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Buckling of laser-welded sandwich panels. Part II: elastic buckling normal to the webs

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Abstract

This is the second of three companion papers which examine the elastic buckling and collapse of laser-welded sandwich panels with an adhesively bonded core and uni-directional vertical webs. In the first paper it has been shown that the critical load in compression parallel to the webs can be accurately predicted by using a Pasternak foundation to model the core and by using a simple clamping factor to describe the boundary conditions at the laser weld. In this paper the same model is adapted to predict the critical load in compression normal to the webs. As the face plate is now short and wide rather than long and narrow as before, a ‘constrained wrinkling’ approach is outlined which recognises that the face plate is limited in its ability to wrinkle naturally and can only deform in certain mode shapes. The clamping factor and the effective wave length are derived for different buckling mode shapes and boundary conditions at the laser weld, and the constrained wrinkling load is determined by evaluating the buckling load for successive modes and simply picking the smallest value. It is shown that for low modulus core materials the constrained wrinkling load is equal to the bare plate buckling load and for high modulus materials the solution converges toward anti-symmetrical wrinkling, as has been found for the case of compression parallel to webs. The constrained wrinkling load agrees well with finite element results but as before the proportional limit of the face plate material is usually reached well before elastic buckling occurs.

Keywords: Sandwich; Laser welding; Web; Core; Pasternak foundation; Constrained wrinkling; Mode shape

1. Introduction

This is the second of three companion papers which examine the elastic buckling and collapse of laser-welded sandwich panels with an adhesively bonded core and uni-directional vertical webs. In the first paper [1] it has been shown that the local buckling strength of the

face plate is not independent of structural boundary conditions (as assumed in wrinkling theory) but is a function of the elastic properties of the core and the rotational restraint provided by the laser weld. A simplified model has been developed that can demonstrate the effect of different boundary conditions at the laser weld and different core properties by the variation of a clamping factor and two foundation parameters. By evaluation of this buckling model it has been shown that for low modulus core materials the elastic buckling load in compression parallel to the webs is equal to the plate buckling load with clamped boundary conditions, while for high modulus core materials the solution converges toward anti-symmetrical wrinkling. As this increases the elastic buckling strength by an order of magnitude and often raises it above the proportional limit of the face plate material, a logical step in design is to reduce the face plate thickness and/or increase web-spacing. Although that makes for a more optimised and elastic sandwich it also increases the risk of buckling in bending or compression normal to the webs.

In bending or compression normal to the webs, as shown in Fig. 1, the face plate is short and wide rather than long and narrow and because the distance between the supports is fixed ‘natural’ wrinkling in direction of the applied load is no longer possible. Instead, it is reasonable to assume that the face plate only buckles in certain mode shapes, dependent on the size of the natural wrinkling wave length λ with respect to the width of the plate b , i.e. the web-pitch $2p$. As shown in Fig. 2, as long as $\lambda > p$ the face plate only buckles in a single wave with wave length $2p$, whereas if $\lambda < p$ the face plate buckles in two waves or more, dependent on the boundary conditions at the laser weld and the elastic properties of the core. Because that implies that the face plate is no longer free to adopt a natural wave length for which the wrinkling stress is minimum, this type of buckling behaviour is referred to as ‘constrained wrinkling’.

As elastic buckling in bending or compression normal to the webs is practically relevant to the design of larger deck structures, and the concept of constrained wrinkling appears to capture the essence of the problem, this paper outlines a constrained wrinkling approach that can be incorporated into the buckling model developed in [1]. The resulting solution is evaluated numerically for simply supported and clamped boundary conditions at the laser weld, and symmetrical and anti-symmetrical wrinkling. The buckling stress and mode shapes are compared to 2D and 3D linear finite element results and modeling implications are discussed.

2. Analytical modeling

Consider a laser-welded sandwich plate element with face thickness t_f , core thickness t_c , distance between the centroids of the faces d , web thickness t_w , web-pitch $2p$, and elastic moduli E_f , E_w , and E_c , as shown in Fig. 3.

2.1. Pasternak foundation model

In general, the buckling stress for a plate with thickness t_f , bending stiffness D_f , and width b can be written as

$$\sigma_{cr} = k \frac{\pi^2 D_f}{b^2 t_f} \quad (1)$$

Fig. 1. Laser-welded sandwich panel in compression/bending normal to the webs.

Fig.2. Constrained wrinkling in compression/bending normal to the webs.

