

A Decentralized Stabilization Scheme for a Class of Large-scale Interconnected Systems^{*}

Omid Khorsand, Ather Gattami

*KTH - Royal Institute of Technology, School of Electrical Engineering,
Automatic Control, SE-100 44 Stockholm, Sweden. (e-mail:
khorsand@kth.se, gattami@kth.se)*

Abstract: In this paper, a decentralized control approach for a class of large-scale interconnected systems is proposed. A set of local decentralized controllers is constructed to guarantee the stability of the global system utilizing the notion of superstability. The control scheme and design are based on local state information and model knowledge of the adjacent subsystems. The approach leads to a computationally efficient method where the design procedure is performed on the subsystem level using linear programming. The solutions have been optimized and it is shown that the scheme has some useful features such as monotonic convergence and robustness under interval matrix uncertainty.

1. INTRODUCTION

One of the recent and most challenging problems in system theory and control is dealing with the ever growing size and complexity of mathematical models of real world processes. This kind of problems may become costly or even impossible in practice to solve by simply using faster computers and larger memories. Typical motivating examples of this kind arise, for instance, in control of large interconnected power distribution systems which have strong interactions, transportation and traffic systems with lots of external signals and water systems which are widely distributed in space (see Bakule [2008], Wu [2003], Ikeda [1989]). For the purpose of stabilization analysis and control design, it has been widely accepted that newer and more efficient methods should be developed. Therefore, the notion of "large-scale" systems has been introduced for more than three decades as it became clear that there are practical control problems that cannot be tackled by classical methods (see Siljak [1978], Jamshidi [1997]). The general approach has been to partition the system into a number of interconnected subsystems for which, instead of a single centralized controller, a set of independent decentralized controllers is constructed. This work belongs to the class of methods which aim to obtain the stability of the autonomous closed-loop subsystems as well as the entire system. The first time domain method for stability was based on state-feedback in individual subsystems by Davison [1974]. Studying different time-domain methods has continued with efforts to extend the class of stabilizable systems, and it should be noted that conditions on stabilizability are sufficient but not necessary, (see Siljak et al. [1977], Sezer et al. [1981], Ikeda et al. [1983], Shi et al. [1986], Geromel et al. [1994], Siljak et al. [2002] and references therein). Several sufficient conditions and methods for decentralized control have been developed based on Lyapunov function method

where sufficient conditions are usually given in the form of LMIs.

In fact, the main problem with large-scale systems is that, sometimes exchange of information is not possible among various subsystems, or the transmission can be lossy or delayed, so the best way is that individual agents utilize locally available information for control and estimation. The proposed stabilization scheme requires local state information and the model of the adjacent neighbors. Sufficient conditions to guarantee the feasibility of the approach can be expressed in the form of linear constraints on system matrix entries which is very easy to test. For the purpose of optimization, linear programming is utilized to find the controller which maximizes the degree of stability for each subsystem. Separate independent linear programs have been set up for individual subsystems so the design procedure has been developed in a distributed fashion. In addition to constructing a stabilizer controller, the design has important features such as monotonic convergence of the state norm and robustness under the interval matrix uncertainty.

In the next section, a sufficient condition for the stability is described based on the notion of diagonal dominance and the Geršgorin theorem. In Section 3, we formally present the problem we are addressing. We then propose an stabilization scheme for continuous-time and discrete-time interconnected systems in Sections 4 and 5 respectively. Section 6 introduces some important properties of the designed scheme followed by the concluding remarks in Section 7.

2. MATHEMATICAL BACKGROUND

Let \mathbb{R} be the set of real numbers and \mathbb{C} be the set of complex numbers. The set of $n \times n$ matrices over a field F is indicated by $M_n(F)$.

Definition 1. Let $A = [a_{ij}] \in M_n(\mathbb{C})$, the matrix is said to be diagonally dominant if,

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- $|a_{ii}| \geq \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}| = R_i$ for all $i = 1, \dots, n$,

strictly diagonally dominant if,

- $|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}| = R_i$ for all $i = 1, \dots, n$.

