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Buckling of laser-welded sandwich panels. Part I: elastic buckling parallel to the webs

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Abstract

This is the first of three companion papers which examine the elastic buckling and collapse of laser-welded sandwich panels with an adhesively bonded core and uni-directional vertical webs. In compression parallel to the webs the face sheet in between two webs can be treated as a long rectangular plate supported at the sides and resting on a continuous elastic foundation. Using a generalised form of the Kirchhoff equation the plate buckling load is evaluated for a Winkler and Pasternak foundation, and for a foundation model where the core is characterised by an elastic half-space. Although the elastic half-space model is most sophisticated in the way that the decay of transverse stress through the core is described, it requires that the buckling load is calculated iteratively and is therefore not suitable for engineering applications. In contrast, the Pasternak foundation model is able to demonstrate the effect of different boundary conditions at the laser weld and different core properties by the variation of a simple clamping factor and two foundation moduli. By changing the clamping factor the model can be used to describe plate buckling as well as wrinkling, and by varying the foundation moduli it is possible to distinguish between symmetrical and anti-symmetrical buckling modes. Evaluation of the buckling solution shows that for low modulus core materials the plate buckling strength is determined only by the boundary conditions at the laser weld, while for high modulus materials anti-symmetrical wrinkling dominates. To improve the accuracy of the solution for non-standard configurations the foundation moduli are calibrated using the elastic half-space model and finite element results as a reference. It is found that by using a smaller Winkler foundation modulus the accuracy and consistency of the Pasternak model are improved considerably. However, the results also show that for current sandwich configurations the proportional limit of the face plate material is usually reached well before elastic buckling occurs. All experimental work is therefore delegated to the third paper, which deals with extension of the Pasternak foundation model into the elasto-plastic regime and non-linear finite element modeling.

Keywords: Sandwich; Core; Web; Laser welding; Buckling; Wrinkling; Foundation; Half-space; Finite elements

1. Introduction

In shipbuilding the replacement of stiffened plate structures by laser-welded sandwich panels with uni-directional webs and an adhesively bonded core (Fig.1) offers considerable advantages [1]. High stiffness and excellent surface flatness in particular make laser-welded sandwich panels well suited to span large open areas such as accommodation decks and hoistable car decks, or to be used as supporting bulkheads and partitioning walls. As a result of in-plane compressive loads and/or local bending moments these panels can buckle and ultimately collapse. Because they are relatively slender with thin plating and small closely-spaced stiffeners, in-plane and bending loads are predominantly carried by membrane compression and both elastic local buckling and overall (Euler) buckling are possible. While in overall or “interframe” buckling the panel collapses as a unit and can be treated as an orthotropic plate [2-4], the mechanisms involved in local buckling and collapse are distinctly more complicated.

In the simplest case, if the sandwich panel is left void or a fire retardant liner with negligible mechanical properties is used, the face plate is only supported at the laser weld and the buckling load can be obtained from standard reference works [5,6]. On the other hand, for panels with a high density PVC foam or lightweight concrete core short wavelength ‘wrinkling’ of the face plate will predominate and the boundary conditions at the laser weld are of little or no influence. Thus, it should be possible to determine the buckling load by sandwich wrinkling formulae as presented in [7,8]. In most other cases, e.g., if injected polyurethane foam is used, the face plate is stabilised by an elastic foundation as well as edge supports, and cross-over buckling modes are possible. In compression parallel to the webs the face plate can still wrinkle more or less naturally, but the rotational restraint at the supports will raise the corresponding elastic buckling load. In compression normal to the webs true wrinkling is no longer possible due to the fixed support-span, and the plate will ‘jump’ between successive buckling modes following the minimum buckling load.

Clearly, these are two distinctly different load cases and to come up with a unified solution or a simplified model that correctly predicts the elastic buckling load and the ultimate strength for both of these is well beyond the scope of a single paper. Instead, it was decided to adopt a three-paper strategy, where the first two papers are dedicated to modeling of the elastic buckling behaviour of the face plate parallel and normal to the webs

Fig. 1. In-plane compression of a laser-welded sandwich panel due to hull-girder bending.

respectively, whilst the third paper is used for extension of the buckling model(s) into the elasto-plastic regime, non-linear finite element modeling and experimental work. Although the decision to delegate all experimental work to the last paper was not made until it transpired that the elastic buckling stress frequently exceeds the proportional limit of the face plate material, this has helped to structure the division of work between the three papers considerably.

In compression parallel to the webs, which is the most practically relevant load case and the subject of this paper, it is assumed that the face plate can be treated as a long rectangular plate supported at the sides and resting on a continuous elastic foundation. Different methods for modeling the foundation are discussed and the corresponding buckling solutions are evaluated numerically for one set of sandwich scantlings, clamped boundary conditions at the laser weld, and core moduli representative of linear elastic materials up to solid Polyurethane elastomer. The predicted buckling loads are compared with linear finite element results and one buckling model is selected for further evaluation and discussion.

