Kinematics and Dynamics of a novel 6-DoF TAU Haptic Device

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Abstract—This paper presents the kinematics and dynamics modeling of a novel hybrid kinematic concept, i.e. the TAU haptic device. This new concept was obtained from the modification of TAU-2 structure proposed by Khan et al [1]. First a kinematic model for inverse and forward kinematics was developed and analyzed. Then an algorithm to solve the close form inverse dynamics is presented using Lagrangian formulation. Numerical simulation was carried out to examine the validity of the approach and accuracy of the technique employed. A trigonometric helical trajectory of 5th order spline was developed in Cartesian space for each degree of freedom of the moving platform in order to verify and simulate the inverse dynamics; the motion of the platform is such that the tool centre point remains on this trajectory while its orientation is changing constantly in roll, pitch and yaw.

Keywords—component; Haptic device, Human robot interface, Mathematical Modelling, Simulation and TAU Configuration.

I. INTRODUCTION

Research on haptics focuses on the assistive technology, which create a physical interface between human's sense of touch and computer generated virtual world, or remote environment. Based on manipulation and interaction with objects within the virtual or remote world, these devices provide feedback forces and torques to the user. Research on haptic devices is getting high attention of researchers due to their emergent applications in many fields like medicine, tele-robotics, engineering design, and entertainment [1]. Due to the wide area of application of these devices, the application environment put conflicting design requirements e.g. large workspace, high stiffness, low inertia and compactness on this device. Commercially as well as in research labs currently both serial and parallel mechanism based devices are used for above specified applications [2]. However, these mechanisms have some advantages as well as some drawbacks. Therefore, we have started to develop a new mechanism, called 6-DoF TAU haptic device [2]. The idea behind this development was to obtain sufficient stiffness, enough workspace and compact design to fulfill the user requirements for the simulation of surgical procedures in hard tissues and bone structure [1]. This hybrid mechanism combines the good characteristics of both parallel and serial mechanisms [3, 4]. Zhenqi Zhu [4] worked on the kinematic analysis and error modeling of new 3-DoF mechanism based on serial-parallel mechanism, called 3- DoF TAU parallel robot. However, there is no effort made to

develop kinematics of 6-DoF TAU device. Furthermore, different models of 6-DoF TAU structures has been developed and analyzed by Khan et al [1] to identify their best characteristics.

Dynamic motion simulation plays an important role in many engineering application domains where there is an interaction between user-applied external outside disturbance to the active bodies like in virtual reality, robot motion simulation and haptic devices in tele-operation. These disturbance such as contact, change their dynamic behavior, in other words it changes the course of simulation trajectories by keeping the track of changes in dynamics. When good dynamic performance and precise position are required, the dynamic model is important for their control.

In the last few years the both parallel, serial manipulators have been up to some extent dynamically investigated by many researchers using different approaches like, Newton Euler formulation [5], Lagrangian formulation [6], principle of virtual work [7] and generalized momentum approach [8].

The Lagrange method describes the dynamics of a mechanical system from the concepts of work and energy. This method enables a systematic approach to develop equations of motion of any mechanical system. Nguyen and Pooran [10] use this method to model a Stewart platform, modeling the legs as point masses. Ahmadi, M et al [11] study dynamical modeling of a parallel robot Hexa using Lagrangian equation of the first type. Less effort is made on dynamic modeling of TAU configuration manipulators. Hongliang et al [3] and Zhenqi Zhu et al [4] have developed kinematic and dynamic model of 3 DOF Tau parallel robot.

However, in this research work, first we develop a kinematic model for inverse and forward kinematics. Thereafter we develop a dynamic model of the system, where the desired motion is given in terms of position, velocity and acceleration in Cartesian space and the required actuators torques are calculated. The resulting dynamic model is also very useful for control design process and will be utilized in the future work for the control design and optimization of the device.

The remaining of the paper is organized in sections. Section II presents the conceptual model and kinematics of the proposed 6-DoF haptic device, section III deals with the inverse dynamics. Furthermore numerical simulations, results and discussion are presented in section IV and V.

II. KINEMATIC MODEL OF TAU HAPTIC DEVICE

A. Conceptual model of the proposed haptic device

This novel concept of a 6-DoF haptic device is a modified version of the TAU-2 structures proposed by Khan et al [1]. The back and supporting arm was removed from TAU-2 and a universal joint was replaced by a revolute joint to get this concept. This configuration consists of fixed I-column, a moving platform and three parallel chains (1, 2 and 3) which connect the base frame to the moving platform. Chain 1 and chain 2 are symmetrical while chain3 is unsymmetrical as in Fig 1. Each symmetrical chain has two active rotational actuators, one attached to the I-column while another one is mounted on the upper link U_1 and U_2 . Furthermore chain 1 and 2 have two extra proximal links connecting the platform to upper links U_1 and U_2 to increase the structural stiffness. The third chain, chain 3, has also two active rotational actuators, one attached to the I-column and the other mounted at the top of the device. This concept is an asymmetric serial-parallel configuration that provides 6-DoF motion at moving platform. The advantages of this concept as compared to the TAU-2 are compact structure, large work space, and small actuating torque for unit applied force on the TCP.