Fig. 3. Sandwich plate element and characteristic dimensions.

In [1] the buckling coefficient k for a rectangular plate on a Pasternak foundation in compression parallel to the webs has been derived as

$$k = \left(\frac{mb}{a} + \frac{an^2}{mb} \right)^2 + f_w \left(\frac{a}{mb} \right)^2 + f_p \left(1 + \left(\frac{an}{mb} \right)^2 \right) \quad (2)$$

where a and b are the length and width of the plate (Fig. 1) and m and n are the number of buckling half-waves in x - and y -direction respectively. By simply reversing the problem, the buckling coefficient for compression normal to the webs can be written as

$$k = \left(\frac{mb}{an} \right)^2 \left[\left(\frac{mb}{a} + \frac{an^2}{mb} \right)^2 + f_W \left(\frac{a}{mb} \right)^2 + f_P \left(1 + \left(\frac{an}{mb} \right)^2 \right) \right] \quad (3)$$

Here, the Winkler foundation parameter f_W associated with tension/compression of the core and the Pasternak or shear foundation parameter f_P are defined as

$$f_W = \frac{b^4 k_W}{\pi^4 D_f} \quad f_P = \frac{b^2 k_P}{\pi^2 D_f} \quad (4)$$

and the extensional foundation stiffness k_W and the shear stiffness k_P for symmetrical wrinkling, anti-symmetrical wrinkling and bending (Fig. 5) are given as [1]

$$k_W = 2 E_c / t_c \quad k_P = 1/6 G_c t_c \quad \text{compression, symmetrical wrinkling} \quad (5a)$$

$$k_W = E_c / 6 t_c \quad k_P = 1/3 G_c t_c \quad \text{compression, anti-symmetrical wrinkling} \quad (5b)$$

$$k_W = E_c / t_c \quad k_P = 1/6 G_c t_c \quad \text{bending} \quad (5c)$$

Rewriting Eq.(3) as

$$k = n^2 + \frac{1}{n^2} \left[\left(\frac{mb}{a} \right)^4 + f_W + f_P \left(\frac{mb}{a} \right)^2 \right] + f_P + 2 \left(\frac{mb}{a} \right)^2 \quad (6)$$

and minimising k for n it follows that

$$n^2 = \sqrt{\left(\frac{mb}{a} \right)^4 + f_W + f_P \left(\frac{mb}{a} \right)^2} \quad (7)$$

Now, the critical buckling coefficient can be determined in two ways; by substituting the solution for n^2 in Eq.(7) into Eq.(6), which is straightforward but only correct if the face plate is free to buckle in any given wave length, or by recognising the fact that boundary conditions at the laser weld prescribe that the face plate can only buckle in particular mode shapes and adjusting n^2 accordingly, which is the basis of the constrained wrinkling approach described in Section 2.3.

Fig. 5. Symmetrical wrinkling, anti-symmetrical wrinkling, and bending.

2.2. Free wrinkling

If the face plate is free to buckle in any given wave length and n^2 in Eq.(7) is substituted in Eq.(6) the critical buckling coefficient k_{cr} becomes

$$k_{cr} = 2\sqrt{\left(\frac{mb}{a}\right)^4 + f_W + f_P\left(\frac{mb}{a}\right)^2 + f_P} + 2\left(\frac{mb}{a}\right)^2 \quad (8)$$

If the plate only buckles in direction of the applied load ($m=0$) k_{cr} equals

$$k_{cr} = 2\sqrt{f_W} + f_P \quad (9)$$

which result is independent of any boundary conditions and identical to the buckling coefficient obtained in [1] for the case of free wrinkling in compression parallel to the webs. By substitution of Eq.(9) and Eq.(4) the critical buckling stress in Eq.(1) becomes

$$\sigma_{cr} = \frac{1}{t_f} (2k_W \sqrt{D_f} + k_P) \quad (10)$$

which by the k_W and k_P specified in Eq.(5) is dependent only on the elastic properties and the thickness of the face plate and the core. For example, by using k_W and k_P given in Eq.(5a) the buckling stress equals Hoff's symmetrical wrinkling solution [2].

