Micro-satellite Mission Analysis:
Theory Overview and Some Examples

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Abstract. Rudimentary mission analysis for micro-satellites has been carried out, in particular for ionospheric/thermospheric “dipper” missions. The basic equations of orbital mechanics are summarized and commented on. General properties of near-Earth orbits are discussed and exemplified in figures and tables. In addition, a few specific mission scenarios are described and discussed.

Introduction

The present brief report is largely based on preliminary mission analysis that was performed in 1994 during the early stages of planning for Astrid-2, but also extends the discussion to more generic “dipper” missions as well as to other types of satellite missions, to serve as a reference for micro-satellites in general.

Astrid is a series of micro-satellites flown within the Swedish national space research programme. It is based on the Freja-C platform developed by the Swedish Space Corporation. The first Astrid, Astrid-1, carried a neutral particle imager into a polar orbit with an altitude of approximately 1000 km, and was launched in 1995. The second, Astrid-2, carried a combined field and wave instrument, particle detectors, Langmuir probes, and spin-scanning photometers; and was launched in 1998. Several different orbital scenarios were discussed for Astrid-2. One early idea for the project was to measure high-frequency fields in conjunction with a “heating transmitter” on the ground. These measurements had to be done below the F-layer ionisation peak. The orbital requirements were far from trivial, and the mission idea was abandoned. With the low mass-to-drag area ratio of a micro-satellite the orbital lifetime is a serious limitation. The primary mission objective was to study the electromagnetic field in the auroral zone. The implied requirements were to some extent in conflict with those discussed above.

For “heating” measurements, satellite passes at low altitude above the transmitter are needed. Furthermore, because of limited physical size of the ground antennae intended, the maximum plasma frequency in the F-layer had to exceed 4 MHz. Calculations by Thidé et al. (private communication) showed that for solar minimum (1996-1997), these requirements imply that passes must be at 250 km altitude or less, and that most of the measurements must be made in a sunlit ionosphere. Passing over a heating transmitter at low altitude is used as an example because it was the motivation for the original study. The analysis outlined in this report may also be applied to, for example, thermospheric missions where the aim is in-situ measurements of the unperturbed plasma or atmosphere. Although we focus here on micro-satellites for cost reasons, a heavier spacecraft has the advantage of a longer orbital lifetime. The formulas given are generally valid and the analysis can easily be adapted to arbitrary spacecraft.

Introductory background to orbital mechanics

To zeroth-order approximation the trajectory of an Earth-orbiting satellite is the solution of a two-body problem, an ellipse with the centre of the Earth at one of the foci. In the first-order approximation, which is good enough for most practical purposes, a few corrections are needed. One such correction arises from the oblateness of the Earth, which means that the satellite is not moving in a central force field. The effect of this is a rotation (precession) of the orbital plane, and also a rotation in the orbital plane of the location of perigee and apogee. Another correction is that of the disturbing forces from the Moon and the Sun. These are not discussed
further here since they are small in comparison for near-Earth orbits. A brief summary of the effects of Lunar and Solar gravity is found in Appendix 1. What is highly important to near-Earth orbits, though, is atmospheric drag. The effect of this is to reduce the apogee altitude to begin with and later on also the perigee altitude, eventually resulting in re-entry. A brief description of these effects, together with approximate formulae for the variations are given in the following.

Basic constants

\[ R_e = 6.371 \cdot 10^6 \text{ m} \]  \hspace{1cm} (1)
\[ G = 6.670 \cdot 10^{-11} \text{ Nm}^2\text{kg}^{-2} \]  \hspace{1cm} (2)
\[ M_e = 5.997 \cdot 10^{24} \text{ kg} \]  \hspace{1cm} (3)
\[ \mu = G \cdot M_e = 4.00 \cdot 10^{14} \text{ Nm}^2\text{kg}^{-1} \]  \hspace{1cm} (4)
\[ J_2 = 1.083 \cdot 10^{-3} \]  \hspace{1cm} (5)

Basic definitions and equations

Satellite orbits are described using a set of orbital elements. The classical elements (some of them illustrated in Figures 1 and 2) are:

\[ a \] : semi-major axis
\[ e \] : eccentricity
\[ i \] : inclination
\[ \Omega \] : longitude of the ascending node
\[ \omega \] : argument of perigee

Fig. 1. The vernal equinox.
O is given relative to an inertial frame of reference since the orbit is unaffected by the rotation of the Earth. Figure 1 illustrates the "vernal equinox" direction which is used in the definition of O. These five elements completely describe the size, shape and orientation of the orbit. To specify the position of a satellite in this orbit a sixth element is needed.