Fig. 1. Conceptual model of the new TAU 6-DoF haptic device.

B. Kinematic model

The considered kinematic structure is an asymmetric serial-parallel (hybrid) configuration where the pose (position and orientation) of the platform is determined by the position of each joint point C_i of the platform. Furthermore the position of each joint point C_i of the platform is the same as the position of the end point of each serial manipulator. Due to this configuration, it is advantageous to the kinematic analysis to consider the serial and parallel structure separately as proposed by S.Lee [12] for serial-parallel manipulator.

To solve the kinematics, the kinematic model was simplified by considering three serial chains $A_i B_i C_i$ ($i=1, 2, 3$) as shown in Fig 2. Furthermore five different coordinate systems were used, i.e. base frame $\{N\}$, local leg frames $N_i = \{N_1, N_2, N_3\}$ and platform frame $\{P\}$. The leg frames for parallel chain 1 and chain 2 are positioned at $1.5d$, $3d$ and $4.5d$ respectively along z-axis with respect to base frame $\{N\}$ according to the preliminary analysis done for the maximum workspace and minimum actuator torques in ADAMS [13]. In this paper intermediate rotation of frames were used to find the coordinates of C_i with respect to leg frame $\{N_i\}$ according to the rotations given in equation 1 for ($i=1, 2$) and equation 2 for ($i=3$).

$${}^{N_i}T_{C_i} = \text{rot}(Z, \theta_{i1}) \text{rot}(X, \theta_{i2}) \text{trans}(X, L_i) \text{rot}(Z, \phi_{i1}) \text{rot}(Y, \phi_{i2}) \text{trans}(X, L_2) \quad (1)$$

$${}^{N_i}T_{C_i} = \text{rot}(Z, \theta_{i1}) \text{rot}(Y, \theta_{i2}) \text{trans}(X, L_1) \text{rot}(Y, \phi_{i1}) \text{trans}(X, L_2) \quad (2)$$

Now the joint positions C_i of platform for chain 1, 2 according to equation to (1) are given in (3-5), while for chain3 the joint position C_i according to equation 2 are given in (6-8).

$$\begin{aligned} p_{L_{xi}} &= \cos \theta_{i1} (L_1 + L_2 \cos \phi_{i1} \cos \phi_{i2}) \\ &\quad - L_2 \sin \theta_{i1} (\cos \theta_{i2} \sin \phi_{i1} \cos \phi_{i2} + \sin \theta_{i2} \sin \phi_{i2}) \end{aligned} \quad (3)$$

$$\begin{aligned} p_{L_{yi}} &= \sin \theta_{i1} (L_1 + L_2 \cos \phi_{i1} \cos \phi_{i2}) \\ &\quad + L_2 \cos \theta_{i1} (\cos \theta_{i2} \sin \phi_{i1} \cos \phi_{i2} + \sin \theta_{i2} \sin \phi_{i2}) \end{aligned} \quad (4)$$

$$p_{L_{zi}} = L_2 (\sin \theta_{i2} \sin \phi_{i1} \cos \phi_{i2} - \cos \theta_{i2} \sin \phi_{i2}) \quad (5)$$

$$p_{L_{xi}} = \cos \theta_{i1} ((L_1 + L_2 \cos \phi_{i1}) \cos \theta_{i2} - L_2 \sin \theta_{i2} \sin \phi_{i1}) \quad (6)$$

$$p_{L_{yi}} = \sin \theta_{i1} ((L_1 + L_2 \cos \phi_{i1}) \cos \theta_{i2} - L_2 \sin \theta_{i2} \sin \phi_{i1}) \quad (7)$$

$$p_{L_{zi}} = -((L_1 + L_2 \cos \phi_{i1}) \sin \theta_{i2} + L_2 \cos \theta_{i2} \sin \phi_{i1}) \quad (8)$$

Where $\{\theta_{i1}, \theta_{i2}\}$ are orientations of active revolute joint angles while $\{\phi_{i1}, \phi_{i2}\}$ for ($i=1, 2$) and $\{\phi_{i1}\}$ for ($i=3$) are orientation of passive universal and revolute joint respectively.

