DETERMINATION OF NON-LINEAR ACOUSTIC PROPERTIES OF PERFORATES USING SINGLE TONE EXCITATION

Hans Bodén and Armin Eslami
Linné Flow Centre, MWL, KTH, SE-10044 Stockholm, Sweden
e-mail: hansbod@kth.se

This paper discusses experimental techniques for obtaining the acoustic properties of in-duct samples with non-linear acoustic characteristics. The methods developed are intended for studies of non-linear energy transfer to higher harmonics for samples accessible from both sides such as perforates or other material used as top sheets in aircraft engine liners and automotive mufflers. New double sided multi-port techniques, using sinusoidal excitation, for characterisation of samples with non-linear properties are developed and experimentally tested. The results of the preliminary experimental tests show that these new techniques can give results which are useful for understanding non-linear energy transfer to higher harmonics.

1. Introduction

When designing perforated mufflers or aircraft engine liners models of the acoustic properties of perforates or other top sheets used are needed. Semi-empirical models for perforates including the effect of flow and high excitation levels have been published see for instance\textsuperscript{1,3}. The quantity used to characterize the sample is in most cases the normal incidence sample impedance defined as the ration between pressure difference over the sample and the particle velocity through the sample. In the linear case the impedance is independent of the sound field but when the sound pressure level is high the perforate impedance will be dependent on the acoustic particle velocity in the holes. The semi-empirical models indicate that the sample impedance is proportional to the peak particle velocity through the sample in the non-linear regime. For pure tone excitation it is obvious that the impedance will be controlled by the acoustic particle velocity at that frequency. If the acoustic excitation is random or periodic with multiple harmonics the impedance at a certain frequency may depend on the particle velocity at other frequencies. Many investigations of nonlinear effects occur-
ring when high amplitude sound waves are incident on perforated plates or orifice plates have been published, see e.g., Refs. 4-9. In many of the early works a standing wave tube with single frequency excitation was used. Mao has studied linear and nonlinear behaviour of so-called micro-perforates. It is generally agreed that the non-linear losses are associated with vortex shedding at the outlet side of the orifice or perforate openings. The effect of sample non-linearity when performing single and multi tone excitation impedance tube measurements were studied in Ref.10. In Ref.11 one-sided and two-sided multi-port techniques for characterization of samples with non-linear properties were outlined. The single sided multi-port techniques applicable when the sample is only accessible from one side such as a complete was further developed and tested in Ref. 12. The two sided multi-port techniques are further developed and experimentally tested in the present paper.

This paper starts with a discussion of the plane wave sound field in a duct where a sample with non-linear properties is mounted. Then new multi-port models for obtaining the acoustic properties of such a sample are developed. These models are inspired by two-port measurement techniques used for characterising linear in-duct components, see e.g. Ref. 13. The new techniques are tested and discussed.

2. Problem formulation

The problem of interest is outlined in Fig. 1 where the sample under test is mounted in a duct with at least two microphones on each side. A loudspeaker mounted in a side branch provides acoustic excitation on side A. The termination at this side is kept fixed during the test. The termination on side B can be changed by varying the length of the termination pipe. Only plane wave propagation is considered. If the incident sound wave to a sample with non-linear characteristics is single frequency there will be reflected and transmitted waves at that frequency but also at other frequencies. The strongest harmonic interaction should be a transfer of signal energy to frequencies three times higher than the excitation frequency. There will also be predominately higher order odd terms, that is 5, 7, 9 times higher than the excitation frequency. For the perforate samples studied here there we will therefore consider excitation at the odd harmonics of the excitation frequency. In Fig. 2 the variation in sound pressure level as the level of the single frequency excitation at 210 Hz is increased is shown. It can be seen that the odd harmonics (1 and 3) dominates the spectrum even though the second harmonic also appears at higher excitation levels. The results are plotted against inverse Strouhal number (1/\(St = \tilde{u}_A/(2\pi fd)\)) based on \(\tilde{u}_A\), the mean peak particle velocity at the sample, where \(f\) is the excitation frequency and \(d\) is the perforate hole diameter.

