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Piecewise Linear Road Grade Estimation

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ABSTRACT

Emerging heavy duty vehicle control systems increasingly rely on advance knowledge of the road topography, described by the longitudinal road grade. Highway road grade profiles are restricted by road design specifications to be piecewise affine. This characteristic is used herein to derive a method for road grade estimation based on standard on-vehicle sensors and optimal piecewise linear estimation through dynamic programming. The proposed method is demonstrated with on-road experiments. It is able to represent the road grade profile for two studied 15 km road sections, by 20 linear segments for each, with a root mean square error between 0.42 % and 0.55 % grade.

INTRODUCTION

Knowledge of the upcoming road topography can be used in automated speed control systems for a heavy duty vehicle (HDV) to optimize the speed profile to avoid unnecessary braking and gear shifting. Due to the large mass of a HDV, such a system can save considerable amounts of energy, particularly where braking is frequently necessary to avoid overspeed. A prerequisite is that precise road grade information is available onboard the vehicle. Vehicles commonly drive the same routes frequently, thus estimating the necessary road grade information locally on the vehicle may be an attractive option to externally sourced road grade data. Although no optimal speed control can be performed on the first drive over unknown roads, a future complete road grade estimation system may be able to recall estimated road grade information on successive passes over the same road.

There are design guidelines for highways which state that the vertical road grade profile shall be laid out as a series of linear segments. The lengths and slopes of those segments depend on the design speed and quality of the road, as well as the surrounding terrain. In this paper the assumption that the true road grade profile consists of a series of piecewise linear segments is used to design and evaluate a road grade estimation method based on data from standard mounted sensors in HDVs. The results of the developed method are compared, based on experimental data, both to a high quality reference road grade measurement and to the results of a previous method that does not use the assumption of a piecewise linear road grade profile.

RELATED WORK

Road grade estimation is carried out in many different contexts. Many of today's vehicles include sensors and software to estimate the instantaneous road grade at the present position. The estimated road grade is useful in many on board control algorithms, although getting good estimates with the tight sensor cost restrictions of production vehicles is hard. A patent application for such a system was filed already in 1971 [1]. Road grade estimation is closely related to vehicle mass estimation. In this work the vehicle mass is assumed to be known, it could also be estimated through a number methods. A current survey of the vehicle mass and road grade estimation state-of-the-art can be found in [2]. Methods focused on mapping the road grade for future use generally employ one or more Global Positioning System (GPS) receivers and/or high quality inertial measurement systems, one example is [3]. In this paper the low cost sensors of a production vehicle are used to achieve the best possible piecewise linear road grade estimate after having completed a full measurement of the road section of interest.

The authors have previously developed a filtering method for combining vehicle sensor and GPS data to produce a road grade estimate [4]. The method is focused on storing road grade estimates such that they can be improved whenever new data become available. The work generally assumes a road grade model where the grade is constant. Recently, that method has been extended to utilize a sub-optimal piecewise linear segmentation of the road grade profile [5]. In this work, the previous results are compared to an optimal piecewise linear segmentation.

CONTRIBUTION

This paper presents a method to estimate the road grade based on sensors that are standard mounted in HDVs, and background information on how highways are designed and built. It is known from road design practices that the broad character of highway road grades can be described by piecewise linear functions. In the proposed method the optimal piecewise linear approximation of the recorded sensor data is found, using an increasing number of segments. The accuracy of the estimated road grade is evaluated experimentally, and the results are compared to a previous method.

METHODOLOGY

The proposed estimation method uses a road model based on design specifications for highways. It also depends on a longitudinal vehicle model with physical parameters, and a GPS sensor. This section describes these models and, the sensed signals, and how they are used to arrive at a road grade estimate.

ROAD MODEL

Roads are built according to specifications that vary somewhat between regions. A common trait to at least Swedish and U.S. highway specifications is that changes in road grade should be carried out as a linear transition, i.e., the altitude profile should be described by a parabola where the vertical offset is proportional to the square of the distance [6,7]. Mathematically, concave curves going into more uphill gradients, are described by the parabola $\Delta z_l = l^2/(2 \cdot R)$ where Δz_l is the relative altitude, l is the horizontal distance measured relative to the lowest point of the parabola, and R is a design parameter. The part of the parabola to use is determined by the road grades at the start and end of the vertical curve. For convex vertical curves, i.e., hilltops, the parabola is flipped upside down.

The value of the design variable R depends on a number of factors, such as traffic safety, driving dynamics, visibility conditions, terrain and esthetics. The chosen vertical arc length and radius parameter have to match the surrounding terrain, and provide sufficient visibility for drivers to be able to stop before obstacles. In Sweden, for a major highway to be considered to have a good visibility standard when designed for a speed of 110 km/h the minimum convex vertical curve radius, as listed in Table 1, is $R = 16\,000$ m.

Table 1 – Minimum convex vertical curve radius R for roads with at least two lanes, with regard to sight distance for passenger cars. High, medium, and low refers to the chosen visibility standard for the road. Excerpt of table 11-1 in [7].

