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IONOSPHERIC ELECTRIC FIELD AND
CURRENT DISTRIBUTION ASSOCIATED
WITH HIGH ALTITUDE ELECTRIC FIELD
INHOMOGENEITIES

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Abstract

A quantitative theoretical analysis of electric field and current distributions in the ionosphere is given assuming certain time variable convection field profiles at an altitude of 1250 km. A number of idealized assumptions regarding the ionospheric characteristics are defined and discussed. A qualitative discussion of a quasi-stationary configuration with an approximately curl free electric field is also given. Geomagnetically field aligned current densities $i_{\parallel}$ of the order $10^{-5} - 10^{-4}$ A/m² are consistent with quite reasonable assumptions about the convection field $E_{\perp}$. Oscillations in $E_{\perp}$ with periods of the order of 10 seconds should readily be generated when $\sigma_{\parallel}$ is large. In the quasi-stationary case there may be a mechanism that strengthens and concentrates $i_{\parallel}$ locally under certain conditions. It is found that a number of recent high altitude observations of convection field reversals may be consistent with large potential drops along the magnetic field lines. The solutions obtained as well as some of the basic assumptions are compared with observations.

Introduction

It is now well established experimentally that strongly inhomogeneous electric fields, directed transverse to the Earth's magnetic field, often appear in the high latitude ionosphere, especially in the neighbourhood of the auroral zones. Observations of such fields at altitudes about 500-2500 km over the auroral zones have been made by, for instance, Frank and Gurnett (1971), Cauffman and Gurnett (1972) and Heppner (1972); for recent reviews, see Haerendel (1972) and Fahleson (1972). When such a transverse electric field is established at an altitude of e.g. 1000 km by some magnetospheric source the lower part of the ionosphere acts as a heavy load connected to the driving field by means of the conductivity along the geomagnetic field lines. In terms of an equivalent circuit the situation can be interpreted as follows.

The ionospheric load is equivalent to two capacitors of quite different magnitudes. Charging the smaller condenser is equivalent to increasing the $E \times B$ -drift velocity of the ions and electrons while the larger one is charged when the neutral gas particles are accelerated in the $E \times B$ -direction by collisions with the drifting charged particles. Equivalently, the acceleration of the charged particle gas may be interpreted as performed by the $i \times B$ -force due to the polarization current while the neutral gas is accelerated by the $i \times B$ -force due to the Pedersen current. Both capacitors are fed through an equivalent resistor along the magnetic field lines and, in addition, the current to the neutral gas condenser must pass through a transverse resistor corresponding to the Pedersen conductivity. The time constant of the circuit containing the smaller condenser is of the order of seconds while the other time constant is of the order of hours or even days. The currents create secondary magnetic fields which affect the distribution of electric fields and currents. The vacuum permeability, μ_0 , and the capacitance per unit length due to the ions, $n m_i / B^2$, are responsible for the hydromagnetic wave propagation along the magnetic field lines characterizing the present phenomena to a varying extent. The deviation from ideal hydromagnetic conditions should be ascribed, primarily, to the parallel resistance which causes a diffusion of the magnetic field relative to the plasma. Hence,

this effect is more pronounced the more concentrated the parallel current is and becomes especially important in cases with anomalously high parallel resistance. Throughout this paper only a normal value of $\sigma_{\parallel}$ will be used, however.

The main problem to be discussed in this paper is the distribution of the parallel current when the behaviour of the transverse electric field is given as a boundary condition at a certain altitude. Special attention will be paid to the inhomogeneities and time variations of $E_{\perp}$ necessary for the parallel current to attain densities large enough to cause double layers or anomalous resistivity in the upper ionosphere. Such critical current densities are reached when the local ordered velocity of the electrons exceeds the thermal velocity.

The present problem has been discussed and analysed in a qualitative manner by Block (1969).

The Geophysical Configuration Considered

The configuration studied is shown in Figure 1. It is typical of the situation at the plasmapause when the plasmasphere is shrinking. This configuration is chosen for reason of simplicity. The present analysis should in essence be applicable also to more violent electric field variations e.g. in the auroral zone with appropriate boundary conditions. The current system in Figure 1b should not be mistaken for the usually discussed polar cap current systems. This figure shows just a most schematic drawing of the currents that are associated with the acceleration of the plasma at the plasmapause. The horizontal dimensions are strongly exaggerated, and the Pedersen current should, for instance, return gradually along the magnetic field lines as the neutral gas is accelerated or it should return where the transverse electric field decreases again.

During disturbed times the plasmasphere is shrinking. The plasmapause is then moving rapidly equatorwards in the lower magnetosphere. The plasma within the plasmasphere has been found experimentally to be essentially co-rotating with the Earth. Poleward of the plasmapause the plasma is participating in the general magnetospheric convection. The contraction of the plasmapause is thought of as being due to erosion by the sunward convection on the polar sides which causes sheets of plasma to be peeled off the plasmasphere. This is illustrated in Figure 1a, where the plasmapause is moving equatorwards with the velocity u which may be of the order of 100 m/s according to Carpenter (1966, p. 701, Figure 8). Although the plasma above the ionosphere is much denser within the plasmasphere than outside, this horizontal gradient in the density is neglected in the calculation of the southward spreading of the electric field in the topside ionosphere. Rycroft and Burnell (1970) have shown that on the nightside the projection of the plasmapause onto the ionosphere along the magnetic field hits the ionosphere about in the middle of a trough. With a broad and flat trough minimum (cf. e.g. Muldrew, 1965, Fig.6) the plasma density may be considered independent of latitude within a latitude interval that is much larger than the thickness of the projected plasmapause boundary layer. Additional arguments in favour of disregarding this gradient will be given below.

It is important to notice that in the configuration assumed the plasma drift is directed east-west while the driving electric field pattern is moving transverse to this direction. This means that the magnetospheric dynamo is moving independently of the ionospheric drift motion. This state of affairs should, especially during more disturbed conditions, be a feature of the auroral zone fields, too, as can be seen, for instance, from results obtained by Chase (1970) and Kelley, Starr and Mozer (1971).

Initial and Boundary Conditions

The geophysical situation in Figure 1 will be analyzed in a frame of reference in rest relative to the neutral gas within the plasmasphere, i.e. roughly at rest relative to the surface of the rotating Earth.

The parallel current is confined to the regions where the transverse electric field $E_{\perp}$ varies along and transverse to the geomagnetic field lines (this will be clarified later but it is a natural consequence of Maxwell's equations when $i_{\parallel}$ is proportional to $E_{\parallel}$ and $i_{\perp}$ consists simply of polarization, Pedersen and Hall currents). As long as the time variations are slow compared to the ion gyrofrequencies ($\sim 200\text{Hz}$) these regions are the same as those where the ion drift motion varies in space. Along a certain magnetic flux tube these variations of the drift motion are eliminated within times ranging from a few seconds to about one minute, depending on the degree of short-circuiting the field lines by the Pedersen conductivity. This means that times characteristic of the present problem are very short compared with the acceleration time of the neutral gas in the ionosphere. As seen from the frame of reference here the neutral gas velocity should then be zero also within the latitude interval of interest outside the plasmasphere, i.e.

$$v_n \equiv 0 \quad (1)$$

As the local frame of reference a cartesian coordinate system will be used with the x-axis pointing to the magnetic north, the y-axis to the magnetic west and the z-axis vertically upwards. In a later section the different approximations will be listed and discussed, but it must be anticipated here that in the

idealized model applied locally the geomagnetic field lines will be considered as straight and vertical through the ionosphere and the Earth as flat. Thus, the z-axis is antiparallel to the geomagnetic field (on the northern hemisphere). In this frame it may be assumed that all components of the electric field are zero on the equatorward side of the projected plasma-pause boundary layer. The magnetic field lines connecting to the plasmapause boundary layer defines the region of inhomogeneous E_x as well as of nonzero values of $E_{||}$. This region moves in the negative x-direction with the velocity u . On the polar side of this region E_x is positive and approximately constant (in space as well as in time).

