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VERTICAL PROPAGATION OF TIME-
DEPENDENT ELECTRIC FIELDS IN
THE ATMOSPHERE AND IONOSPHERE

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VERTICAL PROPAGATION OF TIME-DEPENDENT ELECTRIC FIELDS
IN THE ATMOSPHERE AND IONOSPHERE ^{x)}

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Abstract

Basic methods are investigated for calculation of the vertical propagation of time-dependent electric disturbances in the atmosphere. It is shown how problems involving time-dependent fields can be solved taking into account displacement currents and induction (non-potential) fields as well as the anisotropy and frequency dependence of the complex ionospheric conductivity. Results referring to fields in the form of sinusoidal travelling or standing waves are given and it is shown that fields of a horizontal scale of about 100 km or more penetrate from above to balloon altitudes (30-40 km) with little damping provided the temporal variation is slower than 1 second. Fields of a horizontal scale of 1 km penetrate from above only to the E-layer with little damping for time-independent fields and the damping becomes substantial for time-dependent fields even for periods as long as 1 hour. The field response upon a transient change in the source is also studied. The often quoted result that the temporal variation of the field at each point is governed by the local relaxation time is shown to have only a limited applicability.

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1. Extraterrestrial influence on the atmospheric electric field

Studies of electric fields in the near space indicate that substantial horizontal potential differences are a common feature of the high-latitude ionosphere. Measurements using polar orbiting satellites show spatial potential differences of up to 50 kvolt (Cauffman and Gurnett, 1972). Obviously the classical picture of atmospheric electricity where the upper atmosphere or ionosphere is regarded as a potential equalizing layer must be abandoned. Fields from atmospheric sources are certainly severely damped when they spread upwards into the highly conducting layers, but powerful extraterrestrial sources can produce strong currents and fields in the conducting ionosphere. Clearly the latter fields will spread downwards into the less conducting atmosphere with little damping. Thus, we should expect to be able to trace this extraterrestrial influence on the atmospheric electric field and during the last few years numerous balloon experiments aimed at such studies have been successfully performed.

Obviously it is important to study theoretically the problem of how potential variations produced in one region spread to other regions. Some time-independent problems related to the mapping of ionospheric fields into the lower atmosphere were studied by Kellogg and Weed (1969), Atkinson et al. (1971), Volland (1972), and Chiu (1974). The time-dependent case for low frequency variations was analyzed by Boström et al. (1973) and Boström and Fahleson (1973), while a more general case where induction (non-potential) fields and the anisotropy and frequency dependence of the ionospheric conductivity was taken into account, was studied by Boström (1974). In this paper we will discuss the methods of analysis and present a few results on the vertical propagation of time-dependent electric fields.

2. Various classes of solution

The basic equations to be used are the Maxwell equations

$$\text{curl } \vec{E} = - \mu_0 \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\text{curl } \vec{H} = \sigma \vec{E} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (2)$$

Ohm's law $\vec{i} = \sigma \vec{E}$ has been used in (2) to express the conduction current $\vec{i}$ in terms of the electric field $\vec{E}$. We then neglect other means of charge transport such as convection currents. $\vec{H}$ is the magnetic field associated with the time-varying electric field, ϵ_0 and μ_0 are the vacuum permittivity and permeability, and σ the conductivity which will be considered time independent although it may vary spatially. For a start we will also consider σ to be isotropic although we will in Section 4 treat the case of anisotropic conductivity. For time-dependent problems we have to take both the $\vec{E}$ and $\vec{H}$ fields into account in general, and the solutions to (1) and (2) are forms of electromagnetic waves.

We will study temporal variations of low frequency. For sufficiently low frequencies we may neglect one or both of the time-derivatives of equations (1) and (2). Assuming sinusoidal temporal variations of an angular frequency ω the displacement current $\epsilon_0 \frac{\partial \vec{E}}{\partial t}$ is negligible compared to the conduction current $\sigma \vec{E}$ provided $\omega \ll \sigma / \epsilon_0$. Using the minimum conductivity of the atmosphere, about 10^{-13} S/m, this condition gives $\omega \ll 10^{-2} \text{ s}^{-1}$.

