Identification of Dominant Inter-Area Modes in the Eastern Interconnection from PMU data of the FRCC 2008 Disturbance: an Eigensystem Realization Algorithm Illustration

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Introduction
In this Section we identify two of the main electromechanical modes in the US Eastern Interconnection (EI) and examine of the nature of power system oscillations emerging after a system-wide disturbance. The disturbance in question was a major event that occurred on February 26, 2008 within the service area of the Florida Reliability Coordinating Council (FRCC) Bulk Electrical system [1], [2], [3], [4], [5]. The disturbance data as measured by several PMUs across the Eastern Interconnection were made available for analysis by several operators and were obtained from TVA’s Super Phasor Data Concentrator (SPDC).

The application of the ERA here does have the goal of obtaining a system realization of the same order of the system from where the measurements were obtained, which would be impractical given the dimension of the US EI. Also, we do not attempt to obtain a system realization that exactly reproduces the sequence of data measured by the PMUs. Our true goal is to obtain a system realization that closely represents the underlying electromechanical dynamics of the system. To this aim, the important issue of model order estimation is relevant. We provide general guidelines that aid in this respect.

We apply the Eigensystem Realization Algorithm to decompose the PMU measurements into the different modal components contained within the available PMU measurements, and examine the dominant features of inter-area mode distribution in power networks. Two dominant oscillatory modes are identified and the individual behavior of each mode component is analyzed.

The 2008 Florida Disturbance
Major disturbances involving significant amounts of generation and load losses are precursors of power system oscillations that propagate throughout the whole system via the high-voltage transmission network. One of such events took place on February 26, 2008 when the Florida Reliability Coordinating Council (FRCC) Bulk Electrical system experienced a disturbance whose effect was spread across the U.S. Eastern Interconnection. The inception of the disturbance was the result of delayed clearing of a three-phase fault on a 138 kV switch in one of the substations of Florida Power & Light near Miami. Local primary protection and local back-up breaker failure protection were previously disabled for troubleshooting. As a result, remote backup protection relays performed delayed clearing of the 138 kV fault.

In summary [2], [3], the outcome of this disturbance was a generation loss of approximately 2,500 MW near the fault and 1,800 MW in the FRCC system. In addition, 2,300 MW of load were shed between under-frequency load shedding zones [2], [3]. With the exception of the trip of 25 transmissions lines involved in the remote

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clearing, and other minor transmission line trips, the FRCC system did not experience additional line outages and remained intact. The FRCC system remained connected to the EI during the course of the disturbance. As a result, this large disturbance excited several interarea oscillatory modes across the Eastern Interconnection [1], [4], [5].

Next, we study the main features of the Florida disturbance by analyzing archived phasor measurement data. The PMUs considered in this analysis are Manitoba, near the city of Winnipeg, Canada. Maine near Bangor, Maine. Florida near Jacksonville, Florida. West Tennessee (W. Tenn.), near Memphis, Tennessee, and East Tennessee (E. Tenn.), near Knoxville, Tennessee, as shown in Fig. 1a. In Fig. 1b we plot the bus frequency measured during the disturbance by the PMUs, showing the wide-area impact of the disturbance. The steady state frequency deviation is approximately $\Delta f = 30$ mHz, while the electromechanical swing is propagated from Florida, to E. Tenn. and W. Tenn., and subsequently to Manitoba, and finally Maine. We aim to analyze the oscillatory components and characteristics contained in these measurements.

![PMU Locations and recorded measurements during the 2008 Florida Disturbance](image)

**System Realization**

We aim to analyze the oscillatory components and characteristics contained within the measurements discussed in the last section. For this purpose, we use the Eigensystem Realization Algorithm (ERA) [7], [6] to identify the individual modal components in each measurement. The ERA was applied to the voltage magnitude, voltage angle, and active power flow measurements available from each PMU. A general outline of the different steps used in the system realization process is the following:

- **Step 1.** Data pre-processing and conditioning to generate appropriate signals for ERA
- **Step 2.** Convert the pre-processed data (impulse response signals) into Markov parameters
- **Step 3.** Construct a block Hankel matrix $H(0)$ by arranging the Markov parameters into blocks
- **Step 4.** Determine the order of the system by computing the singular values of the Hankel matrix $H(0)$
- **Step 5.** Using a Hankel matrix shifted at $k=1$, i.e. $H(1)$, obtain a system realization
- **Step 6.** Quantify the system noise and select modes for a reduced system realization.
- **Step 7.** Obtain a reduced model of the system using the eigenvalues selected Step 6, compute the impulse response of the reduced model and compare them with the pre-processed data.
  - Repeat Step 6 and Step 7 until an acceptable reduced model has been obtained.