The initial and boundary conditions for E_x will be given in an interconnected way by defining a transverse field pattern at an altitude of 1250 km above Earth, travelling toward the south without changing shape. If the velocity u has been constant for at least a few minutes it is obvious, in view of the relaxation time mentioned above, that this relation between x-coordinate and time will hold also in the underlying ionosphere, if the ionospheric parameters do not vary with latitude. That is, a function $f(x,t)$ of x and t is a function of the combination $x + u \cdot t$ alone. Hence,

$$\frac{\partial}{\partial t} = u \cdot \frac{\partial}{\partial x} \quad (2)$$

The driving dynamo, in terms of altitudes where transverse currents are fed into the parallel current system, is assumed to be situated far above the upper boundary. The altitude 1250 km has been chosen as upper boundary partly because it is typical of the altitudes where satellite measurements of convection fields have been performed and partly because some of the approximations used in the analysis become inaccurate at higher altitudes.

Although the plasmapause deviates from a constant L-contour (magnetic shell) it is a quite fair approximation to assume that all gradients in the magnetic east-west direction are negligible, because the characteristic dimensions in that

direction should be several hundred times as large as the north-south dimensions. Hence,

$$\frac{\partial}{\partial y} \equiv 0 \quad (3)$$

The boundary conditions at $y=-\infty$ and $y=+\infty$ may then be considered as an independent problem (edge effects). This problem will not be treated here. The only externally imposed boundary and initial conditions may then be the distribution of E_x at a certain altitude and time $t=0$. The following smooth function is chosen

$$E_x = \frac{E_0}{2} \cdot \left\{ \exp\left(\frac{x}{x_0}\right) \cdot H(-x) + \left[2 - \exp\left(-\frac{x}{x_0}\right) \right] \cdot H(x) \right\} \cdot \exp\left(-\frac{x}{x_1}\right) \quad (4)$$

where $x_1 \gg x_0$. Cf. the curve $E_x(z=1.0)$ in Fig.2.

The functions $H(-x)$ and $H(x)$ are Heaviside's unit step functions while $E_0/(2x_0)$ is the maximum horizontal gradient of E_x at this upper boundary. The time dependence at a fixed point is obtained simply by substituting $x+u \cdot t$ for x .

It will be seen from the solutions (or may be realized intuitively) that $2 \cdot x_0$ usually is much smaller than the thickness of the vertical slab where $\partial E_x / \partial z$ differs from zero and where $i_{\parallel}$ flows. The factor $\exp(-\frac{x}{x_1})$ has been included to make the function (4) Fourier integrable in the classical sense ($E_x=0$ at $x=-\infty$ and $x=+\infty$).

Any corresponding boundary conditions for E_y and E_z or for any component of the current or the magnetic field would overdetermine the problem when added to (4). The necessary restrictions at the bottom side of the ionosphere may be deduced from Maxwell's equations.

Basic Equations

In the present problem the topside and F2 layers must be analysed from the dynamical point of view, while the lower ionosphere may be treated primarily as a region with high Pedersen and Hall conductivities but negligible inertia of the charged particles. (As seen from high altitudes the ion density drops to zero in the downward direction within a comparatively thin altitude layer below the F2 maximum, particularly on the night side. Hence, the capacity of the lowest layers due to the charged particles is negligible, while the conducting properties of these layers, i.e. their quasi-stationary properties, are important.) With the upper boundary at 1250 km altitude it is thus justified to consider the dynamics of a plasma consisting simply of neutral gas, singly charged monoatomic oxygen (mass m_i and charge $+e$) and electrons (mass m_e and charge $-e$). (That is, $n_e \approx n_i = n$). As a matter of fact, the solutions may easily be modified to account for an altitude variation in m_i .

With these simplifications as well as equation (1) Boltzmann's equation gives the following approximate momentum equations for the ions and electrons, respectively,

$$nm_i \frac{d}{dt} \underline{v}_i = ne(\underline{E} + \underline{v}_i \times \underline{B}) + nm_i \underline{g} - \text{grad} p_i - nm_i \underline{v}_i n \underline{v}_i + nm_e \underline{v}_{ei} (\underline{v}_e - \underline{v}_i) - m_i n \underline{v}_i + 2nm_i \underline{v}_i \times \underline{\omega} \quad (5)$$

$$nm_e \frac{d}{dt} \underline{v}_e = -ne(\underline{E} + \underline{v}_e \times \underline{B}) + nm_e \underline{g} - \text{grad} p_e - nm_e \underline{v}_e n \underline{v}_e - nm_e \underline{v}_{ei} (\underline{v}_e - \underline{v}_i) - m_e n \underline{v}_e + 2nm_e \underline{v}_e \times \underline{\omega} \quad (6)$$

In a similar manner the conservation equations of mass can be derived,

$$\frac{\partial}{\partial t} nm_i + \text{div}(nm_i \underline{v}_i) = (n - J) m_i \quad (7)$$

$$\frac{\partial}{\partial t} nm_e + \text{div}(nm_e \underline{v}_e) = (n - J) m_e \quad (8)$$

The corresponding equations for the neutral gas molecules are replaced by the simple relation (1). Most of the symbols in these expressions have their conventional meaning, but some of them may need some clarification.

$\underline{B}$ is the total magnetic field, i.e. it may be written in an alternative form with $\underline{B}_0$ symbolizing the undisturbed geomagnetic field

$$\underline{B} = \underline{B}_0 + \underline{b} \quad (9)$$

Furthermore, $\underline{g}$ is the gravitational acceleration corrected for the effect of the Earth's rotation.

$\underline{\omega}$ is the Earth's vector of angular velocity.

$p_i = nkT_i$ and $p_e = nkT_e$ are the pressures of ions and electrons respectively. The pressures are assumed to be isotropic, although this is of minor importance for the subsequent derivations. These two relations will be supplemented by the following assumption.

$$\left. \begin{matrix} T_i \\ T_e \end{matrix} \right\} \sim \text{constant in time} \quad (10)$$

ν_{in} , ν_{en} and ν_{ei} are the collision frequencies for ion-neutral, electron-neutral and electron-ion collisions, respectively.

η is the ionization frequency, i.e. the number of ion pairs formed per m^3 and second.

$\Re$ is the recombination frequency, i.e. the number of recombined ion pairs per m^3 and second.

Several terms of equations (5) - (8) are, of course, quite negligible. Justifications for the ionization and recombination terms as given in equations (5) and (6) have been given in an earlier paper (Lennartsson, 1972), referred to as paper I in the remainder of this paper.

The completing basic relations are provided by Maxwell's equations. The vacuum displacement current is quite negligible here compared to the conduction and polarization currents. The plasma is considered as a collection of particles in vacuum and hence the relative dielectric constant ϵ and the relative permeability μ may be set equal to unity. In view of equation (9) Maxwell's equations may then be written in the following form.

$$\text{curl } \underline{b} = \mu_0 \underline{i} \quad (11)$$

$$\text{div } \underline{b} = 0 \quad (12)$$

$$\text{curl } \underline{E} = - \frac{\partial \underline{b}}{\partial t} \quad (13)$$

$$\text{div } \underline{i} = 0 \quad (14)$$

Equation (14) is a direct consequence of equation (11) and replaces the equation for the divergence of $\underline{E}$. The polarization of the plasma is taken into account by (14). Although this equation is a good approximation, it is, of course, not possible to regard $\text{div } \underline{E} = q/\epsilon_0$ equal to zero (q being the charge density).