Theorem 1. (Geršgorin) Let $A = [a_{ij}] \in M_n(\mathbb{C})$ and

$$R_i(A) \equiv \sum_{\substack{j=1 \\ j \neq i}}^n |a_{ij}|, \quad 1 \leq i \leq n.$$

denote the absolute row sum of A . Then all the eigenvalues of A are located in the union of n discs :

$$\bigcup_{i=1}^n \{z \in \mathbb{C} : |z - a_{ii}| \leq R_i(A)\} \equiv G(A).$$

Corollary 1. Let $A = [a_{ij}] \in M_n(\mathbb{C})$ be strictly diagonally dominant. Then if all main diagonal entries of A are negative, all the eigenvalues of A will have negative real parts and hence A will be Hurwitz, see Horn et al. [1985]. In this case all the discs lie in left half plain. This is a special family of stable matrices and was called *superstable* first by Polyak et al. [2001].

In order to satisfy the superstability the following inequality can be considered,

$$\mu = \mu(A) \doteq \min \left(-a_{ii} - \sum_{i \neq j} |a_{ij}| \right) > 0, \quad i = 1, \dots, n, \quad (1)$$

where the parameter μ is called the degree of superstability. The inequality (1) can also be expressed as follows:

$$-a_{ii} > \sum_{i \neq j} |a_{ij}|, \quad i, \dots, n. \quad (2)$$

In addition, the condition for discrete superstability is expressed in the following:

$$\sum_{j=1}^n |a_{ij}| < 1, \quad i = 1, \dots, n. \quad (3)$$

The inequality in (3) is based on Geršgorin discs, i.e. satisfying the condition guarantees that all the discs lie inside the unit circle.

Based on the definition of the ∞ -norm for matrices, the condition (3) can be described as follows,

$$\|A\| = \max_{1 \leq i \leq n} \left(\sum_{j=1}^n |a_{ij}| \right) < 1.$$

3. STATEMENT OF THE PROBLEM

3.1 Static State-feedback Control

Consider a large-scale linear system for which we would prefer to avoid centralized schemes to control the whole system and hence we start our design by decomposing the system into the *input-decentralized* representation. The

final result of a decomposition algorithm will generally be a set of N interconnected subsystems,

$$\dot{x}_i = A_{ii}x_i + B_i u_i + \sum_{\substack{j=1 \\ j \neq i}}^N A_{ij}x_j, \quad i = 1, \dots, N,$$

where $x_i \in \mathbb{R}^{n_i}$ and $x_j \in \mathbb{R}^{n_j}$ are the states of the i th and the j th subsystems respectively and $u_i \in \mathbb{R}^{m_i}$ is the input to the i th subsystem. The total state vector is $x = (x_1^T x_2^T \dots x_N^T)^T$ and the total number of states is $n = \sum_{i=1}^N n_i$. A_{ii}, B_i and A_{ij} are constant matrices with proper dimensions, for $i, j = 1, \dots, N$

It should be noted that this representation may also be derived by physical modeling of the plant.

The overall matrices of the system can be constructed in a partitioned form as follows:

$$\dot{x} = Ax + Bu$$

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{21} & A_{22} & \dots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NN} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & B_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & B_N \end{bmatrix},$$

where $A \in \mathbb{R}^n$ and $B \in \mathbb{R}^{n \times m}$ and $m = \sum_{i=1}^N m_i$.

Using global state-feedbacks, we get the closed-loop subsystem dynamics as the following:

$$\dot{x}_i = (A_{ii} - B_i K_i)x_i + \sum_{\substack{j=1 \\ j \neq i}}^N (A_{ij} - B_i K_{ij})x_j, \quad i = 1, \dots, N,$$

this is a general structure for the closed-loop interconnected system. However, assuming a constrained information structure, we will continue by using only local state-feedbacks in our design.

Hence, we get the closed-loop subsystem dynamics as the following,

$$\dot{x}_i = (A_{ii} - B_i K_i)x_i + \sum_{\substack{j=1 \\ j \neq i}}^N A_{ij}x_j, \quad i = 1, \dots, N. \quad (4)$$

Based on *Corollary 1*, we aim to stabilize the continuous-time system by moving all the Geršgorin discs associated with the overall closed-loop matrix, A_{cl} , to the left-half plane,

$$\dot{x} = A_{cl}x, \quad A_{cl} = A - BK, \quad K = \begin{bmatrix} K_1 & 0 & \dots & 0 \\ 0 & K_2 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & K_N \end{bmatrix}. \quad (5)$$

Finding an stabilization approach for the above problem will also introduce a method to construct state estimators for the system by duality.