The time derivative of equation (1) can be neglected if the wavelength $2\pi/(\omega\sqrt{\epsilon_0\mu_0})$ and damping length (skin-depth) $2/\sqrt{\omega\mu_0\sigma}$ are large compared to the characteristic dimensions of the problem. Assuming the latter to be less than 100 km,

and using the conductivity $\sigma < 10^{-8}$ S/m valid below 70 km we find the condition $\omega < 2 \times 10^4 \text{ s}^{-1}$. In Section 4 we will return to the question of defining this frequency limit at higher altitudes. When this condition is fulfilled the electric field may be described (approximately) by the gradient of a scalar potential V . However, neglecting the time derivative of equation (1) does not mean that we assume that the magnetic field is strictly time independent.

We may now characterize a problem as belonging to one of the following three classes:

Class 1. Problems where both time derivatives can be neglected. For such cases of very slow temporal variations the solutions have the same character as time-independent ones. As only conduction currents are important an analog circuit model would consist of only resistive elements.

Class 2. Problems where the time derivative is negligible in equation (1) but not in (2). An analog circuit model for such cases would consist of both resistive and capacitive elements, and signals will be both damped and phase shifted.

Class 3. Problems where the time-derivatives of both equation (1) and (2) must be retained. An analog circuit model would then consist of resistive, capacitive and inductive elements, and resonance effects are possible.

We will not consider the solution of Class 1 problems explicitly as they may be regarded as a special and straightforward case of Class 2 problems.

We are interested primarily in the electric field and not in the very minute variations in $\vec{H}$ which are superposed on the dominating permanent field of the Earth and variation fields from currents in the ionosphere. Thus we want to eliminate the $\vec{H}$ -field and solve equations (1) and (2) for $\vec{E}$.

3. Solution of Class 2 problems

By taking the divergence of the equation (2) $\text{curl } \vec{H}$ can be eliminated giving

$$\text{div}(\sigma \vec{E} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}) = 0 \quad (3)$$

This is a form of the continuity equation saying that the total current (conduction plus displacement) is divergence-free. In general, this scalar equation together with the boundary conditions will not suffice to determine the vector field $\vec{E}$, especially since equation (1) has not been taken into account. For Class 2 problems the time derivative in equation (1) can be neglected, that is, $\text{curl } \vec{E}$ is small compared to other derivatives of $\vec{E}$. Then the potential field approximation may be used, thus

$$\vec{E}(x, y, z, t) = -\text{grad}V(x, y, z, t) \quad (4)$$

Using (4) the scalar equation (3) does suffice to determine uniquely the scalar potential V and thus the electric field $\vec{E}$. This approach was used by Boström et al. (1973) and Boström and Fahleson (1973) for studies of some field-mapping problems belonging to Class 2 and has also been used in studies of time-dependent electric fields from thunderclouds, for example by Anderson and Freier (1969), Mann (1970), and Illingworth (1972).

Assuming that σ depends only on the altitude coordinate z and introducing (4) in (3) we find

$$\Delta V + \frac{1}{\sigma} \frac{d\sigma}{dz} \frac{dV}{dz} + \frac{\epsilon_0}{\sigma} \frac{\partial}{\partial t} \Delta V = 0 \quad (5)$$

As the equation is linear an arbitrary field can be decomposed into harmonic temporal and spatial oscillations using Fourier analysis and each component can be studied separately. We can also disregard the permanent steady field and confine our study to the part of the field that varies with time. Thus a basic problem is to find the field distribution in the atmosphere that derives from a source in the form of a harmonic oscillation at a certain altitude for various angular frequencies $\omega=2\pi f$ and wave-numbers $k=2\pi/\lambda$. We will study solutions in the form of travelling waves

$$V(x,y,z,t) = U(z) \exp[j(\omega t - kx)] \quad (6)$$

We use a complex representation where the magnitude of the complex function U defines the amplitude of the oscillation at the altitude z , and the argument of U defines the phase relative to the source field. As the equation is linear we may normalize the field to unit amplitude at the source level $z = z_0$, thus $U(z_0) = 1 + j0$. The ground is a good conductor so we must have $U(0) = 0 + j0$ giving another boundary condition on U . Introducing (6) in (5) gives the equation for determining $U(z)$.