The equations (5) - (8) can be combined to give some comparatively complex expressions for the parallel and perpendicular components (referring to $\underline{B}$) of the current density,

$$\underline{i} = ne(\underline{v}_i - \underline{v}_e)$$

The detailed derivation of these expressions are given in paper I. When proper high latitude values of ionospheric parameters are considered (Boström, 1964; Hanson, 1961; Taylor et al., 1964) and the different variables are estimated to their order of magnitude it is seen that many terms may be dropped from these expressions (cf. next section of this paper). The resulting approximate equations for $\underline{i}_\perp$ and $\underline{i}_\parallel$ are

$$\underline{i}_L \approx \underline{i}_p + \underline{i}_c + \underline{i}_g \quad (15)$$

where

$$\underline{i}_p = \frac{nm_i}{B_0^2} \cdot \frac{d}{dt} \underline{E}_L \quad (16)$$

$$\underline{i}_c = \sigma_p \underline{E}_L + \sigma_H B_0^{-1} B_0 \times \underline{E}_L \quad (17)$$

and

$$\underline{i}_g = B_0^{-1} \cdot B_0 \times \text{grad}_L (p_e + p_i) \quad (18)$$

$$\underline{i}_\parallel + \tau_e \frac{d'}{dt} \underline{i}_\parallel = \sigma \underline{E}_\parallel \quad (19a)$$

The following notations are used,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \underline{v} \cdot \text{grad}, \quad \text{where } \underline{v} = \underline{v}_i + \frac{m_e}{m_i} \underline{v}_e,$$

$$\frac{d'}{dt} = \frac{\partial}{\partial t} + v_{e\parallel} \text{grad}_\parallel,$$

$$\text{and } \sigma = \frac{ne^2}{m_e} \tau_e = \sigma_\parallel, \quad \text{where } \tau_e \approx (v_{en} + v_{ei})^{-1}$$

σ_p and σ_H are the Pedersen and Hall conductivities, the explicit expressions of which are given by, for instance, Alfvén and Fälthammar (1963).

The time derivative in (16) may be linearized, while this is not true of the time derivative in (19a). On the other hand τ_e is sufficiently small throughout the ionosphere to justify the neglect of the time derivative in (19a),

$$\underline{i}_\parallel \approx \sigma \underline{E}_\parallel \quad (19b)$$

The equations (15) - (19b) do not presuppose any particular shape of the geomagnetic field B_0 , but they will be inserted into Maxwell's equations only in the idealized case of parallel and vertical geomagnetic field lines.

As will be discussed in the next section, the relation $|\underline{b}| \ll B_0$ may be assumed. In the frame of reference chosen above it is then a fair approximation to use the relation,

$$i_{\parallel} \approx -i_z \quad (20)$$

The minus sign is due to the downward direction of B_0 . By somewhat more complicated arguments (cf. paper I) it can be seen that

$$E_{\parallel} \approx -E_z \quad (21)$$

as long as $E_{\parallel}$ or E_z is of any importance to the propagation of E_x .

The relation $i_z = \sigma E_z$ may then be used.

The equation for propagation of E_x can be deduced from $\text{curl}(\text{curl}\underline{E}) = -\text{curl}(\partial \underline{b}/\partial t)$. The x-component of this equation is considered together with equations (3), (11), (12), (14), (15) - (18), (19b), (20) and (21). The linearized form of (16) is used. The derivation is simplified by the fact that $\sigma_{\parallel}$ may be considered as a constant in the topside ionosphere, as discussed later on. The resulting propagation equation is

$$\frac{\partial^2}{\partial z^2} E_x = \{V_A(z)\}^{-2} \cdot \frac{\partial^2}{\partial t^2} E_x + \mu_0 \sigma_P(z) \cdot \frac{\partial}{\partial t} E_x - (\sigma \mu_0)^{-1} \cdot \{V_A(z)\}^{-2} \cdot \frac{\partial^2}{\partial x^2} \frac{\partial}{\partial t} E_x - \sigma^{-1} \sigma_P(z) \frac{\partial^2}{\partial x^2} E_x \quad (22)$$

The Alfvén velocity $V_A = B_0 / \sqrt{\mu_0 n m_i}$ has been introduced here. This equation presupposes vertical and parallel geomagnetic field lines but the variation with altitude of the absolute magnitude $|B_0|$ can be accounted for in the altitude variation of V_A and σ_P . The Hall current $\sigma_H E_x$ and the current i_g given by (18) do not appear in (22) because of (3). A component E_y is also induced in the configuration by the magnetic field, but it is so very much smaller than E_x that the current $\sigma_H E_y$ is completely negligible. When the relation (2) holds the number of independent variables are reduced to only two, namely x and z .

The parallel current density can be formally derived from (14), (16) and (17) when E_x is known as a function of x and z ,

$$i_z = - \int_{z_0}^z \frac{\partial}{\partial x} i_x dz + i_z(x, z_0) \quad (23)$$

The function $i_z(x, z_0)$ will be calculated as a function of E_x when certain approximations in the following section have been introduced.

Discussion and Completion of the Approximations

The only approximation in Maxwell's equations (11) - (14) is the well-founded neglect of the displacement current. Throughout the topside ionosphere $\epsilon_0 \lesssim 10^{-3} \cdot n m_i / B_0^2$.

On the other hand, the derivation of the current equations (15)-(18) and (19a) has included a great number of approximations of different accuracy. From the lengthy expression obtained by combining equations (5)-(8) a large number of terms, listed in paper I, have been dropped. They are seen to be negligible when typical ionospheric data are inserted and the orders of magnitude of currents and velocities are estimated. The ionizations and recombinations are thus neglected as well as the Earth's gravitation and the Coriolis's force. The very small inertia of the electrons compared with the ions justifies the neglect of a number of nonlinear terms. Furthermore, $(en)^{-1} \text{grad}_{\perp} p_e$ is negligible compared with $\underline{E}_{\perp}$.

The assumption (1) might be abolished and $\underline{E}_{\perp} + \underline{v}_n \times \underline{B}$ inserted into (17), but as long as the times studied are at most a few minutes the relation (1) is accurate. The current expression (18) should be modified when applied to the low ionosphere but this current is small and is of negligible interest here, as mentioned above.

All of the approximations are consistent with the solutions obtained and the formulas (15) - (19a) should be accurate to within about 1% as long as

$$\left| \frac{d}{dt} \right| \lesssim 10^{-1} \omega_i \quad \text{and}$$

$$|\text{grad}_{\perp}| \lesssim (10^{-2} \text{ s/m}) \cdot \left| \frac{d}{dt} \right|$$

These restrictions are fulfilled in the calculations, except for a schematic analysis of a case with $d/dt=0$.

Because of the second of these restrictions the time derivative in (16) may be linearized with an error of at most a few percent. The linearity should then hold within this error, apart from the time derivative in (19a), because the coefficients are very slow functions of the solutions (via n , T_i and T_e). The relation (10) is possible because there are negligible compression effects and the ohmic heating will not have sufficient time to affect T_e and T_i appreciably in this problem. The high parallel electron velocities at high altitudes might possibly affect $\sigma_{\parallel}$ in cases with extreme values of $i_{\parallel}$. It is generally assumed that $|\underline{b}| \ll E_0$. This is consistent with the solutions ($b \leq 10^{-2} B_0$) and it seems to be a general result of ionospheric measurements near strong parallel current sheets (Zmuda et al. 1970; Armstrong and Zmuda, 1970).

When the nonlinear inductive term in (19a) is dropped, leading to (19b), the resulting error is somewhat more important at high altitudes than the former errors. The result is a slight smoothing of the time variations in $E_{\perp}$ and $i_{\parallel}$ at these altitudes.