2. Analytical modeling

Consider a laser-welded sandwich panel in compression parallel to the webs (Fig. 2a), with face thickness t_f , core thickness t_c , distance between the centroids of the faces d , web thickness t_w , web-pitch $2p$, and elastic moduli E_f , E_w , and E_c (Fig. 2b). Without much simplification, the face plate in between two webs can be treated as a long, rectangular plate resting on a continuous elastic foundation and supported at the unloaded edges. For this particular load case a number of analysis methods are available, which can be identified by the way that the elastic foundation is characterised, i.e., by the way that the decay of transverse stress through the core is described. In order of sophistication these are the Winkler foundation (no shear lag), the two-parameter or Pasternak foundation (linear decay), and the elastic half-space model (exponential decay). In conjunction with this, an important modeling parameter is whether tension/compression and shear of the core are accounted for separately, and thus whether it is possible to distinguish between symmetrical and anti-symmetrical wrinkling (Fig. 3).

Fig. 2. Compressive buckling (a) and characteristic dimensions (b) of a laser-welded sandwich panel.

Fig. 3. Symmetrical and anti-symmetrical wrinkling (Winkler foundation model)

2.1. Winkler foundation approach

The simplest way to describe the behaviour of the core is the Winkler foundation model [8,9], which assumes that the core consists of an array of continuously distributed extensional springs with stiffness $2E_c/t_c$, provided that the faces buckle symmetrically with respect to the mid-plane (Fig. 3). In the anti-symmetrical case the springs remain unloaded even after buckling, and, as the core deforms in shear rather than tension/compression which the set of springs is unable to model, no solution can be derived. Because shear deformations of the core are completely neglected ($G_{cxz} \equiv 0$), the Winkler model becomes inadequate for higher modulus core materials and shorter wavelength deformations, where shear lag effects will be present.

For a uniaxially compressed Kirchhoff plate the buckling stress can be written as

$$\sigma_{cr} = k \frac{\pi^2 D_f}{b^2 t_f} \quad (1)$$

Here, D_f is the plate bending stiffness and the buckling coefficient k for a plate on a Winkler foundation and the foundation parameter f_W are defined as [10,11]

$$k = \left(\frac{mb}{a} + \frac{an^2}{mb} \right)^2 + f_W \left(\frac{a}{mb} \right)^2 \quad f_W = \frac{b^4 k_W}{\pi^4 D_f} \quad (2)$$

a and b are the length and width of the plate (Fig. 2a), m and n are the number of buckling half-waves in x - and y -direction respectively, k_W is the Winkler foundation stiffness mentioned earlier, and D_f is the bending stiffness of the face plate. By minimising k for m the critical buckling half-wavelength l_{cr} and the corresponding buckling coefficient k_{cr} become

$$l_{cr} = b / \sqrt[4]{n^4 + f_W} \quad k_{cr} = 2(n^2 + \sqrt{n^4 + f_W}) \quad (3)$$

If the plate is long and narrow, k_{cr} is smallest for $n=1$ (one buckle in transverse direction) or

$$l_{cr} = b / \sqrt[4]{1 + f_W} \quad k_{cr} = 2(1 + \sqrt{1 + f_W}) \quad (4)$$

which is the same result as obtained by Seide [12] and Ariman [13].

2.1.1. Sandwich wrinkling

If the side supports are removed altogether and the plate only buckles in direction of the applied load (cylindrical bending, $n=0$)

$$l_{cr} = b/\sqrt[4]{f_W} \quad k_{cr} = 2\sqrt{f_W} \quad (5)$$

which is identical to the two-dimensional buckling solution for an infinitely long bar on an elastic foundation derived by Hetenyi [9]. Substitution of (5) and (2) in (1) yields

$$\sigma_{cr} = \sqrt{\frac{2}{3} \frac{E_f E_c t_f}{t_c}} \approx 0.82 \sqrt{\frac{E_f E_c t_f}{t_c}} \quad (6)$$

which is also the first part of Hoff's sandwich wrinkling solution, that associated with out-of-plane extension of the core [14].

2.1.2. Boundary conditions

To evaluate the buckling coefficient for a plate with boundary conditions other than simply supported, which is not unrealistic for laser-welded joints, consider (3) once more. For an ordinary simply supported plate ($a/b \geq 4$) it is known that the buckling coefficient is equal to 4.0. Thus, with the foundation in (3) removed ($k_s=0$) k_{cr} should be equal to 4.0 and hence $n^2=1$. Similarly, for a plate with clamped longitudinal edges k_{cr} should equal 6.98, or $n^2 \sim 1.745$. Hence, it is argued that n^2 can be interpreted as a 'clamping' factor, representing the ratio of the actual buckling coefficient with respect to the coefficient for simple supports. As n^2 is also the square of the number of half-sine waves in transverse direction, this means that the wavelength $l_{cr} = b/n$ equals b and approx. $0.76b$ for the simply supported plate and the clamped plate respectively. As expected, the clamped plate buckles in comparatively short waves. Although this result is not exact ($l_{cr,exact} = 0.65b$) it allows for a useful and fairly reliable comparison of the buckling wavelength with, e.g., finite element results. Using the work by Lundquist and Stowell [15], the influence of intermediate degrees of rotational restraint along the unloaded edges of the plate can be evaluated in a similar fashion.