		High	Medium	Low
v_{ref}	Environment	R (m)	R (m)	R (m)
50	Urban, main	1200	400	300
70	Countryside	3000	1800	1200
90	Countryside	7000	6000	5000
110	Countryside	16000	13000	9000

Assuming that the vertical road profile can only consist of segments with constant road grade, or parabolic segments as described above, we obtain a road model with the distance along the road as the independent variable. For the magnitude of road grades of interest $\alpha_{\mathrm{d}} = \arctan(\mathrm{d}z/\mathrm{d}l) \approx \mathrm{d}z/\mathrm{d}l = l/R$. The change in the road grade α can thus be approximated as $\mathrm{d}\alpha/\mathrm{d}l \approx 1/R$. Since the approximation of $\mathrm{d}\alpha/\mathrm{d}l$ does not depend on l, the distance from the start of a local parabola, l can be replaced by s, the distance along the road. Let $\dot{\alpha} = \mathrm{d}\alpha/\mathrm{d}s$. Dividing the road into N segments we then have local models $\dot{\alpha}(s) = \mathrm{d}\alpha_i/\mathrm{d}s = \pm 1/R_i = c_i$ for convex and concave vertical curves between the knot points, with $i = 1, \dots, N$.

SENSING THE ROAD GRADE

The proposed road grade estimation method relies on two methods for deriving the road grade from sensor data available in today's stock production HDVs. The first method is based on driveline sensors and a longitudinal model that relates modeled forces to measured vehicle acceleration through Newton's second law. The second method computes a numerical derivative from the altitude measurement provided by the on-board GPS unit.

Vehicle model

The engine in a HDV delivers torque to the gearbox. The gearbox output torque is distributed through the final gear to the wheels. Friction in the contact point between the tires and the ground causes the vehicle to move. The propulsive force thus caused by the engine torque, F_{engine}, is the first part of the vehicle model. The other longitudinal forces depend mainly on the vehicle speed, the road grade and the brake pedal position. A number of vehicle and environment parameters also enter the equations; most notable is the vehicle mass. The main longitudinal forces acting on a HDV are shown in Figure 1.

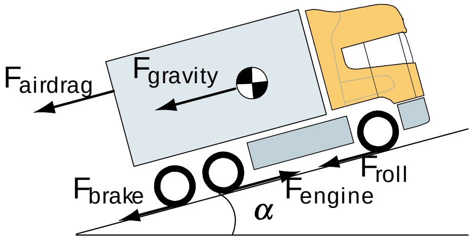


Figure 1 - Longitudinal forces acting on a vehicle traveling on a road with the road grade α. The forces generally vary with the distance along the road; the distance dependence has been left out of the figure for clarity.

The forces shown in Figure 1 are modeled as follows, where the distance dependence is indicated only in the explanations, and left out of the equations. The net engine force $F_{\mathrm{engine}} = i_{\mathrm{t}} i_{\mathrm{f}} \eta_{\mathrm{t}} \eta_{\mathrm{f}} / r_{\mathrm{w}} \cdot T_{\mathrm{e}}$ is given by the transmission gear ratio $i_{\mathrm{t}}(s)$ and efficiency $\eta_{\mathrm{t}}(s)$, final gear ratio $i_{\mathrm{f}}(s)$ and efficiency $\eta_{\mathrm{f}}(s)$, and the wheel radius r_{w}. The engine torque is denoted by $T_{\mathrm{e}}(s)$. $F_{\mathrm{airdrag}} = 1/2 \cdot c_{\mathrm{d}} A_{\mathrm{a}} \rho_{\mathrm{a}} v^2$ is known through the measured vehicle speed $v(s)$ and the air drag coefficient c_{d}, vehicle frontal area A_{a}, and air density ρ_{a}. A very simple model $F_{\mathrm{roll}} = mgc_{\mathrm{r}}\, cos\, \alpha \approx mgc_{\mathrm{r}}$ gives the rolling resistance from the vehicle mass m, gravity g, coefficient of rolling resistance c_{r}, for small values of the road grade $\alpha(s)$. The road grade also appears in the gravity induced force $F_{\mathrm{gravity}} = mg\, sin\, \alpha$. The brake force F_{brake} is excluded from the model since it is generally unknown in a standard HDV. The total dynamic vehicle mass $m_t(s)$ is expressed as $m_{\mathrm{t}} = J_{\mathrm{w}}/r_{\mathrm{w}}^2 + m + i_{\mathrm{t}}^2 i_{\mathrm{f}}^2 \eta_{\mathrm{t}} \eta_{\mathrm{f}} J_{\mathrm{e}} / r_{\mathrm{w}}^2$ where J_w and J_e represent the inertia of the engine and the wheels respectively. By solving for the road grade in the expression $F_{\mathrm{total}} = F_{\mathrm{engine}} + F_{\mathrm{airdrag}} + F_{\mathrm{roll}} + F_{\mathrm{gravity}} = m_{\mathrm{t}} \cdot \mathrm{d}v/\mathrm{ds}$, discretizing and replacing the derivative with a one step forward difference approximation we get the road grade signal from

$$\begin{gathered}
\alpha(s) = \arcsin[\frac{1}{mg}(i_{\mathrm{t}}(s) i_{\mathrm{f}}(s) \eta_{\mathrm{t}}(s) \eta_{\mathrm{f}}(s) / r_{\mathrm{w}} \cdot T_{\mathrm{e}}(s) \\
- m_{\mathrm{t}}(s) v(s) \frac{v(s+\Delta s) - v(s)}{\Delta s} \\
- 1/2 \cdot c_{\mathrm{d}} A_{\mathrm{a}} \rho_{\mathrm{a}} v(s)^2 - mgc_{\mathrm{r}})]\ (1)
\end{gathered}$$

This vehicle model is commonly used when a longitudinal model without the dynamic effects of various components in the driveline is desired. A thorough derivation can be found in [8].