C. Inverse kinematics

The inverse kinematics problem for the proposed 6-DoF TAU configuration determines all orientation (joint angles) $[\theta_{i1}, \theta_{i2}]$ of all active actuators while pose $[p_x, p_y, p_z, \alpha, \beta, \gamma]$ of the platform is given. A close form solution for inverse kinematic can be found by using geometric method as shown in Figure 2, and using equations (3-8). The closed-loop vector equation for each chain $i=1, 2$ and 3 is given in equation (9).

$$p_{Li} = P_d + {}^N R_P P_{ui} - {}^{N_i} H \quad (9)$$

Where $p_{Li} = [p_{L_{xi}}, p_{L_{yi}}, p_{L_{zi}}] = f(p_x, p_y, p_z, \alpha, \beta, \gamma)$ is the position vector of the junction point C_i of the platform in local leg frames $N_i = \{N_1, N_2, N_3\}$. And vector P_d represents position of moving frame $\{P\}$ with respect to the base frame $\{N\}$, ${}^{N_i} H$ denotes the linear transformation to the local leg frame from the base frame. Where ${}^N R_P$ is a rotational matrix based on roll-pitch-yaw (α, β, γ) angles of platform. It defines the orientation of the moving platform w. r. t the base coordinate $\{N\}$. The active joint angle for link chain1, 2 and chain 3 are derived by using transformation of coordinates of the junction point C_i from leg coordinate $\{N_i\}$ to intermediate coordinate $\{N_{i1}\}$ using equations (10-12) for ($i=1, 2$).

$$p_{L_{xi}} \cos \theta_{i1} + p_{L_{yi}} \sin \theta_{i1} = L_1 + L_2 \cos \phi_{i1} \cos \phi_{i2} \quad (10)$$

$$-p_{L_{xi}} \sin \theta_{i1} + p_{L_{yi}} \cos \theta_{i1} = L_2 (\cos \theta_{i2} \sin \phi_{i1} \cos \phi_{i2} + \sin \theta_{i2} \sin \phi_{i2}) \quad (11)$$

$$p_{L_{zi}} = L_2 (\sin \theta_{i2} \sin \phi_{i1} \cos \phi_{i2} - \cos \theta_{i2} \sin \phi_{i2}) \quad (12)$$

Geometric method is used to find the passive angles ϕ_{i1}, ϕ_{i2} for link chain $i=1, 2, 3$ as shown in Fig. 2.

$$a_1 \cos \theta_{i1} + b_1 \sin \theta_{i1} = c_1 \quad (13)$$

Solution of equation (13) will yield two solutions for θ_{i1} as shown in equation (14).

$$\theta_{i1} = 2 \cdot \tan^{-1} \left(\frac{b_1 \pm \sqrt{b_1^2 + a_1^2 - c_1^2}}{a_1 + c_1} \right) \quad (14)$$

Similarly θ_{i2} can be found from either equation (11) or (12), simplifying equation (12), such that $p_{zi} = c_2$, $-L_2 \sin \phi_{i2} = a_2$ and $L_2 \cos \phi_{i2} \sin \phi_{i1} = b_2$. This gives

$$a_2 \cos \theta_{i2} + b_2 \sin \theta_{i2} = c_2 \quad (15)$$

Solution of equation (15) yield two solutions for θ_{i2}

$$\theta_{i2} = 2 \cdot \tan^{-1} \left(\frac{b_2 \pm \sqrt{b_2^2 + a_2^2 - c_2^2}}{a_2 + c_2} \right) \quad (16)$$

The active joint angles for link chain 3 can be found by squaring and adding equation (6) and (7) and by simplification we will get equation (17).

$$\sqrt{p_{xi}^2 + p_{yi}^2} = (L_1 + L_2 \cos \phi_{i1}) \cos \theta_{i2} - L_2 \sin \theta_{i2} \sin \phi_{i1} \quad (17)$$

Simplifying equation (17) such that

$$\sqrt{p_{xi}^2 + p_{yi}^2} = c_3, (L_1 + L_2 \cos \phi_{i1}) = a_3 \text{ and } -L_2 \sin \phi_{i1} = b_3$$

So equation (17) becomes.

$$c_3 = a_3 \cos \theta_{i2} + b_3 \sin \theta_{i2} \quad (18)$$

Solution of equation (19) yields two solutions for θ_{i2} as shown in equation (19).

$$\theta_{i2} = 2 \cdot \tan^{-1} \left(\frac{b_3 \pm \sqrt{b_3^2 + a_3^2 - c_3^2}}{a_3 + c_3} \right) \quad (19)$$

Active joint θ_{i1} for $i=3$ can be find by using equation (6)

$$\theta_{i1} = \cos^{-1} \left(\frac{p_{xi}}{(L_1 + L_2 \cos \phi_{i1}) \cos \theta_{i2} - L_2 \sin \theta_{i2} \sin \phi_{i1}} \right) \quad (20)$$