2.2. Two-parameter or Pasternak foundation approach

As mentioned before, the Winkler foundation approach neglects shear deformation of the core and becomes inadequate for higher modulus core materials and shorter wavelength deformations due to shear lag effects. Thus, a logical extension of the Winkler model is the Pasternak foundation model [16,17] which allows the springs to interact by the introduction of a second, shear foundation constant f_P .

Analogous to (2) the buckling coefficient k for a simply supported plate on a Pasternak foundation can be expressed as [10,11]

$$k = \left(\frac{mb}{a} + \frac{an^2}{mb} \right)^2 + f_W \left(\frac{a}{mb} \right)^2 + f_P \left(1 + \left(\frac{an}{mb} \right)^2 \right) \quad (7)$$

where the foundation parameter f_P and the foundation shear stiffness are defined as [11,18]

$$f_P = \frac{b^2 k_P}{\pi^2 D_f} \quad k_P = \frac{1}{6} G_c t_c \quad (8)$$

By minimising k for m as before the critical wavelength and buckling coefficient become

$$l_{cr} = b \sqrt[4]{n^4 + f_W + n^2 f_P} \quad k_{cr} = 2 \sqrt{n^4 + f_W + n^2 f_P + f_P + 2n^2} \quad (9)$$

2.2.1. Symmetrical wrinkling

If the plate only buckles in direction of the applied load ($n=0$) l_{cr} and k_{cr} reduce to

$$l_{cr} = b \sqrt[4]{f_W} \quad k_{cr} = 2 \sqrt{f_W + f_P} \quad (10)$$

By substituting (10), (8) and (2) into (1) the natural wrinkling wavelength and wrinkling stress become

$$l_{cr} = \frac{\pi}{\sqrt[4]{24}} \sqrt[4]{\frac{E_f}{E_c} t_f^3 t_c} \approx 1.424 \sqrt[4]{\frac{E_f}{E_c} t_f^3 t_c} \quad (11)$$

$$\sigma_{cr} = \sqrt{\frac{2}{3} \frac{E_f E_c t_f}{t_c}} + \frac{1}{6} \frac{G_c t_c}{t_f} \approx 0.82 \sqrt{\frac{E_f E_c t_f}{t_c}} + 0.166 \frac{G_c t_c}{t_f} \quad (12)$$

which now also includes the second, shear related part of Hoff's wrinkling solution [14]. It is important to remember that this buckling solution is for symmetrical wrinkling and relies on the premise that the core-zone affected by wrinkling has a depth $h=t_c/2$. If $h \leq t_c/2$, as can be the case for highly optimised sandwich configurations with thin or flexible facesheets and high-modulus core materials, (12) overestimates the wrinkling stress and the actual depth of the wrinkling zone has to be calculated. For the symmetrical wrinkling case the depth h is given by [14]

$$h = t_f \sqrt[3]{\frac{3}{4} \sqrt{\frac{E_f E_c}{G_c^2}}} \stackrel{\text{isotropic core}}{=} t_f \sqrt[3]{3(1+\nu_c)^2 \sqrt{\frac{E_f}{E_c}}} \approx 1.72 t_f \sqrt[3]{\frac{E_f}{E_c}} \quad (13)$$

For a sandwich panel with mild steel face plates ($E_f = 205$ GPa) and core materials with a compressive modulus up to 1000 MPa this means that the zone affected by wrinkling stretches a minimum distance of roughly $10 \cdot t_f$ into the core. As the face plates and the core are typically 2mm and 40-60mm thick respectively this distance will generally be close to $t_c/2$, and throughout this paper the depth h in symmetrical wrinkling is therefore assumed to be equal to $t_c/2$, unless stated otherwise.

2.2.2. Anti-symmetrical wrinkling

Using (7), anti-symmetrical wrinkling (Fig. 3) can be modeled in two straightforward ways; either by assuming the extensional springs remain unloaded and the shear springs are active

across the entire cross-section,

$$k_W = 0 \quad k_P = \frac{1}{3} G_c t_c \quad (14)$$

or by assuming the extensional springs are active side-by-side across the entire cross-section as well

$$k_W = \frac{E_c}{2t_c} \quad k_P = \frac{1}{3} G_c t_c \quad (15)$$

In the first case, k_{cr} equals f_P and the wrinkling stress becomes

$$\sigma_{cr} = \frac{1}{3} \frac{G_c t_c}{t_f} \quad (16)$$

and in the second case the wrinkling stress is

$$\sigma_{cr} = \sqrt{\frac{1}{6} \frac{E_f E_c t_f}{t_c}} + \frac{1}{3} \frac{G_c t_c}{t_f} \approx 0.41 \sqrt{\frac{E_f E_c t_f}{t_c}} + 0.33 \frac{G_c t_c}{t_f} \quad (17)$$

as compared to Hoff's anti-symmetrical wrinkling solution [14]

$$\sigma_{cr} = 0.59 \sqrt{\frac{E_f E_c t_f}{t_c}} + 0.387 \frac{G_c t_c}{t_f} \quad (18)$$

