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MECHANISMS FOR DRIVING BIRKELAND
CURRENTS

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INTRODUCTION

The hot magnetospheric plasma interacts electro-dynamically with the cold ionospheric plasma by means of Birkeland currents flowing along geomagnetic field lines connecting the two plasmas. The fully ionized, collision-free plasma of the magnetosphere and the partially ionized, collisionally dominated plasma of the ionosphere behave differently and in general there will be an electric mismatch between the two regions resulting in Birkeland current flow. The set of relations governing the system is summarized in Figure 1. We have a closed system of equations that in principle can be solved given appropriate boundary conditions (cf. Vasyliunas 1970). Most studies thus far, including this one, consider only some links of the framework. Here we will not use the approach of a boundary value problem. Rather, we will assume that some of the physical parameters are known from measurements and we will ask what information about the other quantities can be extracted from this knowledge using the governing equations. In particular we will consider possible conclusions regarding mechanisms for driving the Birkeland currents, that can be obtained from the characteristics of these currents as observed by Zmuda and Armstrong (1974).

For the magnetospheric plasma we will use MHD theory. Certainly this has shortcomings which must be borne in mind, for example it cannot be used to study phenomena

Fig. 1. Interrelationships between magnetospheric and ionospheric parameters.

that depend on the energy distribution of the particles. It may well be that the formation of forbidden regions in the particle flow pattern, of a different size for particles of different energy, is significant for the Birkeland current system. Nevertheless a discussion in terms of MHD theory might be useful as we believe that those phenomena that are predicted using the MHD equations would occur also in a more refined treatment.

In the magnetosphere there is a balance between pressure and inertial forces and a magnetic $\vec{J} \times \vec{B}$ force due to the flow of transverse currents. Birkeland currents originate in the magnetosphere when this transverse current varies spatially so that it has a divergence, caused by spatial variations of the magnetospheric parameters. Thus the magnetosphere may act as an electric generator and the ionosphere is then the load. However, Birkeland currents may also originate from the ionosphere. Wind systems in the ionosphere may drive currents there which could have a divergence giving rise to Birkeland currents. Here we will not consider this link involving the motion of the neutral gas. Also secondary Birkeland currents of ionospheric origin occur when the horizontal ionospheric currents driven by the magnetospheric generator flow through regions of varying ionospheric conductivity. Then polarization electric fields tend to build up, but they also tend to discharge to the magnetospheric load by means of Birkeland currents.

BASIC EQUATIONS

For our discussion we will use the very simplest form of equation for the plasma motion

$$\vec{E} + \vec{v} \times \vec{B} = 0 \quad (1)$$

Although field-aligned electric fields might appear in regions, and at times, of intense Birkeland currents we do not believe that they would occur as a regular feature over large areas affecting the magnetosphere-ionosphere mapping of the gross-scale electric fields. Thus we neglect the link of Figure 1 involving parallel electric fields. We will consider relatively quiet (non substorm) conditions for which we can use an electrostatic potential. If this potential is given over a surface, such as the ionosphere, cutting all field lines, then the potential, electric field and plasma velocity are determined everywhere in the magnetosphere, assuming the magnetic

field to be known.

The transverse current in the magnetosphere is related to the plasma pressure and inertial forces by a momentum balance equation which, if solved for $\vec{j}_\perp$, reads

$$\vec{j}_\perp \left(1 - \mu_0 \frac{p_\parallel - p_\perp}{B^2} \right) = \frac{\vec{B}}{B^2} \times \left[\rho (\vec{v} \cdot \nabla) \vec{v} + \nabla p_\perp + \right. \\ \left. + (p_\parallel - p_\perp) \frac{\nabla B}{B} \right] \quad (2)$$

This expression for $\vec{j}_\perp$ may be derived by summing the contributions from the gradient, curvature and polarization drifts of the individual particles and the magnetization current (cf. Parker, 1957). We will use the approximation $\epsilon = \mu_0(p_\parallel - p_\perp)/B^2 \ll 1$. Thus the results will be precise if the pressure is isotropic, while for the case of anisotropic pressure the results are quantitatively correct only if the kinetic energy density of the random ("thermal") particle motion is small compared to the magnetic energy density which implies that the effect of the pressure-driven currents on the magnetic field should be negligible.

The momentum equation (2) primarily gives information only on the transverse component of the current. However, we can use this equation also to obtain information on the Birkeland component using the requirement that the total current is divergence-free, that is, $\text{div } \vec{j}_\parallel = -\text{div } \vec{j}_\perp$. Integrating along a fluxtube we can then find $\vec{j}_\parallel$.