3.2 Estimation

Consider the linear time invariant system equation

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx, \end{aligned} \quad (6)$$

where $y \in \mathbb{R}^p$ is the measurement vector.

Decomposition: forming the dual of (6) we get

$$\begin{aligned}\dot{x} &= A^T x + C^T u \\ y &= B^T x,\end{aligned}$$

where we have used the same variable names for the states and measurements for consistency, but note that now they are different variables, $x \in \mathbb{R}^n, u \in \mathbb{R}^p, y \in \mathbb{R}^m$. Now if we apply the input decentralized scheme we get,

$$\dot{x}_i = A_{ii}^T x_i + C_i^T u_i + \sum_{\substack{j=1 \\ j \neq i}}^p A_{ij}^T x_j \quad (7)$$

$$y = \sum_{i=1}^p B_i^T x_i,$$

forming the dual of (7) gives us the output decentralized version of the plant,

$$\begin{aligned}\dot{x}_i &= A_{ii} x_i + B_i u_i + \sum_{\substack{j=1 \\ j \neq i}}^p A_{ij} x_j \\ y &= C_i x_i,\end{aligned} \quad (8)$$

where $y_i \in \mathbb{R}^{p_i}$ is the measurement of the i th subsystem. In order to construct state estimators, we will form the following dynamic equation:

$$\dot{\hat{x}}_i = F_i \hat{x}_i + G_i y_i + \sum_{\substack{j=1 \\ j \neq i}}^p F_{ij} \hat{x}_j + B_i u_i, \quad i = 1, \dots, p, \quad (9)$$

where the unknown matrices F_i, G_i and F_{ij} are to be calculated.

We form the estimation error equation,

$$e_i = x_i - \hat{x}_i, \quad (10)$$

taking the derivative of both sides of (10) and using (9) and (8) we get,

$$\dot{e}_i = (A_{ii} - G_i C_i) e_i - F_i \hat{x}_i + \sum_{\substack{j=1 \\ j \neq i}}^p A_{ij} x_j - \sum_{\substack{j=1 \\ j \neq i}}^p F_{ij} \hat{x}_j,$$

we choose,

$$A_{ii} - G_i C_i = F_i, \quad A_{ij} = F_{ij}.$$

So the error dynamics become,

$$\dot{e}_i = F_i e_i + \sum_{\substack{j=1 \\ j \neq i}}^p F_{ij} e_j. \quad (11)$$

To have an asymptotically stable estimator we need to stabilize (11) or equivalently stabilize the dual form,

$$\dot{\tilde{e}}_i = F_i^T \tilde{e}_i + \sum_{\substack{j=1 \\ j \neq i}}^p F_{ij}^T \tilde{e}_j,$$

we get,

$$\dot{\tilde{e}}_i = (A_{ii}^T - C_i^T G_i^T) \tilde{e}_i + \sum_{\substack{j=1 \\ j \neq i}}^p A_{ij}^T \tilde{e}_j,$$

which has the dual form of the control problem in (4). Finally we obtain the final decentralized observer equations in (12),

$$\dot{\hat{x}}_i = A_{ii} \hat{x}_i + G_i (y_i - C_i \hat{x}_i) + \sum_{\substack{j=1 \\ j \neq i}}^p A_{ij} \hat{x}_j + B_i u_i. \quad (12)$$

4. STABILIZATION OF CONTINUOUS-TIME SYSTEMS

Let us consider the linear continuous-time system equation (5). We are going to design a stabilizing K based on *Corollary 1*.

The closed-loop matrix of the system is:

$$A_{cl} = \begin{bmatrix} A_{11} - B_1 K_1 & A_{12} & \cdots & A_{1N} \\ A_{12} & A_{22} - B_2 K_2 & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & \cdots & \cdots & A_{NN} - B_N K_N \end{bmatrix}.$$

Now let us name the rectangular blocks of A_{cl} as follows,

$$\begin{aligned}A_1 &= [A_{11} - B_1 K_1 \quad A_{12} \quad \cdots \quad A_{1N}], \\ A_2 &= [A_{22} - B_2 K_2 \quad A_{21} \quad \cdots \quad A_{2N}], \\ &\vdots \\ A_N &= [A_{NN} - B_N K_N \quad A_{N1} \quad \cdots \quad A_{N(N-1)}].\end{aligned} \quad (13)$$

where we have re-arranged the sequence of some blocks in order to simplify the formulation of the problem.