Although for anti-symmetrical wrinkling the Pasternak or two-parameter foundation model does not agree as well with Hoff's wrinkling solution as for symmetrical wrinkling, both (14) and (15) are plausible solutions and in the worst case merely constitute a conservative estimate of Hoff's wrinkling stress. In particular the simple assumption of two parallel extensional springs to model the foundation stiffness k_W of the core (15) matches surprisingly well with the foundation stiffness defined by Hoff for the anti-symmetrical wrinkling case with $h < t_c/2$

$$k_{Hoff} = \frac{2\pi - 3}{2\pi} \frac{E_c}{h} \approx 0.52 \frac{E_c}{h} \quad (19)$$

where

$$h = t_f \sqrt[3]{\frac{3\pi(2\pi - 3)}{2(\pi - 1)^2} \frac{E_f E_c}{G_c^2}} \stackrel{\text{isotropic core}}{=} t_f \sqrt[3]{\frac{6\pi(2\pi - 3)(1 + \nu_c)^2}{(\pi - 1)^2} \frac{E_f}{E_c}} \approx 2.83 t_f \sqrt[3]{\frac{E_f}{E_c}} \quad (20)$$

As the zone affected by wrinkling now stretches a minimum distance of roughly $17 \cdot t_f$ into the core for core moduli up to 1000 MPa, the assumption that the depth h is equal to $t_c/2$ is

realistic for the symmetric as well as anti-symmetrical wrinkling case.

An important conclusion that can be derived from (12), (17) and (18) is that for given sandwich scantlings and increasing core-modulus anti-symmetrical wrinkling will precede symmetrical wrinkling unless boundary conditions prescribe otherwise. Moreover, as the plate thickness decreases the cross-over from anti-symmetrical wrinkling to symmetrical wrinkling will occur at lower core-moduli, which could be relevant in weight optimisation.

2.2.3. Boundary conditions

By removing first the shear stiffness of the foundation ($f_p=0$) and subsequently the extensional stiffness of the foundation ($f_w=0$), k_{cr} in (9) reduces to (3) and finally to $4n^2$. Hence, the interpretation of n^2 as a clamping factor is valid here as well.

2.3. Elastic half-space model (exponential decay)

Although the Pasternak model accounts for extension and shear in the core, and in special cases yields results similar to those obtained by Hoff, the assumption of a linear decay of transverse deformation and stress through the core is known to give an increasingly conservative estimate of the buckling stress, as the foundation modulus rises. For this reason, shear lag problems are frequently described using an exponential decay function, assuming the stress waves in the core damp out exponentially away from the surface.

A buckling model for a simply supported plate on an elastic foundation where such a method of exponential decay is used is presented by Kech [19]. Analogous to (2) the buckling coefficient is herein given as

$$k = \left(\frac{mb}{a} + \frac{an^2}{mb} \right)^2 + f_H \left(\frac{a}{mb} \right)^2 \quad (21)$$

where the foundation parameter f_H and the ‘composite’ foundation stiffness k_{comp} are defined as [19,20]

$$f_H = \frac{b^4 k_{comp}}{\pi^4 D_f} \quad k_{comp} = K\lambda \quad (22)$$

Here, the parameters K and λ are

$$K = \frac{1}{f(\lambda, t_c)} \frac{2(1-\nu_c)E_c}{(1+\nu_c)(3-4\nu_c)} \quad \lambda = \sqrt{\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2} \quad (23)$$

and the correction factor $f(\lambda, t_c) \leq 1$ is derived in [20] by first solving the 3D stress-state in the core for the displacements u , v , and w , and by subsequently solving the *Lamé* elastic continuum equations for various boundary conditions.

2.3.1. Symmetrical wrinkling - 'thick' core

For the special case of (symmetrical) wrinkling of a wide plate ($b \rightarrow \infty$, $n=0$) on an infinitely thick core no correction is necessary ($f(\lambda, t_c)=1$) and the buckling stress can be calculated directly. By minimising k for m as before the critical wavelength and buckling coefficient become

$$l_{cr} = \pi \sqrt[3]{\frac{2D_f}{K}} \quad k_{cr} = \frac{3}{2} \left(\frac{b}{\pi} \right)^2 \sqrt[3]{\frac{2K^2}{D_f^2}} \quad (24)$$