To specify the anisotropic pressure we need at least two scalar parameters, one giving the thermal energy density and the other, or others, characterizing the form of the pitch-angle distribution of the energetic particles providing the pressure. If these are known for example for the equatorial plane the pressure may be evaluated at any point along the field lines assuming that all particles in their bounce motion pass this plane. We will use a simple form of anisotropy corresponding to a pitch-angle distribution where the number of particles per steradian in the equatorial plane for all energies is proportional to $\sin^{2\gamma} \Theta$ with Θ being the pitch angle. Then the form of the pitch-angle distribution remains the same at all points along the field line, although the number density varies. We find

$$p_\perp = (\gamma + 1) p_0 \left(\frac{B}{B_N} \right)^{-\gamma}; \quad p_\parallel = p_0 \left(\frac{B}{B_N} \right)^{-\gamma} \quad (3)$$

where the parameter $p_o = n_o m \langle w^2 \rangle (B_o/B_N)^\gamma / (2\gamma+3)$, for a given γ , gives a measure of the kinetic energy density. n_o is the number density and B_o the magnetic field at a reference level taken to be the ionosphere, while B_N is a constant magnetic field, taken to be the field at the pole, introduced for the purpose of normalization. The parameters γ and p_o are constant along each field line but may vary from one field line to another. Obviously the functional form $\sin^{2\gamma} \Theta$ can be used to model a wide range of pitch-angle distributions: $\gamma = 0$ corresponds to an isotropic pressure $p_\perp = p_\parallel = p_o$, which is constant along the field lines, while the limit $\gamma \rightarrow \infty$ corresponds to $p_\parallel/p_\perp \rightarrow 0$ and the limit $\gamma \rightarrow -1$ to $p_\perp/p_\parallel \rightarrow 0$.

Our problem of ionosphere-magnetosphere interaction is evidently a two-dimensional one as knowledge of parameters over a surface suffices to determine the field variables at all points. By performing integrations along the field lines of variables deriving from (1) and (2) it is then possible to reduce the study to this surface. We will show in the next section how this reduction appears for the case of currents driven by the pressure terms. It is well known that for the ionosphere a similar reduction can be performed (for phenomena of a spatial scale exceeding a few kilometers) by integrating currents and conductivities along the highly conducting field lines, reducing the influence of the ionosphere to that of a conducting sheet.

BIRKELAND CURRENTS DRIVEN BY THE PLASMA PRESSURE

We will discuss the effects of the pressure and inertial forces separately and we start with the pressure. As the magnetic field lines play a central role for our problem it is convenient to use coordinates that explicitly refer to these. Such coordinates are Euler potentials α and β , labelling the field lines, and the length of arc l measured along the field lines from the ionosphere in the northern hemisphere. With these coordinates

$$\vec{B} = - \nabla \alpha \times \nabla \beta \quad (4)$$

Let us consider the current continuity in a fluxtube bounded by the surfaces $\alpha = \alpha_o$, $\alpha = \alpha_o + d\alpha$, $\beta = \beta_o$, $\beta = \beta_o + d\beta$, the ionosphere ($l = 0$), and the equatorial plane. Figure 2 shows a segment of this fluxtube. The total current flowing out from the fluxtube through the

Fig. 2. Segment of fluxtube showing coordinates α , β , and l and surface elements.

surface $\alpha = \alpha_0 + d\alpha$, with the surface element $d\vec{S}_\alpha = \nabla \alpha d\beta d\mathbf{l} / B$, is

$$J_\alpha d\beta = d\beta \int_0^L \frac{\vec{j}_\perp \cdot \nabla \alpha}{B} d\mathbf{l} \quad (5)$$

Using (2), (3), and (4) we find after some reductions, neglecting a term of order ϵ ,

$$J_\alpha d\beta = d\beta \int_0^L \frac{1}{B} \frac{\partial p_\perp}{\partial \alpha} d\mathbf{l} - d\beta \left[\frac{p_\parallel - p_\perp}{B^2} \frac{\vec{B} \times \nabla l \cdot \nabla \alpha}{B^2} \right]_0^L \quad (6)$$

A corresponding expression applies to the integrated current $J_\beta d\alpha$ flowing through the surface $\beta = \beta_0 + d\beta$. The net inflow through the four sides of the fluxtube due to $\vec{j}_\perp$, which is $-(\partial J / \partial \alpha) d\beta d\alpha - (\partial J / \partial \beta) d\alpha d\beta$, must be balanced by the flow of vertical current of density j_v through the bottom surface element $dS_l = |\nabla l| d\alpha d\beta / B_0$. We assume that the equatorial plane is a plane of symmetry so that there is no current flow through this plane into the fluxtube considered. With the vertical current counted positive if flowing into the ionosphere we obtain

$$j_v = - \frac{B_0}{|\nabla l|} \left\{ \frac{\partial}{\partial \alpha} \int_0^L \frac{1}{B} \frac{\partial p_\perp}{\partial \beta} d\mathbf{l} - \frac{\partial}{\partial \beta} \int_0^L \frac{1}{B} \frac{\partial p_\perp}{\partial \alpha} d\mathbf{l} \right\} \quad (7)$$