Introducing the slack variables, q_{ij} , the conditions to obtain strict diagonal dominance can be expressed as in *Definition 1*. This will be shown for instance for A_1 in the following:

$$\begin{aligned}\mu_1 &> 0 \\ -[A_1]_{ii} - \sum_{j \neq i} q_{ij}^1 &\geq \mu_1, \quad i = 1, \dots, n_1, \quad j = 1, \dots, n \\ -q_{ij}^1 &\leq [A_1]_{ij} \leq q_{ij}^1, \quad i = 1, \dots, n_1, \quad j = 1, \dots, n, \quad i \neq j\end{aligned} \quad (14)$$

And similar conditions should hold for A_2, \dots, A_N . If the linear inequalities in (14) have a solution K_1 , for some $\mu_1 > 0$ then the conditions (2) are fulfilled for the first block-row, A_1 . These conditions should be satisfied for the other block-rows accordingly in order to make the total A_{cl} superstable. To test the existence of a solution for (14), the following LP can be set up for A_1 ,

$$\begin{aligned}\max_{K_1} \quad &\mu_1 \\ -[A_1]_{ii} - \sum_{j \neq i} q_{ij}^1 &\geq \mu_1, \quad i = 1, \dots, n_1, \quad j = 1, \dots, n \\ -q_{ij}^1 &\leq [A_1]_{ij} \leq q_{ij}^1, \quad i = 1, \dots, n_1, \quad j = 1, \dots, n, \quad i \neq j.\end{aligned} \quad (15)$$

Similarly, different LPs should be set up for A_2, \dots, A_N . Provided that a solution K_1^* exists, the LP in (15) gives the maximal value of μ_1 over all K_1 . Also for A_2, \dots, A_N similar LPs can be solved to find K_2^*, \dots, K_N^* . It will be shown that these feedback gains together impose the strictest constraint on system's state variables by satisfying the following set of inequalities:

$$\begin{aligned}
-[A_1]_{ii} &> \sum_{i \neq j} |[A_1]_{ij}|, \quad i = 1, \dots, n_1, \quad j = 1, \dots, n, \\
&\vdots \\
-[A_N]_{ii} &> \sum_{i \neq j} |[A_N]_{ij}|, \quad i = 1, \dots, n_N, \quad j = 1, \dots, n,
\end{aligned} \tag{16}$$

Hence, the static feedback K^* satisfies the inequality in (16) and decentrally stabilize the total system. In addition, we define the overall maximal μ as,

$$\mu^* = \min\{\mu_1^*, \dots, \mu_N^*\}. \tag{17}$$

We will summarize the results in the following Theorem:

Theorem 2. Let μ_l^*, K_l^* be the solution of the LP given by:

$$\begin{aligned}
&\max_{K_l} \mu_l \\
&-[A_l]_{ii} - \sum_{j \neq i} q_{ij}^l \geq \mu_l, \quad i = 1, \dots, n_l, \quad j = 1, \dots, n \\
&-q_{ij}^l \leq [A_l]_{ij} \leq q_{ij}^l, \quad i = 1, \dots, n_l, \quad j = 1, \dots, n, \quad i \neq j, \\
&\text{for } l = 1, \dots, N.
\end{aligned}$$

If $\mu_l^* > 0$, for $l = 1, \dots, N$, then the state-feedback $K^* = \text{diag}(K_1^*, \dots, K_N^*)$ decentrally stabilizes the system (5).