$$\frac{d^2 U}{dz^2} + \frac{1}{\sigma_1} \frac{d\sigma_1}{dz} \frac{dU}{dz} = k^2 U \quad (7)$$

Here $\sigma_1 = \sigma + j\omega\epsilon_0$ is the complex conductivity (admittivity) referring to the combination of conduction and displacement currents. If we have a model of the variation of σ_1 with altitude we can solve equation (7) numerically by a straightforward stepwise integration.

Figure 1 shows an example of the computed damping in the atmosphere of the field from an ionospheric source in the form of a travelling wave. A disturbance field with a horizontal wavelength of 400 km (or characteristic scale of field variation $\lambda/4 = 100$ km) or more penetrates down to balloon altitudes (30-40 km) with little damping if $f < 1$ Hz, but for much smaller wavelengths or higher frequencies the damping becomes substantial.

4. Solution of Class 3 problems

For the more general case of higher frequency temporal variations where induction fields cannot be neglected we can eliminate $\vec{H}$ between equations (1) and (2) by taking the curl of equation (1) and using (2) giving

$$\text{curl curl } \vec{E} = -\mu_0 \sigma \frac{\partial \vec{E}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \quad (8)$$

This is a vector form of the "telegraph equation" describing the propagation of damped waves. We will again study solutions in the form of travelling waves but this time we must consider the three components of the electric field

$$\vec{E}(x,y,z,t) = [U(z)\hat{x} + V(z)\hat{y} + W(z)\hat{z}] \cdot \exp[j(\omega t - kx)] \quad (9)$$

Again this is a wave travelling horizontally in the x-direction with the velocity ω/k , and with a certain vertical propagation described by the complex functions U , V and W . We want to apply the solutions also to the ionosphere where the conductivity is anisotropic, or described by a tensor

$$\sigma = \begin{pmatrix} \sigma_P & \sigma_H & 0 \\ -\sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{\parallel} \end{pmatrix} \quad (10)$$

For simplicity we have assumed that the direction of the magnetic field is vertical (2) which is a good approximation at high latitudes where we would expect the most significant influence from extraterrestrial sources. The Pedersen, Hall and parallel conductivities are frequency dependent complex quantities. While in the E-layer the conductivities are given by frequency independent real numbers for a wide range of frequencies from roughly $\omega = 10^{-6} \text{ s}^{-1}$ to $\omega = 1 \text{ s}^{-1}$, this is not so at higher altitudes or at other frequencies. Thus, above 1000 km the Pedersen conductivity is imaginary for $\omega > 0.001 \text{ s}^{-1}$, that is, the current is a capacitive (displacement) current. The conductivity model that we have used for our examples is given by Boström (1974).

The problem of field spreading in the ionosphere with anisotropic conductivity has been considered earlier for the case of time-independent fields by Farley (1959), Spreiter and Briggs (1969), Park and Dejnakintra (1973), and Chiu (1974), and for the case of time-dependent fields by Boström (1974). Dejnakintra and Park (1974) have also recently studied the mapping of time-dependent fields but they restricted their analysis to such low frequencies that the potential field approximation could be used in the anisotropic region (Class 2 problems).