Substituting for K and assuming the core is isotropic ($\nu_c=0.3$) the natural wavelength and wrinkling stress can be approximated by

$$l_{cr} = 1.28\pi \sqrt[3]{\frac{D_f}{E_c G_c}} \quad \sigma_{cr} = \frac{3}{2t_f} \sqrt[3]{2D_f K^2} \approx \frac{3}{2t_f} \sqrt[3]{1.86D_f E_c G_c^2} \quad (25)$$

which are virtually identical to the expressions derived by Plantema [21] for the two-dimensional wrinkling case. Assuming the face plate is isotropic as well ($\nu_f=0.3$) the wrinkling stress simply reduces to

$$\sigma_{cr} \approx 0.83 \sqrt[3]{E_f E_c G_c} \quad (26)$$

which is close to the result obtained by Hoff [14] for the symmetrical case and thick core, with the only difference that another decay function has been used. A similar result as in (26) was obtained by Allen [22] using a differential equation method.

2.3.2. Symmetrical wrinkling - 'thin' core

If the core has finite thickness this will have a stiffening effect on its response as an elastic foundation through the ratio E_c/t_c appearing in, e.g., (12) and (17), and $f(\lambda, t_c)$ will have to be determined iteratively.

For the case of symmetrical wrinkling $f(\lambda, t_c)$ is derived as [20]

$$f(\lambda, t_c) = \frac{\sinh^2 \varepsilon_1 - \varepsilon_2^2}{\varepsilon_2 + \sinh \varepsilon_1 \cosh \varepsilon_1} \quad \varepsilon_1, \varepsilon_2 \ll 1 \quad = \quad \frac{2(1-2\nu_c)}{3-4\nu_c} \frac{\lambda t_c}{2} - \frac{1-4\nu_c}{12(1-\nu_c)} \left(\frac{\lambda t_c}{2} \right)^3 \quad (27)$$

where $\varepsilon_1 = \lambda h$ and $\varepsilon_2 = \lambda h/(3-4\nu_c)$ and $h = t_c/2$ for symmetrical wrinkling. Introducing $\alpha = an/mb$ and the parameter χ

$$\chi = \frac{K}{2D_f} \left(\frac{b}{\pi m} \right)^3 \quad (28)$$

k in (21) can be minimised for α yielding the following equation which has to be solved iteratively.

$$\frac{1 - \alpha^4}{\alpha^3} = \chi \frac{1 + 2\alpha^2}{1 + \alpha^2} \quad (29)$$

By substitution of α and the desired clamping factor n^2 in (21) the buckling coefficient k_{cr} for the pure wrinkling case ($n=0$) and the plate buckling case ($n^2=1, 1.745, \dots$) can be calculated directly.

2.3.3. Anti-symmetrical wrinkling

For the anti-symmetrical wrinkling case $f(\lambda, t_c)$ is derived as [20]

$$f(\lambda, t_c) = \frac{(\varepsilon_2 + \sinh \varepsilon_1)(1 + \cosh \varepsilon_1)}{\sinh^2 \varepsilon_1} \stackrel{\varepsilon_1 \ll 1}{=} \frac{8(1 - \nu_c)}{3 - 4\nu_c} \frac{1}{\lambda t_c} + \frac{1 - 2\nu_c}{3(3 - 4\nu_c)} \lambda t_c \quad (30)$$

and the buckling coefficient k_{cr} is determined by same procedure as described for the symmetrical wrinkling case.

2.3.4. Approximate solution

Although the calculation described in 2.3.2. is fairly uninvolved, for practical purposes it is obviously more convenient to be able to calculate the buckling stress directly, even for sandwich configurations with particular boundary conditions and cores with finite thickness. To accomodate such a direct calculation, Hassinen [23] has presented an approximate solution for the wrinkling problems in 2.3.2. and 2.3.3. based on numerically solved points and subsequent curvefits. Thus, the approximate buckling coefficient for symmetrical wrinkling of a long, uniaxially compressed plate with simply supported longitudinal edges can be written as

$$k_{cr} = 0.703 R^2 - 0.415 R + 4.00 \quad R = \frac{b}{t} \left(\frac{E_c}{E_f} \right)^{1/3} \quad (R < 200) \quad (31)$$

Besides the simplicity of this approximate solution, which makes it an attractive solution for design rules and guidelines, an added advantage is that the contribution of the core (the first two terms) and of the boundary conditions at the sides of the plate (the last term) have been conveniently separated. Unfortunately, the drawback of curve-fitting is that it is generally only valid for a limited range of variable combinations, and the coefficients used in (31) for example only give a proper curvefit for sandwich cladding with thin face plates and light core materials as those used in building applications.