Example 1. As an illustrative example, let us consider the following system composed of two subsystems each one affected by a scalar input,

$$\begin{aligned}
\dot{x} &= Ax + Bu, \quad A = [a_{ij}], \quad i, j = 1, \dots, 4, \\
B &= \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad u = Kx, \quad K = \begin{bmatrix} k_1 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & k_4 \end{bmatrix}.
\end{aligned}$$

then

It can be shown that the corresponding LP for A_1 are feasible if and only if one of the following inequalities holds,

$$\begin{aligned}
a_{11} - a_{21} &< -|a_{12} - a_{22} - |a_{23}| - |a_{24}|| \\
&\quad -|a_{13}| - |a_{14}|, \\
a_{22} - a_{12} &< -|a_{21} - a_{11} - |a_{13}| - |a_{14}|| \\
&\quad -|a_{23}| - |a_{24}|.
\end{aligned} \tag{18}$$

Similarly for A_2 we get:

$$\begin{aligned}
a_{33} - a_{43} &< -|a_{34} - a_{44} - |a_{41}| - |a_{42}|| \\
&\quad -|a_{31}| - |a_{32}|, \\
a_{44} - a_{34} &< -|a_{34} - a_{33} - |a_{31}| - |a_{32}|| \\
&\quad -|a_{41}| - |a_{42}|.
\end{aligned} \tag{19}$$

Therefore, the conditions in (16) reduce to the linear constraints (18) and (19), provided that these constraints on entries are satisfied, decentralized stability of the total system is attainable by dominating the main diagonal of the total system matrix A_{cl} and optimal solutions can be obtained by linear programming accordingly.

Similarly, for systems of other dimensions the sufficient conditions reduce to a number of linear constraints on matrix entries. Hence, checking the feasibility of the scheme is an straightforward task.

Specially, for the case of no interactions among subsystems (completely decoupled), the conditions (18) and (19) reduce to the following:

$a_{11} - a_{21} + a_{22} - a_{12} < 0$, $a_{33} - a_{43} + a_{44} - a_{34} < 0$, this shows that for the decoupled structure, controllability of the subsystems is still not sufficient and the stabilizable family of systems by this method reduces to a narrower class. However, it adds some useful features to the scheme such as monotonic convergence and robustness which will be explained in Section 6.

Remark 1. No stabilizability check or check for existence of unstable fixed modes is required, i.e. fulfilling the corresponding linear constraints, guarantees the stabilizability of system.

Remark 2. Trivially, if the number of inputs on each subsystem is equal or greater than the number of states, superstability of the total system is always attainable due to the possibility of modifying each entry of all local closed-loop subsystem matrices freely and hence satisfying the conditions (16).

Remark 3. If it is necessary to add limitations on the magnitude of feedback gain to bound the magnitude of control input, it suffices to include these constraints in the corresponding LPs, for instance,

$$K = [k_{ij}], \quad |k_{ij}| \leq \bar{k}_{ij}, \quad i, j = 1, \dots, n.$$

5. STABILIZATION OF DISCRETE-TIME SYSTEMS

So far, we have studied the stabilization problem for continuous-time interconnected systems. A similar study can be done for discrete-time systems. Consider the linear discrete-time system,

$$x(k+1) = Ax(k) + Bu(k),$$

where $x(k) \in \mathbb{R}^n$ is the state of the system and $u(k) \in \mathbb{R}^m$ is the input to the system. Both A and B are constant matrices with proper dimensions. Similar to the continuous case, system can be decomposed into several dynamical partitions:

$$x_i(k+1) = A_{ii}x_i(k) + B_i u_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^N A_{ij}x_j(k),$$

where $x_i(k) \in \mathbb{R}^{n_i}$, $x_j(k) \in \mathbb{R}^{n_j}$ are the states of the i th and j th subsystems respectively. $x = (x_1^T \dots x_N^T)^T$, $n = \sum_{i=1}^N n_i$, $u_i(k) \in \mathbb{R}^{m_i}$. Here the control algorithm which guarantees the stability should keep the eigenvalues of the closed-loop system inside the unit circle. The procedure is very similar to the continuous-time case. We use N local state-feedback controllers,

$$u_i = -K_i x_i, \quad i = 1, \dots, N, \quad K = \begin{bmatrix} K_1 & 0 & \dots & 0 \\ 0 & K_2 & \dots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & \dots & K_N \end{bmatrix},$$

$$x_i(k+1) = (A_{ii} - B_i K_i)x_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^N A_{ij}x_j(k).$$

Hence the closed-loop equation for the dynamical interconnected system can be written,

$$x(k+1) = A_{cl}x(k).$$

Similar to continuous-time systems, closed-loop matrix partitions A_1, \dots, A_N are considered as in (13) and for

each subsystem a linear inequality should be satisfied that altogether satisfy the inequality in (3).