Introducing (9) and (10) in (8) we obtain three coupled differential equations for U, V, and W, one from each vector component of (8). After some rearrangements and including the contribution from the vacuum displacement current in the complex Pedersen and parallel conductivities these equations may be written

$$\frac{d^2 U}{dz^2} + \frac{k^2}{j\omega\mu_0\sigma_{\parallel}+k^2} \frac{1}{\sigma_{\parallel}} \frac{d\sigma_{\parallel}}{dz} \frac{dU}{dz}$$

$$= (j\omega\mu_0\sigma_{\parallel}+k^2) \left(\frac{\sigma_P}{\sigma_{\parallel}} U + \frac{\sigma_H}{\sigma_{\parallel}} V \right) \quad (11)$$

$$\frac{d^2 V}{dz^2} = (j\omega\mu_0\sigma_P+k^2)V - j\omega\mu_0\sigma_H U \quad (12)$$

$$W = \frac{jk}{j\omega\mu_0\sigma_{\parallel}+k^2} \frac{dU}{dz} \quad (13)$$

The equations (11) and (12) may be solved for U and V by straightforward numerical integration using as boundary conditions $U(0) = V(0) = V(z_0) = 0 + j0$, $U(z_0) = 1 + j0$. These conditions correspond to zero horizontal field at the ground and a wave of unit amplitude at the source level. The vertical component of the field is then determined by (13).

For the case of isotropic conductivity ($\sigma_P = \sigma_{\parallel} = \sigma_1$ and $\sigma_H = 0$) and for $\omega \ll k^2/\mu_0|\sigma_{\parallel}|$ the equation (11) will reduce to (7) although U of (7) refers to the potential rather than to the horizontal field. However, for the harmonic oscillation considered these two quantities are proportional. Thus the same results as given by the method described in Section 3 are obtained from equation (11) for Class 2 problems which has been verified also by numerical computations. The frequency limit for applying the approximate equation (7) in the ionosphere and magnetosphere may be quite small as the parallel conductivity that enters into the inequality is quite high. Assuming $k = 2\pi \times 10^{-5}$ (100 km horizontal wavelength) and $\sigma_{\parallel} = 30$ S/m we find $\omega \ll 10^{-4} \text{ s}^{-1}$. Thus it seems that even for periods longer than 10 hours it would not be possible to use Class 2 solutions. However, for such small values

of k that $\omega > k^2/\mu_0 |\sigma_n|$ the coefficient in front of U in the right hand member of (11), which then is $\omega\mu_0\sigma_p$, will be a very small number unless ω is high. Assuming $\omega = 0.01$ and $|\sigma_p| < 10^{-6}$ S/m (cf. Boström, 1974) we find $|\omega\mu_0\sigma_p| = 1/(10000 \text{ km})^2$ which means that U is nearly constant over the height range of several thousand kilometers. The same result that U is nearly constant in the ionosphere would be obtained from equation (7) for small values of k . Although equations (7) and (11) predict different rates of change of U with altitude, the results for U are virtually identical.

Regarding the coupling between the two horizontal field components U and V this will vanish if either $\sigma_H = 0$ or $\omega = 0$. Then equation (12) does not contain U , and the only solution to (12) that satisfies the condition that V should vanish at both boundaries is $V \equiv 0$ everywhere. Assuming that $|V| < |U|$ we find that we may neglect the coupling term containing U in (12) if either i) $|\sigma_H| \ll |\sigma_p|$ which is true throughout large height intervals, but not in the E-layer or ii) $\omega \ll k^2/\mu_0 |\sigma_H|$. If we study wavelengths shorter than 1000 km ($k > 2\pi \times 10^{-6}$) the latter condition gives $\omega < 6 \text{ s}^{-1}$ as $|\sigma_H| < 5 \times 10^{-5}$ S/m, so for such ω and k values the coupling may be neglected and the equations simplified.

