Linear Analysis of a two-area system including a linear model of a Thyristor-Controlled Series Capacitor

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Abstract: This paper describes a method to construct a linear model of a Thyristor-Controlled Series Capacitor (TCSC) by analyzing its response in a time domain simulation wherein the TCSC is represented by a detailed instantaneous value model.

To create the linear model Matlab's System Identification ToolBox is used.

The linear model in the classic ABCD-form is included in both time simulations and linear analysis of the power system.

With this technique, dynamic behaviors of the TCSC are included in linear analysis of the power system.

The studied power system is a simplified model of the TCSC compensated south-north link in Brazil. Comparisons between the linear and the instantaneous value model of the TCSC are performed.

By using the linear model, simulations can be done much faster (the computing time is reduced) and still include the dominant behavior of the TCSC.

Index terms: Linear analysis, linearization, TCSC.

I. INTRODUCTION

Linear Analysis is a useful tool to study whether a power system is stable and how to tune controllers. The location of a power system's eigenvalues in the complex plane shows whether the power system is stable or not.

As long as the behavior of each component is linear in the current working point (and in a close time interval to the chosen working point) linearization is straight forward. But nowadays the number of non-linear components in power systems is steadily increasing. Such a component is a Thyristor-Controlled Series Capacitor (TCSC).

This component switches constantly between different working points and therefore it behaves in a non-linear manner.

In a TCSC, all components in Fig. 1 are included. By varying the conducting time for each thyristor, the fundamental

reactance between Node A and Node B can be changed.

If linearizing a power system that includes a TCSC, the linearization will be done for a particular instant with the control algorithms linearized for that particular moment. Therefore the dynamic of the TCSC would not be described. With 'dynamic' is meant the response of the TCSC in the time frame of seconds. Such a low-frequency model will be extracted in this paper.

Fig. 1. Thyristor-Controlled Series Capacitor

The TCSC is built under the assumption that the reactor for each phase is constantly switched on and blocked at the next zero-crossing and that the TCSC, with its control algorithm, passes several different working points. For linearization of such a component, the method shown in this paper is useful.

II. THE FORM OF A LINEAR MODEL

The aim is to create a linear model that corresponds to,

$$\Delta \dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x} + \mathbf{B} \Delta \mathbf{u} \quad (1)$$

$$\Delta \mathbf{y} = \mathbf{C} \Delta \mathbf{x} + \mathbf{D} \Delta \mathbf{u} \quad (2)$$

where $\Delta \mathbf{x}$ contains the n state variables of the linear model, $\Delta \mathbf{u}$ is the input vector, and $\Delta \mathbf{y}$ is the output vector from the model, see Fig. 2. The input, output, and state vectors show the deviation of the corresponding values from their reference value.

$\mathbf{A}$ is the state matrix of size $[n \times n]$, $\mathbf{B}$ describes the relation of each state variable to the input signals, $\mathbf{C}$ describes the relation of the output signals to the state variables, and $\mathbf{D}$ is the feed forward matrix that describes the relation of each output signal directly to the input signals. The sizes of the matrices are $\mathbf{B}$ $[n \times r]$, $\mathbf{C}$ $[m \times n]$, and $\mathbf{D}$ $[m \times r]$ where r is the

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number of input signals and m is the number of output signals [2].

Fig. 2. A linear model

The **A**, **B**, **C**, **D** matrices will not change with time, i.e. the differential equations describing the relations between the input vector $\Delta \mathbf{u}$, the state vector $\Delta \mathbf{x}$, and the output vector $\Delta \mathbf{y}$ will be time independent.

Once those matrices have been built, the component can be represented as linear and included to the rest of the power system that is ready for linear analysis.

III. REPRESENTATION OF THE TCSC

The control of the TCSC in this paper is following the Synchronous Voltage Reversal-control algorithm developed in [3]. Here, that control algorithm is only briefly discussed since the paper is not focused on it.

The aim of the TCSC is to create a desired value of the fundamental reactance between the buses on each side of the series capacitor. To do so, the control algorithm calculates the instant at which the thyristors should start conducting the phase current through the series reactor. The Synchronous Voltage Reversal-control algorithm consists of three parts:

- Phase Locked Loop, PLL
- Booster, BOO
- Thyristor Pulse Generator, TPG

Each phase has a similar control system as shown in Fig. 3.

Fig. 3. Control system of the TCSC

The input signal to the Phase Locked Loop (PLL) is a phase current into the TCSC, i_A , as described in Fig. 3. Output signals from the PLL are signals to activate the Thyristor Pulse Generator (TPG).

