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EFFECTS OF BIRKELAND CURRENT LIMITATION ON HIGH-LATITUDE CONVECTION PATTERNS

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Abstract

It is shown how the high-latitude convection pattern may be modified by substorm-enhanced polarisation electric fields. These are generated whenever the flow of those Birkeland currents which are associated with ionospheric conductivity gradients is limited. Such Birkeland currents are fed mainly by the enhanced Pedersen current in the evening and morning sectors of the auroral oval and by the enhanced Hall current around local midnight. As the current limitation increases, the ionospheric potential, represented here by a symmetric two-cell pattern, will rotate clockwise and deform, just as the associated Birkeland current distribution. The resulting patterns are shown to agree well with observations. A pronounced westward intrusion of the equipotential contours occurs in the auroral oval, and may be associated with the Westward Travelling Surge. This feature does not however require any assumed longitudinal conductivity gradients. Rather it falls out naturally when the limitation of the enhanced Pedersen current is taken into account.
1. Introduction

The electric field or potential distribution of the high-latitude ionosphere is modified considerably during magnetospheric substorms and responds sensitively to variations in the interplanetary magnetic field (IMF). It is now well established that a two cell convection pattern exists during periods of southward IMF or if the IMF has been northward for not more than about two hours (Heelis and Reiff 1984, Wygant et al. 1983). The IMF $B_y$-component is known to cause an asymmetry between the two cells and a shift of the zero-potential line separating them (Heelis and Hanson, 1980). A clockwise rotation of the potential pattern seems however to be a more persistent feature during a substorm. Nopper and Carovillano (1979) found that the orientation of the polar cap electric field varied with the relative intensity between the region 1 and region 2 currents. As the substorm activity increased, this ratio decreased and the polar cap electric field rotated clockwise. Yasuhara et al. (1983) calculated the ionospheric potential pattern for an impressed field-aligned current distribution and found that it rotated clockwise as the Hall-conductivity in the auroral oval increased above its value in the polar cap.

Kan et al. (1984) studied the case of an electric field impressed on the high-latitude ionosphere under the assumption that the Hall current was partially limited from closure in the magnetosphere. The resulting potential pattern was distorted and rotated clockwise, effects which increased with the blockage parameter $\beta$, or limitation factor which is the term that will be used here. The westward intrusion of the distorted potential pattern, on the nightside that resulted from increasing the limitation factor was according to them a possible explanation for the Westward Travelling Surge.

The assumption that only the enhanced Hall current is limited applies generally only to the limited region around local midnight where the Hall currents flow across the high-conducting oval such as in the Westward Travelling Surge. In the evening
and morning sectors of the auroral oval it is instead the enhanced Pedersen currents that may be limited, which will create polarisation electric fields in order to maintain current continuity.

We therefore treat here a more general case where polarisation electric fields are created as a result of limiting the enhanced meridional ionospheric closure currents which may be Pedersen or Hall currents or contain components of both depending on the local time sector. This also agrees with most auroral arc observations made by rockets, satellites and ground-based radar. These observations show clearly that polarisation electric fields always tend to be oriented in the direction of maximum conductivity gradients, i.e. on a global scale, perpendicular to the auroral oval (cf. Fig.1).

The model, used here is a modification of that used by Kan et al. (1984). It is described in Section 2 and the numerical results presented in Section 3. We examine the role of varying the limitation factor $\beta$ and compare the results with the case where only the Hall current is limited as assumed by Kan et al., 1984. In Section 4 the basic assumptions and characteristic features of the results are discussed in more detail followed by a brief discussion on the mechanism causing the Birkeland current limitation and on possible applications to the Westward Travelling Surge.

2. Model

Consider an electric field $E_O$ impressed on the ionosphere driving a Pedersen current $I_P E_{OL}$ and a Hall current $I_H \hat{B}_O \times E_{OL}$ where $\hat{B}_O$ is a unit vector in the direction of $\hat{B}_O$. From the condition that the total current should be divergence-free the Birkeland current flowing into the ionosphere can be written:

$$j_\parallel = \nabla \cdot (I_P E_{OL} + I_H \hat{B}_O \times E_{OL})$$  \hspace{1cm} (1)
which can be rewritten as
\[ j_{\parallel} = \Gamma_P \mathbf{v} \cdot \mathbf{E}_{\parallel} + \mathbf{E}_{\parallel} \cdot \mathbf{v} \mathbf{L}_P + \mathbf{B}_0 \times \mathbf{E}_{\parallel} \cdot \mathbf{v} \mathbf{L}_H \]  

(2)

According to the definition by Boström (1976) the first term in (2) corresponds to primary Birkeland currents "of magnetospheric origin" and the second and third terms to secondary Birkeland currents "of ionospheric origin" the latter occurring because of spatial variations in the ionospheric conductivity.

If these secondary Birkeland currents are partially limited to flow by some mechanism then charges will accumulate and secondary polarisation electric fields will be produced. The assumption used here, is that a factor \( \beta \) of these secondary Birkeland currents is limited from closure in the magnetosphere, which results in the generation of a polarisation electric field, \( \mathbf{E}_P \). This can be expressed mathematically in the following way:

\[ \mathbf{v} \cdot (\Gamma_P \mathbf{E}_P) + \beta (\mathbf{E}_{\parallel} \cdot \mathbf{v} \mathbf{L}_P + \mathbf{B}_0 \times \mathbf{E}_{\parallel} \cdot \mathbf{v} \mathbf{L}_H) = 0 \]

(3)

Thus, the divergence of the Pedersen polarisation current cancels the fraction \( \beta \) of the secondary Birkeland currents. \( \beta = 0 \) corresponds to no limitation and thus no polarisation electric field. \( \beta = 1 \) corresponds to complete limitation and thus a large polarisation electric field.