2.5. Numerical comparison and model selection

Having discussed various ways to describe the ‘supporting’ role of the core, how to account for its finite thickness in symmetrical and anti-symmetrical wrinkling, and how to evaluate the influence of different boundary conditions, it is convenient now to make a numerical comparison. Hence, the buckling coefficient has been calculated for one set of sandwich scantlings (Table 1), simply supported and clamped boundary conditions at the laser-weld, and generic core moduli representative of linear elastic isotropic material up to solid

Table 1
Sandwich scantlings (see Fig. 2b)

t_f	t_c	t_w	$2p$
2.0 mm	40.0 mm	4.0 mm	120.0 mm

polyurethane elastomer. To make for an even distribution of datapoints, 41 core moduli from 0.1 to 1000 MPa have been generated at equal spacing on a $\log(E_c)$ scale. The material used for the webs and the plating is mild steel ($E_{f,w} = 205$ GPa, $\nu_{f,w} = 0.3$) and a perfect bond between the core and the plating is assumed. The results for the symmetrical and anti-symmetrical wrinkling case are presented in Figs. 6 and 7 respectively. A direct comparison between the symmetrical and anti-symmetrical wrinkling case is presented in Fig. 8.

As shown in Fig. 6, the buckling coefficient calculated from (2), (7) and (21) is approximately equal to the bare plate buckling coefficient for low modulus core materials and for high modulus materials both solutions converge toward symmetrical wrinkling; the Pasternak foundation model converging toward Hoff's wrinkling solution and the Winkler foundation model converging toward Hetenyi's beam solution. Clearly visible is also that the Winkler model yields increasingly conservative results for higher modulus core materials, as shear deformation of the core is not accounted for. More importantly, the agreement of the Pasternak model and the elastic half-space model in Figs. 6 and 7 underlines the significance of taking into account the finite thickness of the core, so as not to underestimate the buckling stress. This is supported by the significant difference between the anti-symmetrical buckling coefficient for a complete two-parameter foundation (15) and a foundation with zero extensional stiffness and shear stiffness only (14). As can be seen from Fig. 8 and similar parameter studies the critical buckling mode for standard laser-welded sandwich configurations in compression will always be anti-symmetrical wrinkling.

In general, there is surprisingly good agreement between the Pasternak foundation model and the elastic half-space model, and the fact that the Pasternak model can be evaluated directly, without iteration, obviously speaks in its favour. Hence, the Pasternak foundation model is selected as a basis for a simplified buckling rule, as it is physically robust and able to demonstrate the effect of core property variations and changing boundary conditions, by the simple variation of a clamping factor and two foundation parameters. Moreover, it allows direct calculation of the critical buckling load as a function of those parameters, without iteration, and can easily be adapted to model various buckling and wrinkling modes in compression parallel and normal to the webs. The possibility for direct calculation in particular is an advantage over the elastic half-space model, although in extreme cases it may not be as accurate. For reference purposes the elastic half-space model is therefore taken along for further examination by finite elements.

2.6. Global buckling

Many of the existing rules and guidelines for classification of marine structures are based on the principle that elastic local buckling is accepted, provided that this does not jeopardise overall stability. For redundancy a global buckling check should therefore be performed using, for instance, the buckling solution for an orthotropic simply-supported sandwich plate

Fig. 6. Buckling coefficient for a uniaxially compressed plate on elastic foundation with simply supported longitudinal edges (symmetrical wrinkling).

Fig. 7. Buckling coefficient for a uniaxially compressed plate on elastic foundation with simply supported longitudinal edges (anti-symmetrical wrinkling).

Fig. 8. Buckling coefficient for a uniaxially compressed plate on an elastic half-space with simply supported longitudinal edges (symmetrical and anti-symmetrical wrinkling).

provided by Robinson [24]. For calculating the global buckling load of a sandwich panel with an adhesively bonded core and uni-directional vertical webs the required equivalent stiffness parameters D_x , D_y , D_{xy} , S_x , S_y , ν_{xy} and ν_{yx} have been derived in [1].

3. Finite element model

In the finite element modeling of wrinkling problems the size of the FE-model is typically related to the way that the critical buckling pattern and the lowest buckling eigenvalue are determined [25,26]. One possibility is to use a model with large dimensions which are a multiple of the expected critical wavelength, but in 3D problems this approach can rapidly become unpractical due to the amount of computation involved. Another possibility is to use

a smaller model and vary its dimensions, in order to minimize the calculated eigenvalue. In this case, the maximum critical wavelength is physically fixed by the distance between the webs, and a relatively small, periodic model can be constructed.

Looking at the compressive buckling coefficients for rectangular plates with various edge conditions as presented in, e.g., Timoshenko [6] it is evident that these are almost constant for plate aspect ratio $a/b \geq 4$. A model length of 4 times the web-pitch $2p$ is therefore considered sufficient. Furthermore, due to the bending stiffness of the face plates and the rotational restraint at the laser-weld an outward buckle on one side of a web is likely to be accompanied by an inward buckle on the other side, regardless of the applied load. Thus, at least two cells or a multiple thereof have to be modeled, if periodicity is to be maintained. And last, while it is sufficient to consider only one half of the sandwich for symmetrical wrinkling, the possibility of anti-symmetrical wrinkling requires that both face plates are modeled.