The output signals from the PLL are sent both to the Booster (BOO) and the TPG.

Into the BOO also the series capacitor voltage drop for one phase is an input signal. The BOO is boosting (improving) the predicted instants for the next coming zero-crossing of the series capacitor voltage from the PLL. These instants are sent to the TPG.

The TPG calculates the instants at which the thyristors should start conducting and pulse signals are generated to fire the thyristors.

The TCSC-model in this paper does not contain any unit to damp oscillations. In order to do so, the TCSC-model must be completed with a reactance unit that provides a reactance reference value to the control algorithm. In this paper the reference value is fixed.

From the TCSC internal control this means that there are two input signals, i_A and u_{CA} , and one output signal, the firing angle for the thyristor.

IV. SYSTEM FOR LINEARIZATION OF THE TCSC

To focus on the dynamic behavior of the TCSC, the linearization is done excluding the dynamics of the rest of the power system. Therefore, a load is connected to Node B and the machine is modeled as an infinite production source, see Fig. 4.

Fig. 4. System used for linearization of the TCSC

The line to the left of the TCSC is a low-impedance line. The base power is 1300 MVA and the base voltage is 500 kV.

The control algorithm of the TCSC is in the linearized case set to create a fundamental reactance two times the fundamental reactance of the series capacitor, i.e., $2 \cdot X_C$. The reactance of the series capacitor is $X_C = -13.27 \Omega$.

All power system simulations are performed with the power system software SIMPOW, see [4] and [5].

In the simulations the electrical state is described with the instantaneous value mode using $dq0$ -representation (direct-, quadrature-, and zero-axis, in the following text abbreviated as d -, q -, and $zero$ -axis or -components).

V. SIGNALS USED TO CREATE THE LINEAR MODEL

To obtain an appropriate linear model of the TCSC it has to be disturbed in order to observe its dynamic response.

The load in Fig. 4 will be modeled as a pure d -axis constant current load with $i_d = 1.0$ p.u. and $i_q = 0.0$ p.u. until the TCSC is in steady state condition. It should be mentioned that in steady state the voltage drop over the series capacitor contains harmonics. The load is then varied as;

- The current in d -axis is increased to 1.10 p.u. at $t = 2$ s and the current in q -axis is kept constant to 0.0 p.u. Then the relations of $u_{C d}$ and $u_{C q}$ to i_d can be seen in Fig. 5 and 6.
- The current in d -axis is kept constant to 1.0 p.u. and the current in q -axis is increased to $i_q = 0.1$ p.u. at $t = 2$ s. Then the relations of $u_{C d}$ and $u_{C q}$ to i_q can be seen in Fig. 7 and 8.

$u_{C d}$ is the voltage drop in d -axis and $u_{C q}$ is the voltage drop in q -axis of the series capacitor. The current in the load at Node B is equal the d - and q -component of the current i into the TCSC, see Fig. 4.

The response of the TCSC is then analyzed by using the d - and q -components of the series capacitor voltage drop and the d - and q -components of the current into the TCSC, i , see Fig. 4.

Since there are no un-symmetries outside the TCSC in the example we will study in this paper, there will be no zero-components in the current into the TCSC.

Also the zero-component voltage drop over the series capacitor can be omitted since it contains no frequencies below the third harmonic and since we are interested in building a linear model for low frequencies, there is no need of representing that voltage.

The signals in Fig. 5 - 8 are low-pass filtered with a filter constant of $T = 0.01$ s. That can be done since we have no interest in frequencies above fundamental frequency.

Hence, in the **ABCD**-model for the whole TCSC, including internal control, there will be two input signals and two output signals;

$$\Delta \mathbf{u} = \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix} \quad \Delta \mathbf{y} = \begin{bmatrix} \Delta u_{C d} \\ \Delta u_{C q} \end{bmatrix} \quad (3)$$

where;

Δi_d = deviation from ($t < 2$ s) in d -axis of the current to the TCSC

Δi_q = deviation from ($t < 2$ s) in q -axis of the current to the TCSC

$\Delta u_{C d}$ = deviation from ($t < 2$ s) in d -axis of the series capacitor voltage, see Fig. 7.

$\Delta u_{C q}$ = deviation from ($t < 2$ s) in q -axis of the series capacitor voltage, see Fig. 6.

The following four figures shows the voltage drop of the series capacitor during the variations of the load mentioned above.

Fig. 5. $u_{C d}$ when i_d is increased to 1.10 p.u.

Fig. 6. $u_{C q}$ when i_d is increased to 1.10 p.u. The arrow represents $\Delta u_{C q}$.

