a mixing user  $n \in \mathcal{M}$  transmits.
- $\mathcal{X} = \bigcup_{i \in \mathcal{M}} \mathcal{X}_i$  is the set of all the channels where users are mixing their strategies.
- $\mathcal{M}_k$  is the set of mixing users that transmit on channel  $k \in \mathcal{X}$ .
- $\mathcal{N}_w = \mathcal{N} \setminus (\mathcal{M} \cup \mathcal{N}_m)$  is the set of users who do not transmit and wait in this slot.

In our analysis we treat the Nash equilibria in three cases: Nash equilibria in pure strategies, fully mixed Nash equilibria (FMNE) and partially mixed Nash equilibria (PMNE). For each of these cases we provide the necessary and sufficient conditions that make a strategy profile  $\mathbf{S}$  a Nash equilibrium of the MRAG.

### A. Nash Equilibria in Pure Strategies

Pure strategy Nash equilibria are the equilibria where all users take either one of the actions (transmit or wait) with probability 1.

*Theorem 1:* A strategy profile  $\mathbf{S}$  is a Nash equilibrium in pure strategies if and only if *all* of the following conditions are met:

- 1) If a user monopolizes a channel, then the monopolizing user is the only one to transmit on that channel.
- 2) User  $n \in \mathcal{N}_m$  who is monopolizing channel  $k$  has  $e_{nk} < 1$  and does not have lower transmission cost on any one of the non-monopolized channels. That is,  $e_{nk} \leq e_{nj}$ ,  $j \in \mathcal{K}_f$ .
- 3) Waiting users obtain negative utility from transmitting on any one of the channels. That is,  $T_{nk}(\mathbf{S}) - e_{nk} < 0$  for  $n \in \mathcal{N}_w$  and  $k \in \mathcal{K}_m$  and  $e_{nj} > 1$  for  $j \in \mathcal{K}_f$ .

*Proof:* The conditions given above can be derived by considering the definition of a Nash equilibrium, which states that at a Nash equilibrium no user can increase its utility unilaterally by following a strategy  $\mathbf{S}_n$  other than its equilibrium strategy  $\mathbf{S}_n^*$ . That is,  $U_n(\mathbf{S}_n, \mathbf{S}_{-n}) \leq U_n(\mathbf{S}_n^*, \mathbf{S}_{-n})$  [9]. For conciseness of presentation we refer the reader to [10] for the proof of this theorem. ■

### B. Fully Mixed Nash Equilibrium

A fully mixed Nash equilibrium (FMNE) occurs when all  $N$  users in the system transmit on all of the  $K$  channels with probability strictly between 0 and 1.

*Theorem 2:* In the MRAG with  $N$  users and  $K$  channels there can be one unique Nash equilibrium where all users employ nondegenerate strategies (the fully mixed Nash equilibrium) which is given by  $\mathbf{S}_n = (p_{n1}, \dots, p_{nk}, \dots, p_{nK})$  where

$$p_{nk} = 1 - \frac{\sqrt[N-1]{\prod_{i=1}^N e_{ik}}}{e_{nk}}. \quad (8)$$

In order for the FMNE be feasible, the following  $N(K+1)$  inequalities must be satisfied:

$$0 < \frac{\sqrt[N-1]{\prod_{i=1}^N e_{ik}}}{e_{nk}} < 1 \quad (9)$$

and

$$0 < \sum_{k=1}^K \frac{\sqrt[N-1]{\prod_{i=1}^N e_{ik}}}{e_{nk}} < 1 \quad (10)$$

for all  $n \in \mathcal{N}$  and  $k \in \mathcal{K}$ .

*Proof:* When a user's best response is to mix between its pure strategies, it will be indifferent to adopting either one of its pure strategies with probability 1. This is called the *indifference principle* [9].

In the FMNE, the expected utility that user  $n$  obtains from transmitting on channel  $k$  will be

$$U_{nk} = U_{nk}(\text{Transmit}) = U_{nk}(\text{Wait}) = 0 \quad (11)$$

due to the indifference principle. Using this reasoning the strategy profile  $\mathbf{S}$  defined by the equilibrium probabilities in (8) can be obtained. A detailed proof is given in [10]. ■

### C. Partially Mixed Nash Equilibria

In this section we consider the equilibria where there are one or more users who are mixing on some channels and there are one or more users monopolizing some channels.

*Theorem 3:* A strategy profile  $\mathbf{S}$  is a partially mixed Nash equilibrium if and only if *all* of the following conditions are met in addition to the conditions listed for pure strategy Nash equilibria.