2.3. Constrained wrinkling approach

If free wrinkling of the face plate is unlikely due to boundary conditions at the laser weld, consider Eq.(6) once more. Assuming the plate is very wide ($b \ll a$) and buckles only in direction of the applied load ($m=0$) the buckling coefficient can be written as

$$k_{cr} = n^2 + \frac{f_W}{n^2} + f_P \quad (11)$$

Obviously, with the foundation removed ($f_W = f_P = 0$) k_{cr} must equal the critical buckling coefficient of a conventional compressively loaded plate and hence n^2 can be interpreted as a 'clamping' factor representative of the governing boundary conditions at the laser weld, as proposed in [1]. However, in compression normal to the webs n^2 is linked to the boundary conditions at the laser weld as well as the buckling mode shape, which requires some additional explanation.

Constrained wrinkling hypothesis

Suppose that the face plate is simply supported at the laser weld ($n^2=1$) and the foundation stiffness is such that the natural wrinkling wave length λ is larger or equal to the web-pitch $2p$ and the plate buckles in a single wave ($n=1$) as shown in Fig. 6(a). Although it is evident that λ becomes smaller as the foundation stiffness increases it is unlikely that the face plate will deform at precisely that particular wave length. Instead, the boundary conditions at the laser

weld will force the face plate to buckle at a wave length $l \geq \lambda$, which is a natural fraction (1/2, 1/3, 1/4, etc.) of the web-pitch $2p$, as shown in Fig. 6(b-d).

As long as the face plate buckles in a single wave in between the webs, as in compression parallel to the webs, there is no conflict between the definition of n as a wave-number and the interpretation of n^2 as a clamping factor. However, in Fig. 6(b) the face plate buckles in two half-waves, i.e. $n=2$, whereas the simply supported boundary conditions at the laser weld suggest $n^2=1$. To correct this error for buckling modes with two or more waves in between the webs the interpretation of n^2 as a clamping factor is linked to the buckling wave length as follows.

Assuming the compressive stress is constant throughout the face plate the buckling stress can also be calculated for smaller parts with the size of a single wave length, provided that the boundary conditions match. Consider, for example, the left hand side (the solid drawn part) of the buckling wave in Fig. 6(b). Basically, this part of the face plate constitutes a simply supported plate on its own, with the laser weld as one support and the nodal point in the middle as the other. Thus, the buckling coefficient in Eq.(11) can be rewritten as

$$k = n^2 \left(\frac{b}{l} \right)^2 + \frac{1}{n^2} f_w \left(\frac{l}{b} \right)^2 + f_P \quad (12)$$

where n^2 and l are now the clamping factor and effective wave length for the solid part of the face plate, and by which Eq. (1) can be retained in its original form.

Simply supported boundary conditions

For simply supported boundary conditions at the laser-weld n^2 and l can be determined instantly. As the plate is wide in compression normal to webs its behaviour resembles a column rather than a plate and the clamping factor n^2 is equal to 1 for all buckling modes (Fig. 7). The wave length l is obviously equal to the width of the face plate b divided by the number of waves.

Fig. 6. Buckling of a simply supported plate on elastic foundation, mode 1-4.

Fig. 7. Clamping factor and effective wave length for simply supported plate on elastic foundation, mode 1-4.

Clamped boundary conditions

For clamped boundary conditions at the laser weld n^2 is different for mode 1, mode 2, and mode 3 onward and the effective wave length l has to be determined correspondingly. If the face plate buckles in mode 1, Fig. 8(a), it behaves as a column with clamped ends, i.e. $n^2 = 1/0.5^2 = 4$, and l is obviously equal to the full width of the face plate $b (=2p)$. In mode 2, Fig. 8(b), the left hand side (the solid drawn part) of the buckling wave is simply supported at the nodal point in the middle and clamped at the laser-weld, i.e. $n^2 = 1/0.7^2 = 2.05$, and l is equal to $b/2$. For mode 3 and higher the effective wave length for any part of the buckling wave can be determined from the fact that the effective wave length of all parts combined must be equal to the width of the face plate.

Assuming equal compressive stress in the face plate the Euler buckling load P_1 for the solid drawn part of the plate in Fig. 8(c) must be equal to the buckling load P_2 for either end (the dotted parts), or

$$P_1 = \frac{n_1^2}{l_1^2} \pi^2 D_f = \frac{n_2^2}{l_2^2} \pi^2 D_f = P_2 \quad (13)$$

For $n_1^2 = 1.0$ (simply supported) and $n_2^2 = 1/0.7^2$ (simply supported, clamped) the effective wave length l_1 is equal to $0.7 l_2$ and as $l_1 + 2l_2$ must equal b it follows that

$$l_{l, mode 3} = 1/(1+2/0.7) b \quad (14)$$

Similarly, the wave length for the solid drawn part in Fig. 8(d) is equal to

$$l_{l, mode 4} = 1/(2+2/0.7) b \quad (15)$$

and by regarding ‘simply supported’ buckling waves only ($n^2=1$) the effective wave length for mode 3 and higher can be written as

$$l = 1/(mode-2 + 2/0.7) b \quad (16)$$

Summarised, the clamping factor and effective wave length for clamped boundary conditions at the laser weld are as shown in Fig. 9.