GPS altitude

A GPS sensor was used to derive a road grade measurement as an approximate derivative of an altitude measurement through a central difference formula

$$\frac{dz}{ds}(s) \approx \alpha_{\mathrm{GPS}}(s) = \frac{z(s+\Delta s) - z(s-\Delta s)}{2\Delta s}\ (2)$$

In the experiments it was noted that the random noise in the GPS derived road grade signal was significant, while the average during an entire segment was very stable. Therefore, only the average road grade calculated for each experiment was used in the final estimate. The information from the GPS was incorporated by adding a bias to the vehicle model derived road grade signal such that its mean value matched that of the GPS derived signal.

PIECEWISE LINEAR ESTIMATION

Given the road model presented above the road grade profile of a properly engineered road should consist of linear segments. The validity of this model is studied by finding the optimal piecewise linear representation based on measured data. The road grade data can be regarded as a time series, although the independent variable in this case represents distance along the road instead of time.

Formally the road grade profile consists of a sequence of points $(s_0, \alpha_0), \dots, (s_{N-1}, \alpha_{N-1})$ where s, the distance values, are ordered such that $s_i > s_{i-1}$. A segmentation of the profile is defined as the sorted set of $k+1$ segmentation indices $z_0, \dots, z_k$ such that $z_0 = 0$ and $z_k = N$. The profile is divided by the segmentation points into intervals $S_1, \dots, S_k$ defined by the segmentation indices as $S_j = \{(s_i, \alpha_i) | z_{j-1} \leq i < z_j\}$. For each interval $S_1, \dots, S_k$, there is an associated linear model defined by a_j, b_j. The total segmentation error is computed from $\sum_{j=1}^{k} Q(S_j)$ where Q is the total squared error between the piecewise linear approximation and the measured data. The squared error Q is given by $Q(S_j) = \min_{a_j, b_j} \sum_{r=z_{j-1}}^{z_j - 1} \left(a_j s_r + b_j - \alpha_r\right)^2$. Other measures than the squared error, such as the l_∞-norm (maximum error) are also possible in the same framework, by replacing the $\sum$ operators by $\max$ operators. When reporting errors in this paper the RMSE e defined as $e(k, N) = \sqrt{\sum_{j=1}^{k} Q(S_j) / N}$ is used.

Finding linear segments in time series data is a problem that arises in many different fields and that has been extensively studied. When the number of segments in the data as well as their start and end positions are unknown, finding the optimal piecewise linear approximation is computationally intensive. Straight on evaluation of all $\binom{N-1}{k-1}$ possible segmentations of N data points into $k$subintervals is not feasible, even a short data set with $N = 200$ can be divided into $k = 6$ segments in over $2 \cdot 10^9$ different ways. Fortunately, there are more efficient algorithms available. It has been known for a long time that dynamic programming can be used to solve the problem for a fixed number of segments with computational complexity $O(N^3)$ [9].

Using an improved dynamic programming algorithm where the cost function is partially pre-computed, as presented in [10] the optimal segmentation and associated linear segments can be found with computational complexity $O(N^2 k)$. With this algorithm, the optimal segmentation can be found for data sets large enough to provide insights into the limits of piecewise linear road grade estimation (in this work $N = 1200$, $k = 40$) within a few hours of computation time on a standard office PC (single threaded execution, ~ 2 GHz CPU). The piecewise linear approximations used in this paper have been determined using a Matlab implementation of the algorithm proposed in [10] provided on the homepage of the author.

In a large scale implementation of the method significant performance gains can be achieved by dividing the road into sections of approximately 10 km. The resulting piecewise linear road grade profile for the entire road will not be globally optimal for the chosen number of segments, but the approximation within each section will. At the cost of a slight deviation from the optimal solution the computation complexity for estimating M such sections of X km with k_x segments and N_X samples each can be reduced from $O(M^3 N_X^2 k_X)$ to $O(M N_x^2 k_x)$. On a 100 km road, this gives a solution in only 1% of the original computation time.

EXPERIMENTAL SETUP

The proposed road grade estimation method has been implemented and applied to real world measurements recorded south of Södertälje, Sweden. Data were collected along a 15 km road section in both the southbound and northbound directions of highway E4. The location of the test site is illustrated in Figure 2.

Figure 2 – The experiments were conducted on highway E4 south of Södertälje, Sweden. The section in the northeast of the map is the southbound experiment, and the section in the middle of the map is the northbound experiment.

