IN-DUCT SOURCE CHARACTERIZATION FOR MULTIPLE SOURCES

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This paper discusses experimental techniques for detecting if there are multiple sources in a duct and obtaining the acoustic characteristics of these sources. Experimental techniques for in-duct source characterization under plane wave conditions in ducts, when we know the location of the source, are well established. In some cases there can however be sources at both ends of a duct. The paper starts with discussing the possibility to, by using a number of flush mounted microphones in the duct, detect sources located on both sides of the test section and to extract the acoustic source characteristics of the sources. First the sound field in a duct with sources at both ends is discussed and described. The theory for experimental determination of source data is then described. A discussion of the consequences of source correlation is included. The methods are first tested using loudspeakers in a duct.

1. Introduction

Techniques for low frequency in-duct source characterisation are well developed for fluid machines [1-2]. There are also some publications on characterisation on flow generated noise sources such as constrictions in ducts, see e.g. [3]. The latter problem is more difficult since the source levels are usually lower and there may be high background levels caused by flow generated noise from other parts of the test rig. This fact inspired the work presented in this paper to detect if there are multiple sources in a duct and obtaining the acoustic source characteristics of these sources. The idea is to use a number of flush mounted microphones in the duct and detect sources located on both sides of the test section. Plane wave propagation in the duct will be assumed which means that sound field decomposition can be performed using the two-microphone technique [4-7].

2. Some duct acoustics

Let us assume that we have at least two microphones placed in a duct between two sources, see Figure 1. The cross section used for the source characterisation is \( x = 0 \) independently of if there is a source to the left, to the right or at both ends. We will here only use two microphones since there is no reason to use more microphones other than to increase the frequency range covered or to measure additional quantities such as the wave number [8]. The sound pressure at microphones 1 and 2 caused by the source to the left will be given by
\( p_1^L(\omega) = p_+^L(\omega)e^{jk_1} + p_-^L(\omega)e^{-jk_1} = p_+^L(\omega)(e^{jk_1} + R^R(\omega)e^{-jk_1}), \quad (1) \)

\( p_2^L(\omega) = p_+^L(\omega)e^{jk(L+s)} + p_-^L(\omega)e^{-jk(L+s)} = p_+^L(\omega)(e^{jk(L+s)} + R^R(\omega)e^{-jk(L+s)}), \quad (2) \)

where \( p_+^L, p_-^L \) are the amplitudes of the pressure waves in positive and negative x-direction, \( R^R \) is the reflection coefficient at \( x=0 \) in the right direction and \( k \) is the wave number. The sound field caused by the source to the right will be given by

\( p_1^R(\omega) = p_+^R(\omega)e^{-jk_1} + p_-^R(\omega)e^{jk_1} = p_+^R(\omega)(e^{-jk_1} + R^L(\omega)e^{jk_1}), \quad (3) \)

\( p_2^R(\omega) = p_+^R(\omega)e^{-jk(L+s)} + p_-^R(\omega)e^{jk(L+s)} = p_+^R(\omega)(e^{-jk(L+s)} + R^L(\omega)e^{jk(L+s)}), \quad (4) \)

where \( p_+^R, p_-^R \) are the amplitudes of the pressure waves in positive and negative x-direction and \( R^L \) is the reflection coefficient at \( x=0 \) in the left direction. The sound field caused by both sources is therefore given by

\( p_1(\omega) = p_+^L(\omega)(e^{jk_1} + R^R(\omega)e^{-jk_1}) + p_-^L(\omega)(e^{-jk_1} + R^L(\omega)e^{jk_1}), \quad (5) \)

\( p_2(\omega) = p_+^L(\omega)(e^{jk(L+s)} + R^R(\omega)e^{-jk(L+s)}) + p_-^R(\omega)(e^{-jk(L+s)} + R^L(\omega)e^{jk(L+s)}). \quad (6) \)

In these expression we have four unknowns: \( p_+^L(\omega), p_-^R(\omega), R^R(\omega) \) and \( R^L(\omega) \) which we want to determine.