Figure 1 Two-port test rig.
Figure. 2 Variation in sound pressure level (re. 2 $10^{-5}$ Pa) with inverse Strouhal number as the level of the single frequency excitation at 210 Hz is increased for one of the test cases used in the present study: solid line – 210 Hz, dashed line – 420 Hz, dashed dotted line – 630 Hz.

It is assumed that although there is a sample with non-linear properties at $x=0$ in the duct the non-linearity is only local and linear theory applies for the sound propagation on side A and B of the sample. In the plane wave region we can by measuring the pressure at two positions in the duct decompose the sound field into the waves going in positive and negative x-direction and determine the reflection coefficients.

$$p_{A+}(f) = e^{-jKL} \frac{p(-L,f) + p(-(L+s),f)e^{jks}}{1 - e^{j2ks}},$$

$$p_{A-}(f) = e^{jKL} \frac{p(-L,f) + p(-(L+s),f)e^{-jks}}{1 - e^{-j2ks}},$$

$$p_{B+}(f) = e^{-jKL} \frac{p(L,f) + p((L+s),f)e^{jks}}{1 - e^{j2ks}},$$

$$p_{B-}(f) = e^{jKL} \frac{p(L,f) + p((L+s),f)e^{-jks}}{1 - e^{-j2ks}},$$

$$R_A(f) = \frac{p_{A-}(f)}{p_{A+}(f)},$$

$$R_B(f) = \frac{p_{B+}(f)}{p_{B-}(f)},$$

The sound power towards or away from the sample on sides A and B can now also be expressed as

$$W_{A+}(f) = S \frac{|p_{A+}(f)|^2}{\rho c},$$

$$W_{A-}(f) = S \frac{|p_{A-}(f)|^2}{\rho c},$$

$$W_{B+}(f) = S \frac{|p_{B+}(f)|^2}{\rho c},$$

$$W_{B-}(f) = S \frac{|p_{B-}(f)|^2}{\rho c},$$
where \( \rho \) is the density of air, \( c \) is the speed of sound, \( p(x,f) \) is the sound pressure at \( x \), \( u(x,f) \) is the particle velocity at \( x \), \( p_{A,B+}(f) \) is the amplitude of the pressure wave in the positive \( x \) direction, \( p_{A,B-}(f) \) is the amplitude of the pressure wave in the negative \( x \) direction, \( R_{A,B}(f) \) is the reflection coefficient at \( x = 0 \), \( k \) is the wave number and \( S \) is the duct cross section area.

3. Multi-port models

The basic idea is now to analyze the problem described in section 2 using multi-port models to describe the non-linear energy transfer. Two different versions of the model can be used depending on if the sample is just a perforated or a facing sheet so that both reflection and transmission of acoustic energy is of interest or if the sample is accessible only from one side, such as a complete liner, in which case only reflections will be of interest. In the first case techniques similar to those applied for measuring acoustic two-port data for mufflers and other duct discontinuities will be developed in the following section. The second case has been discussed in a separate paper\(^{12} \).

3.1 Scattering matrix model

The traditional scattering matrix two-port relationship between side A and B of the sample in Fig. 1 can then be written

\[
\begin{bmatrix}
  p_{A-}(f) \\
  p_{B-}(f) \\
  p_{A-}(3\cdot f) \\
  p_{B-}(3\cdot f)
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} & S_{13} & S_{14} \\
  S_{21} & S_{22} & S_{23} & S_{24} \\
  S_{31} & S_{32} & S_{33} & S_{34} \\
  S_{41} & S_{42} & S_{43} & S_{44}
\end{bmatrix}
\begin{bmatrix}
  p_{A+}(f) \\
  p_{B+}(f) \\
  p_{A+}(3\cdot f) \\
  p_{B+}(3\cdot f)
\end{bmatrix},
\]

(11)

giving the relation between the pressure wave amplitudes on each side of the sample on a frequency by frequency basis. If we now assume that the excitation is single frequency an that there is a coupling, transfer of signal energy, to begin with only to a frequency three times the excitation frequency we instead get the following two-port matrix

\[
\begin{bmatrix}
  p_{A-}(f) \\
  p_{B-}(f) \\
  p_{A-}(3\cdot f) \\
  p_{B-}(3\cdot f)
\end{bmatrix} =
\begin{bmatrix}
  S_{11} & S_{12} & S_{13} & S_{14} \\
  S_{21} & S_{22} & S_{23} & S_{24} \\
  S_{31} & S_{32} & S_{33} & S_{34} \\
  S_{41} & S_{42} & S_{43} & S_{44}
\end{bmatrix}
\begin{bmatrix}
  p_{A+}(f) \\
  p_{B+}(f) \\
  p_{A+}(3\cdot f) \\
  p_{B+}(3\cdot f)
\end{bmatrix}.
\]