In a two-body problem there are two fundamental constants of motion, energy ($E$) and angular momentum ($L$). These are given by:

$$E = \frac{mv^2}{2} + V$$

where $m$ is spacecraft mass, $v$ its velocity, and $V$ is the gravitational potential which to zeroth order equals $-\mu/r$, where $r = |\mathbf{r}|$ is geocentric distance; and

$$L = \mathbf{r} \times \mathbf{p}$$

where $\mathbf{p}$ is the momentum of the spacecraft.

The non-central gravitational forces change the direction but not magnitude of the angular momentum but not in the mean the energy, while atmospheric drag changes the energy and the magnitude of the angular momentum but not its direction. The reason for the former effect is that with an azimuthally symmetrical mass distribution the gravitational force has no azimuthal component. Since this is true by definition also for the radius vector, the torque has an azimuthal component only, i.e., the torque is perpendicular to the angular momentum vector. The reason for the latter effect is that the drag force lies in the orbital plane so that the associated torque is anti-parallel to the angular momentum vector.
Finally, the orbital period, \( T \), is given by:

\[
T = \frac{2\pi}{\sqrt{\mu}} a^{3/2} = \pi \cdot 10^{-7} \cdot a^{3/2}
\]  

(8)

**Nodal regression and apsidal rotation**

Because of the oblateness of the Earth, the orbital plane will precess in inertial space. For the same reason, perigee will rotate relative to the equatorial plane. As illustrated in Figure 3, the Earth can be regarded as a homogeneous sphere with an added “waist-belt” (Figure 3). This means that in addition to the dominant central force there is also an out-of-the-orbital-plane force which gives a torque on the orbit. This torque is the cause of the perturbations discussed above. Quantitatively, including a first-order correction, but still assuming azimuthal symmetry, the gravitational potential, \( V \), can be written:

\[
V = -\frac{\mu}{r} \left( 1 + \frac{J_2}{2} \left( \frac{R_E}{r} \right)^2 (1 - 3\sin^2 \phi) \right)
\]  

(9)

![Fig. 3. Earth's "waist-belt".](image)

In this approximation for the gravity field, integration over one revolution yields the following expressions for the change of \( \Omega \) and \( \omega \) per revolution:

\[
\Delta \Omega_{\text{rev}} = -3\pi J_2 R_E^2 \frac{\cos i}{a^2 (1-e^2)^2} \approx -4.14 \cdot 10^{11} \cdot \frac{\cos i}{a^2 (1-e^2)^2}
\]  

(10)

\[
\Delta \omega_{\text{rev}} = \frac{3\pi}{2} J_2 R_E^2 \frac{5 \cos^2 i - 1}{a^2 (1-e^2)^2} \approx 2.07 \cdot 10^{11} \frac{5 \cos^2 i - 1}{a^2 (1-e^2)^2}
\]  

(11)

We see that to first order for a polar orbit \( (i = 90^\circ) \) there is no nodal regression, thus the orbital plane is fixed in inertial space. We also see that for \( i \approx 63.4^\circ \) there is no apsidal rotation, thus apogee and perigee remain at fixed latitudes.
Atmospheric braking

The effect of atmospheric drag on satellites in near-Earth orbits is to reduce the total energy or in other words to reduce the semi-major axis. The apogee altitude decreases faster than the perigee altitude, i.e., the eccentricity is continuously reduced. For any reasonable atmosphere the density decreases with increasing altitude, thus the drag maximises at perigee. To simplify the picture, replace the drag with a Dirac-shaped reduction of the kinetic energy at perigee. This will reduce apogee altitude, but not affect the perigee altitude since the impulse is horizontal and thus there is no added vertical acceleration at perigee. For eccentric orbits in atmospheres with realistic scale-heights (10’s of km’s) this simple picture is rather representative.

<table>
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<th>Atmospheric density (kg m$^{-3}$)</th>
<th>Scale height (km)</th>
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Table 1. Atmospheric density and scale height used in the example calculations.

Integrating the change of momentum caused by the drag force on the satellite over one revolution results in the following approximate expressions for the change per revolution of semi-major axis and eccentricity:

$$
\Delta a_{\text{rev}} = -2\pi(C_D A/m)a^2 \rho_p e \frac{ae}{H} \left(I_0\left(\frac{ae}{H}\right) + 2eI_1\left(\frac{ae}{H}\right)\right) \tag{12}
$$

$$
\Delta e_{\text{rev}} = -2\pi(C_D A/m)a^2 \rho_p e \frac{ae}{H} \left(I_1\left(\frac{ae}{H}\right) + e\left(I_0\left(\frac{ae}{H}\right) + I_2\left(\frac{ae}{H}\right)\right)\right) \tag{13}
$$

where $C_D$ is the drag coefficient, $A$ is the spacecraft drag area, $\rho_p$ is the atmospheric (mass) density at perigee, and $H$ is the atmospheric scale height.