![Figure 1: Sketch of measurement configuration](image)

The sources are described by linear time-invariant frequency domain one-port models which can be completely characterised by two complex parameters: the source strength \( (p^s) \) and the source reflection coefficient \( (R_s) \) (or alternatively the source impedance). The behaviour of the one-port (see Figure 2) can in the frequency domain, be described by \([1-2]\):

\( p_+(\omega) = R_s(\omega)p_- (\omega) + p_{s_1}(\omega) = R_s(\omega)R(\omega)p_- (\omega) + p_{s_1}(\omega), \quad (7) \)

where \( p_+ , p_- \) again are the amplitudes of the pressure waves in positive and negative x-direction, \( R_s \) is the source reflection coefficient and \( p_{s_1} \) is the source strength at \( x=0 \). The source strength can be interpreted as the pressure generated by the source when the system is reflection free. Equation (7) also gives the relation between the source strength and the amplitude of the pressure wave
\[ p_{Ss}(\omega) = p_s(\omega)(1 - R_s(\omega)R(\omega)), \]  
\[ q_s(\omega) = \frac{Z_s(\omega)q(\omega) - q_s(\omega)}{Z(\omega)} = \frac{Z_s(\omega)}{Z(\omega) + Z_s(\omega)}, \]  
where \( p \) and \( q \) are acoustic pressure and volume velocity, respectively, and \( Z_0 \) is the characteristic impedance of the fluid divided by the duct cross section area. The source impedance \( Z_s \) represents the normalised acoustic impedance seen from the reference cross-section towards the source. Figure 3 shows the equivalent acoustic circuit for a linear and time invariant source expressed in these alternative forms. The representations of the source described by (7) and (9)-(10) are theoretically equivalent and it is possible to go from one representation to the other by using the relations given below:

\[ q_s(\omega) = \frac{p_s(\omega)}{Z_0 Z_s(\omega)}, \]  
\[ q_s(\omega) = \frac{2p_s(\omega)}{Z_0(1 + R_s(\omega))}, \]  
\[ Z_s(\omega) = \frac{(1 + R_s(\omega))}{(1 - R_s(\omega))}, \]  
\[ R_s(\omega) = \frac{(Z_s(\omega)-1)}{(Z_s(\omega)+1)}. \]

It should be noted that the pressure source formulation in Figure 3 a) is not valid if \( Z_s \to \infty \) (\( R_s = 1 \)) which gives a constant volume velocity source and the volume velocity source formulation in Figure 3 b) is not valid when \( Z_s = 0 \) (\( R_s = -1 \)) which gives a constant pressure source.
3. Source characterisation

The methods for source characterisation are divided into two main categories: direct or external source methods and indirect or multi-load methods [1-2]. In the direct techniques the passive properties are first measured using an external source and the source strength is then determined in a separate measurement. For the indirect techniques a number of acoustic loads are applied and the sound pressure is measured at the source cross section. The active and passive source data is then determined by solving a linear system of equations. Equation (15) and (16) are the basis for the source characterisation where we will consider three different cases. In the first case we assume that it is possible to first measure $R_S^L(\omega)$ and $R_S^R(\omega)$ separately for instance by using only a source on the left side and then turn this off and run a source on the right side. In the second case we will assume that it is only possible to measure $R_S^L(\omega)$ since we control the source on the left but not the source on the right. In the third case we will assume that it is not possible to measure the reflection coefficient separately so that all the four unknowns have to be determined from the same set of measurements.