Based on this, the commercial finite element code ABAQUS [27] has been used to construct a fully periodic 3D finite element model, representative of a sandwich plate element which is two 'cells' wide and four cells long (Fig. 9). As only bifurcation buckling is considered and the buckling mode shapes will be fairly smooth, the face plates and the webs have been modeled by 8-node, second-order shell elements with reduced integration (S8R). Compared to first-order elements, these elements are able to capture stress concentrations and model geometric features more effectively, and reduced integration generally yields more accurate results than the corresponding fully integrated elements. Above all, the use of reduced integration significantly reduces running time, especially in 3D problems, which is of considerable importance with regard to the 20-node brick elements (C3D20R) used to model the core.

Given the choice of elements and the advantage of reduced integration it was found that satisfactory convergence of the bifurcation load was achieved for a mesh containing 4 brick elements through the thickness, 6 brick elements between two webs, and 24 in length direction, giving an element aspect ratio of 2:2:1. This offered a convenient margin for shrinking or stretching of the model without the need for remeshing or the risk of exceeding an element aspect ratio of 4:4:1. As a result, it has been possible to automate much of the computational work and post-processing, and due to the rapid convergence of the buckling solution parameter studies could be performed almost as quickly as with the analytical model.

Part of the reason why the solution converges so rapidly is the way that boundary conditions have been applied. To achieve continuity without restraint periodic boundary conditions have been used to couple the displacement and/or rotation of opposite sides and edges, including the corner edges, while retaining the possibility for lateral contraction. Looking at Fig. 9 rigid body motion has been locked in the left hand corner, while the two corner nodes at the far right are used to 'drive' the model in compression parallel or normal to webs. In addition, a third node has been incorporated by which the middle web can be 'locked' to avoid global (shear) buckling in compression normal to the webs in particular.

As a matter of interest; although a laser-weld-bead in reality will rarely cover more than half the thickness of the web, by the way that the sandwich is modeled here the laser-weld constitutes a perfect T-joint using the full thickness of the shell elements. Unless special measures are taken to model the laser-weld, e.g., by using connector elements, coupling constraints or the spot-weld capability available in ABAQUS, its rotational stiffness will always be close to clamped. Hence, it is not only for practical purposes that it is convenient to

Fig. 9. Finite element model of periodic sandwich plate element with core removed [ABAQUS]

have the facility of a clamping factor, but also for the comparison of analytical and finite element results.

4. Results and discussion

As concluded earlier, for standard sandwich configurations in compression anti-symmetrical wrinkling will always be the critical buckling mode and both the Pasternak foundation model and the elastic half-space model are in close agreement on that. Due to its ability to demonstrate the effect of parameter variations by the simple variation of a clamping factor and two foundation parameters, the Pasternak foundation model is selected as a basis for a simplified buckling rule. On the other hand, the elastic half-space model is based on a more accurate description of the stress-state in the core and is generally more accurate, although it does not allow the buckling load to be calculated directly. Also, a considerable amount of computational effort and experience is required to perform parameter studies using finite elements, even for a relatively straightforward linear buckling problem as this. Thus, it is decided not to compare the Pasternak model to the finite element model directly, but to use finite element results to validate the elastic half-space model, and subsequently to use this to adjust the foundation parameters of the Pasternak model. If necessary, in case of drastic configuration changes this last step should be easy enough to repeat without advanced programming skills.

4.1. Validation of elastic half-space model with finite elements

A comparison between the elastic half-space model and finite element results for different thickness of the face plate is presented in Fig. 10, which shows that the half-space model accurately predicts the buckling stress for all types of filling material. The practical value of the clamping factor n^2 is demonstrated by the fact that for $t_f = 4\text{mm}$ the boundary conditions at the laser-weld are representative of an intermediate state in between clamped and simply supported (Fig. 11), and by adjusting the clamping factor accordingly ($n^2=1.375$) the half-space solution is still able to make an accurate prediction. To be sure, the presence of such an intermediate support condition has been verified by visual inspection of the deformed shape of the finite element model in the area surrounding the web-face joint. Visual inspection of the buckling patterns in Fig. 12 also shows that anti-symmetrical wrinkling is the critical buckling mode for all core materials, as predicted earlier.

Due to the ‘fixed’ and relatively short length of the face plate the measured half-wavelength does not follow the calculated wavelength as smoothly as the buckling stress does, and it is natural for the finite element model to lag behind until the wavelength becomes small. Nonetheless, if the wavelength calculated from (29) is used to determine the expected number of half-waves in the finite element model, the jump from 6 to 8 waves between $E_c \sim 40\text{MPa}$ and 50MPa is predicted correctly. Here, it must be remembered that because of periodicity only an even number of half-waves can be expected, and the number of waves is determined by dividing the length of the model over the calculated wavelength and rounding toward the nearest even integer above.

Similar comparisons between the half-space model and finite element results for different core thickness ($t_c = 20, 40, 60\text{mm}$) and different web-pitch ($2p = 120, 240, 360\text{mm}$) are presented in Figs. 13 and 14, with the same good agreement. These figures also show that for relatively slender sandwich configurations ($t_c = 20\text{mm}$, $2p = 240, 360\text{mm}$) and high modulus core materials global buckling becomes the critical buckling mode, where both Euler buckling and mixed local-shear buckling are a possibility (Fig. 15).