In order to find the $\min_K \|A_{cl}\|$, an LP can be considered for each subsystem. For the first subsystem the following LP is set up,

$$\begin{aligned} & \min_{K_1} \mu_1 \\ & \sum_j q_{ij}^1 \leq \mu_1, \quad i = 1, \dots, n_1, \quad j = 1, \dots, n \\ & -q_{ij}^1 \leq [A_1]_{ij} \leq q_{ij}^1, \quad i = 1, \dots, n_1, \quad j = 1, \dots, n, \quad i \neq j. \end{aligned} \quad (20)$$

Similar LPs should be set up for A_2, \dots, A_N .

The result is summarized in the following Theorem:

Theorem 3. Let μ_l^*, K_l^* be the solution of the LP given by:

$$\begin{aligned} & \min_{K_l} \mu_l \\ & \sum_j q_{ij}^l \leq \mu_l, \quad i = 1, \dots, n_l, \quad j = 1, \dots, n \\ & -q_{ij}^l \leq [A_l]_{ij} \leq q_{ij}^l, \quad i = 1, \dots, n_l, \quad j = 1, \dots, n, \quad i \neq j. \end{aligned}$$

for $l = 1, \dots, N$.

If $\mu_l^* < 1$, for $l = 1, \dots, N$, then the state-feedback $K^* = \text{diag}(K_1^*, \dots, K_N^*)$ decentrally stabilizes the discrete-time system.

Remark 4. It should be noted that thus far we have used state feedback controllers for both continuous-time and discrete-time systems. However, it is straightforward to modify these methods for output feedback design, by simply inserting matrix C into the equations.

6. PERFORMANCE PROPERTIES

This chapter gives some insight into the useful features of the diagonally dominant or superstable systems. These properties like monotonic decrease of the state infinity norm or the robustness issues are discussed in some works, see Polyak et al. [2002]. Similar statements hold for large-scale interconnected systems for which the whole closed-loop system matrix is made superstable by means of a decentralized controller.

6.1 Monotonic Convergence and BIBO Stability

Consider the system $\dot{x} = Ax + Bu$, where matrix A is superstable,

- For $u \equiv 0$,
$$\|x\| \leq \|x_0\| e^{-\mu t}, \quad t \geq 0 \quad (21)$$

where $\mu = \mu(A)$ as in (17) and the infinity norm of the vector x has been used,

$$x \in \mathbb{R}^n : \|x\| = \max_{1 \leq i \leq n} |x_i|$$

This shows that the infinity norm of the autonomous system converges monotonically to the origin.

- For $\|u\| \leq 1, t \geq 0$ with the initial condition x_0 ,
$$\|x\| \leq \alpha + e^{-\mu t} (\|x_0\| - \alpha)_+, \quad t \geq 0$$

where $\alpha \doteq \|B\|/\mu$. This result is related to the BIBO stability and implies that the trajectories originates and

stay within an invariant set around the origin. The proofs can be found in Polyak et al. [2002]. However for a stable system the equation (21) is replaced with the following,

$$\|x\| \leq C(A, \nu) \|x_0\| e^{-\nu t}, \quad 0 < \nu < \min_i -\text{Re } \lambda_i,$$

where C is a constant parameter and in general can be large. So in this case the norm does not monotonically decrease and may have undesirable peaks within the first time instants. Hence, such overshoot does not occur in superstable systems as expressed in (21).

6.2 Robustness

Thus far, we have assumed to have the exact model of the plant, where the exact system matrices A and B are given. However, in practice control design should be reliable in the presence of model uncertainties. In fact model uncertainty has been an important issue in the evolution of automatic control and as a result this problem has received substantial attention, see Åström [2000]. Here we address a class of robustness problems in which the system is affected by interval matrix uncertainties.