D. Forward kinematics

The forward kinematics determines the pose $[p_x, p_y, p_z, \alpha, \beta, \gamma]$ of the platform while given the joint angles $[\theta_{i1}, \theta_{i2}]$ of all active actuators. No closed form analytical solution is available for forward kinematics of 6-DoF TAU configuration, due to the complex non-linear equations. Therefore Newton Raphson numerical approximation method is used to solve this forward kinematics problem. The Newton Raphson approximation to the solution is given by equation (21).

$$X_{n+1} = X_n - [F'(X_n)]^{-1} \cdot F(X_n) \quad (21)$$

Where X_n is the initial guess value of platform pose, while X_{n+1} is the solution that is to be determined through a function $F(X_n)$ and its derivative $F'(X_n)$. $F(X_n)$ is defined separately for each serial manipulator and parallel structure by the method proposed by Soo S. Lee [12]. Two different functions were defined for each serial chain and parallel configuration. For each serial link this function is defined from the end junction point C_i of the chain with platform and the joints angles as $F_i(X_n) = \theta_i = F_{pi}(p_i)$, and its differentiation can be obtained as

$$\dot{\theta}_i = J_{si} \dot{p}_i \quad (22)$$

Where for each serial manipulator $\theta_i = [\theta_{i1} \ \theta_{i2}]$, $p_i = [p_{xi} \ p_{yi} \ p_{zi}]$ are the coordinate of the junction point C_i in base coordinates where $(i=1, 2, 3)$, and J_{si} can be defined as.

$$J_{si} = \begin{bmatrix} \frac{\partial F_{pi}}{\partial p_{xi}} & \frac{\partial F_{pi}}{\partial p_{yi}} & \frac{\partial F_{pi}}{\partial p_{zi}} \\ \frac{\partial F_{p2i}}{\partial p_{xi}} & \frac{\partial F_{p2i}}{\partial p_{yi}} & \frac{\partial F_{p2i}}{\partial p_{zi}} \end{bmatrix}, J_s = \begin{bmatrix} J_{s1} & 0 & 0 \\ 0 & J_{s2} & 0 \\ 0 & 0 & J_{s3} \end{bmatrix} \in R^{6 \times 9} \quad (23)$$

For the parallel case the function is defined by the relation between the junction vector of the platform p_i and the generalized coordinates can be expressed as $p_i = F_{xi}(X_i)$, and it differentiation is obtained as

$$\dot{p}_i = J_{pi} \dot{X}$$

$$J_{pi} = \begin{bmatrix} \frac{\partial F_{xi}}{\partial p_x} & \frac{\partial F_{xi}}{\partial p_y} & \frac{\partial F_{xi}}{\partial p_z} & \frac{\partial F_{xi}}{\partial \alpha} & \frac{\partial F_{xi}}{\partial \beta} & \frac{\partial F_{xi}}{\partial \gamma} \end{bmatrix}, J_p = \begin{bmatrix} J_{p1} \\ J_{p2} \\ J_{p3} \end{bmatrix} \in R^{9 \times 6} \quad (24)$$

Where $F_{xi} = [p_{xi} \ p_{yi} \ p_{zi}]$, and $X = [p_x \ p_y \ p_z \ \alpha \ \beta \ \gamma]$. The components of Jacobian matrix J_{pi} for each junction are calculated on the base of equation (2). The Jacobian for the whole system is derived by putting equation (24) into equation (22) as given,

$$\dot{\theta}_i = J_{si} J_{pi} \dot{X} = J \dot{X}, J = J_{si} J_{pi} \in R^{6 \times 6} \quad (25)$$

Now the forward kinematics, using equation (21) can be calculated by suppose an initial guess of pose X_n and update its value by ΔX that is calculated from the inverse of the Jacobian matrix and tolerance (error) $d\theta$ between the required and calculated joint angle.

$$\Delta X = [J(X)]_{6 \times 6}^{-1} \cdot d\theta \quad (26)$$

$$X_{n+1} = X_n + \Delta X \quad (27)$$

III. WORKSPACE

The approach used in this paper, utilizes a searching technique for locating the workspace boundary on planes parallel to the Y-Z plane at discrete values of X location using constant orientation of the platform. Potential platform locations are determined to be inside or outside the workspace by calculating the inverse kinematics and then testing if each of the joint angles θ_{i1} , and θ_{i2} , has real values.

Fig. 3. Workspace of 6 DoF TAU haptic device.