3.1 Case 1: Source characterisation by the direct method

The reflection coefficients are first determined using the conventional two-microphone technique. This means that the passive termination or source data ($R_S^L(\omega)$ and $R_S^R(\omega)$) on the left and right sides are known. In this case we have two equations and two unknowns so it is sufficient to measure the sound pressure at two positions in the duct. Equation (15) and (16) gives the equation system according to (17) for determination of the unknown source strengths $p_S^L(\omega)$ and $p_S^R(\omega)$.
The usual frequency range limitations [6-7] for two-microphone impedance measurements apply for this expression. It should be noted that this formulation require a phase reference so that the complex pressures can be measured. This is fine as long as a reference signal can be measured and the two sources are correlated. In the general case they must be assumed to be uncorrelated and the appropriate form is then the source cross spectrum matrix

\[
G_{SS}(\omega) = \begin{pmatrix}
p_{S+}^L(\omega) & p_{S+}^L(\omega) & p_{S+}^L(\omega) & p_{S+}^L(\omega) \\
p_{S-}^L(\omega)^* & p_{S-}^L(\omega)^* & p_{S-}^L(\omega)^* & p_{S-}^L(\omega)^* \\
G_{LL}(\omega) & G_{RL}(\omega) \\
G_{LR}(\omega) & G_{RR}(\omega)
\end{pmatrix},
\]

which can be obtained by matrix operations on (17). It can also be used to calculate the source coherence giving information about the correlation of the two source terms

\[
\gamma_{SS}^2(\omega) = \frac{G_{RL}(\omega)G_{LR}(\omega)}{G_{LL}(\omega)G_{RR}(\omega)}.
\]

### 3.2 Case 2 and 3: Source characterisation by a combination of direct and in-direct methods

If we only can apply a loudspeaker source to the left and measure the reflection coefficient on the right side \(R_S^L(\omega)\) we have two equations and three unknowns. By changing the system on the left side giving another reflection coefficient \(R_S^R(\omega)\) we get one more unknown but two more equations from measuring the pressure at the two microphone positions. It should therefore be possible to solve the system of equations and obtain the unknowns. The system of equations will however be non-linear which means that the number of linear unknowns grows to six as seen in (20)

\[
\begin{bmatrix}
p_1(\omega) \\
p_2(\omega) \\
p_3(\omega)
\end{bmatrix} = \begin{bmatrix}
1 \\
1 - R_S^L(\omega)
\end{bmatrix} \begin{bmatrix}
e^{i\beta L} + R_S^R(\omega)e^{-i\beta L} \\
e^{i\beta(L+\pi)} + R_S^R(\omega)e^{-i\beta(L+\pi)} \\
e^{-i\beta L} + R_S^L(\omega)e^{i\beta L}
\end{bmatrix} \begin{bmatrix}
e^{-i\beta L} + R_S^L(\omega)e^{i\beta L} \\
e^{-i\beta(L+\pi)} + R_S^L(\omega)e^{i\beta(L+\pi)} \\
e^{i\beta L} + R_S^R(\omega)e^{-i\beta L}
\end{bmatrix} \begin{bmatrix}
p_{S+}^L(\omega) \\
p_{S-}^L(\omega) \\
p_{S-}^L(\omega)
\end{bmatrix} + \begin{bmatrix}
p_{S+}^R(\omega) \\
p_{S-}^R(\omega) \\
p_{S-}^R(\omega)
\end{bmatrix} \begin{bmatrix}
p_1(\omega) \\
p_2(\omega) \\
p_3(\omega)
\end{bmatrix} = \begin{bmatrix}
p_1(\omega) \\
p_2(\omega) \\
p_3(\omega)
\end{bmatrix},
\]

Increasing the number of loads further will not help as the number of unknowns will grow at the same rate as the number of equations. It is therefore not possible to solve this problem by linear system theory. If the reflection coefficient on the right side is also considered as an unknown quantity in (20) the situation will obviously be worse. It can therefore be concluded that the only option is using direct source characterisation techniques according to section 3.1.

