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INTERFERENCE ALIGNMENT IN FREQUENCY — A MEASUREMENT BASED PERFORMANCE ANALYSIS

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ABSTRACT

The idea of interference alignment (IA) has shown great promise in many theoretical studies. Yet it is not clear under what operating conditions it will perform better than traditional multiple access schemes under realistic network conditions. Here, we use measured channels to evaluate a number of different IA schemes and related methods, focusing on wideband single-antenna transmission, using frequency extensions for the IA. The radio channels were measured jointly from three base station sites in an urban macrocell scenario, over a 20 MHz bandwidth, allowing to emulate a scenario with 3 interfering cells (i.e. 3 interfering transmit/receive pairs). The numerical results show clear gains using IA related methods, compared to frequency planning with frequency reuse 3 and to uncoordinated transmission, even at low to moderate SNR.

Index Terms— interference alignment, frequency extension, channel measurements, performance analysis

1. INTRODUCTION

Out-of-cell interference is a performance bottleneck in current cellular systems. The interference is especially detrimental to cell-edge users, which have weak channels to the base stations serving them. Traditional methods to limit interference is by orthogonalizing the cells in some dimension, e.g. by time or frequency planning. By allowing base station cooperation, these methods can be outperformed.

Here, we study interference alignment (IA), a new concept springing out of the information theoretic study of the degrees of freedom (DoF) of the interference channel (IC) [1]. For the K-user single-input single-output (SISO) IC, the achievable DoF per user is $1/2$ [1], which corresponds to half the available DoF of a point-to-point link. IA can achieve this, and can consequently scale optimally in terms of sum rate as the signal-to-noise ratio (SNR) grows large.

Most performance evaluations of IA in the literature is performed using synthetically generated channels [2–6]. Such generated channels often correspond to scattering which is too rich to be experienced in nature. Therefore, it is important to study channels corresponding to realistic conditions, e.g. by using channels obtained from measurements. This was done in [7] for the the multiple-input multiple-output (MIMO) IC with frequency-selective channels, where spatial IA was shown to perform well. In [8], a real-time MIMO testbed was set up, and the performance for an indoor scenario was evaluated in terms of error vector magnitude and sum rate. A large-scale outdoor measurement operation was done in [9], which was used to study the loss in performance due to interference in a downlink scenario. IA was however not considered.

In this paper, we evaluate the performance of interference alignment and related precoding methods using channel measurements. The measurements correspond to a wideband urban macrocell scenario with single-antenna nodes, a practically relevant setting for which few performance analyses of IA exist. Using the measurements, a 3-user frequency-selective SISO IC is emulated. The performance of IA and the related methods is analyzed in terms of achievable rate, and compared to orthogonalization through frequency planning, and uncoordinated transmission from the base stations. Our numerical results show that IA achieves more degrees of freedom than frequency planning, but that performance is benefited by allowing some residual interference and treating it as noise, especially at low and intermediate SNR.

2. INTERFERENCE ALIGNMENT USING FREQUENCY EXTENSIONS

We consider interference alignment in the downlink of a network consisting of K base stations (BSs) and K mobile stations (MSs). All nodes are equipped with one antenna, and the channels are frequency-selective. Each BS transmits data to one desired MS, yielding interference at the other MSs. This scenario is modeled by the SISO IC, as seen in Fig. 1. Using orthogonal frequency-division multiplexing (OFDM), the frequency-selective links are converted to a set of parallel flat fading channels, over which IA is performed.

To facilitate the analysis, we assume perfect channel state information at the BSs, as well as perfect synchronization among them. Also, the channels are assumed to be block fading. For a given time instant, the channel between BS l and MS k over N_c subcarriers is

$$\mathbf{h}_{kl} = \begin{pmatrix} h_{kl}(0) & h_{kl}(1) & \cdots & h_{kl}(N_c - 1) \end{pmatrix}^T$$

and can equivalently be written as a diagonal matrix $\mathbf{H}_{kl} = \operatorname{diag}(\mathbf{h}_{kl})$. The received signal is modeled as

$$\mathbf{y}_k = \mathbf{H}_{kk}\mathbf{V}_k\mathbf{x}_k + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{H}_{kl}\mathbf{V}_l\mathbf{x}_l + \mathbf{n}_k$$