Accurate modelling of the atmosphere is difficult. The values in Table 1 have been used as a basis for the estimations made for this report.

Example results of mission analysis

Figures 4 and 5 show two examples of orbital decay. Figure 6 gives the nodal regression and apsidal rotation rates as a function of inclination for near-Earth orbits.
Fig. 4. Example of orbital evolution in a braking atmosphere, initial perigee is 320 km, initial apogee 480 km.

Fig. 5. Example of orbital evolution in a braking atmosphere, initial perigee is 280 km, initial apogee 1000 km.
Fig. 6. Nodal regression and apsidal rotation rates as a function of inclination for a near-Earth orbit, perigee 320 km, apogee 480 km.

Tables 2-4 show the orbital lifetime for different combinations of initial perigee and apogee altitudes. In all three tables the atmospheric scale height at perigee was taken to be 50 km, the spacecraft mass 25 kg, the drag area 0.5 m², and the drag coefficient 2.2. The latter is a rather weak function of atmospheric composition. To first order the life-time is proportional to spacecraft mass and inversely proportional to drag area. Thus, orbital lifetimes for other combinations of mass and drag area may be obtained by linear scaling of the values given in the tables. The dependence on scale height is more complicated but weaker.

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Table 2. Orbital lifetimes (days), upper limits, for apogee altitudes given in the upper row and perigee altitudes given in the left column. The second column gives atmospheric density. Atmospheric scale height 50 km, drag coefficient 2.2.

In Table 2 the atmospheric mass density at 300 km was taken to be 5x10⁻¹² kg m⁻³. This is representative of night time at solar minimum. In Table 3 the density is 4.7x10⁻¹¹ kg m⁻³, representative of day time at solar maximum. Thus, Tables 2 and 3
give upper and lower limits to the orbital lifetime. Note that lifetime is inversely proportional also to the density so the same scaling law as for the drag area applies.

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<th>800</th>
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Table 3. Orbital lifetimes (days), lower limits, for apogee altitudes given in the upper row and perigee altitudes given in the left column. The second column gives atmospheric density. Atmospheric scale height 50 km, drag coefficient 2.2.

Table 4 is based on the day time density for solar minimum which is probably a realistic average density for the 2007-2008 time period which is close to solar minimum.

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Table 4. Orbital lifetimes (days), averages, for apogee altitudes given in the upper row and perigee altitudes given in the left column. The second column gives atmospheric density. Atmospheric scale height 50 km, drag coefficient 2.2.

In polar coordinates the equation for the orbit ellipse may be written:

\[
r = \frac{a(1-e^2)}{1+e \cos \nu}
\]  

(14)

where \(\nu\) is known as true anomaly. Thus, for small eccentricities:

\[
r - r_{\text{per}} \approx ae(1-\cos \nu)
\]

(15)

Since velocity is fairly constant for near-circular orbits, the fraction of a revolution, \(e_{\text{below}}\), that a spacecraft spends at an altitude within \(x\) above perigee is given by \((ae = (h_{\text{apo}} - h_{\text{per}}) / 2)\) :
\[
\varepsilon_{\text{below}} = \frac{1}{\pi} \arccos \left(1 - \frac{2x}{h_{\text{apo}} - h_{\text{per}}}\right) \tag{16}
\]

As examples, for a 220 x 1200 km orbit \( \varepsilon_{\text{below 250km}} \) is 0.11 and for a 230 x 1200 km orbit \( \varepsilon_{\text{below 250km}} \) is 0.09.

![Spherical triangle](image)

Fig. 7. The spherical triangle used for estimating \( t \).

For non-polar orbits the time spent in a certain latitude interval is a function of latitude. Using the notation of Figure 7, the spherical sine theorem applied to the spherical triangle in the figure gives:

\[
\frac{\sin x}{\sin 90^\circ} = \frac{\sin(90^\circ - i)}{\sin(90^\circ - \varphi)} \tag{17}
\]

which simplifies to

\[
\sin x = \frac{\cos i}{\cos \phi} \tag{18}
\]

If \( i \) is not too close to \( \varphi \) planar trigonometry may be used to estimate the time spent in a latitude interval centred on \( \varphi \). This time exceeds that of a polar orbit by a factor \( t \), given by:

\[
\tau = \sec x = \left(1 - \frac{\cos^2 i}{\cos^2 \phi}\right)^{-1/2} \tag{19}
\]
Discussion, summary and conclusions

For the heating requirements to be fulfilled, a perigee of less than 250 km is necessary. To get a reasonable lifetime this implies an apogee of at least 1000 km. Using the average atmosphere and with a perigee of 225 km, a 1200 km apogee gives a lifetime of seven months. In this orbit, 10% of the time is spent below 250 km, or alternatively, at a given latitude the spacecraft is below 250 km within an angular interval of 36° around perigee. Northbound and southbound passes may “overlap” depending on inclination.