IV. INVERSE DYNAMICS

The Lagrange method describes the dynamics of a mechanical system from the concepts of work and energy. The general equation for Lagrange dynamic formulation is

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \tau \quad (28)$$

Where $L = K - P$, is the Lagrange function, K is total kinetic energy of the system, P is total potential energy of the system, q_i is generalized coordinates $[p_x, p_y, p_z, \alpha, \beta, \gamma]$, $\dot{q}_i$ is corresponding velocity of generalized coordinates.

A. Kinetic Energy

1) Kinetic Energy of platform

For simplicity the platform is modeled as a point mass at the centre of plate, the kinetic energy of the platform can be written in the form

$$K_p = \frac{1}{2} \dot{q}^T M_p \dot{q} \quad (29)$$

Where $\dot{q} = [\dot{p}_x \ \dot{p}_y \ \dot{p}_z \ \dot{\alpha} \ \dot{\beta} \ \dot{\gamma}]^T = [\dot{v}_{p,1 \times 3}^b \ \omega_{p,1 \times 3}^b]^T$ is the vector of generalized velocities of platform and M_p is the mass matrix of the platform .

$$M_p = \begin{bmatrix} m_p \times I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & R_p^b I_p (R_p^b)^T \end{bmatrix} \in R^{6 \times 6} \quad (30)$$

Where I_p the platform inertia tensor at the centroid, and R_p^b is the rotation matrix of the platform coordinates with respect to the base frame presented.

2) Kinetic energy of serial links

Since the 6 DoF TAU haptic device consist of 3 serial link chains as shown in Figure 2, so we need to find the kinetic energy of each link of the three serial chains. But to simplify the analysis each link is assumed as a rod with mass M and length L and cross-sectional area A, the moment of inertia of the rod can be computed as the limiting case of the moment of inertia of a cylinder such that radius $R \rightarrow 0$.

Sine chain 1 and 2 are symmetrical so kinetic energy of chain1 will be presented here. The kinematic chain i ($i=1, 2$) consist of four links $A_i B_i$, $B_i C_i$, $B_i D_i$ and $D_i E_i$, for serial manipulator the mass of each link exist at a centre point in the form of point mass.

B. Kinetic energy of link $A_i B_i$

For serial manipulator the links can be considered as slender rod where mass of each link exist at a centre point in the form of point mass. Position of the centre of mass of link $A_i B_i$ can be expressed in the local frame as

$$r_{A_i B_i} = [\frac{L_i}{2} \cos \theta_i, \frac{L_i}{2} \sin \theta_i, 0]^T \quad (31)$$

Linear velocity of the centre of mass can be obtained equation.

$$v_{A_i B_i} = \omega_{A_i B_i} \times r_{A_i B_i} \quad \text{where} \quad \omega_{A_i B_i} = [0 \ 0 \ \dot{\theta}_i]$$

where $\dot{\theta}_i$ is related to the generalized velocities through the jacobian matrix given in equation (25). Therefore the kinetic energy of the link can be obtained as

$$K_{A_i B_i} = \frac{1}{2} V_{A_i B_i}^T M_{A_i B_i} V_{A_i B_i} \quad (32)$$

Where $V_{A_i B_i}$ and $M_{A_i B_i}$ are the mass matrix and velocity vector respectively as given in equation (33) and (34).

$$v_{A_i B_i} = [v_{A_i B_i x} \ v_{A_i B_i y} \ v_{A_i B_i z} \ \omega_{A_i B_i x} \ \omega_{A_i B_i y} \ \omega_{A_i B_i z}]^T \quad (33)$$

$$M_{A_i B_i} = \begin{bmatrix} m_{A_i B_i} \times I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{A_i B_i, \in R^{3 \times 3}}^b \end{bmatrix} \in R^{6 \times 6} \quad (34)$$

Where mass moment of inertia of the link $A_i B_i$ in the fixed frame.

C. Kinetic energy of link chain $B_i C_i$

The kinetic energy of link $B_i C_i$ can be obtained as using a closed loop equation. Since link $B_i C_i$ is connected to the moving platform through a spherical joint so the velocity at that point C_i can be obtained though the translation velocity

of platform and relative velocity of point C_i to the platform coordinate system.

$$V_{C_i} = V_p + \omega_p \times PC_i^T \quad (35)$$

Where $V_p = [\dot{p}_x \ \dot{p}_y \ \dot{p}_z]$ and $\omega_p = [\dot{\alpha} \ \dot{\beta} \ \dot{\gamma}]$ are the angular velocity of platform, Linear velocity V_{B_i} of point B_i can be found from angular velocity of link $A_i B_i$,

$$V_{B_i} = \omega_{A_i B_i} \times r_{B_i} \quad (36)$$

Where $r_{B_i} = [L_i \cos \theta_i \ L_i \sin \theta_i \ 0]$ and linear velocity of point C_i can be computed from the linear velocity of B_i and angular velocity of link $B_i C_i$.