Introducing the expression (3) for $p_{\perp}$ we find after some reductions

$$j_v = \nabla_o p_o \times \nabla_o V_1 \cdot \hat{r} - \nabla_o \gamma \times \nabla_o (p_o V_2) \cdot \hat{r} \quad (8)$$

Here

$$V_1 = \int_0^L \frac{B_N^{\gamma} d\ell}{B^{1+\gamma}} \quad (9)$$

$$V_2 = \int_0^L \frac{B_N^{\gamma} \ln(B/B_N) d\ell}{B^{1+\gamma}} \quad (10)$$

and ∇_o is the gradient evaluated for $\ell = 0$ and $\hat{r}$ is a vertical unit vector. In deriving equations (7) and (8) we have neglected terms of order ϵ and terms which are at most of the same order as the flow of $j_{\perp}$ through the end surfaces of the fluxtube, which we can neglect as we are interested in cases where the contribution to j_v from the Birkeland current $j_{\parallel}$ dominates over the contribution from $j_{\perp}$. For isotropy ($\gamma=0$) all neglected terms would vanish identically.

As the variables p_o , γ , V_1 , and V_2 depend only on α and β but not ℓ , equation (8) demonstrates that it is possible to derive the vertical current density j_v from a study of the variation of these variables in the ionosphere only.

For the particular case of isotropy ($\gamma=0$) Vasyliunas (1970) has given an expression analogous to what (8) and (9) would give for $\gamma=0$ (although off by a factor 2). For the special case of a dipolar field and isotropy we may solve (9) analytically and derive the expression

$$j_v = \frac{4R_e^2}{\mu_o m} (1 + 4 \cos^2 \theta_o - 2 \cos^4 \theta_o + \frac{4}{5} \cos^6 \theta_o - \frac{1}{7} \cos^8 \theta_o) \sin^{-10} \theta_o \frac{\partial p}{\partial \phi} \quad (11)$$

Here m is the magnetic moment of the earth, R_e the radius of the ionosphere, and θ_o the colatitude. A similar expression has been given by Kern (1967) although it differs from (11) by a very small term due to the contribution from the vertical component of $j_{\perp}$ at the ionosphere, neglected in Kern's analysis but included here. This term, although smaller than the contribution from $j_{\parallel}$ by a factor 5×10^4 for $\theta_o = 20^\circ$, has a certain significance in that it makes the structure of the analytic expression simpler. Only the azimuthal gradient of the pressure is

significant for driving Birkeland currents according to (11). Evidently j_v integrated along a curve $\theta_0 = \text{constant}$ around the globe vanishes. Thus there is as much Birkeland current flowing to the earth as away from the earth in each strip of constant θ_0 . We will show below that a similar conclusion applies under more general conditions.

Returning now to the more general case of anisotropic pressure and non-dipolar field geometry, we should first of all point out that if we have a model of p_0 and γ and of the magnetic field we may of course use (8), (9), and (10) to evaluate j_v . However, a more interesting question is to ask what information about the pressure parameters p_0 and γ we can obtain from a model, or measurements, of j_v . Clearly knowledge of one variable, j_v , is not sufficient to derive the two independent quantities p_0 and γ . Thus, different distributions of p_0 and γ could give the same j_v . However, if one of the quantities p_0 and γ is given we should be able to derive some information about the other from measurements of j_v . The problem is particularly simple if the anisotropy factor γ is constant. Then equation (8) reduces to

$$j_v = \nabla_0 p_0 \times \nabla_0 V_1 \cdot \hat{r} \quad (12)$$

Equation (12) shows that if j_v is known it is possible to derive the component of $\nabla_0 p_0$ along contours $V_1 = \text{constant}$, and by integration, the variation of p_0 along each such contour. Integrating around a closed curve $V_1 = \text{constant}$ the final p_0 should equal the initial value, that is, the total variation of p_0 should be zero. However, in general this would not be the case for an arbitrary distribution of j_v . Thus there exists a constraint on the possible forms of distribution of vertical current if this is driven by a distribution of pressure with constant anisotropy factor. This can be used as a test if the observed currents are driven in this way. If we evaluate the total current flowing into the ionosphere within a contour $V_1 = \text{constant}$ we find

$$\begin{aligned} J_v &= \iiint j_v dS_r = \iiint -\nabla_0 p_0 \times \nabla_0 V_1 \cdot \vec{dS}_r = \\ &= \iiint \text{curl} (V_1 \nabla_0 p_0) \cdot \vec{dS}_r = - \int V_1 \nabla_0 p_0 \cdot \vec{ds} = \\ &= -V_1 \int \nabla_0 p_0 \cdot \vec{ds} = 0 \end{aligned} \quad (13)$$

Thus the net current to the ionosphere is zero in the region enclosed by each contour $V_1 = \text{constant}$ and thus also within any strip bounded by contours of constant V_1 . Also in the ionosphere there could be no net flow of the

Fig. 3 a, b.