(12)

We have four equations and 16 unknowns in Eq.(12) so we need to make measurements using at least four independent acoustic states in order to get sufficient information for solving the equation. With \( N \) harmonics we have \( 2N \) equations and \( (2N)² \) unknowns so \( 2N \) acoustic test states would be needed. If the sample is symmetric and there is no mean flow through the sample some of the elements of Eq. (12) should be identical, since the same reflection or transmission should take place from side A or B. This means that the number of unknowns in principle can be reduced. The following identities apply: \( S_{22} = S_{11}, \ S_{21} = S_{12}, \ S_{24} = S_{13}, \ S_{23} = S_{14}, \ S_{42} = S_{31}, \ S_{41} = S_{32}, \ S_{44} = S_{33} \) and \( S_{43} = S_{34} \). This gives a system of four equations with 8 complex unknowns. In this case we therefore only need measurements for two independent acoustic states to solve for the unknowns. If we generalise to \( N \) harmonics we need \( N \) acoustic test states.
We could also assume that the non-linear energy transfer only takes place in the direction from lower frequencies to higher harmonics. In this case a number of terms in Eq. (12) would be equal to zero leading to

\[
\begin{bmatrix}
  p_A(f) \\
  p_B(f) \\
  p_A(3f) \\
  p_B(3f)
\end{bmatrix}
= \begin{bmatrix}
  S_{11} & S_{12} & 0 & 0 \\
  S_{21} & S_{22} & 0 & 0 \\
  S_{31} & S_{32} & S_{33} & S_{34} \\
  S_{41} & S_{42} & S_{43} & S_{44}
\end{bmatrix}
\begin{bmatrix}
  p_A(f) \\
  p_B(f) \\
  p_A(3f) \\
  p_B(3f)
\end{bmatrix},
\]

(13)

which means that this equation can be solved in two steps. First we solve for the ordinary linear two-port for the terms \(S_{11}, S_{12}, S_{21}\) and \(S_{22}\). Then the remaining part gives the non-linear coupling terms and the linear two-port at \(3f\). Here we need at least four independent acoustic state test cases to solve for the latter part. If we in addition assume the same symmetry properties mentioned above we now need just one acoustic test case to solve for \(S_{11}\) and \(S_{12}\). To determine the remaining terms we need two independent acoustic test cases. One further assumption which can be made is that the level of excitation at \(3f\) and higher harmonics is low and that we are in the linear regime at these frequencies. Then the terms \(S_{33}, S_{34}, S_{43}\) and \(S_{44}\) can be determined by a separate measurement using low level excitation at these frequencies. The validity of this assumption can be checked by comparing the level at \(3f\) when the system is excited at frequency \(f\) with the level used for the test with excitation at \(3f\). Equation (13) can then be divided into three separate equations

\[
\begin{bmatrix}
  p_A(f) \\
  p_B(f)
\end{bmatrix}
= \begin{bmatrix}
  S_{11} & S_{12} \\
  S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
  p_A(f) \\
  p_B(f)
\end{bmatrix},
\]

(14)

\[
\begin{bmatrix}
  p_A(3f) \\
  p_B(3f)
\end{bmatrix}
= \begin{bmatrix}
  S_{31} & S_{32} \\
  S_{41} & S_{42}
\end{bmatrix}
\begin{bmatrix}
  p_A(f) \\
  p_B(f)
\end{bmatrix},
\]

\[
\begin{bmatrix}
  p_A(3f) \\
  p_B(3f)
\end{bmatrix}
= \begin{bmatrix}
  S_{33} & S_{34} \\
  S_{43} & S_{44}
\end{bmatrix}
\begin{bmatrix}
  p_A(3f) \\
  p_B(3f)
\end{bmatrix}.
\]

(15)

Here we obviously need two independent acoustic test cases to solve for the unknowns. If we again make the symmetry assumptions we now only need one measurement at \(f\) and one measurement at \(3f\) to determine the data.