where $\mathbf{V}_k \in \mathbb{C}^{N_c \times d_k}$ denotes the precoder at BS k and $\mathbf{x}_k \in \mathbb{C}^{d_k \times 1}$ is the symbol vector transmitted from BS k, intended for MS k. The symbol vectors intended for different MSs are independent and the number of data streams allocated to MS k is d_k. A Gaussian codebook is assumed such that $\mathbf{x}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{x_k})$ and the transmit covariances and precoders are normalized such that $\mathbb{E}\left(||\mathbf{V}_k\mathbf{x}_k||_2^2\right) \leq PN_c$. The noise is assumed to be $\mathbf{n}_k \sim \mathcal{CN}\left(\mathbf{0}, \sigma^2\mathbf{I}_{N_c}\right)$, giving an

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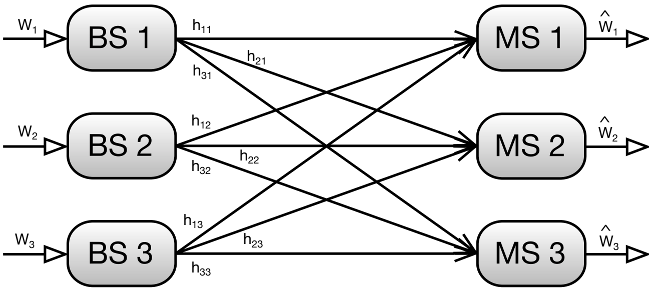

Fig. 1. 3-user interference channel with frequency-selective links. Each transmitter transmits a message to its desired receiver, which appears as interference to the other receivers.

SNR for MS k

$$\text{SNR}_k = \frac{P\,\mathbb{E}\left(|h_{kk}(n)|^2\right)}{\sigma^2}.$$

If interference suppression is performed at the receiver using a linear receive filter, the effective channel for MS k becomes

$$\widehat{\mathbf{x}_k} = \mathbf{U}_k^H \mathbf{y}_k = \mathbf{U}_k^H \mathbf{H}_{kk} \mathbf{V}_k \mathbf{x}_k + \sum_{\substack{l=1 \\ l\neq k}}^{K} \mathbf{U}_k^H \mathbf{H}_{kl} \mathbf{V}_l \mathbf{x}_l + \mathbf{U}_k^H \mathbf{n}_k.$$

Interference alignment then corresponds to finding a solution to:

$$\mathbf{U}_k^H \mathbf{H}_{kl} \mathbf{V}_l = 0 \quad \forall l \neq k \qquad (1)$$

$$\text{rank}\left(\mathbf{U}_k^H \mathbf{H}_{kk} \mathbf{V}_k\right) = d_k \qquad (2)$$

where (1) is the requirement for no residual interference, and (2) forces the effective channel to be able to transmit d_k parallel data streams.

We are interested in the performance of interference alignment as a precoding method, but not the influence of the design of the receive filters. Therefore, neglecting the impact of the cyclic prefix, the achievable rate (in bits per channel use) for MS k is determined as

$$R_k = \frac{1}{N_c} \log_2 \det\left(\mathbf{I}_{N_c} + \mathbf{Q}_k^{-1} \mathbf{H}_{kk} \mathbf{V}_k \mathbf{R}_{x_k} \mathbf{V}_k^H \mathbf{H}_{kk}^H\right)$$

where $\mathbf{Q}_k = \sigma^2 \mathbf{I}_{N_c} + \sum_{l\neq k} \mathbf{H}_{kl} \mathbf{V}_l \mathbf{R}_{x_l} \mathbf{V}_l^H \mathbf{H}_{kl}^H$ is the interference and noise covariance matrix at MS k. Note that this definition corresponds to using MMSE receive filters instead of the zero-forcing receiver imposed by (1), which gives a fair comparison to the iterative MMSE/SINR based methods described below.

In this paper, we measure performance by the individual user rates R_k, as well as the system sum rate $R = \sum_{k=1}^{K} R_k$.

2.1. Interference Alignment Algorithms and Related Methods

Several closed-form solutions for finding IA precoders exist [1,4,5]. Here, the original method for the $K = 3$ user SISO case [1] is applied, and called *IA (Cadambe 2008)*. This scheme shows achievability in terms of DoF, but is not optimized for finite SNR.