Data were gathered using two vehicles in a total of ten experiments, five in the southbound direction and five in the northbound direction along the test road. Most of the signals needed for the road grade estimation are available on the CAN bus of stock production trucks. These are the vehicle speed, engine torque (calculated based on fuel injection times), current gear, gearshift status, and brake utilization. The CAN bus signals were recorded using a laptop. There was no GPS data available on the vehicle bus, instead an external single frequency standard positioning service VBOX GPS receiver with a CAN interface was used. The absolute position obtained from the GPS was used to synchronize data from the different measurements. Data were recorded while driving at normal highway speed, with different sample rates depending on the source sensors. All signals were later resampled to provide one data point every 12.5 m. Experiments 1—3 in each direction were conducted with a tractor semi-trailer combination with a gross vehicle weight of 39 000 kg, while experiments 4—5 were conducted with a tractor with safety ballast but no trailer. The second vehicle weighed 12 000 kg.

EXPERIMENTAL RESULTS

The presentation of the results has been divided into three parts. First the reference road grade profile used for evaluating the proposed method is described. This includes a study of how well the reference is described by its optimal piecewise linear approximation. Next, the recorded sensor signals are analyzed, to provide a baseline for the performance of the proposed estimation method. The estimation results are presented using two example experiments, in addition to aggregate results. Finally, the tradeoff between storage requirements and road grade profile accuracy is treated briefly. The estimation results have been evaluated mainly based on the root mean square error (RMSE) in the estimated road grade.

REFERENCE ROAD GRADE

In order to evaluate the accuracy of the road grade estimation a reference road grade profile has been obtained using a high quality combination inertial navigation system and GPS receiver. To further improve the accuracy the road was driven multiple times. The northbound reference profile is based on three passes, and the southbound profile is based on five passes.

To investigate the hypothesis that the true road grade can accurately be described as a piecewise linear function an optimal piecewise linear representation has been determined, with an increasing number of segments. The RMSE between the segmented representations and the measured reference road grade profile as a function of the number of linear segments used is shown in Figure 3.

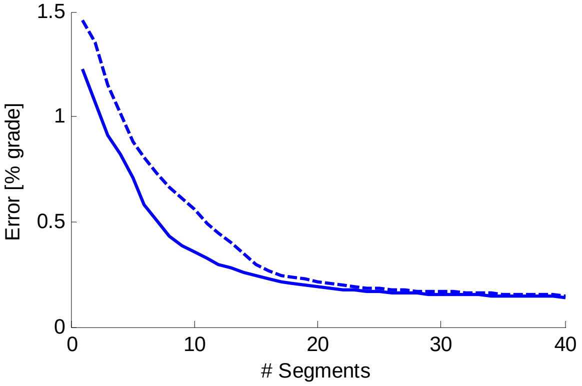
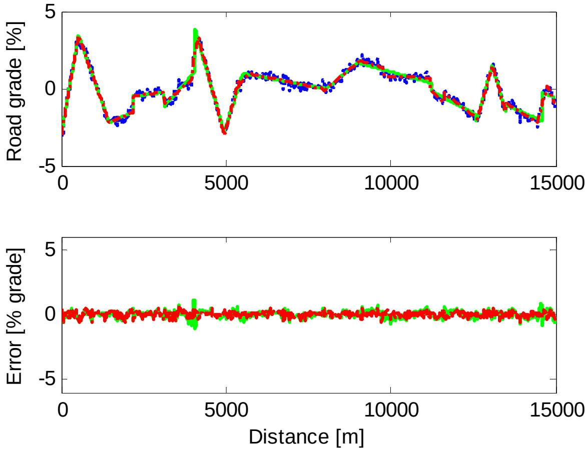


Figure 3 - Difference (RMSE) between the reference road grade measurement and its piecewise linear approximation. Results for both the northbound (solid line) and southbound (dashed line) test roads are shown.

It is clearly seen in Figure 3 that the estimation performance improves, but also that the rate of improvement decreases, as more segments are added. Each added segment increases the computation cost of the method. Choosing the best number of segments to use is often referred to as a model order selection problems, and there exist a large number of methods to choose from in the literature. A simple method would be to assign a weight w to the number of segments, k, used to represent N_0 data points, and minimize a summation cost function including the RMSE, $e(k, N_0)$, on the form $\min_k C(k) = e(k, N_0) + w \cdot k$. Here we are satisfied by noting that $k = 20$ segments is enough to get an RMSE compared to the measurement of approximately 0.2 % grade, or 3 % of the range of the road grade signal. Adding a 21^{st} segment decreases the RMSE by less than 4 % in both the experiments. The measured reference road grade for both the north- and southbound directions, as well as different piecewise linear approximations and resulting errors, are shown in Figure 4.



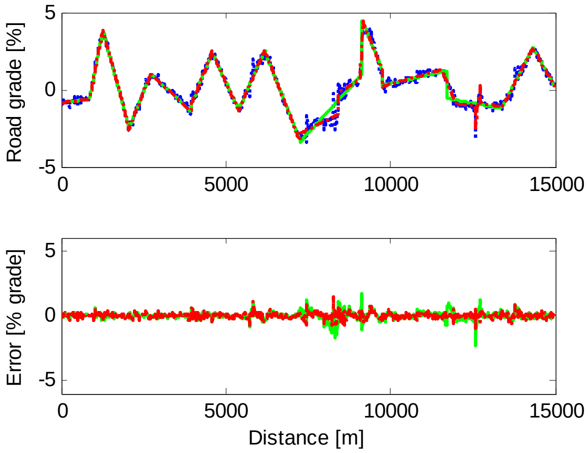


Figure 4 - The top half of each figure shows the reference road grade profile for the northbound (top) and southbound (bottom) test road sections (dotted) together with the optimal piecewise linear approximation with 15 segments (solid) and 20 segments (dash-dotted). The bottom half of each figure shows the RMSE for the approximation with 15 segments (solid) and 20 segments (dash-dotted).