Figure 2 shows the magnitude $|U|$ as function of altitude for the case of a source far out in the magnetosphere. We can see that fields of a wavelength longer than about 100 km penetrate far down into the atmosphere while fields of a wavelength of 1 km penetrate only to the ionospheric E-layer. As expected the damping increases with increasing frequency. For the atmosphere below 100 km this frequency dependence occurs for $\omega > 0.01 \text{ s}^{-1}$, when the displacement current becomes important. It is interesting to note that

in the magnetosphere this frequency dependence appears at much lower frequencies. For $\omega = 10^{-4} \text{ s}^{-1}$ the displacement current (or imaginary part of σ_p) dominates over the conduction current (or real part of σ_p) at altitudes above 1000 km. Thus, as seen in Figure 2, a field with a wavelength of 1 km, which is not much damped between 10000 and 1000 km for $\omega = 10^{-6} \text{ s}^{-1}$ is severely damped even at such a low frequency as $\omega = 10^{-3} \text{ s}^{-1}$. At high frequencies ($\omega > 1 \text{ s}^{-1}$) and high altitudes ($z > 500 \text{ km}$) where the damping length is long compared to the hydromagnetic wavelength the solutions for U will show rapid oscillations. In order not to make Figure 2 unintelligible some of the curves are not shown in this upper region where they oscillate. Note that Figure 2, by normalization, can be applied to the downward field mapping also for sources at lower altitudes, as long as the Hall field V is not of importance in the source region.

Strictly speaking our equations, which are based on an assumption of straight and parallel geomagnetic field lines, cannot be applied in the outer magnetosphere. However, qualitatively we expect a similar behaviour for fields in the real magnetosphere (except for changes in field intensity due to converging field lines). Furthermore, we must emphasize that in the real high-latitude ionosphere small-scale horizontal variations of the conductivity may occur which would complicate the field propagation.

5. Switching transients

5a. The validity of the classical, exponentially varying solution

In textbooks on atmospheric electricity (Chalmers 1967, Israel 1973, p.323 and 392) one may find an expression for the way in which the atmospheric electric field varies with time. This expression

$$\vec{E} = \vec{E}_e \exp(-\sigma t / \epsilon_0) + \vec{E}_s [1 - \exp(-\sigma t / \epsilon_0)] \quad (14)$$

is meant to describe the changes in the field that occur

upon a transient change in the source at time $t = 0$. It says that everywhere in space the initial state $\vec{E}_e$ decays and the final state $\vec{E}_s$ builds up exponentially with a time constant determined by the local relaxation time $\epsilon_0/\sigma(x,y,z)$. We will show here that this expression is not generally applicable.

In the history of atmospheric electricity this exponential solution appeared first in the studies of the classical, spherically symmetric capacitor model with no voltage or current generators in the atmosphere (Benndorf 1925, 1927, see also Israël 1973, p. 323). For this case the field has only a radial component E_r and varies only with the radial coordinate and equation (8) gives

$$\sigma \frac{\partial E_r}{\partial t} + \epsilon_0 \frac{\partial^2 E_r}{\partial t^2} = 0 \quad (15)$$

Integrating equation (15) we find

$$E_r(r,t) = E_{re}(r) \exp(-\sigma t/\epsilon_0) + E_{rs}(r) [1 - \exp(-\sigma t/\epsilon_0)] \quad (16)$$

as the only possible form for temporal variations for strictly spherically symmetric problems. A generalization of (16) to the form

$$\vec{E}(x,y,z,t) = \vec{E}_e(x,y,z) \exp(-\sigma t/\epsilon_0) + \vec{E}_s(x,y,z,t) [1 - \exp(-\sigma t/\epsilon_0)] \quad (17)$$

meant to be valid when $\vec{E}$ also has horizontal components and varies with all three coordinates was attempted by Kasemir (1950, 1963). The expression (17) was derived (Kasemir 1950) by formally integrating equation (3) and assuming that the unknown function of zero divergence that enters upon integration had certain properties. It was verified (Kasemir 1963) that the solution (17) satisfies the

boundary conditions and equation (3) (when $\vec{E}_s$ is assumed to vary with time slowly compared to the relaxation time). However, as mentioned above (Section 3) equation (3) with boundary conditions only does not define a unique solution. A further condition has to be imposed on $\vec{E}$ which for Class 2 problems is $\text{curl } \vec{E} = 0$. Forming the curl of $\vec{E}$ as given by (17) we find

$$\text{curl } \vec{E} = \frac{t}{\epsilon_0} \exp(-\sigma t / \epsilon_0) (\text{grad } \sigma) \times (\vec{E}_s - \vec{E}_e) \quad (18)$$

as $\vec{E}_s$ and $\vec{E}_e$ are assumed to be curlfree fields.