Fig. 3. Contours of constant V_1 (in units of Re/nT) which encircle regions of zero net Birkeland current flow to the ionosphere shown for three different values of γ . Differences in absolute values of V_1 between the three plots have no significance.

horizontal currents across these contours. In itself, the existence of contours with this property is not unexpected. However, it is worth noting that we have been able to predict the shape of these contours, so that we can use this as a test on a measured distribution of j_v . Figure 3 shows the contours $V_1 = \text{constant}$ evaluated for three different values of γ , namely $\gamma = -0.5$ ($p_{\parallel} = 2p_{\perp}$), $\gamma = 0$ ($p_{\parallel} = p_{\perp}$), and $\gamma = 1$ ($p_{\perp} = 2p_{\parallel}$), using the recent magnetic field model of Olson and Pfitzer (1974). The shape of the contours, although somewhat dependent on γ , in general resembles that of the auroral oval.

Figure 4 shows the average characteristics of the gross-scale Birkeland current flow as observed by Zmuda and Armstrong (1974). There are two separate systems, one in the evening and one in the morning sector of the auroral oval, each consisting of two broad sheets of current.

Fig. 4. Regions and directions of Birkeland current flow observed by Zmuda and Armstrong (1974). In the region of the Harang discontinuity, A, both types of current patterns are found, while in region B no fixed pattern is observed.

The Birkeland current sheets close by means of horizontal Pedersen currents, flowing northward in the evening sector and southward in the morning sector. In addition there are horizontal, electrojet, Hall currents flowing eastward in the evening and westward in the morning auroral oval, driven by the same electric field as the Pedersen currents. In each one of the two systems there definitely seems to be a net current flow across contours $V_1 = \text{constant}$ shown in Figure 3. Thus these current systems could be driven by a magnetospheric pressure distribution of constant γ only if the total current of the evening and morning systems at all times are equal so that there is no net current flow across the contours $V_1 = \text{constant}$. As long as we have observations from one satellite alone this can only be checked on a statistical basis. However, as these sheets of Birkeland currents are the driving agents for the eastward and westward electrojets we should also expect that these jets then should vary in intensity synchronously. Studies of electrojet variations reported in the literature do not always clearly separate substorm variations, which could be of a different nature, from quiet-time variations. If we in-

clude substorm events it is obvious that the eastward and westward electrojets do not develop synchronously and, therefore, we would hardly expect that the two systems of Birkeland currents at all times carry the same total current. At quiet times the two systems are separated by the Harang discontinuity and it seems that they are two separate entities. Thus there may be some difficulties explaining the observed current system as being driven by the magnetospheric pressure.

We may also make a quantitative estimate of the pressure variations in the magnetosphere required to drive the observed Birkeland currents. According to Zmuda and Armstrong (1974) the Birkeland current densities lie mainly between 3×10^{-7} and 4×10^{-6} A/m². Comparing Figure 4 with Figure 3 for $\gamma = 0$ we find that the currents flow in regions where V_{V_1} is of the order $1.5-5 \times 10^9$ T⁻¹. Using equation (12) we then find that the component of $V_0 P_0$ along the V_1 -contours, which approximately delineate the Zmuda-Armstrong current sheets, amounts to 6×10^{-17} - 3×10^{-15} Pa/m. The total pressure drop along a sheet extending over about 3000 km is then 1.8×10^{-10} - 9×10^{-9} Pa. For the case of isotropic pressure we would have the same pressure variation in the magnetosphere. A magnetic field of corresponding energy density would be 30-180 nT, thus the magnitude of the required pressure variation is not unrealistic. The form of the pressure variation needed is remarkable. The pressure should increase by the amount evaluated from the night-side to the dayside along the poleward sheets, but decrease along the equatorward sheets in both the evening and morning systems. Thus, somewhere in the systems there must exist even stronger gradients of pressure perpendicular to the sheets, but these cannot be evaluated from the Birkeland currents. Figure 5 shows the contours $V_1 = \text{constant}$ of Figure 3 for $\gamma = 0$ mapped onto the equatorial plane, with the projection of the Birkeland current regions of Figure 4, and the inferred directions of pressure gradients.