Robustness of interval matrix family Consider the state-space representation:

$$\dot{x} = A(\theta)x$$

where θ is a vector of real uncertain parameters in a set Θ . A family of interval matrices A_I can be defined:

$$\{A_I = A | a_{ij} = \theta_{ij}, \theta_{ij}^- \leq \theta_{ij} \leq \theta_{ij}^+\}$$

This shows that the system is described by a dynamic matrix where the entries take values in the interval $[\theta_{ij}^-, \theta_{ij}^+]$. An uncertain matrix belonging to this family can be defined by a nominal matrix A_0 and uncertainties Δ_{ij} ,

$A = [a_{ij}]$, $a_{ij} = a_{ij}^0 + \gamma \Delta_{ij}$, $|\Delta_{ij}| \leq m_{ij}$, $i, j = 1, \dots, n$, where $\gamma \geq 0$ is a numerical parameter and $m_{ij} \geq 0$ are the entries of the matrix $M = [m_{ij}]$.

Assuming A_0 is Hurwitz we aim to find the largest stability radius, γ_{max} , implying that the family will be stable for $\gamma < \gamma_{max}$. For the case where A_0 is stable in a general sense the problem has been proved to be NP-hard in Nemirovskii [1994]. However for a superstable system the problem is simple, see Polyak et al. [2002].

If the nominal matrix A_0 is superstable we have,

$$\mu(A_0) \doteq \min_i (-a_{ii}^0 - \sum_{i \neq j} |a_{ij}^0|) > 0,$$

so for the interval family the following inequality should be satisfied,

$$-(a_{ii}^0 + \gamma \Delta_{ii}) - \sum_{i \neq j} |a_{ij}^0 + \gamma \Delta_{ij}| > 0, \quad i = 1, \dots, n.$$

This inequality will be satisfied if,

$$-(a_{ii}^0 + \gamma m_{ii}) - \sum_{i \neq j} (|a_{ij}^0| + \gamma m_{ij}) > 0, \quad i = 1, \dots, n, \quad (22)$$

and hence,

$$\gamma < \gamma_{max} = \min_i \frac{-a_{ii}^0 - \sum_{i \neq j} |a_{ij}^0|}{\sum_j m_{ij}},$$

specifically for $m_{ij} = 1$,

$$\gamma_{max} = \frac{\mu(A_0)}{n},$$

and for discrete case,

$$\gamma_{max} = \frac{1 - \|A_0\|}{n}.$$

6.3 Robust Stabilization

Consider an uncertain large-scale system,

$$\dot{x} = Ax + Bu,$$

assuming uncertainty is collocated in matrix A , the task is to find a decentralized state-feedback controller that stabilizes the entire matrix family. The closed-loop system takes the form,

$$A_{cl} = A - BK = A_0 + \gamma\Delta - BK = A_{cl}^0(K) + \gamma\Delta,$$

where $\Delta = [\Delta_{ij}]$ and A_{cl}^0 is the nominal closed-loop matrix. Similar to (22) the uncertain family will be stabilized if,

$$-(a_{ii}^0(K) + \gamma m_{ii}) - \sum_{i \neq j} (|a_{ij}^0(K)| + \gamma m_{ij}) > 0, \quad i = 1, \dots, n.$$

If the decentralized superstabilization problem for nominal matrix A_{cl}^0 admits a solution K^* and μ^* , then

$$\gamma < \gamma_{max} = \frac{\mu^*}{n}, \quad \text{for } m_{ij} = 1.$$

This gives the maximal radius of robustness because the parameter μ has been maximized over controller K by linear programming. Similar results hold for the discrete case.

Therefore, the problem of robust stability for interval matrix family admits a simple solution for superstable systems and the maximum robustness radius is relative to parameter μ which is maximized in the process of optimization.

7. CONCLUSION

This paper studies the problem of stability of large-scale interconnected systems. The considered information structure assumes that each agent has information about its own state variables and the model of adjacent neighbors. Using some well-known results from matrix theory a sufficient condition for stability of the total system is introduced. Stabilization of both continuous-time and discrete-time interconnected systems was studied and sufficient conditions, in terms of linear constraints on matrix entries, were derived. We use the linear programming technique to find the stabilizable class of systems. This distributed method reduces the complexity and dimensionality of the problem. Some performance properties of the designed controllers were discussed such as monotonic convergence of state infinity norm and robustness in the interval matrix family.

This study can be further extended to deal with delayed measurements of adjacent subsystems. In addition, robustness in presence of external disturbances is another important issue to be studied.

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