In [5], a closed-form solution which is improved in terms of sum rate for finite SNR is proposed, here called *IA (Sung 2010)*. The improvement comes from optimizing the chordal distance between the desired signal space and the interference signal space and subsequently orthogonalizing the precoders. Both closed-form solutions require global channel knowledge at a central node of the network, and can therefore be characterized as centralized.

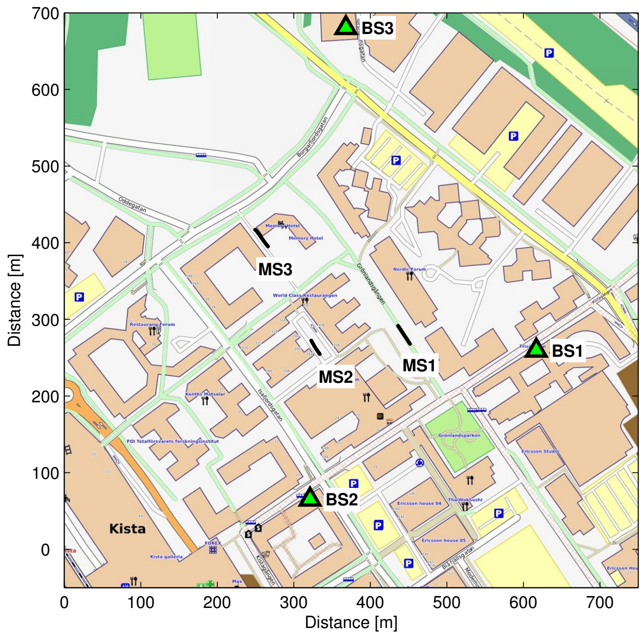

Fig. 2. Map over Kista with possible user positions marked in black. ©www.openstreetmap.org contributors, CC-BY-SA, http://creativecommons.org/licenses/by-sa/2.0

A multitude of iterative methods for the MIMO IC have been proposed [3, 10–12]. The *MIN_WLI* algorithm [10] iteratively minimizes a weighted interference leakage criterion, updating the precoders and receive filters in each iteration. It converges to some point, which may be an IA solution. The *MAX_SINR* [10] on the other hand sequentially updates the filters to maximize the signal-to-interference-and-noise ratios (SINRs) of all streams in the network. This algorithm is not proven to converge, and does in general not give rise to an IA solution.

In [11], a *sum rate maximization* procedure is proposed. Precoders are iteratively found as the solution to a set of weighted MMSE problems, where the weighting matrices are selected to maximize the sum rate. The receivers are found as MMSE receive filters. This method was developed for the MIMO interference broadcast channel, where each BS transmits to several MSs, but here it is applied to the case of each BS serving only one MS.

Finally, an iterative *SINR balancing* method [13] is applied. In each iteration, receive filters are found as MMSE filters, and precoders are obtained by finding the optimal solution to an SINR balancing problem where each user is weighted equally. Also, this method allows for a redistribution of the transmit power between BSs, resulting in a relaxed power constraint $\sum_k \mathbb{E}\left(||\mathbf{V}_k \mathbf{x}_k||_2^2\right) \leq KPN_c$. This formulation gives all streams in the network the same SINR, and hence the same rate.

The presented iterative methods were proposed for the MIMO case, but here they are applied using diagonal channel matrices corresponding to SISO channels. With the initial points used, they generally converge to good solutions. This may not be the case for any initial point, as further elaborated in [10]. Except for MIN_WLI, they all accept some residual interference at the receivers, which is

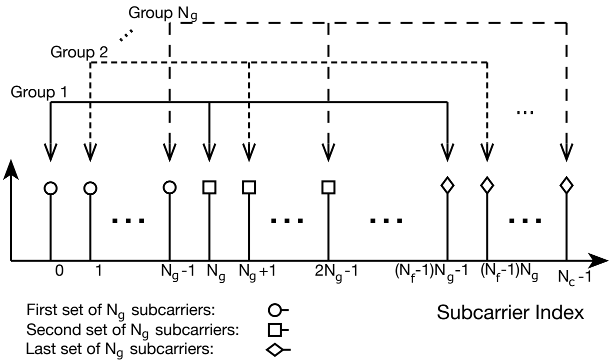

Fig. 3. Alignment group structure. Each alignment group contains N_f subcarriers, and there are N_g alignment groups.

treated as noise. This is beneficial in terms of sum rate for finite SNR, but does not correspond to IA.