Fig. 7. $u_{C d}$ when i_q is increased to 0.10 p.u. The arrow represents $\Delta u_{C d}$.

Fig. 8. $u_{C q}$ when i_q is increased to 0.10 p.u.

The y-axis in the diagrams above are plotted in p.u. of the base voltage $500 \frac{\sqrt{2}}{\sqrt{3}}$ kV.

VI. CREATING THE LINEAR MODEL

By using the results in section V, the four relations

$$\Delta u_{Cd} / \Delta i_d, \Delta u_{Cq} / \Delta i_d, \Delta u_{Cd} / \Delta i_q, \Delta u_{Cq} / \Delta i_q \quad (4)$$

can be scheduled. By using Matlab's System Identification ToolBox [1], transfer functions for them can be found.

For instance, the relation $\Delta u_{Cd} / \Delta i_q$ is identified with the following transfer function,

$$\frac{\Delta u_{Cd}(s)}{\Delta i_q(s)} = d_2 \frac{(s + a_2)}{(s - (b_2 + jc_2))(s - (b_2 - jc_2))} \quad (5)$$

where, $a_2 = 175.3$, $b_2 = -14.8$, $c_2 = 26.0$, and $d_2 = 0.71$.

It is found that an eleventh-order model is enough to describe the behavior of the four relations (4).

The damping for each pole in the four transfer functions is put on the diagonal of the **A**-matrix and the imaginary part for each pole is put on the corresponding sides of the diagonal, as in the matrix below.

By identifying $C_{32} = (a_2 + b_2)/c_2$ the reader can see how equation (5) can be described in the **ABCD**-model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \\ \dot{x}_{11} \end{bmatrix} = \begin{bmatrix} b_1 & -c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_1 & b_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & b_2 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_2 & b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & b_3 & -c_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_3 & b_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{76} & -a_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_5 & -c_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_5 & b_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{109} & b_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_{119} & c_6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \end{bmatrix} + \begin{bmatrix} C_{11} & 0 \\ 1 & 0 \\ 0 & C_{32} \\ 0 & 1 \\ C_{51} & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & C_{82} \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \Delta u_{Cd} \\ \Delta u_{Cq} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & d_2 & 0 & d_3 & d_3 & 0 & 0 & 0 & 0 \\ 0 & d_1 & 0 & 0 & 0 & 0 & 0 & 0 & d_4 & 0 & d_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta i_d \\ \Delta i_q \end{bmatrix} \quad (7)$$

Values of the elements $b_1, b_3, \dots, b_6, c_1, c_3, \dots, c_6, d_1, d_3, d_4, C_{11}, C_{51}, C_{82}, A_{76}, A_{109}$, and A_{119} are omitted in this paper.

With this, the system in Fig. 3 can be modeled as an eleventh-order system as in Fig. 2.

VII. AN INTERNAL dq -REFERENCE INSIDE THE LINEAR TCSC-MODEL

The linear TCSC-model contains an internal reference system that refers the incoming deviations in the currents to the steady state current. The steady state current is assumed to be a pure direct-axis current, see Fig. 9.

Fig. 9. Internal reference system in the linear TCSC-model

Also the output signals (the change in voltages) are referred to the same reference and they are added with their signs to the steady state voltages.

VIII. SYSTEM USED FOR COMPARISONS

The system used for checking the accuracy of the linear model, is presented in Fig. 10. The system is a simplified model of the TCSC compensated south-north link in Brazil. The two synchronous machines, S1 and S2, are modeled as constant voltages behind a transient reactance and equipped with turbines and governors, see Fig. 11 and 12. SN = 900 MVA, H = 6.5 MWs/MVA, $X_D = X_D' = X_Q = X_Q' = 1.7$ p.u.

The machines have different rated voltages and are assumed to be connected directly to the 500 kV level; $UN_{S1} = 500$ kV and $UN_{S2} = 530$ kV. No transformers are included in the system.

The TCSC has the same settings as in section IV.

Fig. 10. A simplified model of the TCSC compensated south-north link in Brazil

The load at Bus A is $P = 500$ MW and $Q = 0$ Mvar and it is modeled with constant current character, the impedance of the line between Bus D and Bus E is $R = 17.7 \Omega$ and $X = 266 \Omega$. It represents a 1000 km long transmission line.