$$V_{C_i} = V_{B_i} + \omega_{B_i C_i} \times B_i C_i \quad (37)$$

$$B_i C_i = [x_{c_i} - L_i \cos \theta_i \ y_{c_i} - L_i \sin \theta_i \ z_{c_i} - 2d_i] \quad (38)$$

where the coordinates of junction points $[x_{c_i}, y_{c_i}, z_{c_i}]$ and PC_i can be given by equation (1). Equating the above two equations (35) and (37).

$$\omega_{B_i C_i} \times B_i C_i = V_p + \omega_p \times PC_i - \omega_{A_i B_i} \times A_i B_i \quad (39)$$

Performing cross product of vector $B_i C_i$ on both sides of equation (39) from right and using the property of triple cross product, $\omega_{B_i C_i}$ will be found as

$$\omega_{B_i C_i} \times B_i C_i \times B_i C_i = [V_p + \omega_p \times B_i C_i - \omega_{A_i B_i} \times A_i B_i] \times B_i C_i \quad (40)$$

Since the term $(B_i C_i \cdot \omega_{B_i C_i})$ is equal to zero because link 2 is between a universal and spherical joint and is not allowed to have any rotation in the direction of itself, sorting the terms, $\omega_{B_i C_i}$ is computed as

$$\omega_{B_i C_i} = \frac{1}{L_i^2} [B_i C_i \times (V_p + \omega_p \times B_i C_i - \omega_{A_i B_i} \times A_i B_i)] \quad (41)$$

Using $\omega_{B_i C_i}$ linear velocity of centre of mass of link $B_i C_i$ can be calculated as.

$$V_{B_i C_i} = \omega_{A_i B_i} \times A_i B_i + \frac{1}{2} \omega_{B_i C_i} \times B_i C_i \quad (42)$$

So the kinetic energy of link 2 can be calculated in the compact form as

$$K_{B_i C_i} = \frac{1}{2} [V_{B_i C_i}^T \ \omega_{B_i C_i}^T] M_{B_i C_i} \begin{bmatrix} V_{B_i C_i} \\ \omega_{B_i C_i} \end{bmatrix} \quad (43)$$

$$\text{Where } M_{B_i C_i} = \begin{bmatrix} m_{B_i C_i} I_{3 \times 3} & 0_{3 \times 3} \\ 0_{3 \times 3} & R_{B_i C_i}^b I_{B_i C_i} (R_{B_i C_i}^b)^T \end{bmatrix} \quad (44)$$

$R_{B_i C_i}^b$ is the orientation matrix for link $B_i C_i$ in a frame attached to link $B_i C_i$, the frame attached to link $B_i C_i$ is chosen as : axis $X_{B_i C_i}$ coincides with the link $B_i C_i$ and points from the fixed length link to the platform, meaning that it is coincident with vector $B_i C_i$, axis $Y_{B_i C_i}$ is perpendicular to $X_{B_i C_i}$, axis $Z_{B_i C_i}$ complete the frame following the right-hand rule and its projection along the p_z is always positive, so the rotation matrix becomes.

$$R_{B_i C_i}^b = [X_{B_i C_i} \ Y_{B_i C_i} \ Z_{B_i C_i}] \in R^{3 \times 3} \quad (45)$$

$$X_{B_i C_i} = [B_i C_{ix} \ B_i C_{iy} \ B_i C_{iz}]^T, \ Z_{B_i C_i} = X_{B_i C_i} \times Y_{B_i C_i}$$

$$Y_{B_i C_i} = \begin{bmatrix} \frac{B_i C_{ix}}{\sqrt{B_i C_{ix}^2 + B_i C_{iy}^2}} & -\frac{B_i C_{iz}}{\sqrt{B_i C_{ix}^2 + B_i C_{iy}^2}} & 0 \end{bmatrix}^T$$

D. Kinetic energy of link chain B_iD_i

Velocity of link B_iD_i can be found by using the assumption that the two links B_iC_i and D_iE_i are parallel to each other, so there angular velocity will be same.

$$\omega_{D_iE_i} = \omega_{B_iC_i} \quad (46)$$

Velocity of point E_i can be found from the closed loop equation.