Critical buckling coefficient and buckling mode shape

Using Eq.(12) the critical buckling coefficient k_{cr} for simply supported or clamped boundary conditions at the laser weld can be determined by evaluating k for different values of n^2 and l and picking the smallest value, which is referred to as the constrained wrinkling solution. By doing so, it is also possible to predict what buckling mode shape is critical or when one buckling mode shape changes into another, e.g., as the influence of boundary conditions decreases and wrinkling predominates. Although the actual mode shape is of little practical significance it is convenient as a reference for the comparison of the buckling model with finite element result, as is done later in this paper.

Fig. 8. Buckling of a clamped plate on elastic foundation, mode 1-4.

Fig. 9. Clamping factor and buckling wave length for the solid-drawn part of the plate - constrained wrinkling.

2.4. Numerical evaluation

The buckling coefficient has been calculated for one set of sandwich scantlings (Table 1), simply supported and clamped boundary conditions at the laser weld, and core moduli representative of linear elastic isotropic material up to solid Polyurethane elastomer. To make for an even distribution of datapoints, 41 core moduli from 0.1 to 1000 MPa have been generated at equal spacing on a $\log(E_c)$ scale. The material used for the webs and the plating is mild steel ($E_{f,w} = 205$ GPa, $\nu_{f,w} = 0.3$) and a perfect bond between the core and the plating is assumed. For reference, the constrained wrinkling solution is compared to the corresponding free wrinkling solution in Eq.(9).

Simply supported vs. clamped boundary conditions

In Fig. 10(a) the buckling coefficient in Eq.(12) is evaluated for simply supported boundary conditions at the laser weld and the first five symmetrical wrinkling modes (note: in Fig. 10 m denotes the mode number). As expected, for very light foundations ($E_c < 1$ MPa) the face plate buckles in a single wave ($m=1$) and the critical buckling coefficient is approximately 1, which is equal to the buckling coefficient of a wide plate with no foundation at all. As the foundation stiffness increases, the constrained wrinkling solution follows the path of least resistance, shifting from one buckling mode to the next in search of the smallest buckling coefficient. For simply supported boundary conditions constrained wrinkling obviously bares

Table 1
Sandwich scantlings (see Fig. 2b)

t_f	t_c	t_w	$2p$
2.0 mm	40.0 mm	4.0 mm	120.0 mm

close resemblance with free wrinkling and it is not surprising therefore that only the mode shifts from $m=1-2$ and $m=2-3$ can be clearly distinguished.

For clamped boundary conditions, Fig. 10(b), the buckling behaviour is not much different than for simply supported boundary conditions, but as mode shifts occur at higher foundation stiffness where the buckling coefficient varies little from one mode to another, only the first mode shift from $m=1-2$ is recognisable. By comparison of, e.g., Fig. 7(c) and Fig. 9(c) it can also be seen that higher buckling modes ($m \geq 3$) the mode shapes for simply supported and clamped boundary conditions are not much different. Although for $m=3$ the effective wave lengths are not the same ($1/3b$ and $\sim 1/4b$ resp.), it must be remembered that for higher foundation stiffness and shorter wave length deformations shear lag effects become dominant and by Eq.(12) these are independent of the effective wave length. Thus, it is not surprising that the constrained wrinkling solution for clamped edges matches closely with the solution for simply supports as well as free wrinkling, except for light foundations ($E_c < 10$ MPa) where the buckling coefficient is obviously higher due to clamping at the laser weld ($k_{cr} = 4$).

Symmetrical vs. anti-symmetrical wrinkling

Disregarding the influence of boundary conditions, the buckling coefficient is also dependent on the type of constrained wrinkling that occurs, i.e. symmetrical or anti-symmetrical (bending is a special case of symmetrical wrinkling). In [1] it has been shown that for the standard sandwich configuration used here the critical buckling coefficient in compression parallel to the webs is always obtained for the anti-symmetrical wrinkling case. In particular, it has been concluded that by using a reduced Winkler foundation stiffness as specified in Eq.(5b) the accuracy and generality of the Pasternak foundation model is improved up to a point that it is almost exact or slightly conservative with respect to an elastic half-space solution.