It can be seen in the RMSE error plots in the bottom of Figure 4 that using only 15 linear segments leads to rather large errors locally, where there are not enough segments to represent the behavior of the signal. With 20 segments the situation improves, and the RMSE is 0.19 % grade in the northbound direction and 0.21 % grade in the southbound direction. This error between the piecewise linear approximation and the reference profile itself forms a baseline for the estimation of a piecewise affine road grade profile based on measured data, as it would be impossible to find a profile with 20 segments that has a lower estimation error.

SENSOR DATA ACCURACY

The standard mounted on-board sensors on a HDV give a road grade signal of significantly lower quality than the reference equipment. It is of interest to relate the performance of the proposed method to the quality of the measurements, and therefore the measured signals are analyzed separately. Since the road grade cannot be measured directly using the available sensors, two direct unfiltered estimates are used instead. These virtual sensors are based on the vehicle model and the GPS altitude, as described above.

The road grade root mean square (RMS) and mean errors compared to the reference road grade profile, for each of the virtual sensors and conducted experiments, are shown in Figure 5. From the figure it is clear that the virtual sensor based on the GPS data has a much lower average error, while the RMSE is similar or worse than for the vehicle model-based virtual sensor. The mean grade of the virtual sensor based on the vehicle model has therefore been shifted to match the mean road grade of the GPS based sensor. The RMS and mean errors of this third signal have also been included in Figure 5.

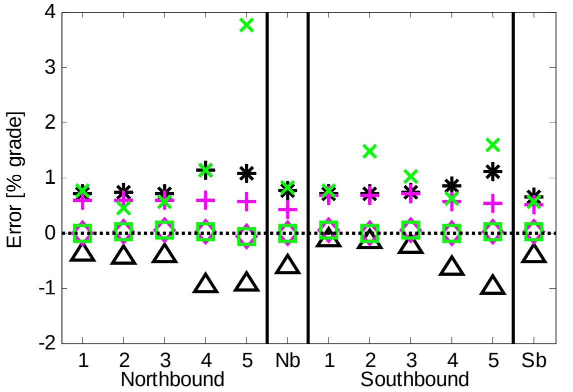


Figure 5 – The error characteristics of the virtual road grade sensors are shown for each of the ten experiments, five in the northbound direction and five in the southbound direction. To the right of each experiment series the results from averaging all measurements in each data point are shown. The vehicle model-based RMSE is indicated by (∗), and the mean error by (△). The

GPS based RMSE is denoted by (×) and the mean error by (□). The RMSE of the vehicle model-based sensor, with its mean updated to match the GPS based virtual sensor is denoted by (+). The mean of this combined signal of course coincides with the mean of the GPS based signal, and is denoted by (◊).

An illustration of each of the virtual sensor signals for two different experiments are shown in Figure 6. In the first example, representing experiment 4 in the northbound direction, the vehicle model-based sensor signal shows a clear bias. The measured road grade stays noticeably below both the reference grade and the average of the GPS based sensor signal. This confirms what could be seen in Figure 5. When comparing the two examples it is apparent that the random noise in the GPS signal was much worse during experiment 4 in the northbound direction than during experiment 1 in the southbound direction. The large variations in the random GPS noise between experiments, which can also be seen clearly in the large differences in RMSE in Figure 5, were dealt with by only considering the average value of the GPS sensor during each experiment, as described in previously.