We next explain the steps above through an application example using the Florida disturbance data.
Step 1. Data pre-processing and conditioning to generate appropriate signals for ERA

Data pre-processing and conditioning is a crucial step for a successful application of different realization techniques. This step has not been entirely documented in different studies and may be an unattended aspect affecting the estimations obtained from the subsequent realizations. Here we perform two simple steps for pre-processing the data prior to apply the Eigensystem Realization Algorithm, for this we use the voltage magnitude measurements at the different buses shown in Fig. 2a (which have already been per unitized), and the voltage phase angles (not shown in Fig. 2a). To detrend the data we compute the mean of each measurement and subtract it from each corresponding voltage, as a result the pre-processed data in Fig. 2b is obtained.

![Detrending the PMU measurements of the voltage magnitude at each bus](image1)

(a) Measured voltage magnitudes

(b) Detrended measurements

Figure 2: Detrending the PMU measurements of the voltage magnitude at each bus

![Selected window of data for system identification](image2)

Figure 3: Selected window of data for system identification

Perhaps the most difficult step in pre-processing the data is to select a proper window of data that contains the ringdown which will be used for system realization, and at the same time, does not longer contain non-linearities which are evident after the on-set of the disturbance. This is more of an art than a science, and the selection of this window is done by careful judgment of the analyst. For this data set, we have selected a window shown in Fig. 3 which has produced good estimates, as discussed below.
Step 2. Convert the pre-processed data (impulse response signals) into Markov parameters

Markov parameters can be obtained by different means [6],[9], here we obtain it from the response of the power system conceptualizing it as an “impulse response”. This conceptualization is important because from the Markov parameters, the embedded system matrices $A$, $B$, $C$, and $D$ can be extracted. The impulse response used for this example is a measurement matrix $y$ which contains the bus voltage magnitudes (5) and the bus voltage angles (5), hence $y$ is of size $361 \times 10$, as shown in Fig. . The corresponding time vector if of size $1 \times 361$, and the number of inputs is 1 and the number of outputs is 10.

![Impulse Response for Sys. Id.](image1)

Time in seconds with origin at: 26/02/2008: 26/02/2008 - 18:10:04.333h

![Impulse Response for Sys. Id.](image2)

Time in seconds with origin at: 26/02/2008: 26/02/2008 - 18:10:04.333h

Figure 4: Input Impulse Response used in ERA

Let the Markov parameters, $\beta_0^{(k)}$, be given by

\[
\begin{align*}
Y_0 &= D = \beta_0 \\
Y_1 &= C B = \beta_0^{(1)} \\
Y_2 &= C A B = \beta_0^{(2)} \\
&\vdots \quad \vdots \quad \vdots \\
Y_k &= C A^{k-1} B = \beta_0^{(k)}
\end{align*}
\]  

(1)

where $Y_k$ is the corresponding measurements from the impulse response matrix $y$, whose four first rows ($k = [0-3]$) are given by
Markov parameters in $\beta_0$, can be obtained directly from the impulse response at $k=0$, these correspond directly to the $D$ matrix:

$$
\begin{bmatrix}
0.6362 & -0.4942 & 0.4804 & 0.1891 & 0.1874 & 0.0044 & -0.1037 & -0.0065 & -0.0062 & 0.0069 \\
0.6442 & -0.4875 & 0.4986 & 0.1638 & 0.1694 & 0.0034 & -0.0935 & -0.0081 & -0.0056 & 0.0071 \\
0.6507 & -0.4800 & 0.5166 & 0.1382 & 0.1502 & 0.0019 & -0.0832 & -0.0091 & -0.0053 & 0.0072 \\
0.6555 & -0.4715 & 0.5332 & 0.1123 & 0.1302 & 0.0007 & -0.0729 & -0.0091 & -0.0053 & 0.0072 \\
0.6588 & -0.4625 & 0.5495 & 0.0864 & 0.1099 & -0.0006 & -0.0626 & -0.0096 & -0.0049 & 0.0074 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\end{bmatrix}
$$