The subsequent derivation of equation (22) includes apparently more drastic approximations. The most important is, perhaps, the assumption of straight and parallel geomagnetic field lines (a flat Earth). This is reasonable because all dimensions involved in the calculations are rather small compared to the Earth's radius, even the vertical dimensions. Including the dipole character of B_0 would, to first order, just increase $i_{\parallel}$ at low altitudes by some 30-50% and decrease $i_{\parallel}$ at altitudes above the upper boundary, but it would complicate the calculations tremendously (see paper I). For simplicity the field lines are assumed to be vertical at these latitudes, although it is not crucial for the solutions. Furthermore, the parallel conductivity σ is assumed independent of altitude, that is, equation (22) is valid only above about 250 km altitude (F2 layer). Above this altitude $\tau_e \approx \nu_{ei}^{-1}$ (Boström, 1964) which means that τ_e is nearly inversely proportional to n , that is, σ is nearly independent of n . Actually, σ is slowly decreasing with decreasing n but this is counteracted by a slow increase in T_e with increasing altitude.

Equation (22) is also based on the absence of x-gradients in σ , σ_p and V_A , although it can be easily changed to include such gradients. As mentioned in connection with the plasmopause displacement, there could possibly be rather large latitudinal variations in n at high altitudes in the topside layer. However, as $\sigma_{||}$ is very insensitive to n and the transverse currents are very small at these altitudes these variations would not affect the solutions in a serious manner.

Finally, the east-west gradients are neglected according to (3).

The propagation equation (22) could be solved by straightforward numerical methods, when (2) and (4) are considered together with some equations for the low ionosphere and when $\sigma_p(z)$ and $V_A(z)$ are given as functions of z . However, the simultaneous calculation of i_z is much more complicated. Actually, it would be necessary to find i_z by numerical differentiations and integrations of the tabulated values of E_x , that is, by a very inefficient method. The reason for this is the lack of explicit boundary values for i_z (requiring knowledge of $\partial E_x / \partial z$ at the upper boundary) that could be used to solve the partial differential equation for i_z . By simplifying the ionospheric model a step further it is possible to get analytic solutions for E_x as well as i_z (and i_x). This final approach introduces the following features.

$z=0$ at 250 km altitude, i.e. at the charge density maximum.

$$\left. \begin{aligned} n &= n_0 e^{-z/h_n}, \text{ for } z \geq 0 \\ n &= 0, \text{ for } z < 0 \end{aligned} \right\} \quad (24)$$

The function $B_0(z)$ is approximated by a slow exponential function in the interval $0 \leq z \leq 1000$ km.

All Pedersen conductivity is concentrated to a conducting surface at $z=0$, where

$$\Sigma_P = \int \sigma_P dz, \quad (25)$$

while $\sigma_P = 0$ elsewhere.

The integral in (25) is carried out across the altitude range with non-negligible σ_P . The corresponding treatment of σ_H is not needed here. The parallel conductivity may then be considered as constant wherever $i_{||}$ flows.

In spite of its extreme simplicity, this model should meet fairly well the requirement of giving adequate distributions of $E_{\perp}$ at all altitudes as well as $i_{||}$ at high altitudes, as discussed in detail in paper I. The main arguments for (24) are that the Alfvén velocity becomes practically infinite and $\sigma_{||}$ zero very soon in the downward direction below $z=0$, at least at night (cf. Hanson, 1961). The main arguments for the use of (25) are the same, and furthermore, that σ_P decreases by more than five orders of magnitude from $z=0$ to $z=1000$ km while n decreases by less than two orders of magnitude (Hanson, 1961).

Solutions

Equation (22) has been solved for $z \geq 0$ according to the simplified model above, that is, by setting $\sigma_P = 0$ and using an exponential function for $V_A(z)$, cf. paper I. The following numerical data are used

$$n(z) = n_0 \cdot e^{-z/h_n} = 5 \cdot 10^{11} \cdot e^{-z/0.226} \quad (\text{particles per m}^3) \text{ and}$$

$$B_0(z) \approx B_0(0) \cdot e^{-z/h_B} = 0.52 \cdot e^{-z/2.27} \quad \text{gauss, where the unit length}$$

for z is 1000 km

m_i = mass of O^+

$u = 100$ m/s and $u = 1000$ m/s

$E_0 = 50$ mV/m

$x_0 = 50$ m and $x_0 = 0.5$ km

$x_1 = 100$ km

$\sigma = \sigma_{||} = 30$ mho/m

$\Sigma_P = 0.56$ mho and $\Sigma_P = 36$ mho

The two values of Σ_P are taken from Boström (1964) and are rather typical of normal night-time conditions and auroral arc conditions (or daytime conditions), respectively.

The conditions (2) and (4) are used

The two values of x_0 are selected to give values of $i_{||}$ comparable to the highest observed (cf. next section). The parallel current density $i_{||}$ is calculated from (23), where i_x is just the polarization current (16) in this model,

$$i_z(x, z_0) = i_z(x, 0) = - \Sigma_P \cdot \frac{\partial}{\partial x} E_x(x, 0) \quad (26)$$

The analytic solutions are the following at time $t = 0$.

$$E_x(x, z) = \frac{1}{\pi} E_0 \int_0^{\infty} \operatorname{Re} \{ (C_1(s) H_0^{(1)}(z, s) - C_2(s) H_0^{(2)}(z, s)) e^{j \frac{x}{\Lambda} s} \} ds \quad (27)$$

$$\begin{aligned} i_z(x, z) = & - \frac{1}{\pi} \sigma E_0 \frac{u}{V_A(z)} \cdot \int_0^{\infty} \operatorname{Re} \{ (C_1(s) H_1^{(1)}(z, s) - C_2(s) H_1^{(2)}(z, s)) \cdot \\ & \cdot \frac{s}{\sqrt{1-j}s} e^{j \frac{x}{\Lambda} s} \} ds + \frac{1}{\pi} \sigma E_0 \frac{u}{V_A(0)} \cdot \int_0^{\infty} \operatorname{Re} \{ (C_1(s) H_1^{(1)}(0, s) - \\ & - C_2(s) H_1^{(2)}(0, s)) \frac{s}{\sqrt{1-j}s} e^{j \frac{x}{\Lambda} s} \} ds - \frac{1}{\pi} \frac{\Sigma_P E_0}{\Lambda} \int_0^{\infty} \operatorname{Re} \{ (C_1(s) H_0^{(1)}(0, s) - \\ & - C_2(s) H_0^{(2)}(0, s)) \cdot j s \cdot e^{j \frac{x}{\Lambda} s} \} ds \end{aligned} \quad (28)$$

The time dependence is obtained by substituting $x+u \cdot t$ for x .

The following notations are used

$$C_1(s) = \frac{D(s)}{D(s)H_O^{(1)}(z_b, s) - H_O^{(2)}(z_b, s)} \cdot T(s)$$

$$C_2(s) = C_1(s)/D(s)$$

$$D(s) = \frac{H_1^{(2)}(0, s) \cdot \frac{u}{V_A(0)} \cdot \sqrt{1-j\epsilon} - H_O^{(2)}(0, s) \frac{\Sigma P}{\sigma} \cdot \frac{s+j}{\Lambda}}{H_1^{(1)}(0, s) \cdot \frac{u}{V_A(0)} \cdot \sqrt{1-j\epsilon} - H_O^{(1)}(0, s) \cdot \frac{\Sigma P}{\sigma} \cdot \frac{s+j}{\Lambda}}$$

$$T(s) = \frac{1}{\Lambda E_O} \cdot \int_{-\infty}^{\infty} E_x(x, z_b) e^{-js \frac{x}{\Lambda}} dx$$

$$z_b = 1.0 \text{ (thousands of kilometers)}$$

$$z=0 \text{ at 250 km altitude}$$

$$\Lambda = (\sigma \mu_O u)^{-1} \text{ (length dimension)}$$

$$V_A(z) = B_O(z) / \sqrt{\mu_O n(z) m_i}$$

j is the imaginary unit

$H_V^{(1)} = J_V + jY_V$ and $H_V^{(2)} = J_V - jY_V$ are the Hankel functions as defined by, for instance, Olver (1965).

$$H_O^{(1)}(z, s) \equiv H_O^{(1)}\left(\frac{2h}{\Lambda} \cdot \frac{u}{V_A(z)} \cdot s \cdot \sqrt{1-j\epsilon}\right) \text{ etc., where } h \text{ is the}$$

scale height of V_A^{-2} , i.e. $h = \left(\frac{1}{h_n} - \frac{2}{h_B}\right)^{-1} = 282 \text{ km}$ (cf. $n(z)$ and

$B_O(z)$ above)

The distributed current density i_x is in this case just the polarization current which may easily be found from (16) (with linearized derivative) and (27), while the Pedersen current flows as a sheet current at $z=0$ (at the altitude 250 km). However, to first order the real altitude distribution of $i_{\perp}$ should be found from (15)-(17) and (27) where E_x at $z=0$ could be considered as adequate also below $z=0$.