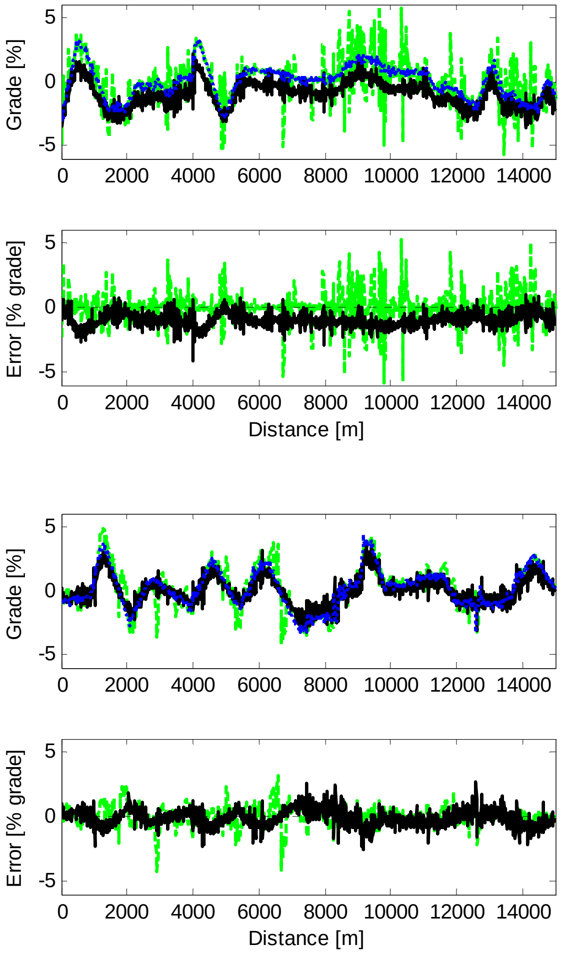


Figure 6 – Recorded road grade sensor data are shown for experiment 4 in the northbound direction (top figure) and for experiment 1 in the southbound direction (bottom figure). The virtual sensor based on GPS data is shown as a dashed line, the virtual sensor based on the vehicle model is shown as a solid line. The top parts of the figures show the road grade, and include the reference road grade as a dotted line. The bottom parts of the figures show the deviation of each sample from the reference profile. It can be clearly seen that the signal quality varies between experiments, and that there is a noticeable bias in the vehicle model-based signal in the top figure.

ESTIMATION RESULTS

The proposed estimation method generates a piecewise linear approximation that can be described by only a few dozen parameters instead of the 1200 original data points, and has a lower RMSE relative to the reference grade profile than the source vehicle model-based road grade virtual sensor, for all the conducted experiments. The residual error is significantly higher than for a previously published method [4], applied to the same input data. The previous method however, does not produce a piecewise linear output. Based on the analysis of the reference road grade above, the majority of results have been computed for a piecewise linearization using 20 segments.

When measurement data from all five experiments in each direction are averaged at each position before the linear segment identification, the RMSE of the linear profile is negligibly better than that of the averaged source data. The RMS errors for the source data and estimated piecewise linear road grade profile for each of the experiments are shown in Figure 7 together with results for the comparison method.

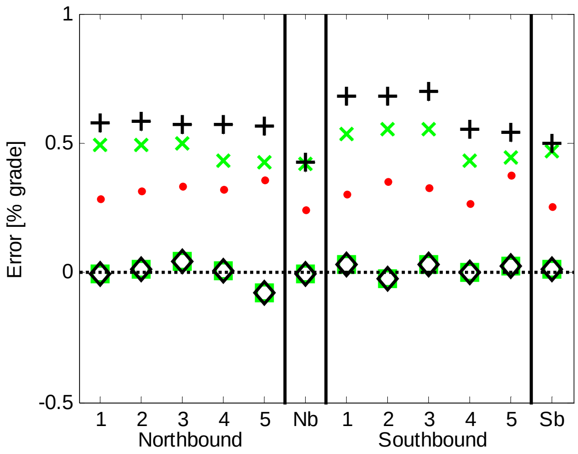
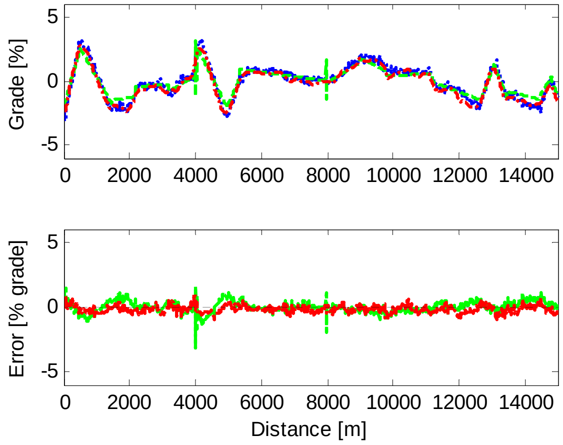


Figure 7 – Estimation RMSE and mean error for source data (+) and estimate (×). After the five experiments in each direction the results based on averaging all five measurements at each data point is shown The mean error for the piecewise linear estimate (□) and the source data (◊) are almost identical The RMSE for each experiment, and using all available measurements, for the comparison estimation method described in the text is denoted by (.). Note that the vertical scale of this figure is different from that of Figure 5, in order to more clearly illustrate the results.

The estimated piecewise affine road grade profiles for the two example experiments discussed above are shown in Figure 8. The figure also contains the road grade estimate obtained from the previously published method. When comparing the residual error to the error in the vehicle model-based source signal it is apparent that while most of the bias has been removed by the use of the averaged GPS based signal, a significant low frequency error component still remains. This error is not observed in the residual obtained when the reference signal is linearized, which leads to the conclusion that it comes from the source data rather than from difficulties representing the true road grade profile as a piecewise affine signal. The comparison method uses the GPS signal throughout the estimation, not just to counteract the vehicle model bias. The results indicate that despite the noisy character of the GPS signal it seems to contain more useful information than just the average road grade over 15 km.