$$V_{E_i} = V_{B_i} + \omega_{B_iD_i} \times B_iD_i + \omega_{D_iE_i} \times D_iE_i \quad (47)$$

$$V_{E_i} = V_p + \omega_p \times PE_i \quad (48)$$

$$B_iD_i = \begin{bmatrix} x_{D_i} - L_{A_i} \cos \theta_1 & y_{D_i} - L_{A_i} \sin \theta_1 & z_{D_i} - 2d_i \end{bmatrix} \quad (49)$$

$$D_iE_i = \begin{bmatrix} x_{E_i} - x_{D_i} & y_{E_i} - y_{D_i} & z_{E_i} - z_{D_i} \end{bmatrix} \quad (50)$$

$$\omega_{B_iD_i} = \frac{1}{d^2} \cdot [B_iD_i \times (V_p + \omega_p \times PE_i - V_{B_i} - \omega_{D_iE_i} \times D_iE_i)] \quad (51)$$

So linear velocity of the centre of mass of link B_iD_i is

$$V_{B_iD_i} = \omega_{A_iB_i} \times A_iB_i + \frac{1}{2} \omega_{B_iD_i} \times B_iD_i \quad (52)$$

The kinetic energy of link B_iD_i can be calculated in the compact form as

$$K_{B_iD_i} = \frac{1}{2} \begin{bmatrix} V_{B_iD_i}^T & \omega_{B_iD_i}^T \end{bmatrix} M_{B_iD_i} \begin{bmatrix} V_{B_iD_i} \\ \omega_{B_iD_i} \end{bmatrix} \quad (53)$$

The orientation matrix $R_{B_iD_i}^b$ for link B_iD_i can be found by the same procured the one used for link B_iC_i .

E. Kinetic energy of link chain D_iE_i

As it is assumed that the two links B_iC_i and D_iE_i are parallel to each other, so there angular velocity will be same from equation (41), $\omega_{D_iE_i} = \omega_{B_iC_i}$, So velocity of the centre of mass of link D_iE_i can be found as

$$V_{D_iE_i} = \omega_{A_iB_i} \times A_iB_i + \omega_{B_iD_i} \times B_iD_i + \frac{1}{2} \omega_{D_iE_i} \times D_iE_i \quad (54)$$

So the kinetic energy of link D_iE_i can be calculated in the compact form as

$$K_{D_iE_i} = \frac{1}{2} \begin{bmatrix} V_{D_iE_i}^T & \omega_{D_iE_i}^T \end{bmatrix} M_{D_iE_i} \begin{bmatrix} V_{D_iE_i} \\ \omega_{D_iE_i} \end{bmatrix} \quad (55)$$

The orientation matrix $R_{B_iD_i}^b$ for link B_iD_i can be found by the same procured the one used for link B_iC_i .

3) Kinetic energy of chain 3

The kinematic chain3 consist of two links A_3B_3 and B_3C_3 , so the kinetic energy can be calculated by the same procedure the one used for A_iB_i and B_iC_i .

F. Potential Energy of Platform

Potential energy of the platform is given by

$$U_p = m_p g p_z \quad (56)$$

Where p_z is the z- component of the position vector of the centre of mass of platform expressed in the base coordinates.

G. Potential energy of chain 1

Since the 6 DoF TAU haptic device consist of 3 serial link chains as shown in Figure 2, so we need to find the potential energy of each link which is shown in Figure 2. For simplicity only the potential energy of link chain 1 and 3 is given.

Potential energy of link A_iB_i , B_iC_i , B_iD_i , D_iE_i , A_3B_3 and B_3C_3 is given by

$$U_{A_iB_i} = m_{A_iB_i} \cdot g \cdot (1.5d) \quad (57)$$

$$U_{B_iC_i} = m_{B_iC_i} \cdot g \cdot \left(\frac{L_{2i}}{2} (\cos \phi_{i2} \sin \phi_{i2} \sin \theta_{i2} + \sin \phi_{i2} \cos \theta_{i2}) + 1.5d \right) \quad (58)$$

$$U_{B_iD_i} = m_{B_iD_i} \cdot g \cdot \left(-\frac{d}{2} (\sin \phi_{i2} \sin \phi_{i1} \sin \theta_{i2} + \cos \phi_{i2} \cos \theta_{i2}) + 1.5d \right) \quad (59)$$

$$U_{D_iE_i} = m_{D_iE_i} \cdot g \cdot \left(\left(\frac{L_2}{2} \sin \phi_{i2} + 1.5d \cos \phi_{i2} \right) \cos \theta_{i2} + \sin \phi_{i1} \sin \theta_{i2} \left(\frac{L_2}{2} \cos \phi_{i2} - 1.5d \sin \phi_{i2} \right) + 1.5d \right) \quad (60)$$

$$U_{A_3B_3} = m_{A_3B_3} \cdot g \cdot \left(\frac{L_1}{2} \sin \theta_{32} + 4.5d \right) \quad (60)$$

$$U_{B_3C_3} = m \cdot g \cdot \left(\left(\frac{L_2}{2} \cos \phi_{31} + L_1 \right) \sin \theta_{32} + \frac{L_2}{2} \cos \theta_{32} \sin \phi_{31} + 4.5d \right) \quad (61)$$