Fig. 10. Buckling coefficient for wide rectangular plate on Pasternak foundation (modes 1-5), constrained wrinkling and free wrinkling solution. (a) loaded edges simply supported. (b) loaded edges clamped.

By comparison of the constrained wrinkling solution for clamped boundary conditions at the laser weld, which is considered most practically relevant, it can be seen in Fig. 11(a) that the critical buckling coefficient is obtained for anti-symmetrical wrinkling, as is the case for compression parallel to the webs. As anti-symmetrical wrinkling in compression normal to the webs is basically the same as global (shear) buckling this is not surprising, but the practical consequences are considerable. In particular, the point at which the constrained wrinkling solution and the free wrinkling solution yield approximately the same result has shifted to considerably higher foundation stiffness; $E_c \sim 100$ MPa as opposed to $E_c \sim 10$ MPa for symmetrical wrinkling. Although this point will shift back to lower foundation stiffness as the web-pitch increases, Fig. 11(b), this means that for the majority of core materials it is essential that boundary condition effects are accounted for.

Fig. 11. Buckling coefficient for wide rectangular plate on Pasternak foundation with clamped loaded edges, constrained wrinkling and free wrinkling solution, symmetrical and anti-symmetrical wrinkling. (a) $2p = 120$ mm. (b) $2p = 240$ mm.

3. Finite element analysis

Because the case of compression parallel to the webs does not lend itself to description in 2D no attempt has been made in [1] to do so using finite elements either. However, as the sandwich in compression normal to the webs is basically in a state of plane stress and constrained wrinkling is strictly a 2D phenomenon it seems logical to make a comparison with 2D finite elements as well. Hence, a modeling scheme has been set up to try and capture the constrained wrinkling behaviour in 2D, before comparison of the solution with the 3D finite element model used in [1]. In particular, this scheme aims to demonstrate how periodic boundary conditions affect the buckling behaviour and the bifurcation load.

Starting from a basic 2D representation of a periodic laser-welded sandwich plate element with two ‘cells’, as shown in Fig. 12(a), a simple finite element mesh has been constructed using the commercial finite element code ABAQUS [3], with quadratic beam elements (B22) for the face plate and webs and quadratic membrane elements (M3D8R) through the core, Fig. 12(b). Based on the experiences of Fagerberg [4] a reliable estimate of the bifurcation

load can be obtained if the element length is shorter than half the wrinkling wave length, but for evaluation of the wrinkling mode shape at least four elements per wave length should preferably be used. In this case the highest mode of interest is $m=4$, Fig. 10(b), and by compromise 12 beam elements have been used to model the face plate in between two webs, which also makes for a convenient element length of 10mm. To allow the modeling of ‘thick’ cores 8 membrane elements have been used through the thickness.

Fig. 12. (a) 2D representation and (b) finite element mesh of periodic laser-welded sandwich plate element.

Using the mesh in Fig. 12(b) the first step has been to model constrained wrinkling of the upper face for symmetrical wrinkling and clamped boundary conditions at the laser weld. This has been achieved by locking the vertical displacement of the lower face and the rotation at the laser weld in the upper face, while linking the vertical displacements and all remaining rotations of both sides by means of a linear multi-point constraint to maintain periodicity, Fig. 13(a). An example of what the resulting deformation looks like is shown in Fig. 14, which obviously bares close resemblance to Fig. 8(a).

By unlocking the rotation at the laser weld and linking the rotation of both upper corners the second model, Fig. 13(b), can be used to demonstrate constrained wrinkling for periodic boundary conditions. Thus, only the difference in bending stiffness between the web and the face plate determines the rotational restraint at the laser weld, which makes for a ‘softer’ structural response and is more realistic (Fig. 15). Here, it must be remembered that adjoining elements of the same type always share the same rotation(s) at a particular node, and hence simply supported boundary conditions at the laser weld can only be simulated by uncoupling the element rotations in the finite element model (e.g., using a connector element) or by making the web thickness very small. Also, as it is the lower face and not the mid-plane that has been locked in vertical direction the bifurcation load of models Fig. 13(a) and (b) corresponds to the symmetrical constrained wrinkling load for double the core thickness.

