On the Concept of Moving Magnetic Field Lines

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More than 99% of the known matter in the universe is ionized into the plasma state. This cosmical plasma is generally permeated by magnetic fields. Hence, magnetic fields, and the concepts of magnetic field lines and magnetic field line motion, are of fundamental interest in the physics of space plasmas, including those that constitute the near environment of the Earth and other planets.

W. Belcher discussed the concept of magnetic field line motion as a useful tool in teaching electromagnetic theory in an article in Eos [Belcher, 2005]. However, there are some important limitations and pitfalls to this concept, none of which is mentioned in the article by Belcher.

The common definition of magnetic field line velocity, used also by Belcher [2005], is

\[ \mathbf{v} = \mathbf{E} \times \mathbf{B}/B^2. \]

This construct is a useful tool for visualizing the temporal evolution of a magnetic field configuration under conditions where this approach produces the same answer as does Maxwell’s equations. However, it has been shown [Newcomb, 1958; Langmuir, 1963; Mozer, 2005] that the same answer is obtained only when the fields satisfy the requirement

\[ \mathbf{B} \times \text{curl} [\mathbf{B} \times \mathbf{E}/B^2] = 0. \]

This equation is most easily satisfied when the parallel electric field (the electric field component parallel to the magnetic field line) is zero.

The purpose of these comments is to urge caution in using the moving magnetic field line construct because two kinds of problems have arisen from its misuse.

The first kind of problem is that the construct of moving field lines is sometimes used to produce the incorrect magnetic field evolution in cases where equation (2) is not satisfied. In fact, the most interesting plasma physics occurs precisely where and because this equation is not satisfied, such as the auroral acceleration region, magnetic field reconnection, turbulence, shocks, and many wave modes.

The second concern is that the construct of moving field lines is sometimes confused with the concept of moving flux tubes. A flux tube can be thought of as an ensemble of field lines that are identified by their low-energy plasma, which moves at the \( \mathbf{E} \times \mathbf{B}/B^2 \) velocity. Some researchers have asserted that as the plasma moves from region A to region B at this velocity, the field lines that were at A are later at B, so the magnetic field lines moved together with the plasma.

This conclusion is wrong for two reasons. First, it is meaningless to assert that a field line that was at A is now at B, because there is no way to identify or distinguish one magnetic field line from another. Second, the concept of moving magnetic field lines is reasonable if it is used only for visualizing the temporal evolution of the magnetic field, and then, only if equation (2) is satisfied. This point is emphasized by the fact that there are an infinite number of field line velocities that produce the correct temporal evolution of the field when equation (2) is satisfied [Vasyliunas, 1972].

Belcher [2005] gives a number of examples where the concept of moving magnetic field lines gives a correct representation of the evolution of the real field line pattern because his examples satisfy equation (2). The following two examples illustrate what happens when this equation is not satisfied.

Consider a time-independent magnetic dipole field in the presence of a homogeneous electric field parallel to the dipole axis, as in Figure 1. Below the equatorial plane, \( \mathbf{E} \times \mathbf{B}/B^2 \) points into the plane of the paper, as indicated, while above the plane, \( \mathbf{E} \times \mathbf{B}/B^2 \) points out of the plane. Thus, field lines that move with this velocity will continually twist out of a meridional plane while, in reality, the field lines remain undistorted and confined to the meridional plane. This is the reason for the difference between reality and the results obtained from equation (1) that is equation (2) is not satisfied in this case.

Another example of how the concept of moving magnetic field lines can be deceptive is that of a homogeneously magnetized conducting sphere surrounded by vacuum and rotating around its axis. For someone thinking of magnetic field lines as entities that can ‘move,’ it is a not an uncommon fallacy to believe that the magnetic field lines outside the spherical magnet ‘rotate with the magnet’ and that this rotating field is capable of exerting a force on a test charge at rest, due to the perceived ‘relative’ motion between the test charge and the magnetic field. What really happens is very different. As the spherical magnet is conducting, a charge on its surface will be subject to a magnetic force. This causes a redistribution of charges until the electric field from these changes gives a force that precisely balances the magnetic force. The result is an electrostatic potential on the surface of the sphere with the equator having a potential opposite to that of the poles. This establishes a
quadrupolar electrostatic potential throughout the surrounding vacuum.

The force on a test particle is purely electrostatic. The corresponding magnetic field line velocity calculated from (1) would twist the magnetic field lines, whereas in reality, they remain undistorted and confined to meridional planes.

Historically, the concept of moving field lines dates to Hannes Alfvén who, in an early paper [Alfvén, 1942] noted that in an infinitely conducting medium “the matter of the liquid is fastened to the lines of force.” This phenomenon became known as ‘frozen-in magnetic field lines’ and is a consequence of equation (2) being satisfied because the magnetic field aligned or parallel electric field is zero in an infinitely conducting medium.

The nonexistence of parallel electric fields was later challenged by Alfvén, who suggested that auroral primary electrons may gain their energy from falling through a parallel potential drop above the ionosphere and described how parallel electric fields can ‘cut’ magnetic field lines. Alfvén’s idea was contrary to contemporary beliefs and was almost universally disregarded, but when in situ measurements in space became possible, they brought the first indications that Alfvén might be right. Since then, an overwhelming amount of empirical data have proven that magnetic field aligned electric fields exist and are of key importance in the physics of auroras [Fälthammar, 2004], in magnetic field reconnection [Mozer, 2005], in shocks [Mozer et al., 2006], and in plasma turbulence and many wave modes.

Alfvén, who had introduced the concept, became a strong critic of ‘moving’ magnetic field lines [Alfvén, 1976], especially in his later years. He warned against use of the concepts of ‘frozen-in’ and ‘moving’ magnetic field lines for the reasons that are emphasized above.

The basic reason for these difficulties with ‘moving’ magnetic field lines is, of course, that motion of magnetic field lines is inherently meaningless. The magnetic field \( B \) is a vector field defined as a function of space coordinates and time. At a fixed time, one may trace a field line from any given point in space. But that field line has no identity, and in a time-dependent magnetic field it cannot be identified with any field line at a different time, except by one convention or another. As we have seen, such conventions are fraught with pitfalls and should only be used with utmost care lest they lead to erroneous conclusions. To paraphrase Ralph Nader, moving magnetic field lines are “unsafe at any speed.”

References


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