4. Experimental results and discussion

In this section results of some first tests of the techniques described in section 3 are presented and discussed.

4.1 Test methodology

Tests of the techniques described in section 3 have been made using a duct as shown in Fig. 1, where the sample was mounted in the centre with microphones on each side and a loudspeaker at one end. The microphone separation was \(s = 0.1\) m and the distance between the sample and the first microphones on each side was \(L = 0.225\) m. This microphone separation gives a measurement range of 172 to 1372 Hz for the two-microphone wave decomposition. The sample studied was a perforate with circular holes with 1 mm diameter, 2 mm plate thickness and 2 % percentage open area. The acoustic load at the downstream (B) side of the sample was varied in order to create dif-
different acoustic test states. A total of 4 different loads consisting of open and closed pipes of different lengths were used. To get the same non-linear forcing at the sample for different downstream acoustic loads it was attempted to keep the particle velocity on the A-side of the sample constant. For the first sample tested a number of different excitation levels were set by varying the input level to the loudspeaker. For the subsequent acoustic loads the particle velocity was set to the same as for the first load by testing with a number of different input level settings until a reasonable agreement was obtained.

4.2 Scattering matrix results

In this section results for the different versions of the scattering matrix models are compared and discussed. First results for the ordinary scattering matrix at the excitation frequency, terms $S_{11}$, $S_{12}$, $S_{21}$ and $S_{22}$ are shown. The results from three different formulations are compared: the full scattering matrix according to Eq. (12), the ordinary scattering matrix according to Eq. (11) and the ordinary scattering matrix assuming symmetry. It can be seen that the agreement between the results obtained using the different models is good. It can also be seen that assumption of symmetry, that is that $S_{11} = S_{22}$, and $S_{12} = S_{21}$, is good.

![Scattering matrix results](image)

**Figure 3.** Absolute value of scattering matrix elements plotted against inverse Strouhal number for excitation at 210 Hz: solid line – ordinary scattering matrix Eq. (11), dashed line – full scattering matrix Eq. (12), dashed dotted line – ordinary scattering matrix with symmetry.
Results for the terms describing the non-linear energy transfer from the excitation frequency to the third harmonic $S_{31}$, $S_{32}$, $S_{41}$ and $S_{42}$ are presented in Fig. 4. The results from four different formulations are compared: the full scattering matrix according to Eq. (12), the full scattering matrix assuming symmetry ($S_{31} = S_{42}$, and $S_{32} = S_{41}$), the scattering matrix with $S_{33}$, $S_{34}$, $S_{43}$ and $S_{44}$ determined from a separate measurement Eq. (15) and the scattering matrix with $S_{33}$, $S_{34}$, $S_{43}$ and $S_{44}$ determined from a separate measurement and symmetry. It can be seen that there are some differences between the results even if all results have the same order of magnitude. These terms are all of the order of magnitude 0.02-0.04 at higher excitation levels, which is an agreement with the level difference of around 30 dB between the fundamental frequency and the third harmonic seen in Fig. 2.

![Figure 4](image_url)

**Figure 4.** Absolute value of scattering matrix elements plotted against inverse Strouhal number for excitation at 210 Hz: solid line – full scattering matrix Eq. (12), dashed line – full scattering matrix with symmetry assumption, dashed dotted line – scattering matrix with $S_{33}$, $S_{34}$, $S_{43}$ and $S_{44}$ determined from separate measurement Eq. (15), dotted line – scattering matrix with $S_{33}$, $S_{34}$, $S_{43}$ and $S_{44}$ determined from separate measurement and symmetry.
5. Conclusions

In the present paper measurements of non-linear acoustic properties of in-duct samples, such as perforates, have been discussed. A number of novel two-port models applicable for single frequency excitation have been suggested and preliminary tests were made for the case of a sample with reflection, transmission and absorption. It was found that the scattering matrix formulation of the two-port gave promising results. Results which can be obtained using these techniques include information about non-linear energy transfer between harmonics for tonal excitation. Preliminary experimental tests were made on a perforate sample showing that these new techniques can indeed give results which are useful for understanding this non-linear energy transfer to higher harmonics.

REFERENCES