### 4. Results and discussion

This section presents results from some tests of the source characterisation techniques described in section 3 using loudspeaker sources in a duct.
4.1 Test setup

A sketch of the test rig was shown in Figure 1. Two \( \frac{1}{4}\)-inch B&K condenser microphones, at a distance of \( s = 100 \text{ mm} \), were used for the measurements. The test duct had an inner diameter of 43 mm. There was one loudspeaker at each end of the duct. The loudspeaker on the right also provided the duct termination on the right side while the loudspeaker on the left side was mounted in a side branch giving the possibility to change the acoustic loading by varying the termination duct. The output signals exciting the loudspeakers were controlled from the data acquisition system and were measured. Cross-spectra, frequency response functions and coherence functions using the two output signals and microphone 1 as the reference along with auto-spectra of all signals were measured and used for the subsequent analysis.

4.2 Experimental results

As described in section 3 the reflection coefficients in the right and left direction at the source cross section are first measured by first running the loudspeaker on the left and then running the loudspeaker on the right. Figure 4 shows the magnitude of these reflection coefficients which is part of the measured source data. A reference estimate of the source strength can also be obtained from the same data. When we know the reflection coefficients and measure the amplitude of the pressure wave in the direction away from the source under test we can obtain the source strength from (8). This result can then be compared to the result obtained when both sources are running simultaneously. Figure 5 shows this comparison for the source strength of the source on the right expressed as sound pressure level. In this case the signal exciting the two was the same random noise that is the two sources were perfectly correlated. It can be seen that the agreement is very good in this case showing that the technique works in principle.

![Figure 4: Magnitude of reflection coefficients; black - \( R^{R}_{S}(\omega) \), red - \( R^{L}_{S}(\omega) \)](image)

If the sources are not correlated it will not be possible to determine complex pressures to apply source characterisation according to (17). Instead the source cross spectrum matrix according to (18) can be determined which is based only on cross-spectra and auto-spectra for the two microphone signals and does not require any reference signal. Figure 6 shows the source strength for the source on the right side for this case. It can be seen that the agreement with the result obtained with only the source on the right running is still good. Figures 7 and 8 shows the source coherence and the ordinary coherence between the two microphone signals (\( \gamma_{12}^{2}(\omega) \)) for correlated and uncorrelated sources. It can be seen that the existence of additional un-correlated sources can be seen from the source coherence as well as from the ordinary coherence between the microphones.
If there is only a source active on the left side we should get a significantly lower source strength when trying to obtain source data for a source on the right side. Figure 9 shows the source strength obtained using (17) and Figure 10 shows the source strength obtained using (18) compared to the source strength obtained when there is a source active on the right. It can be seen that using (17) with the phase information between the reference signal and the microphone signal retained gives the correct result, i.e. that there is no source to the right while using (18) does not indicate this as clearly.

**Figure 5:** Source strength for the source on the right side from (17); black – determined with just right side source running, red – determined with both sources running, correlated sources.

**Figure 6:** Source strength for the source on the right side from (18); black – determined with just right side source running, red – determined with both sources running, uncorrelated sources.

**Figure 7:** Coherence; black – source coherence determined with both sources running, correlated sources, red – coherence between microphone signals.

**Figure 8:** Coherence; black – source coherence determined with both sources running, un-correlated sources, red – coherence between microphone signals.
5. Conclusions

Experimental techniques for detecting if there are multiple sources in a duct and obtaining the acoustic characteristics of these sources have been developed. The techniques developed use at least two flush mounted microphones in the duct to detect sources located on both sides of the test section and to extract the acoustic source characteristics of the sources. It was concluded that the only possibility is to use so called direct or external source characterisation techniques where the active and passive source data are determined in two separate measurements. The consequences of source correlation were discussed and a source characterisation technique valid also for the case of uncorrelated sources was developed. The first tests using loudspeaker sources in a duct showed that the methods develop can yield good results.

![Figure 9](source_strength_right_side_17.png)

**Figure 9:** Source strength for the source on the right side from (17); black – determined with just right side source running, red – just right side source running.

![Figure 10](source_strength_right_side_18.png)

**Figure 10:** Source strength for the source on the right side from (18); black – determined with just right side source running, red – just right side source running.

REFERENCES