BIRKELAND CURRENTS DRIVEN BY THE PLASMA CONVECTION

We start this section with a qualitative discussion of the observed relationships between the directions of quiet time current flow, electric fields and plasma velocity. Figure 6 shows schematically, for a dipolar-like geometry, the current sheets observed by Zmuda and Armstrong (1974). The Birkeland currents close in the ionosphere, presumably by currents transverse to the sheets.

Fig. 5. Projections into equatorial plane of the contours $V_1 = \text{constant}$ of Figure 3 for $\gamma = 0$, and projection of the Birkeland current region of Figure 4 using the field model of Olson and Pfizter (1974). Also shown are inferred directions of pressure gradients along the V_1 contours.

As the electric field in the oval is known to be generally northward in the evening and southward in the morning we conclude that the closure currents dissipate power and are essentially Pedersen currents. In the magnetosphere the currents must also close by currents transverse to the magnetic field. Although it is not known that the currents flow along the most natural, shortest path as shown in the figure, the currents will, whatever path they take, flow from a region of higher potential to a region of lower potential through a generator region. This transverse current is then associated with a $\mathbf{j} \times \mathbf{B}$ force directed in such a way that it will brake the plasma flow associated with the electric field, so that kinetic energy of the convecting plasma is transferred to electric energy.

Fig. 6. Birkeland current sheets observed by Zmuda and Armstrong (1974) with associated closure currents, electric fields and plasma convection shown for a dipolar geometry.

Thus there exists an obvious generator mechanism for Birkeland currents using the plasma convection. However, it has been thought to be less effective than pressure gradients because there seems to be more kinetic energy available in the random (thermal) particle motion than in the ordered (convective) motion. Only if the field lines from the auroral oval extend far back into the tail the plasma velocities and volumes would be large enough to support a current system in which the plasma motion is not stopped too rapidly. Now there is an increasing amount of evidence accumulating showing that the auroral oval maps into the tail plasma sheet, which extends at least as far back as the orbit of the moon. (See Rostoker and Boström (1974) for a discussion and references. Note, however, that the field model of Olson and Pfitzer (1974), which applies to the most quiet state of the magnetosphere, predicts closure of auroral field lines at rather short distances.) Rostoker and Boström (1974) have revised the model of Birkeland current generation taking the tail geometry into account and have demonstrated that with realistic parameters it is quite feasible to drive Birkeland currents by slowing down convection in the magnetotail. Figure 7 shows the presumed electric field pattern. In the plasma sheet the $\vec{E} \times \vec{B}/B^2$ velocity must be directed essentially towards the flanks of the magnetotail. If this velocity is slowed down it could drive the four systems of Birkeland currents flowing to the morning and evening auroral ovals of the northern and southern hemispheres. In the tail these currents would flow along the boundaries between the plasma sheet and tail lobes and, in the opposite direction, in

Fig. 7. Cross section of the magnetotail looking towards the earth showing qualitatively the electric field pattern mapped from the ionosphere, and directions of plasma flow and Birkeland closure current (after Rostoker and Boström, 1974).

the central part of the plasma sheet. It is interesting to note that these currents in each system would give the total magnetic field a skew so that the field lines flare outwards in the plasma sheet in agreement with observations.

In the discussion above we have assumed that the poleward and equatorward sheets of Birkeland current balance all along the auroral oval so that the ionospheric closure currents are meridional. Akasofu (1975) reports, however, that the poleward sheets typically, and particularly during substorms, are more intense than the equatorward sheets implying a westward component of the ionospheric closure current flowing across the midnight meridian from the morning to the evening part of the auroral oval. In the tail a corresponding generator current would flow from the dusk to the dawn part of the boundaries between the plasma sheet and the tail lobes braking the flow of plasma from the tail lobes into the plasma sheet, a flow that we expect to be enhanced during substorms.

Having shown that a mechanism similar to that of a magnetohydrodynamic dynamo qualitatively could explain the generation of the gross scale Birkeland currents we will now study the mechanism in some detail. Due to the quadratic nature and complicated structure of the inertial term $(\vec{v} \cdot \nabla)\vec{v}$, of equation (2), where $\vec{v}$ is related to the magnetospheric electric field by (1), it seems somewhat difficult to discuss this in as general terms as we have done for the pressure terms. Thus we will carry out the analysis using a very much simplified geometry with a homogeneous magnetic field. There are two justifications for this. First of all the inertial term could be of importance only if the field lines carrying the Birkeland current are extended far back into the magnetotail and for that region a homogeneous field may not be too bad an approximation. Secondly, the Birkeland currents driven by the inertial term do not depend critically on the existence of gradients in the magnetic field as the pressure-driven currents do.