Graphs of E_x and i_z versus x or t at different altitudes are shown in Fig. 2-5 for differently combined values of u , x_0 and Σ_p . The parameter z , expressed in thousands of kilometers, is the altitude minus 250 km.

In Fig. 2 E_x appears earlier at low altitudes than higher up at first (to the left). This non-Alfvénic effect is due to the slip of the plasma relative to the magnetic field as well as to the stationarity in a frame of reference moving with the pattern.

In Fig. 4 and 5 the propagation shows a more pronounced hydro-magnetic behaviour because $i_{||}$ is more widely spread, reducing the effect of the parallel resistivity. The oscillations in E_x are not simply a result of the approximation (24), but rather a typical behaviour of the ionosphere in the present case, because the vertical wavelength of these oscillations, $\lambda \approx V_A/v$ (V_A having the value 400 km/s, typical of the F2 layer), is about 4000 km. Thus, λ is about 25 times larger than the thickness of the neglected layers below 250 km altitude.

Fig. 6 shows the current pattern as well as E_x at different altitudes with the same values of u , x_0 and Σ_p as in Fig. 2. The complete pattern should be thought of as moving to the left without changing shape. The sheet current at 250 km altitude is in reality spread over an altitude interval of some hundred km. Above the dotted curve the ordered parallel velocity of the electrons exceeds the thermal velocity.

For comparison, the distribution of E_x and i_z in two further simplified ionospheric models are shown in Fig. 7 and 8. In Fig. 7 $\Sigma_p=0$ and in Fig. 8 the exponential ionosphere is extended to $z = -\infty$ with $\sigma_p \equiv 0$ and $\sigma = 30$ mho/m everywhere. Cf. Fig. 2 and 3 (Fig. 8 is not strictly comparable to Fig. 7 or Fig. 2 because the variation in B_0 with altitude is neglected and, besides, $n_0 = 4 \cdot 10^{11} \text{ m}^{-3}$ and the scale height is 238 km here).

The analysis may quite easily be generalized to a situation without any *a priori* relation between the variables x and t , provided that the boundary function $E_x(x, z_D, t)$ is Fourier integrable with respect to both x and t . In this case the relation (2) should be dropped and a more general boundary function may be used instead of (4). In equation (22) (with $\sigma_p = 0$) $\partial/\partial t$ should be replaced by $j\omega$ and $\partial/\partial x$ by jk giving partial solutions of type $H_0^v \{2h(\omega^2 - j\omega k^2/(\sigma\mu_0))^{1/2}/V_A(z)\} \cdot \exp\{j(kx + \omega t)\}$, where $v = 1$ or 2 . The analysis may then be carried out as a straightforward extension of the detailed calculations in paper I leading to solutions analogous to (27) and (28) but with two-dimensional integrals (and x, z and t as independent variables)

An adequate treatment of field and current configurations above auroral forms must consider latitudinal variations in Σ_p and Σ_H . In principle this may be attained by modifying the boundary condition (26), but then the analysis becomes considerably more cumbersome.

Even without these extended calculations several quite general conclusions can be drawn, however (cf. the section "Conclusions").

The Quasi-Stationary Case, $u \approx 0$

The case $u \approx 0$ or $\partial/\partial t \approx 0$ may be treated by elementary analysis without solving equation (22). The word quasi-stationary here means that the neutral gas may still be considered as immobile, cf. relation (1), but that $\text{curl } \underline{E} \approx 0$ is a fair approximation. Hence, according to the discussion in the introduction the time studied should be at most a few hours. On the other hand the Figures 3 and 5 show that appreciable changes of fields and currents must not take place within times shorter than a few minutes. Obviously, the relation $|\text{grad}_\perp| \leq (10^{-2} \text{ s/m}) \cdot |d/dt|$ does not hold in this case reducing the accuracy of the model. For instance, the neglect of $(en)^{-1} \text{grad}_\perp p_e$, when compared with $E_\perp$, may be less accurate, as $i_\parallel$ and $i_\perp$ are carried by different compositions of electrons and ions. Furthermore, T_e and T_i may change with time and so on. No attention will be paid to these problems here, neither will latitudinal variations of σ be considered. Certain consequences of latitudinal variations in Σ_p will be discussed

briefly, however. The analysis of this case will thus be rather schematic in several respects, and the solutions will only be discussed in a qualitative manner in order to illustrate some interesting features of a narrow convection field inhomogeneity.

Even in this case the approximations (24) and (25) should give a reasonably adequate distribution of $i_{||} \approx i_z$ at high altitudes where negligible Pedersen and Hall currents flow. In this case no polarization current flows and, hence, i_z is independent of altitude above $z=0$. If, for simplicity, $E_x(x, z_b)$ is given as a simple step function of height E_0 at $x=0$ according to Fig. 9 the relation (26) (with constant Σ_p) and $\text{curl } \underline{E}=0$ easily give the following results

$$\text{at } z=0 \begin{cases} E_x(x) = \frac{E_0}{2} e^{x/L} & \text{when } x \leq 0 \\ E_x(x) = \frac{E_0}{2} (2 - e^{-x/L}) & \text{when } x > 0 \end{cases} \quad (29)$$

$$\text{and at } z \geq 0 \begin{cases} i_z(x) = -\sigma' \frac{E_0}{2} e^{x/L} & \text{when } x \leq 0 \\ i_z(x) = -\sigma' \frac{E_0}{2} e^{-x/L} & \text{when } x > 0 \end{cases} \quad (30)$$

$$L = \sqrt{\frac{z_b \Sigma_p}{\sigma}} \quad \text{and} \quad \sigma' = \sqrt{\frac{\sigma \Sigma_p}{z_b}} \quad (\sigma \approx \sigma_{||})$$

The main part of $i_{||}$ is evidently limited to a vertical slab of thickness $2L$ and $|i_{||}|_{\text{max}} = \sigma' \cdot E_0/2$. Cf. Fig. 9 where E_x is sketched at four altitudes. As time goes on (many hours) the neutral gas is accelerated, the currents disappear and the convection field profile becomes equal to $E_x(x, z_b)$ at all altitudes. As can be seen from Fig. 9 $E_x(x)$ attains peak values at the transition zone at altitudes above z_b , larger peaks the higher the altitude is. A smooth function $E_x(x, z_b)$ with a narrow transition zone gives, of course, also these peaks at larger altitudes and, in addition, the maximum gradient of the profile is steeper the larger the altitude is. It is easily seen that according to the simple model used in this section the following relation holds

$$E_x(x,z) = (z/z_b) \{E_x(x,z_b) - E_x(x,0)\} + E_x(x,0) \quad (31)$$

This relation holds also when $\partial\sigma_{||}/\partial x \neq 0$ and $\partial\Sigma_p/\partial x \neq 0$ but must be modified when $\partial\sigma_{||}/\partial z \neq 0$ and when the dipole character of $\underline{B}_0$ is considered.