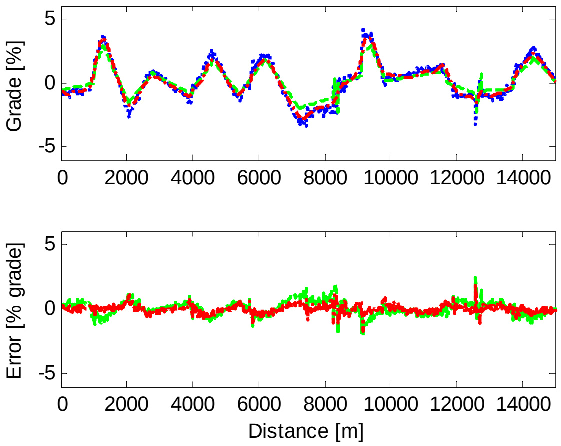
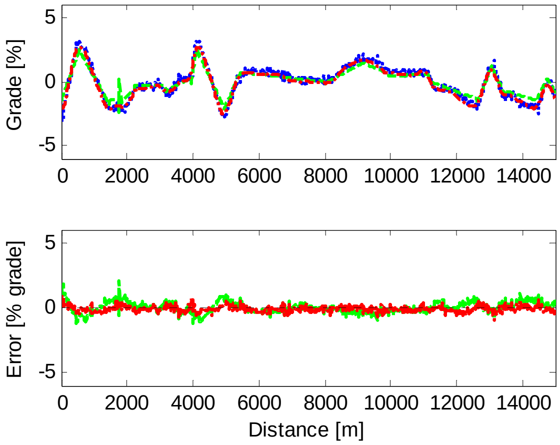


Figure 8 – Estimation results for the same experiments as sensor data are presented for above (dashed), compared with reference road grade (dotted) and the result from the comparison method (dash-dotted). The northbound direction is shown in the top figure, and the southbound in the bottom one. The usage of the GPS signal throughout the estimation in the comparison method seems to have a positive effect. Especially around 8000 m in the bottom figure the linear representation is clearly sub-optimal. The top figure shows some examples of over-fitting linear segments to noise in the input signal.

The piecewise linearization based on the averaged input data is only a marginally better approximation of the reference road grade data than the averaged sensor data itself. For the southbound experiments the estimated profiles from experiments four and five are better than the profile based on averaged input data, indicating significant differences in the measurement noise between experiments. The estimated road grade profile, and associated residual error, presented in the same manner as in Figure 8, are shown in Figure 9.



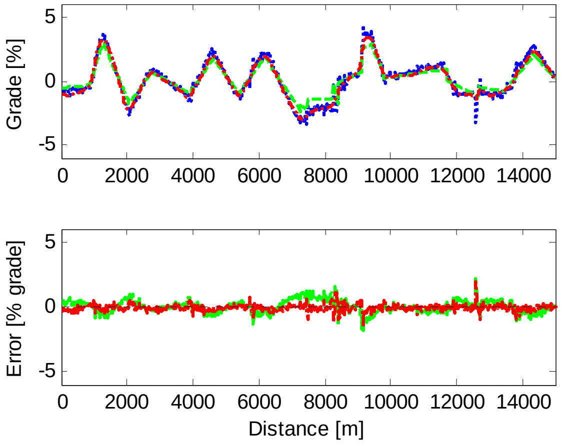


Figure 9 – Piecewise linear estimation results based on the average of the five measurements for each direction, northbound at the top, and southbound at the bottom. The piecewise linear profile (dashed) is based on the average sensor value at each position in the five experiments. The reference road grade (dotted) and result from the comparison method (dash-dotted) are also included. The results for the northbound direction show signs of fitting linear segments to measurement noise around 2000 m, indicating that the optimal number of segments may be lower than 20.

It can be expected that increasing the number of linear segments used when fitting the experimental data would improve the agreement with the reference road grade profile. This is true up to a certain point, after which additional linear segments end up being fitted to measurement noise rather than true road features. The experiments showed that the RMS road grade error decreased significantly when new segments were added up to about 15 linear segments. Additional segments would not give any significant change to the RMSE for the complete profile. Additional segments do however sometimes increase the error locally, where lines previously fitted to a reasonable average are split and adjusted to measurement errors. The number of linear segments required before the estimation quality levels out depends on the actual road profile, and given the relatively modest error increases from choosing too many segments care should be taken not to use too few segments. The obtained RMSE as a function of the number of piecewise linear segments used, for each of the ten experiments, and by using the average of all five measurements at each measurement point, are shown in Figure 10. The piecewise linear segment estimation method does not show significantly improved results when using input data averaged over multiple experiments, the results with averaged data are similar to the best individual experiments. In contrast the comparison method produces averaged results that are better than when data from only one experiment is used.

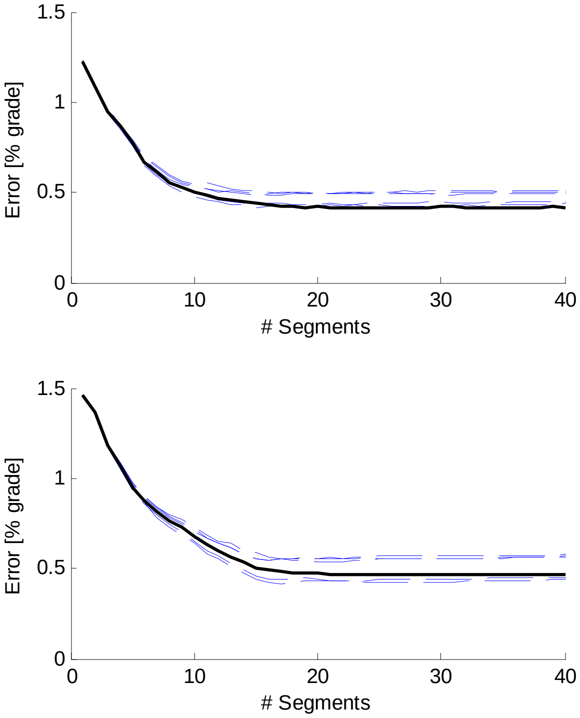


Figure 10 – Error compared to the reference profile, as a function of the number of segments used. The northbound experiment is shown at the top, and the southbound at the bottom. The results based on averaged input data are shown (solid line) together with the individual experiments (thin dashed lines). The values in this figure, when using 20 segments, correspond to those in Figure 7. The individual experiments yield different final errors, but all indicate that slightly less than 20 linear segments is probably ideal for these road sections and this method.