Based on these results it is concluded that the elastic half-space model is able to accurately predict the buckling stress for a variety of practically relevant sandwich configurations and core materials, and can be used to calibrate the more simplified Pasternak model, instead of finite elements.

Fig. 10. Buckling stress for a uniaxially compressed plate on elastic half-space with clamped or semi-clamped longitudinal edges (anti-symmetrical wrinkling).

Fig. 11. Boundary conditions at the laser-weld for different combinations of web and face plate thickness.

Fig. 12. Change in buckling mode shape from 6 to 8 half-waves – Standard configuration (Table 1), $E_c \sim 40\text{MPa}$ and 50MPa respectively.

Fig. 13. Buckling stress for a uniaxially compressed plate on elastic half space with clamped or semi-clamped longitudinal edges (anti-symm. wrinkling).

Fig. 14. Buckling stress for a uniaxially compressed plate on elastic half space with clamped or semi-clamped longitudinal edges (anti-symm. wrinkling).

Fig. 15. Euler and mixed local-shear buckling for slender sandwich panel and high modulus core material.

4.2. Calibration of Pasternak foundation model

Although in anti-symmetrical wrinkling the real foundation parameter k_w obviously has to lie in between the values calculated from (14) and (15), and k_s should not deviate too far from $\frac{1}{3}G_c t_c$, it may be necessary in some cases to adjust one or both foundation parameter(s) for better agreement with the elastic half-space model. Besides, by ‘calibrating’ the foundation parameters for different sandwich configurations it is possible to get an understanding of how

changes in geometry and/or material properties affect the buckling sensitivity. Thus, k_W and k_s can be rewritten as

$$k_W = E_c / a_1 t_c \quad k_s = a_2 G_c t_c \quad (32)$$

and based on a curvefit of the half-space solution for the same values of face plate thickness, core thickness and web-pitch as used before the constants a_1 and a_2 have been determined as 6.1 and 0.368 respectively. As expected a_2 is close to $1/3$ and as can be seen in Fig. 16, a good approximation of the half-space solution is obtained by using the foundation parameters

$$k_W \approx E_c / 6 t_c \quad k_s = 1/3 G_c t_c \quad (33)$$

For the sandwich panels covered by Fig. 16 and practical sandwich configurations in general these foundation parameters are almost exact or slightly conservative. The only exception are highly optimised sandwich configurations ($t_f \leq 1\text{mm}$, $t_c \geq 80\text{mm}$) for which these foundation parameters are increasingly non-conservative with rising core modulus, and recalibration would be required. Nonetheless, it is recommended that if the Pasternak foundation model is accepted as a basis for a simplified buckling rule, it is done using the foundation parameters in (33).

4.3. Practical note

As mentioned in the introduction, elastic local buckling of marine structures can be allowed and exploited, provided that the corresponding buckling load and ultimate strength can be calculated reasonably accurately. In this paper two elastic buckling models - one exact and one simplified - for the case of compression parallel to the webs have been examined and validated and the first issue has been partially resolved. However, one look at the graphs presented throughout this paper also shows that in many cases the predicted buckling stress is far from elastic and even above the proportional limit of high-strength steel. Thus, an

Fig. 16. Buckling stress for a uniaxially compressed plate on elastic half space with clamped or semi-clamped longitudinal edges (anti-symmetrical wrinkling).

extension of the selected buckling models into the elasto-plastic regime is required and hence also the decision to delegate all experimental work to the last paper.

6. Summary and conclusions

In this paper the elastic buckling of laser-welded sandwich panels with an adhesively bonded core and uni-directional vertical webs has been investigated. In compression parallel to the webs, the face plate in between two webs can be treated as a long, rectangular plate resting on a continuous elastic foundation and supported at the unloaded edges. A Pasternak foundation model is selected as a basis for a simplified buckling rule, due to its ability to demonstrate the effect of parameter variations by the simple changing of a clamping factor and two foundation parameters. To calibrate these foundation parameters for new sandwich configurations, a second buckling model is provided in which the foundation is described by an elastic half-space and the finite thickness of the core is accounted for. This second model accurately predicts the buckling stress when compared to finite element results, and is used to evaluate the foundation parameters for a broad range of core materials and sandwich scantlings. Using the newly determined foundation parameters the Pasternak model agrees well with the elastic half-space solution, and it is effectively shown that for low modulus core materials the elastic buckling load is equal to the plate buckling load, while for high modulus materials the solution converges toward anti-symmetrical wrinkling. However, for current sandwich configurations the proportional limit of the face plate or web material is usually reached well before elastic local buckling occurs and thus an extension of the selected buckling model into the elasto-plastic regime is required to determine the ultimate (local) strength of the sandwich and to actually be able to validate any of the numerical results by experiment.

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