From equations (29) and (30) it can be seen that $|i_z|_{\max}$ grows when Σ_p grows although $|\partial E_x/\partial x|$ is reduced at $z=0$ (a short circuiting effect). Hence, if the convection profile at high altitudes is maintained approximately constant it is still possible to enhance and even concentrate the parallel current by locally enhancing Σ_p at the base of a parallel current slab. This is still more evident when the relation (26) is extended to take into account latitude variations in Σ_p ,

$$i_z(x,0) = -\Sigma_p(x) \frac{\partial}{\partial x} E_x(x,0) - \left(\frac{\partial}{\partial x} \Sigma_p(x) \right) \cdot E_x(x,0) \quad (32)$$

Assume that $\Sigma_p = \Sigma_p^n + \Delta\Sigma_p(x)$, where Σ_p^n is the normal background value and $\Delta\Sigma_p(x)$ has a narrow peak at $x=0$. Hence, if the relation (32) as a first approximation is applied to $E_x(x,0)$ according to (29) (cf. Fig. 9) it is seen that the peak value (negative) of $i_z(x,z)=i_z(x,0)$ is sharply enhanced and may move somewhat towards negative x and i_z may even be positive for certain positive x -values (i_z gets an asymmetric profile). In reality, $E_x(x,0)$ or $|\partial E_x(x,0)/\partial x|$ is, of course, locally reduced when $\Delta\Sigma_p(x)$ is superposed on Σ_p^n . This reduction is, however, a consequence of the local enhancement of $|i_z|$ or $|E_z|$ ($\text{curl } \underline{E} \approx 0$ gives $\partial E_x/\partial z \approx \partial E_z/\partial x$) and, hence, the conclusions about the parallel current are still qualitatively valid.

It is interesting to note that there is, at least in principle, a mechanism that may increase Σ_p locally at the base of a parallel current slab. This is due to the different compositions of charge carriers in $i_{\perp}$ and $i_{||}$. While $i_{||}$ is carried mainly by electrons, the Pedersen current is carried mainly by ions. Hence, in an inhomogeneous quasi-stationary convection field configuration with upward parallel current (e.g. if E_0 is made negative in Fig. 9) the charged particles are accumulated at the base of

$i_{\parallel}$ giving rise to enhanced Σ_p . If the ionizations and recombinations are neglected it is seen that $\partial n / \partial t = \partial n_e / \partial t = -\nabla \cdot (n_e \mathbf{v}_e) \approx -\partial(n_e v_{ez}) / \partial z \approx e^{-1} \cdot \partial i_z / \partial z$, where $\partial i_z / \partial z$ may be about $i_z / (100 \text{ km})$ or, in cases with highly ionized E-layer, even about $i_z / (20 \text{ km})$, cf. Boström (1964). Hence, $\partial n / \partial t \sim 10^{14} \cdot i_{\parallel} \text{A}^{-1} \text{s}^{-1} \text{m}^{-1} - 5 \cdot 10^{14} \cdot i_{\parallel} \text{A}^{-1} \text{s}^{-1} \text{m}^{-1}$. If $i_{\parallel} \sim 10^{-5} \text{A/m}^2$ it is seen that $\partial n / \partial t$ may well be of the order n per minute, particularly in the E-layer, and even n per ten seconds when $i_{\parallel} \sim 10^{-4} \text{A/m}^2$. Under certain conditions this may give an accelerated strengthening of $|i_{\parallel}|_{\text{max}}$ ending in a current induced anomalous $\sigma_{\parallel}$ or in double layers (which may spread out $i_{\parallel}$ again) even with a constant dynamo field at high altitudes. Precipitating electrons may, of course, also increase Σ_p locally but the precipitation may be a result of these current induced anomalies (giving a strong $E_{\parallel}$). Hence, the enhancement of Σ_p by this effect may be a later stage of the development of an auroral form. These problems will be more thoroughly discussed in a later paper with special attention to convection field reversals at the auroral zone (cf. next section) and the problem of particle acceleration along the geomagnetic field lines by means of parallel electric fields.

Comparison with Observations.

The observational data on Birkeland currents and convection field gradients at the plasmopause are, unfortunately, very sparse. Cauffman and Gurnett (1971 and 1972) and Frank and Gurnett (1971) have reported spatial variations of 10-20 mV/m in $E_{\perp}$ at the plasmopause and according to Haerendel and Lüst (1970) there seems to exist a sharp boundary in the convection pattern at the plasmasphere.

Stable auroral red arcs have been found on the magnetic field lines through the plasmopause during times of shrinking plasma-

sphere by Chappell et al. (1971) and Hoch and Smith (1971). Under the assumption that the auroral particles are accelerated by strong parallel electric fields this observation may indirectly indicate the occurrence of parallel current densities large enough to induce anomalous resistivity or double layers. The analysis of this resulting situation requires, of course, a more complex model. These same authors have also found topside troughs at the magnetic field lines connecting to the plasma-pause. This observation may be of some interest here as Block and Fälthammar (1968) and Block (1972) have shown that Birkeland currents may lead to density reductions under certain conditions. (This density reduction may also facilitate the creation of anomalous $\sigma_{\parallel}$ or double layers as it forces the local velocity of the electrons to increase in order to maintain $i_{\parallel}$). The topside troughs may appear when the displacement velocity, u , tends to zero, then extending the time during which the topside plasma within a certain flux tube is exposed to the parallel current (flowing to or from the neutral gas condensor).

The experimental data on electric fields and currents in the auroral zone, especially in auroras, are much more numerous. However, simultaneous measurements of both fields and currents are not known to the author.

Strong parallel currents have been observed by, for instance, Armstrong and Zmuda (1970) and Park and Cloutier (1971). According to the former authors Satellites 1963 38 C and Explorer 22 detected magnetic disturbances transverse to the local field direction on about 90% of all passes through the auroral oval at an altitude of about 1100 km. The interpretation of these disturbances in terms of field aligned currents give current densities larger than $1 \times 10^{-4} \text{ A/m}^2$ in some cases, i.e. comparable to the largest current densities calculated in this paper. Park and Cloutier have reported parallel current systems near auroral forms where the Birkeland currents flow in two parallel slabs of

varying thickness, connected at low altitudes by means of horizontal Pedersen and Hall currents. In some cases they have been able to establish the coincidence of the upward Birkeland current with an auroral form as defined by precipitating energetic electrons that carried the upward current to a certain extent. The downward current has flown in the vicinity (within some tens of kilometers), carried by low energy particles, probably thermal electrons.

The reported thickness of parallel current slabs has ranged from some km to several tens of kilometers. With the assumption that $i_{\parallel}$ when integrated across the current slab, with thickness d , approximately equals the height integrated Pedersen current Σ_p is of the order of $i_{\parallel} \cdot d / E_{\perp}$. If $E_{\perp} \approx 0.05$ V/m is a typical value of the convection field in connection with the reported parallel current events it may be seen that the values 0.56 mho and 36 mho used in this paper define the approximate lower and upper limits of the Σ_p values characterizing the observed current systems while $\Sigma_p = 1-5$ mho may be the most typical values.

Some authors have reported very high values of $E_{\parallel}$, suggesting small values of $\sigma_{\parallel}$, within auroral forms (Kelley, Mozer and Fahleson, 1971; Mozer and Fahleson, 1970) indicating current induced anomalies also in the low ionosphere. Some authors have observed reduced values of $E_{\perp}$ within auroras (Potter, 1970; Wescott et al., 1969) and this may be consistent with Fig. 3 and Fig. 5 which illustrate the short circuiting effect of a large Σ_p .