While the fit compared to the reference road grade does not improve when using an increasing number of segments above 15, the fit to the data itself naturally does. The internal fit to the measurement data was significantly better when all experiments were averaged before the identification of linear segments. The local fit for both the investigated road sections, based on averaged data, was 0.24 % grade. This is only about 25 % worse than the local fit of the reference road grade profile. It can thus be concluded that, on the average, the measured data is almost as well approximated by a piecewise linear function as the reference data. This is illustrated in Figure 11. Due to modeling and measurement errors there is however a notable difference between the identified piecewise linear functions.

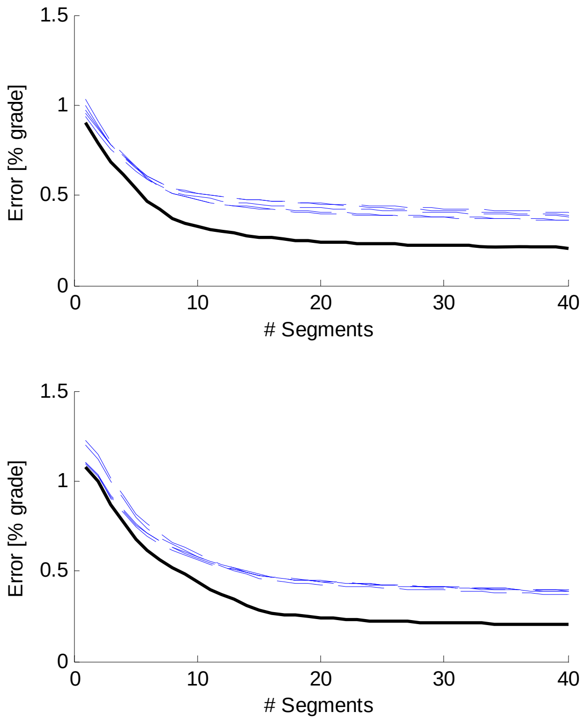


Figure 11 – Fit error between each piecewise affine estimate and the actual data points used in that estimate. Northbound experiments (top figure) and southbound experiments (bottom figure) are shown, and the results based on averaged input data (solid line) are compared with the individual experiments (thin dashed lines). Note that the piecewise linear fit to the averaged data is significantly better than the fit in any of the individual experiments.

The road grade profile obtained from the comparison method consists of 1200 data points that all need to be stored in a map to represent the profile. The piecewise linear profile with 20 segments only requires the storage of 40 line parameters, and 20 segment start positions. If the comparison method result is downsampled such that it is represented by 60 equidistantly spaced data points, and then restored using linear interpolation the resulting RMSE for the northbound profile is 0.39 % grade. The corresponding error for the southbound profile is 0.37 % grade. The proposed method based on averaged measurements from all experiments, the most directly comparable result, yields an RMSE of 0.42 % grade in the northbound direction, and 0.47 % grade in the southbound direction. The comparison method still performs better, but the difference is much smaller than when ignoring storage requirements. The remaining performance gap is likely due to a combination of better utilization of the GPS signal and the added freedom of not being limited to a piecewise linear representation.

CONCLUSIONS

It is possible to estimate highway road grades based on data collected using standard onboard sensors in a HDV. An estimation method representing the road grade signal as a piecewise linear signal, as suggested by road design guidelines, has been proposed and evaluated through experiments. The validity of the piecewise affine assumption has been verified by analyzing reference road grade data. The proposed method was able to estimate the road grade of two 15 km test road sections with RMS errors of 0.42 % grade and 0.47 % grade respectively.

The proposed method represents road grade profiles in a compact way, as parameters describing a piecewise linear function. While both the reference data and the averaged experimental data can be well described by piecewise linear functions, the estimation error for the proposed method is larger than that of a comparison method that does not use the assumption of piecewise linearity, using the same input data. A challenge in the design of the proposed method has been the use of the GPS signal. While it provides vital bias compensation for the vehicle model, it is at times very noisy. It was therefore only included as an average over an entire experiment. It may well be possible to improve the results by adding the GPS data directly to the estimation of linear segments, after suitable and possibly adaptive pre-filtering.

The number of linear segments used to represent a road section must be selected a priori, or determined through some model order selection method. In the conducted experiments, choosing the number of segments too large only produces a small increase in the estimation error while choosing to use too few segments yielded a large increase. The presented method is computationally intensive, but it is useable if computations can be carried out off-line without real-time constraints, and the road is divided into approximately 10 km sections before estimation is carried out.

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