- 1) For every  $k \in \mathcal{X}$  and for every  $n \in \mathcal{M}_k$ , the transmission probability is

$$p_{nk} = 1 - \frac{\sqrt{|\mathcal{M}_k|-1} \sqrt{\prod_{i \in \mathcal{M}_k} e_{ik}}}{e_{nk}} \quad (12)$$

- 2) For every  $k \in \mathcal{X}$  and for every  $m \in \mathcal{M} \setminus \mathcal{M}_k$  we have  $\prod_{n \in \mathcal{M}_k} (1 - p_{nk}) < e_{mk}$ .
- 3) If  $n \in \mathcal{N}_m$  is monopolizing on channel  $k$ , and  $l \in \mathcal{X}$  then  $T_{nl}(\mathbf{S}) - e_{nl} < 1 - e_{nk}$ .
- 4) If  $m \in \mathcal{N}_w$  then for all  $l \in \mathcal{X}$  user  $m$  has  $T_{ml}(\mathbf{S}) - e_{ml} < 0$ .

*Proof:* The partially mixed Nash equilibria are essentially a combination of pure strategy Nash equilibria and fully mixed Nash equilibria; therefore the strategy profiles that occur at these Nash equilibria can be obtained like in the previous two cases using the indifference principle and the principle that at the Nash equilibrium no user can improve its utility by unilaterally changing its strategy [10]. ■

## IV. SIMULATION RESULTS

In this section we present numerical results that compare the distribution of sum- and user-utilities of the selfish MRA system with the three cooperative MRA systems.

### Methodology

In our simulations we assumed  $N = 5$  and  $K = 4$  and performed snapshot simulations. In each snapshot we simulated one time slot of the system. We randomly placed the users across the service area and calculated their transmission costs ( $e_{nk}$ ). Using these transmission costs we calculated all possible operating points of the MRA systems (i.e. Nash equilibria) and then obtained a distribution from the sum- and user-utilities at these operating points.

### Results

Figure 1 shows the PDF of sum utilities of selfish MRA, cooperative scheduling, maxmin-fair and multichannel ALOHA systems that we consider. An interesting observation from this figure is the performance degradation caused by lack of information on channel conditions. The users in the multichannel ALOHA system transmit with probabilities which will maximize system throughput, which is  $p_{nk} = 1/(NK)$  in this system [6]. This transmit behavior leads to a distribution with a heavy tail towards negative utilities because the user utilities are not taken into consideration when determining the transmission probability in this system. For example, even though all users have very high transmission costs, they will still transmit at the fixed probability of  $1/(NK)$ . Therefore

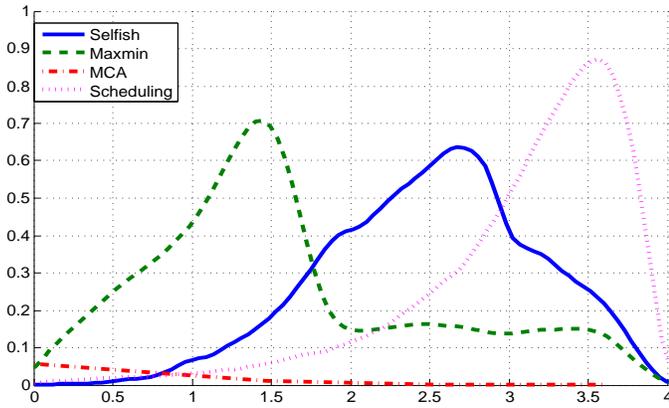


Fig. 1. PDF of sum utilities in selfish, maxmin-fair, cooperative scheduling and multichannel ALOHA systems with  $N=5$ ,  $K=4$ .

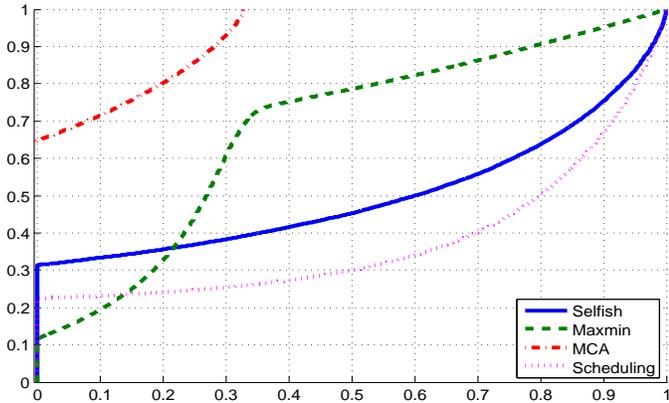


Fig. 2. CDF of individual utilities in selfish, maxmin-fair, cooperative scheduling and multichannel ALOHA systems with  $N=5$ ,  $K=4$ .

not considering the channel conditions in transmission strategy, or not knowing the channel conditions results in severe degradation in performance.