We introduce orthogonal curvilinear coordinates u_1, u_2 in planes perpendicular to the magnetic field, see Figure 8. Here h_1 and h_2 are scale factors for the coordinates u_1 and u_2 respectively, such that the differential arc lengths along the u_1 and u_2 curves are $h_1 du_1$ and $h_2 du_2$. Unit vectors in directions of increasing u_1 and u_2 are denoted $\hat{u}_1$ and $\hat{u}_2$. We define the coordinate system so that the u_2 coordinate lines are equipotentials, then the electrostatic potential V is a function of u_1 solely. Using equation (1) the plasma velocity is found to be

$$\vec{v} = \frac{1}{Bh_1} \frac{dV}{du_1} \hat{u}_2 \quad (14)$$

and using (2) with $\epsilon = 0$ the contribution to $\vec{j}_1$ from the inertial term is

$$\vec{j}_1 = \frac{\rho}{B^3} \frac{1}{h_1^3 h_2} \left(\frac{dV}{du_1} \right)^2 \left[\frac{\partial h_1}{\partial u_2} \hat{u}_1 - \frac{\partial h_2}{\partial u_1} \hat{u}_2 \right] \quad (15)$$

Equation (15) shows that the inertial force gives a current component $\vec{j}_1 = (\rho v^2 / Bh_1 h_2) (\partial h_1 / \partial u_2) \hat{u}_1$ in the direction of $\hat{u}_1$, which is directed oppositely to the electric field provided $\partial h_1 / \partial u_2 > 0$. Thus the region acts as a generator if the separation of the equipotentials increase in the direction of $\vec{v}$, so that the electric field and plasma velocity decrease. The current component j_1 then gives a magnetic force $\vec{j}_1 \times \vec{B}$ which brakes the plasma

Fig. 8. Cross section of fluxtube, looking along the magnetic field lines towards the earth in the northern hemisphere afternoon sector, showing orthogonal coordinates u_1 , u_2 used in study of inertial effect.

convection. In the ionosphere this current component is closed by a Pedersen current dissipating the energy supplied in the MHD dynamo region.

The other current component $\vec{J}_2 = -(\rho v^2 / B h_1 h_2) (\partial h_2 / \partial u_1) \hat{u}_2$, directed parallel or antiparallel to $\vec{v}$, is not involved with any power production or dissipation as it is perpendicular to $\vec{E}$. However, this current component is associated with a magnetic force $\vec{J}_2 \times \vec{B}$ which deflects the plasma flow but does not change its velocity or kinetic energy. This current can be a part of a current circulating in the magnetosphere or it can possibly close in a loop to the ionosphere involving ionospheric Hall currents which are not accompanied by power dissipation. However, as a first order approximation we may neglect the divergence of the Hall currents, compared to that of the Pedersen currents, as the derivatives along the auroral oval (in the direction of the Hall current) generally are smaller than transverse to the oval (in the direction of the Pedersen current).

The power produced by the inertial forces in a fluxtube of length L and cross section $h_1 du_1 h_2 du_2$ is

$$-\vec{J}_1 \cdot \vec{E} h_1 du_1 h_2 du_2 L = \rho v^3 \frac{\partial h_1}{\partial u_2} du_1 du_2 L \quad (16)$$

which also can be seen by considering the difference in kinetic energy of the plasma flowing in and out of the fluxtube (the energy and momentum equations are equivalent). Putting this energy equal to the power dissipated in the ionosphere $\Sigma_P E^2 h_1 du_1 h_2 du_2$ gives

$$\rho v L \frac{1}{h_1 h_2} \frac{\partial h_1}{\partial u_2} = \Sigma_P B^2 \quad (17)$$

Strictly speaking we should here use an effective integrated conductivity mapped from the ionosphere to the tail, which could differ from the ionospheric one if the scale factors relating the east-west and north-south separations of field lines in the ionosphere to corresponding separations in the tail are different. Rostoker and Boström (1974) estimate that this effect might increase the effective conductivity by a factor of 2. We will take as typical values $\Sigma_P = 10S$, $\rho = 5 \times 10^{-21} \text{ kg/m}^3$ (3 protons cm^{-3}), $B = 7 \text{ nT}$ and $v = 50 \text{ km/s}$ (corresponding to $E = 0.35 \text{ mV/m}$). The derivative $(1/h_1 h_2)(\partial h_1 / \partial u_2)$ is interpreted as the relative rate of slowing down of the plasma $-(1/h_2)(\partial v / \partial u_2)/v$ as can be verified using (14). We will take this derivative to be $5 \times 10^{-9} \text{ m}^{-1}$ corresponding to a characteristic length for stopping the plasma of about 30 Re which we consider to be a reasonable value as it is somewhat larger than the radius of the tail. The derivative also describes the divergence of the equipotential contours. $5 \times 10^{-9} \text{ rad/m}$ corresponds to $1.8^\circ/\text{Re}$ or a tilt of the plasma sheet - tail lobe boundary of 14° relative to the plane of symmetry (neutral sheet) assuming a plasma sheet half-width of 7.5 Re. With the parameters given above we may now use equation (17) to evaluate the length L of the fluxtube, and we find $L \approx 60 \text{ Re}$. This is not an unrealistic value but it shows that Birkeland currents could be driven by inertial forces only if the field lines extend far into the magnetotail.