To allow anti-symmetrical wrinkling the vertical displacement of the lower face in Fig. 13(b) is unlocked and, instead, the vertical displacements and rotations at both lower corners of the model are linked by means multi-point constraints as before. In addition, the vertical displacement of the middle web is linked to the displacement of at the sides to avoid global buckling, Fig. 13(c), or released to allow it, Fig. 13(d). Regarding the similarities between anti-symmetrical wrinkling and global buckling, the link between these two models is particularly important from a practical safety point of view.

Before extension of the model in 3D, the sandwich has been remeshed using only 6 beam elements to model the face plate and 4 membrane elements through the thickness, Fig. 13(e), which in this case is the minimum number of elements required for proper convergence of the

bifurcation load. For the extension itself, it has been mentioned in [1] that the buckling coefficient for a long rectangular plate is almost constant if $a/b \geq 4$, and in the 3D model the face plate should therefore stretch $4 \times 6 = 24$ elements in x -direction, Fig. 13(f). Detailed information on the actual 3D model can be found in [1], but in short it uses second-order, reduced integration elements throughout (S8R and C3D20R resp.), it is fully periodic and can be used to model compression parallel and normal to the webs by ‘driving’ the two corner nodes at the far right in Fig. 16. In addition, a third node has been incorporated by which the middle web can be ‘locked’ to avoid global buckling in compression normal to the webs.

Fig. 13. Different ways to model constrained wrinkling in 2D. (a) Symmetrical wrinkling with clamped boundary conditions at the laser weld. (b) Symmetrical wrinkling with periodic boundary conditions. (c) Symmetrical and anti-symmetrical wrinkling. (d) Symmetrical and anti-symmetrical wrinkling, global buckling. (e and f) Mesh adjustment and extension into 3D.

Fig. 14. Deformed shape of face plate (top) for constrained wrinkling with clamped boundary conditions at the laser weld (mode 1) (ABAQUS).

Fig. 15. Boundary conditions at the laser weld for different combinations of web and face plate thickness.

Fig. 16. Finite element model of periodic sandwich plate element with core removed (ABAQUS)

5. Results and discussion

For brevity the finite element models presented in Fig. 13 are referred to as Model (a) to (f).

5.1. Model (a) and (b) - symmetrical wrinkling of half sandwich (reference case)

In Fig. 17 a comparison is presented between the buckling stress as obtained from Model (a) and the constrained wrinkling solution for symmetrical wrinkling and clamped boundary conditions as obtained from Eqs.(5a) and Eq.(12). While there is (reasonably) good agreement of the buckling stress for core moduli up to $E_c=200$ MPa, the constrained wrinkling solution is increasingly non-conservative for higher core moduli. To try an explain this, and why for example the mode shift $m=1-2$ occurs earlier in the finite element model and not later as can be expected, it is convenient to have a look at the actual deformation of the face plate.

In Fig. 18 the deformed shape of Model (a) for the first four buckling modes is presented alongside with the corresponding schematic presented in Fig. 9. Clearly, the deformed shape matches closely with the schematic for Mode I and II, but for Mode III and IV it becomes increasingly non-linear. Consider Mode IV in Fig. 18(d) for example. Although the basic shape is the same as in the schematic, the wave length and amplitude clearly are not. While the amplitude slowly increases toward the middle of the face plate and is larger for an inner (simply supported) part of the buckling wave than for the outer part, the wave lengths of both parts are actually fairly similar. In fact, it can be seen by visual inspection that for Mode III and higher the inner wave length l_1 varies between 0.81 and 1.08 times the outer wave length l_2 (compared to $0.7l_2$ assumed for constrained wrinkling), as shown in Table 2.

Although the rotation at the laser weld has been locked the face plate behaves as if it simply supported, at least when the wave length and (to a lesser extent) the mode shape are

Fig. 17. Buckling stress for beam (wide plate) on elastic foundation with clamped edges. Constrained wrinkling solution and finite element results (ABAQUS).

Fig. 18. Buckling mode shapes for beam (wide plate) on elastic foundation in symmetrical wrinkling with clamped boundary conditions (ABAQUS). (a) Mode I. (b) Mode II. (c) Mode III. (d) Mode IV.

considered. In Table 2 the critical wave length l_{cr} for Model (a) is therefore compared to l_{cr} for free wrinkling and constrained wrinkling for simply supported boundary conditions at the

laser weld. Obviously, the deformed shape resembles constrained wrinkling most closely directly after each mode shift and as can be seen the corresponding l_{cr} is virtually the same. Compare, for example, the finite element result for #30, 39 mm ($l_1/l_2 = 0.97$), with the constrained wrinkling wave length for #28, which is 40 mm (Mode III). Then, as the core modulus increases the influence of the boundary conditions at the laser weld becomes smaller and the critical wave length drops down to 30 mm for #34 ($l_1/l_2 = 0.81$), before the shift to Mode IV. For Mode V the critical wave length for free wrinkling, constrained wrinkling and the finite element model is practically the same and true wrinkling can be assumed. Overall, the mode shifts in the finite element model follow the constrained wrinkling solution more closely as the core modulus increases.