(2)

The next set of Markov parameters, $\beta_0^{(1)}$, are obtained from the impulse response at $k=1$, these being

$$
\begin{bmatrix}
0.6442 & -0.4875 & 0.4986 & 0.1638 & 0.1694 & 0.0034 & -0.0935 & -0.0067 & -0.0061 & 0.0071 \\
\end{bmatrix}^T
$$

(3)

where the superscript {} indicates the input channel from the $y$ measurement matrix.

Similarly for $\beta_0^{(2)}$ at $k=2$ the parameters are

$$
\begin{bmatrix}
0.6507 & -0.4800 & 0.5166 & 0.1382 & 0.1502 & 0.0019 & -0.0832 & -0.0081 & -0.0056 & 0.0071 \\
\end{bmatrix}^T
$$

(4)

For $\beta_0^{(3)}$ at $k=3$ the parameters are

$$
\begin{bmatrix}
0.6555 & -0.4715 & 0.5332 & 0.1123 & 0.1302 & 0.0007 & -0.0729 & -0.0091 & -0.0053 & 0.0072 \\
\end{bmatrix}^T
$$

(5)

and finally for $\beta_0^{(4)}$ the Markov parameters are given by

$$
\begin{bmatrix}
0.6588 & -0.4625 & 0.5495 & 0.0864 & 0.1099 & -0.0006 & -0.0626 & -0.0096 & -0.0049 & 0.0074 \\
\end{bmatrix}^T
$$

(6)

Markov parameters continue to be determined until all measurements in $y$ have been processed. These first few parameters can be used to illustrate other parts of the ERA algorithm.

**Step 3. Construct a block Hankel matrix $H(0)$ by arranging the Markov parameters into blocks**

A Hankel matrix composed by the Markov parameters is now constructed, in general, this Hankel matrix has the following form

$$
H(0) = \begin{bmatrix}
Y_1 & Y_2 & \cdots & Y_p \\
Y_2 & Y_3 & \cdots & Y_{p-1} \\
\vdots & \vdots & \ddots & \vdots \\
Y_p & Y_{p+1} & \cdots & Y_{p+\gamma-1} \\
\end{bmatrix}
$$

(8)

where $p$ and $\gamma$ are integers such that $\gamma r \geq pm$. Note that $m$ is the number of outputs and $r$ is the number of inputs. Observe that $Y_0 = D$ is not included in $H(0)$.

For this example, using the Markov parameters in (4)-(6), the four upper right vectors of the Hankel matrix are
Step 4. Determine the order of the system by computing the singular values of the Hankel matrix $H(0)$

The ERA algorithm uses Hankel matrices composed by the Markov parameters to compute the system matrices $A$, $B$, and $C$. Note that matrix $D$ is readily computed from the Markov parameters $\beta_0$ in (3). Firstly, the ERA starts by factorizing the Hankel matrix $H(0)$ by performing singular value decomposition

$$H(0) = R \Sigma S^T$$

where the columns of $R$ and $S$ are orthonormal and $\Sigma$ is a rectangular matrix containing the singular values $\sigma_i$, $i = 1, 2, \ldots, n$, meaning that $H(0)$ is of rank $n$ which is the order of the system, in the example in this Section the rank is 40. A plot of the computed singular values for this example is shown in Fig. 5.

Although the matrix $H(0)$ is of full rank, which generally speaking would be the order of the system, this might not be desirable for many purposes. Measurement noise, nonlinearities, and computer round off contribute for $H(0)$ to be of full rank. The goal should not be to construct a realization that reproduces the input sequence of data exactly, but, that encapsulates the underlying dynamics of the system. For the purpose of analyzing inter-area modes this is relevant; a realization of a large dimension might fail to capture the true nature of the overall inter-area dynamics, and thus, the insight gained from the system identification would not be very useful. The approach that we have used to accomplish a reasonable system realization described below.
Step 5. Using a Hankel matrix shifted at $k=1$, i.e. $H(1)$, obtain a system realization of large order