The intensity of the field-aligned current at the earthward side of the generator region is, neglecting the possible contributions from j_2 and ionospheric Hall currents,

$$j_{\parallel} = L \text{div } \vec{j}_{\perp} = \text{div } \Sigma_P \vec{E} \quad (18)$$

Assuming that Σ_P is constant at $10S$ and that the characteristic length for variations of $\vec{E}$ across the tail plasma sheet is 5 Re we find, with $E = 0.35 \text{ mV/m}$ as earlier, $j_{\parallel} = 1 \times 10^{-10} \text{ A/m}^2$ or mapped to the ionosphere $j_{\parallel} \cdot (B_{\text{ionosphere}}/B_{\text{tail}}) = 9 \times 10^{-7} \text{ A/m}^2$ in excellent agree-

ment with the range 3×10^{-7} to 4×10^{-6} A/m² quoted by Zmuda and Armstrong (1974).

A more detailed version of the model of the magnetotail MHD generator has been given by Rostoker and Boström (1974).

Hitherto we have considered Birkeland currents associated with steady state convection. However, substorms are also associated with intense Birkeland current flow. During the reconfiguration of the magnetic field in a sector of the magnetotail from a tail-like to a more dipolar geometry, which is an intrinsic feature of the substorm, magnetic energy is released which accounts for the energy dissipated in the ionosphere. Substantial induction electric fields appear, which set the plasma into a motion, which is braked by the Birkeland current loop to the ionosphere. Assuming that the characteristic time for changing the tail field in a sector is 15 minutes, we find from the Maxwell equation $\text{curl } \vec{E} = -\partial \vec{B}/\partial t$ a spatial derivative of $\vec{E}$ of $B \cdot (1.1 \times 10^{-3} \text{ s}^{-1})$ or of the associated plasma velocity of $1.1 \times 10^{-3} \text{ s}^{-1}$. We note that the curl of a transverse electric field is associated with a derivative in the direction of the plasma flow. Assuming the plasma velocity to be 50 km/s as before, the relative rate of slowing down of the plasma is $2.2 \times 10^{-8} \text{ m}^{-1}$ or somewhat larger than in our previous analysis. Evidently the induction process can drive significant Birkeland currents.

SECONDARY BIRKELAND CURRENTS OF IONOSPHERIC ORIGIN

When the horizontal ionospheric currents flow through regions of varying conductivity secondary Birkeland currents of ionospheric origin may be generated. Neglecting contributions to the currents from neutral winds, and using the approximation of vertical magnetic field lines, the height-integrated current is

$$\vec{J} = \Sigma_P \vec{E} + \Sigma_H \vec{B} \times \vec{E} \quad (19)$$

The divergence of $\vec{J}$ must match the Birkeland current, thus

$$j_u = \Sigma_P \text{div } \vec{E} + \vec{E} \cdot \text{grad } \Sigma_P + \vec{B} \times \vec{E} \cdot \text{grad } \Sigma_H \quad (20)$$

The Birkeland current may close in the magnetosphere by inertial forces, accelerating the plasma if the iono-

Fig. 9. Secondary Birkeland currents $j_{||}$ discharging polarization electric field E' developed in region of enhanced ionization in the ionosphere.

sphere acts as a generator. If there is some mechanism resisting the flow of Birkeland current, such as anomalous resistivity, a polarization electric field will develop in the ionosphere so that the term $\Sigma_P \text{div } \vec{E}$ partly balances the contributions from the conductivity gradients. However, we would not expect a complete choking of $j_{||}$. While the opposing term $\Sigma_P \text{div } \vec{E}$ is positive in regions of positive space charge the net $j_{||}$ deriving from conductivity gradients is negative (out of the ionosphere). Considering the ionospheric currents connected to these Birkeland currents and the electric field deriving from the space charges it is clear that the ionosphere acts as a generator with $\vec{E} \cdot \vec{J} < 0$, although using the total ionospheric current and field $\vec{E} \cdot \vec{J}$ is always positive.