3. MEASUREMENTS

In this paper, measured channels are used to to evaluate interference alignment in realistic network conditions. The measurements have been obtained in a setting emulating three BSs in an urban macrocell scenario in Kista in the northern part of Stockholm, Sweden. The three BS sites are geographically separated with an inter-site distance of around 400–600 m, as can be seen in Fig. 2. To allow for simultaneous coherent channel measurements from all three sites, these were connected to a central transmit unit using optical fibres in combination with RF-opto converters [14]. Each site was equipped with a sector-covering antenna aimed towards the central region, in which channel measurements were recorded using a van traveling along different streets. The van was equipped with a 4-channel receiver connected to four receive antennas mounted on the van roof. The base station and receiver were part of a purpose-built channel sounder based on an LTE-like OFDM-based design [15]. Pilot symbols were transmitted on all subcarriers over the full 20 MHz bandwidth, thereby allowing high quality channel estimates to be recorded jointly for every combination of transmit and receive antenna. Thus, in total a 4×3 MIMO channel was measured on 432 subcarriers with a frequency of 190 snapshots per second, which is sufficient for Nyquist sampling of the fast fading variations in the channel.

The measurements were used to emulate a $K = 3$ user IA scenario with spatially separated users by selecting three different segments of the measurement route (MS1, MS2 and MS3 in Fig. 2) from which channel realizations for each user was randomly drawn. Other user location scenarios were also investigated, but are not presented here due to space constraints. Each user is assumed to have one antenna. By assuming channel stationarity in time the channel measurements from the three route segments can be combined into a 3-user interference channel.

Note that the recorded channels will be subject to corruption by thermal noise. The channel impulse response estimates generally had a noise floor around 30 dB lower than the peak values. However, when adding Gaussian noise to the channel in the evaluation step this measurement noise can be neglected. Only at the higher SNR levels will the effect of measurement noise be prominent, and there it will appear as an added channel "richness".

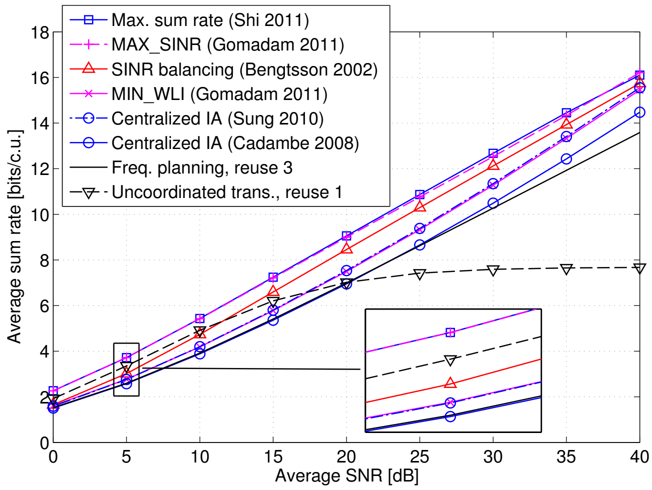

Fig. 4. Sum rate averaged over 500 channel realizations.

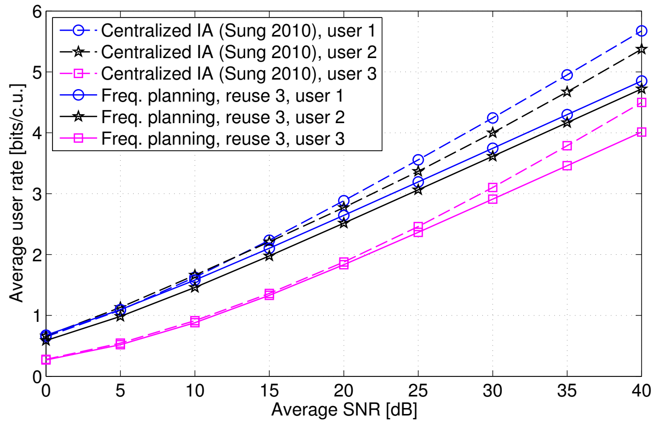

Fig. 5. Individual user rates averaged over 500 channel realizations.