For the atmosphere we may assume that σ varies with altitude only, thus $\text{grad } \sigma$ is vertical. Then $\text{curl } \vec{E}$ will be small only if $\vec{E}_s$ and $\vec{E}_e$ (and then $\vec{E}$) are nearly vertical everywhere. Thus for Class 2 problems where $\text{curl } \vec{E}$ is small expression (14) gives a correct description of the temporal variation of the field only if there are no horizontal field components.

5b. Study of switching transients by Fourier analysis

To study switching transients for the general case with three-dimensional fields we may use Fourier analysis, that is, we build up the solution for a transient source by superposition of the solutions for harmonically oscillating sources discussed earlier. As an example we have investigated how the atmospheric electric field at a certain altitude responds when an ionospheric source is switched on at time $t = 0$. The source is assumed to have a spatial sinusoidal variation with a wavelength $\lambda = 100$ km but is constant in time for $t > 0$. The problem has been solved using 3 different methods. Shown in Figure 3 are 1) a solution of (11), (that is the full Maxwell equations) using the method of Fourier superposition, 2) a solution of (7) (based upon $\text{curl } \vec{E} = 0$) using the same method and 3) a solution according to the exponential expression (17). The Fourier solutions have been obtained by superposition of 101 terms with Lanczos' convergence factors to form a rectangular wave of period 1200 sec at the upper boundary. It can be seen in the figure

that the temporal variation at the upper boundary so obtained is a good approximation to the desired step function.

As expected from the discussion in Section 4, the solution of equation (7) fully agrees (better than the line thickness in the figure) with that of the exact equation (11). The exponential solution, on the other hand, gives a too slow response. For the case illustrated the difference is a factor of 4.7 in the time to reach half amplitude. A similar difference occurs for the solutions of the vertical field component.

6. Concluding remarks

The purpose of this report is not to give a full account of the results on how electric fields produced in one region spread to other regions, but rather to discuss the methods of analysis and give some examples showing the qualitative behaviour of the solutions. The quantitative results on the field intensity and phase shift as a function of altitude, horizontal scale size and frequency, will depend very much on the conductivity model used. Further studies using various conductivity models, as well as more specific models for the temporal variation and spatial geometry of the source, are needed.

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Fig. 1 Amplitude of potential, $|U|$, or horizontal electric field, $k|U|$, at different altitudes for various wavelengths $\lambda = 2\pi/k$ (in km) and frequencies $f = \omega/2\pi$ (in Hz) of the wave originating from an ionospheric source $V = 1 \cdot \exp[j(\omega t - kx)]$ at 112 km. The conductivity model used is $\sigma = 9 \times 10^{-14} \exp(z/7000)$.

Fig. 2 Amplitude of horizontal electric field at different altitudes for various wavelengths λ and angular frequencies ω of the wave originating from a magnetospheric source $E_x = 1 \cdot \exp[j(\omega t - kx)]$ at 35 000 km. The conductivity model takes into account the anisotropy and frequency dependence of the ionospheric conductivities. After Boström (1974).

Fig. 3 Normalized amplitude of horizontal electric field at 7 km altitude due to sudden application of $E_x = 1 \cdot \cos(2\pi \frac{x}{\lambda})$ at 112 km with $\lambda = 100$ km. Fourier method solutions of the exact equation (11) and the approximate equation (7) agree better than line thickness in contrast to a solution by the exponential method, equation (17), which deviates considerably. The temporal variation at the upper boundary used in the Fourier approximation is also shown. Conductivity model same as for Figure 1.

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