As an example we may consider the effect of applying a westward electric field E to an ionospheric region with a slab of enhanced ionization extended in the east-west direction, see Figure 9. If there is some choking of Birkeland currents the excess northward Hall current in the slab builds up a southward polarization electric field E' , which drives a Pedersen current j_P opposing the primary Hall current. However, as the primary northward Hall current dominates, Birkeland currents will flow out of the ionosphere at the northern edge of the slab. Evidently the ionosphere acts as a generator for the current loops involving the Birkeland currents and their closure currents in the ionosphere. Models like this have been proposed for the large scale current and electric field structure of the auroral electrojets. However, they are not consistent with the observations of Zmuda and Armstrong (1974) which show that the polarity of the

Birkeland current sheets is such that the ionosphere acts as a load to these currents. Furthermore, to explain an eastward electrojet one would have to impose an eastward electric field on the ionosphere and this has not been observed to be a characteristic feature of the region of the eastward jet.

On the other hand the results of some rocket experiments (e.g. Park and Cloutier, 1971) may be interpreted as giving evidence that the mechanism discussed here may operate on a smaller scale in association with individual auroral arcs. In discussing findings from such experiments, and performing comparisons with satellite observations, it is important to realize their limited latitudinal coverage of the electrojet region, which may extend over 10° of latitude. Also it is very important to try to separate Birkeland currents of ionospheric origin from Birkeland currents of magnetospheric origin, which could be done if one could decide whether the closure currents are dissipative or generative. The horizontal ionospheric currents can be inferred from measurements of the electric field and from indirect information on the conductivity structure provided by measurements of precipitating particles or observations of visual auroral forms.

SUMMARY AND DISCUSSION

We have considered the basic mechanisms for generating Birkeland currents in the magnetosphere, that is mechanisms for redirecting transverse currents to form Birkeland currents. Using MHD equations we find that both the plasma convection and pressure can account for currents of the observed magnitude. For currents driven by the pressure there exists a constraint on the possible forms of the current distribution implying that the morning and evening systems of Birkeland current flow must balance each other at all times so that there is no net current flow in the ionosphere across certain specified contours. If the currents are driven by the plasma convection the morning and evening current systems may be independent. We have suggested geometries for plasma pressure variations and convective flow that could account for the observed distribution of Birkeland currents assuming that the currents originate on field lines within the magnetosphere extending into the plasma sheet of the magnetotail.

We have not discussed how the required plasma con-

vection or pressure is established. The energy dissipated by the Birkeland currents must primarily derive from the solar wind sweeping by the magnetosphere. Currents flowing through the solar wind plasma will extract kinetic energy from the solar wind which is transferred to electric energy. The current driven in this way closes through the magnetosphere where it can set up the required internal convective flow and pressure distributions which then can drive the internal current system considered in this paper. If the geometry of the current flow is different from that envisaged here, and in particular if the Birkeland currents occur on open field lines extending into the solar wind, some Birkeland currents could be directly linked to the currents in the solar wind plasma.

We have also considered secondary Birkeland currents of ionospheric origin which are currents tending to discharge ionospheric polarization electric fields. In interpreting measurements of Birkeland currents it is important to distinguish these secondary currents from the primary currents of magnetospheric origin which could be done if one can decide whether the ionospheric closure currents are dissipative or generative.

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MECHANISMS FOR DRIVING BIRKELAND CURRENTS

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Basic mechanisms for generating Birkeland currents in the magnetosphere, that is mechanisms for redirecting transverse currents to form field-aligned currents, are considered. Both the plasma convection and the plasma pressure can account for currents of the observed magnitude. For currents driven by the pressure there exists a constraint on the possible forms of the current distribution implying that the morning and evening systems of Birkeland current flow must balance each other at all times so that there is no net current flow in the ionosphere across certain specified contours. If the currents are driven by the plasma convection the morning and evening current systems may be independent. Geometries are suggested for plasma pressure variations and convective flow that could account for the observed distribution of Birkeland currents assuming that the currents originate on field lines within the magnetosphere extending into the plasma sheet of the magnetotail.

Also considered are secondary Birkeland currents of ionospheric origin, which are currents tending to discharge ionospheric polarization electric fields. In interpreting measurements of Birkeland currents it is important to separate these secondary currents from primary currents of magnetospheric origin, which requires a determination of whether the ionospheric closure currents are dissipative or generative.

Key words Birkeland currents, Magnetospheric pressure distribution, Magnetospheric convection, Magnetohydrodynamic theory of magnetosphere, Ionospheric currents.