Probably the most extensive exploration of high altitude (400-2500 km) convection fields has been performed by Cauffman and Gurnett (1971 and 1972), Frank and Gurnett (1971) and Heppner (1972). They have frequently found reversals in the electric field at auroral zone latitudes and high altitude fluctuations in $E_{\perp}$ ("noise") with time scales generally less than 60 seconds (cf. the oscillations in Fig. 4). Often, the latitude of a reversal has been found to change markedly on time scales less

than 2 hours (as much as several degrees as indicated by Fig. 25 on page 404 in the paper by Cauffman and Gurnett, 1972), comparable to the equatorward velocity $u=100$ m/s of the configuration considered in this paper. Furthermore, they have observed inverted "V" type low energy electron precipitation events nearly coinciding with the reversal, cf. the profile of $i_{\parallel}$ ($\approx E_{\parallel}$) in Fig. 9 (E_0 should be negative in this connection in order to give an upward current and in the corresponding equations (29) and (30) an anomalously small σ should be assumed). They have found the convection directions to be generally magnetically eastward or westward (even at local midnight) with essentially sunward components between the plasmapause and the latitude of the field reversals and anti-sunward components over the polar cap. Very abrupt changes in $E_{\perp}$ have been seen which can not unambiguously be identified as preferably spatial or temporal. Changes as large as 230 mV/m within some tens of kilometers or within a few seconds have been observed near the reversals (Cauffman and Gurnett, 1972, p. 402, Fig. 23). If purely temporal, such changes imply values of $\partial E_{\perp}/\partial t$ in good agreement with those assumed in this paper. If purely spatial they may be consistent with $i_{\parallel} \sim 10^{-4}$ A/m² provided Σ_p is rather large and $\sigma_{\parallel} \sim 30$ mho/m which may be seen from a modification of $E_x(x, z_p)$ in Fig. 9, cf. next section. The maximum convection fields have generally been observed near the reversal boundary, which may be compared with the peaks at the top of Fig. 9.

Wescott et al. (1969) have observed rapid reversals of drift directions in barium clouds with and without visual aurora in the proximity of the clouds ($\Delta E_{\perp} \sim 100$ mV/m within tens of seconds). Obviously, this observation is consistent with an electric field pattern moving relative to the plasma. Of interest in this connection is also the observation by these authors that auroral arcs tend to be aligned along the direction of drift motions.

In connection with motions of auroral arcs and electron precipitation structures (with essentially east-west orientation) several authors, as Chase (1970), Cresswell (1968) and Maral (1970), have reported large north-south velocity components that are not consistent with plasma drift motion. Cresswell has reported as large southward velocities as 50 to 300 km/s. These may be compared with the velocity u used in this paper which is at most 1 km/s.

Conclusions

As long as the relation $i_{\parallel} = \sigma_{\parallel} E_{\parallel}$ holds at the altitudes of observations, i.e. the parallel current is not appreciably carried by precipitating energetic particles or $i_{\parallel}$ is the remaining current when precipitating particles are recorded separately, there is, obviously, a close relationship between the disturbances of $i_{\parallel}$ and inhomogeneities in $E_{\perp}$ which is illustrated by Figures 2-9. The geomagnetically field-aligned slabs where $i_{\parallel}$ flows constitute the regions where $E_{\perp}$ varies along the field lines as well as transverse to them. In strictly stationary cases, where the neutral gas has been accelerated, regions of strong transverse gradients in $E_{\perp}$ may exist without any $i_{\parallel}$ (or $i_{\perp}$) flowing, but in these cases $E_{\perp}$ does not vary along the field lines either. As the dynamo field profile has to be constant for many hours or tens of hours in such cases the notion quasi-stationary as defined above may be more relevant to the observed transverse gradients in the convection field.

The above conclusions evidently indicate a method of checking the relation $i_{\parallel} = \sigma_{\parallel} E_{\parallel}$ without measuring $E_{\parallel}$, which is very small in cases of normal $\sigma_{\parallel}$. For instance, in a quasi-stationary configuration with the y-axis in the convection direction, $E_{\perp} = E_x$, and the z-axis parallel to the magnetic field (as in Fig. 9) $\sigma_{\parallel}$ is simply given by $\sigma_{\parallel} = (\partial i_z / \partial x) / (\partial E_x / \partial z)$, provided $\partial \sigma_{\parallel}^{-1} / \partial x$ can be neglected (this follows from $\text{curl } \underline{E} = 0$). Note, that this relation is not affected by $\partial \sigma_{\parallel} / \partial z$. Unfortunately, the measurement of $\partial E_x / \partial z$ should require observations of E_x along a certain magnetic field line at altitudes several hundred kilometers apart in order to get measurable differences ($\Delta E_x \sim 5-10 \text{ mV/m}$) in cases with normal $\sigma_{\parallel}$ (and $i_{\parallel} \sim 10^{-5} - 10^{-4} \text{ A/m}^2$). The measurements may in principle be performed by means of two rockets measuring the electric field simultaneously at different altitudes. Precipitating particles should be recorded by a separate method making it possible to deduce that part of the total parallel current ($\sim \partial b_y / \partial x$) which is carried by thermalized electrons. If visible auroral forms are present they may indicate a homogeneity of the type $\partial / \partial y \approx 0$ and, hence, the two rockets do not have to cross the same geomagnetic field lines. In cases with anomalously small $\sigma_{\parallel}$, $\partial E_x / \partial z$ is much larger. $\partial \sigma_{\parallel}^{-1} / \partial x$ must then be considered, i.e.

may be

$\partial E_x / \partial z = \sigma_{\parallel}^{-1} \partial i_z / \partial x + i_z \partial \sigma_{\parallel}^{-1} / \partial x$, where the term $\sigma_{\parallel}^{-1} \partial i_z / \partial x$ may instead be negligible, as $i_{\parallel}$ will have a tendency towards flowing outside the region of enhanced parallel resistivity. When $\text{curl } \underline{E} \neq 0$ it is, of course, necessary to consider the complete expression $\partial E_x / \partial z = \partial(\sigma_{\parallel}^{-1} i_z) / \partial x - \partial b_y / \partial t$ (b_y being the transverse magnetic disturbance).

Furthermore, it is evident that the shape of the inhomogeneity in $E_{\perp}$ varies with altitude. It becomes steeper closer to the dynamo (cf. e.g. Figures 2 and 3) and less steep near the load at the altitudes where the Pedersen conductivity is large and, in a rapidly time varying case, also where the plasma density and hence the polarization current is large. The location of the dynamo, i.e. the altitude where transverse currents are fed into the parallel current system, is in reality not well-defined but extends over a large altitude range. In the calculations it has been assumed that the dynamo is situated at very high altitudes and the load is, of course, at low altitudes. Hence, the $E_{\perp}$ -inhomogeneity shows a steepening with increasing altitude. This steepening is more pronounced for lower $\sigma_{\parallel}$ (cf. the expression (29)), larger Σ_p (cf. (29) and Figures 2-5) and faster time variations, provided that the profile (and time variation) of $E_{\perp}$ is fixed at some arbitrary altitude. It is, of course, partly counteracted by the outward diverging magnetic field lines.

As may be seen from the expression (31) there will often be a pronounced hump at each side of a narrow transverse inhomogeneity in $E_{\perp}$ at altitudes above z_b (arbitrary reference altitude) even if there are no humps at z_b , cf. Fig. 9. The dynamo must, of course, be situated above the altitude region studied. This state of affairs will be particularly typical of configurations where the potential drop along the magnetic field lines is anomalously high within the parallel current region ($\partial E_x / \partial z \approx \partial E_z / \partial x$). The observed characteristics of field reversals (e.g. Cauffman and Gurnett, 1972) may thus indicate a large parallel potential drop below the altitude of observations. The observed thickness of the regions with inverted "V" low-energy electron precipitation is obviously rather large (several hundred km according to Cauffman and Gurnett, 1972, p. 399, Fig. 21) as compared with the thickness

of the parallel current regions found in this paper. Provided that the associated parallel current due to thermalized electrons flow within a slab of the same or larger thickness (e.g. $i_{\parallel} \sim 10^{-6}$ A/m² or smaller) this observation is consistent with an anomalous resistivity along the field lines within a comparable latitude interval, cf. the quantity L in equation (30).