A near-Earth spacecraft crosses all latitude circles below its inclination some 30 times per day (14-15 times northbound and 14-15 times southbound). Thus, at a given latitude (such as that of a ground-based transmitter) the spacecraft passes, in the mean, within half a degree of all meridians in 12 days. Thus, if the criterion for conjunction is that it shall pass within 100 km of the meridian, there will be 2 conjunctions per week at 60 degrees latitude (since 100 km corresponds roughly to 2 degrees of longitude). Note that this is an ensemble average. “Orbital resonances” may make certain longitudes recur regularly, which could be good as well as bad from the point of view of conjunctions. At this point it is not meaningful to discuss anything but mean values quantitatively. At latitudes close to the orbital inclination the trajectory is fairly oblique to the meridians which increases the probability of conjunction.

An important factor for the orbital lifetime is the drag area. For an Astrid type spacecraft the drag area is significantly different for different attitudes. Since the Astrid platform is roughly Sun pointing, with the solar arrays more or less perpendicular to the Sun, drag is much more severe when the orbital plane has a noon-midnight orientation than when it has a dawn-dusk orientation. To reduce drag it may be desired to use “overdimensioned” solar arrays so that they give enough power even if they are tilted somewhat away from the Sun. Another possibility, although probably impractical, is to have retractable solar arrays so that the drag area can be temporarily reduced close to perigee.

A few sample “dipper mission” scenarios

Based on the discussion above, perigee and apogee altitudes of 225 and 1200 km are taken as a baseline for a sample dipper mission. In this orbit the spacecraft spends at least 10% of the time below 250 km, and the estimated lifetime is seven months. The remaining free parameter is then the inclination, \(i\).

\(i = 63.43°\) gives an orbit whose argument of perigee remains constant. For example, with this orbit perigee can be locked at the latitude of a heating transmitter. A possible drawback is that all potential auroral measurements in this configuration are made over the opposite hemisphere, probably with less ground support coverage. There is also very little cusp and polar cap coverage. For a thermospheric dipper mission aimed at measuring the unperturbed atmosphere, a perigee-locked orbit is probably not optimal. The nodal regression rate relative to the Sun is -4.1° per day. For this inclination, \(t = 1.75\).

\(i = 70°\) gives an apsidal rotation rate of -1.6° per day. A full cycle is then 270 days, with a time window below 250 km of 27 days. The nodal regression rate is -3.3° per day. This orbit is probably a poor compromise. \(t = 1.28\).

\(i = 80°\) gives an apsidal rotation rate of -3.3° per day. A full cycle is 110 days. The time window where measurements below 250 km are possible is 11 days wide. With a
7-month lifetime three such cycles may be completed, if properly initialised. The nodal regression rate is \(-2.2^{\circ}\) per day. From the point of view of auroral measurements this orbit is most likely optimal. \(t = 1.06\), i.e., negligibly different from unity.

\(i = 98.2^{\circ}\) gives a Sun-synchronous orbit, i.e., one where \(O\) changes by \(+360^{\circ}/365\) per day, so that the orbital plane remains fixed relative to the Sun. The apsidal rotation rate is \(-3.1^{\circ}\) per day. A big advantage with a Sun-synchronous orbit is that it allows for continuous cartwheel mode operations, which means that the spacecraft can fly with minimum drag area at all times. This may significantly improve the lifetime. A drawback is that the local time coverage for auroral measurements is limited. \(t = 1.04\), i.e., very close to the value for a polar orbit. Another possible drawback is that this strategy works best for dawn-dusk orbits, which may not be optimal for ionospheric/thermospheric dipper measurements, since neither the dayside ionisation maxima nor the nightside minima are sampled.

Finally, a few comments about launch strategy. Regardless of inclination, for a dipper mission with a Sun-pointing spacecraft, launch into a dawn-dusk orientation is preferable, in order to keep the drag area to a minimum as long as possible. Furthermore the initial argument of perigee should be chosen such that the first “low-altitude window” begins at the end of the experiment check-out phase.

Appendix 1

An upper limit to the effect of Lunar gravity on the nodal regression, for high-altitude Earth-centred orbits, is given by:

\[ \Delta \Omega_{\text{rev}} < 1.2 \cdot 10^{-20} \frac{a^3}{\sin i} \cdot \frac{2 + 3e^2}{\sqrt{1 - e^2}} \]  \hspace{1cm} (20)

The effect of Solar gravity is normally less than that of Lunar gravity.

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Literature