Practically, the influence of boundary conditions on the buckling stress and deformed shape of the face plate is therefore confined to Mode I and Mode II, i.e. for sandwich configurations with relatively light cores and/or small web-spacing. For higher buckling modes ($m \geq 3$) the face plate basically behaves as if it is simply supported at the edges, and the deformed shape and buckling stress can be described accordingly. Proof of that is presented in Fig. 19 which shows that the results for Model (a) with clamped boundary conditions and Model (b) with periodic boundary conditions are more or less coincident for $E_c \geq 50$ MPa. Similarly, it can be shown that the results discussed thusfar are more or less consistent for $t_c = 20$ -100 mm, with the only difference that for ‘thick’ cores mode shifts are less pronounced and occur later. For $m=1-2$ compare $E_c=10, 25$ and 40 MPa for $t_c=40, 80$ and 200 mm resp.

Thus, if the buckling stress for high modulus materials is independent of the boundary conditions at the laser weld and the thickness of the core, the non-conservative result in Fig. 17 has to be caused by a local phenomenon. A plausible explanation is that the depth of the core zone affected by wrinkling in fact becomes smaller than half the thickness of the sandwich, which renders the finite element model increasingly ‘soft’ with respect to the constrained wrinkling solution. For free wrinkling the depth of the wrinkling zone is typically

Table 2
Critical wave length for symmetrical wrinkling (see Fig. 7 for mode shape)

Core material		Free wrnk.	Constrained wrinkling (SS)		Finite elements, Model (a)		
Specimen	E_c [MPa]	l_{cr} [mm]	l_{cr} [mm]	Mode	$l_{cr}^{\dagger}$ [mm]	l_1/l_2	Mode
#1	0.1	515	120	I	120	x	I
#11	1	239	-	-	-	x	-
#18	5	140	60	II	-	x	-
#21	10	111	-	-	60	x	II
#28	50	66	40	III	-	x	-
#30	79	56	-	-	39	0.97	III
#31	100	52	-	-	38	0.94	-
#34	200	41	30	IV	34	0.81	-
#35	251	40	-	-	30	1.07	IV
#38	501	30	24	V	27	0.80	-
#39	631	28	-	-	25	1.08	V
#41	1000	24	-	-	22	0.83	-

[†] For Mode III and higher $l_{cr} = l_1$

in the same order of magnitude as the buckling wave length, and this softening effect should become noticeable when the wrinkling wave length drops down to approx. 20mm. However, in this case the constrained wrinkling solution becomes increasingly non-conservative as $E_c \sim 200$ MPa, i.e. for a wave length of approx. 40 mm (Table 2). Combined with the fact that the core can not be compressed in the vicinity of the webs, this leads to the conclusion that for the deformation energy is concentrated in the middle of each ‘cell’, hence the increasing deformation amplitude and lower buckling stress.

Fig. 19. Buckling stress for beam (wide plate) on elastic foundation. Constrained wrinkling solution (symmetrical) and finite element results for clamped edges and periodic boundary conditions (ABAQUS).

5.2. Model (b), (c) and (d) - (anti-)symmetrical wrinkling and global buckling

A comparison between the constrained wrinkling solution and finite element results for symmetrical and anti-symmetrical wrinkling is presented in Fig. 20(a), with the first three anti-symmetrical wrinkling modes shown in Fig. 21(a) to (c). Clearly, in anti-symmetrical wrinkling there is relatively little extension/compression of the core and using the reduced foundation stiffness k_w obtained in [1] there is very good agreement of the constrained wrinkling solution with finite element results for all practically relevant core moduli. In fact, if global buckling or ‘shear crimping’ of the sandwich is allowed, Fig. 20(b) shows the transition from global buckling for low modulus materials to anti-symmetrical wrinkling for high modulus materials, with equally good agreement of the analytical and finite element results.