A comparison of Figures 2-5 demonstrates clearly the short-circuiting property of a large Σ_p . Hence, reduced values of $E_{\perp}$ within auroral arcs should be a common event, particularly in situations where the convection velocity is growing in the vicinity of the auroral forms (e.g. when precipitating structures move relative to the plasma). This is consistent with several observations (e.g. Potter, 1970; Wescott et al., 1969). On the other hand, these observations of reduced $E_{\perp}$ within auroral forms may also be merely manifestations of the reported field reversals in $E_{\perp}$ as indicated by the coincidences of reversals and inverted "V" events ($E_{\perp}$ being zero in the middle of the reversal anyhow).

Fig. 4 shows that oscillations in $E_{\perp}$ (and oscillations in the magnetic field) with periods of the order of 10 seconds should readily be generated in the upper ionosphere (cf. the "noise" reported by Cauffman and Gurnett (1972)). The specific appearance of the oscillations in Fig. 4 is influenced by the special type of boundary condition (4) chosen at $z=1000$ km. This boundary condition is, of course, only approximately valid and does not account for waves reflected in the low ionosphere.

Obviously, the parameters used in Figures 2-8, first of all $\partial E_x / \partial t$ and $\partial E_x / \partial x$ at the altitude 1250 km, give parallel current densities about the largest ever observed. On the other hand, it seems doubtful whether the observed values of $\partial E_x / \partial x$ at altitudes about 1000-2000 km have ever been of the same order of magnitude as $\partial E_x(x, z_b) / \partial x$ in Figures 2-8 (where the combinations of $\partial / \partial x$ and $\partial / \partial t$ are subject to the assumption (2) and the restrictions of accuracy given in the section "Discussion and Completion of the Approximation"). In Figures 3 and 5 $\partial E_x / \partial x$ at low altitudes,

and hence $i_{||}$, is limited mainly by the maximum growth rate of the induced magnetic field. Evidently, if $\partial E_x / \partial x$ at high altitudes is given the parallel current is most concentrated and $|i_{||}|_{\max}$ is largest when Σ_p is large and $\partial / \partial t$ is small. By modifying $E_x(x, z_p)$ in Fig. 9 and using $\Sigma_p = 36$ mho and $\sigma_{||} = 30$ mho/m it may be seen that $i_{||} \approx 10^{-4}$ A/m² can be achieved with $\partial E_x / \partial x$ at $z = 1000$ km in good agreement with the largest values observed at field reversals (cf. the previous section). This large value of Σ_p may be associated either with a local daytime observation (the extreme value of $\partial E_x / \partial x$ taken from Cauffman and Gurnett, 1972, p. 402, Fig. 23, was observed at 17^h54^m magnetic local time, June 14, 1969) or with a local enhancement of Σ_p within a certain latitude interval, in which case it may be a result of the particle accumulating effect discussed in the quasi-stationary case above (or of intense precipitation).

Above the dotted curve in Fig. 6 favourable conditions should exist for creation of potential double layers or anomalous resistivity. When the outward diverging magnetic dipole field is considered this dotted curve should be raised some hundred km. The possible corresponding region in the low density plasma at low altitudes (below F_2 -maximum) can not be found without refinement of the ionospheric model used here. It might seem that current densities much lower than those presented here should suffice to create critical electron velocities at very high altitudes, but the outward diverging magnetic field lines should make the magnitude $i_{||}/n$ ultimately a decreasing function of altitude within a certain parallel current sheet in the absence of polar wind. Furthermore, T_e is mainly increasing outward. Very roughly, $i_{||}$ at ionospheric altitudes should be at least of the order of 10^{-6} - 10^{-4} A/m² for the critical velocity condition to be fulfilled at some altitude and the appropriate altitudes should range up to a few thousand km. These problems have been thoroughly discussed by Block (1972) for situations with polar wind present as well as without. The simple expressions (29) and (30) show, at least qualitatively, that $i_{||}$ and the region of transverse inhomogeneity in $E_{\perp}$ will spread over a wider latitude interval, and hence $i_{||}$ will decrease, when an anomalous parallel resistivity (or a double layer) is created. These problems will be discussed in a later paper.

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Figure Captions

Fig. 1. a) A situation with shrinking plasmapause (growing magnetic activity) that forms the geophysical base of the analysis. It should be regarded merely as an example of situations when sharp gradients in the convection field and strong geomagnetically field aligned currents are likely to occur.

b) Schematic current pattern associated with acceleration of the ions (polarization current) and the neutral gas (Pedersen and Hall currents).

Fig. 2. Distributions of E_x ($\approx E_{\perp}$) and i_z ($\approx i_{\parallel}$) when $\Sigma_p = 0.56$ mho, $\sigma_{\parallel} = 30$ mho/m and $u = 100$ m/s. $|\partial/\partial t| \leq 1$ s $^{-1}$. The x-axis is northward and the z-axis is upward. $z=0$ at 250 km altitude.

Fig. 3. Distributions of E_x and i_z when $\Sigma_p = 36$ mho and $u = 100$ m/s.

Fig. 4. Distributions of E_x and i_z when $\Sigma_p = 0.56$ mho and $u = 1000$ m/s. $|\partial/\partial t| \leq 1$ s $^{-1}$.

Fig. 5. Distributions of E_x and i_z when $\Sigma_p = 36$ mho and $u = 1000$ m/s.

Fig. 6. The current pattern corresponding to the parameters of Fig. 2. The steplike curves across the current lines represent the convection field at different altitudes. Above the dotted curves $v_{e\parallel} > \sqrt{3kT_e/m_e}$.

Fig. 7. Distributions of E_x and i_z in the fictive case with $\Sigma_p = 0$. $u = 100$ m/s.

Fig. 8. Distributions of E_x and i_z in a still more simplified model with infinitely deep exponential ionosphere, where the only transverse current is the polarization current

($\sigma_p = \sigma_H = 0$ everywhere and $\sigma_H = \text{constant}$). $u = 100$ m/s.

Fig. 9. Distributions of E_x in the quasi-stationary case $u=0$, when E_x is given as a step function at $z=z_b$. The distribution of i_z is indicated by the arrows at the top.

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig.8

Fig. 9

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IONOSPHERIC ELECTRIC FIELD AND CURRENT DISTRIBUTION ASSOCIATED
WITH HIGH ALTITUDE ELECTRIC FIELD INHOMOGENEITIES

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A quantitative theoretical analysis of electric field and current distributions in the ionosphere is given assuming certain time variable convection field profiles at an altitude of 1250 km. A qualitative discussion of a quasi-stationary configuration with an approximately curl free electric field is also given. Geomagnetically field aligned current densities $i_{\parallel}$ of the order $10^{-5} - 10^{-4} \text{ A/m}^2$ are consistent with quite reasonable assumptions about the convection field $E_{\perp}$. Oscillations in $E_{\perp}$ with periods of the order of 10 seconds should readily be generated when $\sigma_{\parallel}$ is large. In the quasi-stationary case there may be a mechanism that strengthens and concentrates $i_{\parallel}$ locally under certain conditions. It is found that a number of recent high altitude observations of convection field reversals may be consistent with large potential drops along the magnetic field lines.

Key words Ionospheric electric field, Plasmapause, Auroral electric field, Magnetospheric convection field, Field-aligned current, Anomalous resistivity, Double layer.