4. PERFORMANCE EVALUATION

The performance of the described methods was evaluated using the defined 3-user scenario. As baselines, the methods of orthogonalizing the users in frequency (i.e. frequency planning, allocating $1/3$ of the subcarriers to each BS) and joint uncoordinated transmission by the BSs (frequency reuse 1), were applied.

In the evaluation, $N_c = 48$ subcarriers were used. Their corresponding channel coefficients were taken from the measurements, evenly spaced over the 432 available frequency points. In order to limit the effect of correlation between subcarriers, the subcarriers were divided into $N_g = 16$ alignment groups, and the interference alignment operation was performed independently and simultaneously for all groups. Each group consisted of $N_f = N_c/N_g = 3$ subcarriers, selected as shown in Fig. 3. This choice of N_f corresponds to the smallest number of frequency extensions necessary to perform IA, based on the original scheme from [1].

The number of data streams allocated per user in each alignment group was defined by $\mathbf{d} = \begin{pmatrix} 2 & 1 & 1 \end{pmatrix}^T$, which is a feasible allocation [1]. Consequently, 4 data streams were transmitted over the 3 subcarriers in each group. For fairness, $\mathbf{d}$ was cyclically shifted for

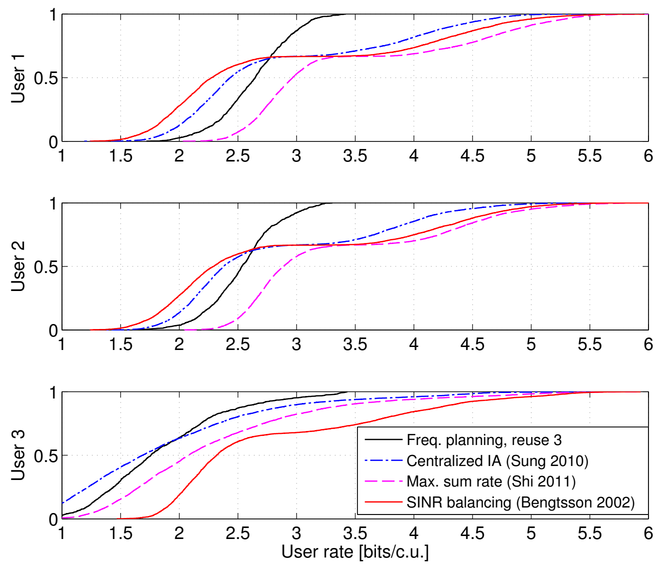

Fig. 6. Empirical cumulative distribution function of individual user rates for $\overline{\text{SNR}} = 20$dB, obtained from 5000 channel realizations.

every channel realization. The iterative algorithms were initialized with truncated DFT matrices and run for 5000 iterations. For the frequency planning reference, each BS was allocated 16 interference-free subcarriers. For the case of uncoordinated transmission, the BSs simply transmit over all subcarriers without any precoding. The average SNR of the network was defined as

$$\overline{\text{SNR}} = \frac{1}{K}\sum_{k=1}^{K}\text{SNR}_k = \frac{P}{K\sigma^2}\sum_{k=1}^{K}\mathbb{E}\left(|h_{kk}(n)|^2\right).$$

The average sum rate is shown in Fig. 4. The methods that accept some residual interference perform better than IA for the full studied SNR range, and also beat frequency planning. Max. sum rate and MAX_SINR consistently perform better than uncoordinated transmission, even for low SNRs. The individual user rates are plotted in Fig. 5. All users are able to achieve more DoF per user by employing IA, than by frequency planning.

The empirical distributions of the individual user rates are shown in Fig. 6. The particular shape of the curves for the iterative methods is due to the cyclic shift in data stream allocation between users. SINR balancing produces similar rates for all users, which as seen in Fig. 4 is not sum rate optimal. Only the sum rate maximization procedure performs better than frequency planning for all users and all data stream allocation permutations.

5. CONCLUSIONS

The theoretical performance gains possible from IA are well known in the literature. Here, we have shown that these performance gains are also possible under realistic channel conditions. Furthermore, our results indicate that performance at finite SNR is benefited from treating some residual interference as noise, rather than removing all interference at the receivers. Altogether, our results show the significance of the evaluated schemes for increasing the data rates in future wireless networks.

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