H. Lagrange equation in Cartesian space

Using the Lagrange's equation, the dynamic equation in the Cartesian space can be obtained with the kinetic and potential energy obtained above. Similarly the total kinetic and potential energy can be simplified by adding the kinetic and potential energy of all components and finally the Lagrange equation will be

$$M(q)\ddot{q} + \underbrace{\dot{M}(q)\dot{q} + \dot{q}^T \dot{M}(q)\dot{q}}_{V(q,\dot{q})} + G(q) = \tau \quad (62)$$

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = \tau_C \quad (63)$$

I. Lagrange equation in joint space

In order to derive the joint torques we will use transpose of the inverse of Jacobian matrix as given in equation.

$$\tau_j = J^T \tau_C \quad (64)$$

The equation of motion that shows the torques on actuators in joint coordinate system as

$$M(\theta)\ddot{\theta} + V(\theta, \dot{\theta}) + G(\theta) = \tau_J \quad (65)$$

$$\text{Where } \dot{\theta}_i = J\dot{q}_i \text{ and } \ddot{\theta}_i = J\ddot{q}_i + \dot{J}\dot{q}_i$$

V. NUMERICAL SIMULATION

A numerical example of the inverse dynamic model developed in the previous section has been implemented in MAPLE® and simulated in MATLAB®. The geometric parameters used during simulation is presented in the Table 1.

A trigonometric helical trajectory is developed in Cartesian space and joint space for each degree of freedom for the moving platform in order to verify and simulate the inverse dynamics; the motion of TCP of platform is such that point P remains on this trajectory while its orientation is changing constantly in roll, pitch and yaw.

TABLE I: GEOMETRIC PARAMETERS

Parameters	values	Parameters	values
R_p	45mm	m_p	0.1 kg
L_1	150 mm	mass of L_1	0.05 kg
L_2	200 mm	mass of L_2	0.05 kg
d_1	60 mm	g	9800 mm/s ²

1) Trajectory in Cartesian space

To generate a simple trajectory in Cartesian space, we let the position vector of point P of TCP be defined in base frame by a vector-valued function that represents the position vector of a point of a helix, defined in (66)

$$\begin{aligned} x &= p_x + b(\beta + \bar{\omega}), y = a \cos(\beta + \omega), z = p_z + a \sin(\beta + \omega) \\ \alpha_z &= a \cos(\beta + \omega), \beta_y = a \cos(\beta + \omega), \gamma_x = a \cos(\beta + \omega) \end{aligned} \quad (66)$$

Where a, b, ω and $\bar{\omega}$ are scalar constants, the first two with units of length, the last two are dimensionless, while β denotes the parameter of the helix that varies with time t according to the quintic polynomial (67).

$$\beta = \sum_{k=0}^5 a_k t^k \quad (67)$$

The coefficient of can be found by using initial condition motion of the manipulator starts at $t=0$ and at time $t=t_f$. To simulate the trajectory for 6 DoF motion in Cartesian coordinate we choose $t=0$ and $t=5$ seconds and the following parameters were assumed. The trajectories are shown in Fig 4.

$$a = 6, b = 3, \omega = \frac{\pi}{2}, \bar{\omega} = \frac{\pi}{10}, \beta_f = 10, \beta_0 = 0, a_0 = 0, a_1 = 0, a_2 = 0$$

Fig.4. Trajectories of pose and orientation and helical trajectory in Cartesian space.

2) Trajectory in joint space

To find the trajectories in joint space we to use the inverse kinematic model developed for joint orientation θ_i , see Fig 5.

Fig. 5. Trajectories of actuators orientation in joint space.

3) Actuator torque in joint space

The resulting actuator torques, given by the inverse dynamic simulation is shown in Fig 6.

VI. CONCLUSIONS

In this paper, the inverse kinematics and inverse dynamics of a novel 6 DoF haptic device based on TAU configuration is presented in closed form and forward kinematics using Newton Rapson approximation. The study presented in this paper provides a framework for future research such as stiffness, control model and structural optimization design of the 6 DoF TAU haptic device.

Fig. 6. Trajectories for on actuator torque.

To study the motion characteristics we introduce a kinematic model that was used to derive the dynamic equation of the system based on lagrangian dynamics. In the kinematic analysis the inverse kinematics has been performed using homogeneous transformation. Particularly, all component equations needed for dynamic analysis have been derived in closed form. A graphic user interface (GUI) was developed for inverse and forward kinematics in Matlab to verify the validity of the mathematical model as shown in Fig 7.

Fig 7. GUI of 6 DoF haptic device

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