In terms of sum utilities, the selfish MRA system performs better than the maxmin-fair system. Although we know from the game theoretic analysis that in some Nash equilibria some users end up receiving zero utility, the occurrence of such equilibria where some channels are monopolized are much more frequent than mixed strategy equilibria, therefore sum utilities are very rarely close to zero in the selfish MRA system.

The cumulative distribution of individual utilities is presented in figure 2. An interesting note is that the median user in the selfish system receives more than double the utility of the median user in the maxmin-fair system. This is also due to the fact that, in the selfish system, Nash equilibria in which some users monopolize channels are much more likely to occur than the equilibria where no users monopolize any channel, as we discussed in the context of the sum utilities.

## V. DISCUSSION

In this paper we analyzed the multichannel random access system with selfish users by using game theory. We

modelled this system as a non-cooperative game and obtained all possible Nash equilibria of this game. We then performed simulations to compare the performance of the selfish multichannel random access system with scheduling, maxmin-fair and multichannel ALOHA cooperative systems. We found that the selfish system can perform comparably to the cooperative systems in question and knowledge of channel conditions improve system performance in terms of sum and individual user utilities.

In this work we assume that no user is able to "capture" the channel and in a collision, i.e. when two or more users transmit on the same channel all of the colliding packets are lost. This no-capture assumption can be valid when the variance of the received powers of all the users in the system is small such that no single user's received power can rise above the others to reach the SNR requirement for successful reception. On the other hand, if a user can achieve this requirement and can be captured, this opens up new and interesting ways of utilizing the channel more efficiently by adjusting transmit powers. As future work, we aim to extend our analysis by incorporating effects of power capture in our system model and also by investigating algorithms to bring the system to a desired operating point.

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## REFERENCES

- [1] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks*, vol. 50, no. 13, pp. 2127 – 2159, 2006.
- [2] H. Inaltekin and S. Wicker, "The analysis of nash equilibria of the one-shot random-access game for wireless networks and the behavior of selfish nodes," *Networking, IEEE/ACM Transactions on*, vol. 16, no. 5, pp. 1094–1107, Oct. 2008.
- [3] A. MacKenzie and S. Wicker, "Selfish users in aloha: A game-theoretic approach," in *IEEE VTS 54th Vehicular Technology Conference, 2001. VTC 2001 Fall*, vol. 3, 2001.
- [4] J. Hultell, "Cooperative and non-cooperative wireless access," Ph.D. dissertation, Royal Institute of Technology (KTH), Dec. 2008.
- [5] D. Wang, C. Comaniciu, H. Minn, and N. Al-Dhahir, "A game-theoretic approach for exploiting multiuser diversity in cooperative slotted aloha," *Wireless Communications, IEEE Transactions on*, vol. 7, no. 11, pp. 4215–4225, November 2008.
- [6] W. Szpankowski, "Packet switching in multiple radio channels: Analysis and stability of a random access system," *Computer Networks (1976)*, vol. 7, no. 1, pp. 17 – 26, 1983.
- [7] A. Ozyagci and J. Zander, "Distributed dynamic spectrum access in multichannel random access networks with selfish users," in *Wireless Communications and Networking Conference, 2010. WCNC 2010. IEEE*, 2010.
- [8] A. MacKenzie and S. Wicker, "Stability of multipacket slotted aloha with selfish users and perfect information," in *INFOCOM 2003. Twenty-Second Annual Joint Conference of the IEEE Computer and Communications Societies. IEEE Societies*, vol. 3, March-3 April 2003, pp. 1583–1590 vol.3.
- [9] D. Fudenberg and J. Tirole, *Game Theory*. MIT Press, October 1991.
- [10] A. Ozyagci, O. Ileri, and J. Zander, "Distributed multichannel random access networks with selfish users," Royal Institute of Technology, Internal Report TRITA-ICT-COS-1002, February 2010, <http://web.it.kth.se/~ozyagci/internalreport/dmranwsu2010.pdf>.