The governors are modeled as in Fig. 11.

$$Y_{MAX} = 1 \quad Y_{MIN} = -1 \quad K = 100 \quad T = 0.1$$

Fig. 11. Block diagram of the governor

P0 in Fig. 11 is the initial mechanical power at $t = 0$ s, PEG is the active power in the machine, W is the speed of the machine and Y is the gate opening that is the input signal to the turbine. The steam turbines are modeled as in Fig. 12.

$$TC = 0.1 \quad TR = 7.0 \quad K_{S1} = 0.6 \quad K_{S2} = 0.3$$

Fig. 12. Block diagram of the steam turbine

TM is the mechanical torque of the turbine and is an input signal to the synchronous machines.

IX. COMPARISONS IN THE TIME DOMAIN

To control the validity of the linear model of the TCSC, comparisons are made between the linear and the detailed instantaneous value model of the TCSC.

When steady state has been reached at $t = 6$ s the load in Bus A is increased with 10% ($\Delta P = 50$ MW).

Two simulations are now done, one with the detailed instantaneous value model and one with the linear model. In Fig. 13 the voltage drop over the series capacitor, $u_{Cq}(u_{Cd})$ is shown for the two simulations, and in Fig. 14 the current into the TCSC, $i_q(i_d)$ is shown.

Fig. 13. $u_{Cq}(u_{Cd})$ for the two models

Fig. 14. $i_q(i_d)$ for the two models

In the following three graphs, characteristic signals for machine S1 are outlined. The results from the simulation wherein the linear model was used are in every diagram smaller than the ones obtained in the simulation including the detailed instantaneous value model.

Fig. 15. Delivered power in p.u. for machine S2

Fig. 16. Rotor angle of machine S2 in radians relative S1

Fig. 17. Speed of the generator S2

The simulations with the linear model are 4000 times faster than the simulation done with the detailed instantaneous value model.

X. COMPARISONS IN LINEAR ANALYSIS

Here follows linear analysis performed to the system. The system is linearized when it has reached steady state, just before the load at Bus A is increased with 10% at $t = 6$ s.

A. System with the detailed instantaneous value model

Depending on when linear analysis is performed, this model gives different answers since the system topology changes with the thyristors and the control algorithm. In the table below, eigenvalues from three different linearizations within 0.05 s are viewed. Complex eigenvalues with real part greater than -3 are listed.

TABLE I
COMPLEX EIGENVALUES FOR DIFFERENT LINEARIZATIONS

Linearization						
1	$-0.29 \pm j1.58$	$-0.88 \pm j32.1$	$-1.16 \pm j0.84$	$-2.87 \pm j0.63$	$-1.75 \pm j0.45$	$-2.23 \pm j0.47$
2	$-0.29 \pm j1.58$	$-0.88 \pm j32.1$	$-1.16 \pm j0.84$	$-2.87 \pm j0.63$	$+0.72 \pm j0.28$	
3	$-0.29 \pm j1.58$	$-0.88 \pm j32.1$	$-1.16 \pm j0.84$	$-2.87 \pm j0.63$	$+0.32 \pm j0.34$	

The imaginary part is given in Hz.

The eigenvalues in the fifth column are changing and with that means, the power system changes working point.

B. System with the linear model

When using the linear model of the TCSC the following complex eigenvalues with real part greater than -3 are found.

$$\lambda_{1,2} = -1.15 \pm j0.84 \text{ and } \lambda_{3,4} = -2.87 \pm j0.63$$

Also the eleven poles (eigenvalues) we added to the system in section VI can be found among the eigenvalues, but all of them have real parts less than -3 and are therefore not viewed here.

The system with the linear model contains the same eigenvalues, no matter when the linearization is done.

XI. CONCLUSIONS

The paper contains an introduction to why it is necessary to take care of non-linear components before linearizing power systems that include them.

It shows a method to create linear models of non-linear components. A TCSC is such a component and it is linearized in the paper.

Finally, the paper compares the detailed instantaneous value model and the linear model of the TCSC in a network, both in time domain simulations and in linear analysis.

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XIII. REFERENCES

- [1] Lennart Ljung: "System Identification, Theory For The User", Prentice Hall, 1999.
- [2] Prabha Kundur: "Power System Stability and Control", The EPRI Power System Engineering Series, 1994.
- [3] Lennart Ångquist, Gunnar Ingeström, Hans-Åke Jönsson, "Dynamical Performance of TCSC Schemes", CIGRE Session – 1996, Paper 14 - 302.
- [4] Power System Simulation & Analysis Software SIMPOW, User Manual, ABB Power Systems, Västerås, Sweden, 2000.
- [5] H. R. Fankhauser, T. Adielson, K. Aneros, A-A. Edris, L. Lindkvist, and S. Torseng, "SIMPOW – a digital power system simulator", *Reprint of ABB Review*, No. 7, 1990.

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