5.3. Model (d), (e) and (f) - mesh enlargement and 3D

Despite good agreement of the anti-symmetrical wrinkling and global buckling results in Fig. 20, it has to be remembered that constrained wrinkling, like free wrinkling, is a local phenomenon for which a certain level of mesh refinement is required. However, as shown in Fig. 22(a) it is virtually impossible to distinguish between the finite element results for the original 2D model, Model (d), the 2D model with a less refined mesh, Model (e), and the 3D model. As the same applies to the results for anti-symmetrical wrinkling and global buckling

in Fig. 20(b), it is concluded that the 3D model is sufficiently accurate for evaluation of the buckling stress in compression normal to the webs. Moreover, as buckling parallel to the webs is a less local phenomenon, the same periodic 3D model can be used to evaluate both cases simultaneously, as is done in the next section.

Fig. 20. Buckling stress for sandwich beam (wide plate). (a) Constrained wrinkling solution and finite element results (ABAQUS) for half-sandwich and full sandwich with periodic boundary conditions. (b) Constrained wrinkling, global (shear) buckling and finite element results for full sandwich with periodic boundary conditions.

Fig. 21. Buckling mode shapes for sandwich beam (wide plate) with periodic boundary conditions (ABAQUS). (a) Anti-symmetrical wrinkling (Mode I). (b) Anti-symmetrical wrinkling (Mode II). (c) Anti-symmetrical wrinkling (Mode III). (d) Global buckling.

Fig. 22. Buckling stress for sandwich beam/plate. Constrained wrinkling solution for clamped boundary conditions and 2D/3D finite element results for periodic boundary conditions (ABAQUS).

5.4. Model (f) - Compression parallel and normal to the webs

In Fig. 23(a) the buckling stress in compression parallel to the webs as treated in [1] is compared to the buckling stress in compression normal to the webs, and clearly there is very good agreement of the analytical and finite element result for both cases. As expected, for high modulus materials the buckling stress converges to the approximately the same value as the critical wavelength becomes small and boundary conditions at the laser weld no longer are of any influence.

Fig. 23. Buckling stress for sandwich plate in compression parallel and normal to the webs. Pasternak foundation model with clamped boundary conditions at the laser weld [1], constrained wrinkling solution, and finite element results for 3D periodic model (ABAQUS).

5.5. Design implications

Assuming the face plate is clamped at the laser weld, the baseline buckling stress (for low modulus core materials) in compression normal to the webs is approx. 40% lower than the buckling stress in compression parallel to the webs. However, the predicted buckling stress for standard sandwich configurations quickly exceeds the proportional limit of the face plate material here as well, and an extension of the constrained wrinkling model into the elasto-plastic regime is required. On the other hand, for more optimised sandwich configurations with reduced face plate thickness, increased web-spacing, or, e.g., composite scantlings elastic buckling in compression parallel to the webs is definitely a possibility. Combined with the Pasternak foundation model developed in [1] and a simple global buckling check [1,5] the constrained wrinkling approach is therefore at least as important for future development as it is present buckling control.

Practically, the fact that for standard sandwich configurations the proportional limit of the face plate material determines the critical strength implies that it should be possible to design for global strength and stiffness only and, if necessary, improve local strength separately. From a manufacturing point of view, a promising aspect of this investigation is that adequate stabilising effect can be obtained by in-situ injection of polyurethane (PU) foam, reducing the need for costly core materials and adhesives and eliminating the problems associated with a combined laser-welding and bonding technique. Of particular interest with regard to future optimisation are the positive experiences with the injection of solid PU elastomer, originally developed for ship repair.

6. Summary and conclusions

In this paper, the second of three companion papers, the elastic buckling of laser-welded sandwich panels with an adhesively bonded core and uni-directional vertical webs has been investigated. A buckling model developed in the first companion paper for the case of compression parallel to the webs has been adapted to predict the critical load in compression normal to the webs. A ‘constrained wrinkling’ approach has been outlined, recognising that the face plate is limited in its ability to wrinkle naturally and can only deform in certain mode shapes. The clamping factor and the effective wave length have been derived and numerically evaluated for different buckling mode shapes and boundary conditions at the laser weld. It has been shown that for low modulus core materials the constrained wrinkling load is equal to the bare plate buckling load and for high modulus materials the solution converges toward anti-symmetrical wrinkling, as has been found for the case of compression parallel to webs. The constrained wrinkling load agrees well with finite element results and although at present the proportional limit of the face plate material is usually reached before elastic buckling occurs, there is certainly potential for practical application.

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