A shifted Hankel matrix at $k=1$ is calculated as

$$
H(1) = \begin{bmatrix}
Y_2 & Y_3 & \cdots & Y_{r+1} \\
Y_3 & Y_4 & \cdots & Y_{r+2} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{p+1} & Y_{p+2} & \cdots & Y_{p+r}
\end{bmatrix}
$$

where the Markov parameters of the four upper left corner elements are given by (5)-(7). As shown in Error! Reference source not found., by writing the Hankel matrix in terms of matrices $A$, $B$, and $C$ the following relationship is obtained

$$
H(1) = R_n \Sigma_n^{1/2} A \Sigma_n^{1/2} S_n
$$

Hence the state matrix can be computed from the Hankel matrix $H(1)$, and as described in [6], matrices $B$ and $C$ can be obtained from the following equations

$$
\hat{A} = \Sigma_n^{-1/2} R_n H(1) \Sigma_n^{-1/2}, \quad \hat{B} = \Sigma_n^{1/2} S_n^T E_r, \quad \hat{C} = E_m^T R_n \Sigma_n^{1/2}
$$

where the $\hat{\cdot}$ indicates a realization of not necessarily order $n$, but of large order. Defining the null matrix of order $i, O$, and the identity matrix $I_i$ of order $i$ also, then $E_r^T = [I_r \ O \ \cdots \ O_r]$ and $E_m^T = [I_m \ O_m \ \cdots \ O_m]$, refer to [6] for details. In general, it can be said that $B$ is formed by the first $r$ columns of $\Sigma_n^{1/2} S_n^T$, and $C$ is formed by the first $m$ rows of $R_n \Sigma_n^{1/2}$ [9].

For this example, we have obtained a large order realization, of order 40. It is possible to compare the impulse response of the realization against the pre-processed data, as shown in Fig. 6 where only the voltage magnitude components are shown. Although this realization fits well the data, a model of order 40 might have too many eigenvalues close to each other which arise when the method is trying to fit the noise exactly. To obtain a better realization we turn to the discussion in the next step.
Step 6, Quantify the system noise and select modes for a reduced system realization.

There are many available techniques for this step, however, a simple technique is to choose an appropriate number of eigenvalues to be included in a reduced order realization. The discrete model computed from (13) can be transformed to a continuous time model, from here mode damping and frequencies can be computed from the eigenvalues of the continuous time state matrix \( A \). This is done for the large order realization, the continuous time state matrix is of order 40 and hence there are 40 eigenvalues. In Table 1 we present only a subset of those that are relevant to the discussion. The complex pairs are ordered in terms of their energy, it can be observed from Mode 1 that one of the issues of using a large model is noise over fitting, this negative damping mode has been added to fit the data exactly, but a realization as such would impose difficulties if the model were to be used for feedback control design. In addition, for inter-area mode analysis the frequency range of analysis is between 0.1-2 Hz, so Mode 2-3 is not necessary for our purpose. Therefore, only the complex pairs in Mode 4-7 have been used to obtain a two-mode realization of the inter-area behaviour in the US Eastern Interconnection.

Table 1: A subset of the estimated mode damping and frequencies from the large order realization
(The values highlighted in green are used to construct the reduced order realization)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Freq. (rad/s)</th>
<th>Freq. (Hz)</th>
<th>Damping</th>
<th>Residue</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0865</td>
<td>0.0000</td>
<td>-1.0000</td>
<td>0.4286</td>
<td>40.4745</td>
</tr>
<tr>
<td>2</td>
<td>0.5919</td>
<td>0.0272</td>
<td>0.9575</td>
<td>1.8385</td>
<td>2.0756</td>
</tr>
<tr>
<td>3</td>
<td>0.5919</td>
<td>0.0272</td>
<td>0.9575</td>
<td>1.8385</td>
<td>2.0756</td>
</tr>
<tr>
<td>4</td>
<td>1.4290</td>
<td>0.2227</td>
<td>0.2026</td>
<td>5.6925</td>
<td>1.0214</td>
</tr>
<tr>
<td>5</td>
<td>1.4290</td>
<td>0.2227</td>
<td>0.2026</td>
<td>5.6925</td>
<td>1.0214</td>
</tr>
<tr>
<td>6</td>
<td>3.0703</td>
<td>0.4868</td>
<td>0.0877</td>
<td>0.1893</td>
<td>0.9725</td>
</tr>
<tr>
<td>7</td>
<td>3.0703</td>
<td>0.4868</td>
<td>0.0877</td>
<td>0.1893</td>
<td>0.9725</td>
</tr>
</tbody>
</table>

Step 7. Obtain a reduced model of the system using the eigenvalues selected Step 6, compute the impulse response of the reduced model and compare them with the pre-processed data

Using the eigenvalues in Table 1 a reduced order realization is obtained, the continuous time state matrices for this realization are

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4 In this illustration we have used MATLAB for our analysis, the conversion from a discrete to continuous time is achieved using the function `d2c` and using the zero order hold method.
\[
A = \begin{bmatrix}
0.9893 & -0.0468 & 0 & 0 \\
0.0468 & 0.9893 & 0 & 0 \\
0 & 0 & 0.9859 & -0.1009 \\
0 & 0 & 0.1009 & 0.98599
\end{bmatrix}
\]

(14)

\[
B = \begin{bmatrix}
0.2878 \\
1.1959 \\
-0.1324 \\
-0.0698
\end{bmatrix},
C = \begin{bmatrix}
-0.0685 & 0.6692 & 0.0491 & -0.0099 \\
0.1475 & -0.3274 & -0.1370 & -0.0509 \\
0.4298 & 0.5781 & 0.0373 & 0.0060 \\
0.2878 & 0.6417 & 0.2315 & 0.0627
\end{bmatrix}, \text{ and } D = \begin{bmatrix}
0.6362 \\
-0.4942 \\
0.4804 \\
0.1891
\end{bmatrix}
\]

(15)

It should be noted that the mode frequencies and damping are those of the selected modes in Table 1, moreover, eigenvectors can be also computed from (14).

The impulse response of (15) overlaid to the pre-processed data is shown in Fig. 7. Observe that this second order realization captures the main dynamics of interest in inter-area oscillations phenomena; this will be further discussed in the next section.

![Impulse Response of the Reduced Order System Realization](image)

**Figure 7: Impulse Response of the Reduced Order System Realization**

**Analysis of the Identified Electromechanical Modes**

The identified low-frequency interarea modes bear a frequency of 0.22 Hz and 0.49 Hz. Similar mode frequencies for this event have been identified by other analysts using other techniques [1], [4], [5]. In Figs. 8 and 9 the voltage measurements of each PMU are shown along with their ERA approximation for the 0.22 and 0.49 Hz components of the signals. Note that the 0.22 Hz component is prominent in Maine, Florida and Manitoba, while the 0.49 Hz component has much more influence in West Tennessee's and East Tennessee's voltage.
All identified components for the 0.22 Hz mode in the voltage angle are shown in Figs. 10 (a), and for the 0.49 Hz mode they are shown in Figs. 10 (b). A snapshot of the voltage angle components is taken at $t=3$ sec., and is used for the projection of the 0.22 Hz mode shown in Fig. 10 (c), and for the 0.49 Hz mode a snapshot of the voltage angle components is taken at $t=1.93$ sec., and used for the projection in Fig. 10 (d). The starting time $t=0$ sec corresponds to 18:10:04.333 hrs. From the voltage angles of the 0.22 Hz mode we note that Florida oscillates against Maine and Dorsey, that is, it is a North vs South mode.
Figure 10: Voltage Angle Inter-area Oscillations and Projections

The 0.49 Hz mode is more difficult to analyze from this limited data set. However, it can be noted that the voltage angle at West Tennessee, East Tennessee, and Florida have the largest oscillations while Dorsey and Maine have a less significant contribution. More important, West Tennessee and East Tennessee are in anti-phase suggesting that the pivot of the oscillation occurs somewhere between those locations.

It is also possible to analyze how the power oscillations for each mode distribute in the different lines measured by the PMUs, this is discussed with more details in.

The single most important observation that could be made about the oscillations discussed above is the following: for all of the network variables, the independent modal components do not peak at the same instants. This is in fact a time delay between the modal components. This time delay can be rationalized as a phase shift between the modal components in the phase domain. In [] the origin of this phase shift is explained by analyzing the mode shapes of a test network and by providing analytical closed form expressions.
References