Since \( \mathbf{E}_P = -\nabla \Phi_P \) Eq.3 can be rewritten as

\[ \nabla^2 \Phi_P + \frac{\mathbf{v} \cdot \mathbf{L}_P}{\Gamma_P} \cdot \Phi_P - \beta \left\{ \frac{\mathbf{v} \cdot \mathbf{L}_P}{\Gamma_P} \cdot \mathbf{E}_\perp + \frac{\mathbf{v} \cdot \mathbf{L}_H}{\Gamma_P} \cdot (\mathbf{B}_0 \times \mathbf{E}_\perp) \right\} = 0 \]

(4)

where we have replaced \( \mathbf{E}_{\parallel} \) by \( \mathbf{E}_\perp \).

The resulting polarisation potential \( \Phi_P \), superposed on the initial potential distribution \( \Phi_0 \) thus gives the modified potential distribution:

\[ \Phi_T = \Phi_0 + \Phi_P \]  

(5)

The \( \Phi_0 \)-distribution is the same as that used by Kan et al. (1984), namely a solution of Laplace's equation \( \nabla^2 \Phi_0 = 0 \) under the boundary conditions that \( \Phi_0(\Gamma_1, \Theta) = \Phi_{in} \sin \Theta \) and
\( \Phi_0(r_2, \theta) = 0 \), implying that there are excess charges at \( r = r_1 \).

The colatitude \( r = r_1 \) is the polar cap boundary and \( r = r_2 \) is the equatorward boundary of the convection. \( \theta \) is the azimuthal angle, zero at local magnetic midnight and positive towards east, and \( \Phi_m \) represents the maximum dawn potential. \( \Phi_0 \) takes the form:

\[
\Phi_0 = \Phi_m \sin \theta \times \begin{cases} 
\frac{r}{r_1} & \text{for } r < r_1 \\
\frac{r_2}{r_1} - \frac{r-r_2}{r_2} & \text{for } r > r_1 \\
\frac{r_2}{r_1} - \frac{r_1}{r_2} & \text{for } r = r_1
\end{cases}
\tag{6}
\]

\( \Phi_p \) is thus solved from Eq.(4) using (6) and the conductivity distribution given below.

\[
\Gamma_p = \Gamma_0 + (\Gamma_{po}^2 + \Gamma_{pa}^2)^{1/2}
\tag{7}
\]

\[
\Gamma_H = \Gamma_0 + (\Gamma_{Ho}^2 + \Gamma_{Ha}^2)^{1/2}
\tag{8}
\]

where \( \Gamma_{po} = \Gamma_{mo} \exp \left\{ -\left( \frac{\theta_0 - \theta_o}{A_o} \right)^2 - \left( \frac{r-r_o}{L_o} \right)^2 \right\} \)

\[
\Gamma_{pa} = \Gamma_{ma} \exp \left\{ -\left( \frac{\theta_0 - \theta_a}{A_a} \right)^2 - \left( \frac{r-r_a}{L_a} \right)^2 \right\}
\]

\[ \Gamma_{Ho} = \Gamma_{po}, \quad \Gamma_{Ha} = k \cdot \Gamma_{pa} \]

This is a slight modification of the Gaussian conductivity model by Kamide et al. (1981) in that we have added a constant background conductivity level \( \Gamma_0 \) in (7) and (8).

\( \Gamma_{po}(pa) \) represents the latitudinal variation of the enhanced height-integrated Pedersen conductivity, having a maximum \( \Gamma_{mo}(ma) \) located at \( r = r_0(a) \) and a latitudinal width of \( L_0(a) \) during quiet (disturbed) conditions. The local-time dependence has been omitted here, (i.e. \( A^2_{o(a)} \gg (\theta - \theta_{o(a)})^2 \))
for the following reason: In this study we have concentrated upon the global aspects of the distortion of the potential pattern during substorms. Generally, the conductivity gradients are much smaller in the longitudinal than in the latitudinal direction, and this is especially true during disturbed conditions. For the relatively localized phenomena of the Westward Travelling Surge east-west conductivity gradients surely exist but even for this case the maximum conductivity gradients are oriented across and not along the oval. This will be discussed in more detail in Section 4.

The field-aligned current \( j_\parallel \) can be calculated from the divergence of the total resulting current.

\[
j_\parallel = \nabla \cdot \left( \Gamma_p (\vec{E}_o + \vec{E}_p) + \Gamma_H \vec{B}_o \times (\vec{E}_o + \vec{E}_p) \right)
\]  \hspace{1cm} (9)

and the excess charge associated with the polarisation electric field is given by

\[
\varrho_c = \nabla \cdot (\varepsilon_0 \vec{E}_p)
\]  \hspace{1cm} (10)

3. Results

In Figs. 2 a-e are shown the equipotential contours \( \Phi_T \), for the case where both the Hall and Pedersen current limitation is taken into account and obtained by solving Eqs.(4-5) above requiring that \( \Phi_P(r_2) = 0 \). Figs. 3 a-e show the corresponding \( \Phi_T \)-contours for the case where the Pedersen contribution to the secondary Birkeland currents has been neglected (first term within the brackets of Eq.(4)). The thicker zero-line separates the regions of positive (+) and negative (-) equipotential contours. The latter are separated by 5 kV.

The values of the parameters in the expressions for \( \Phi_o \) and the conductivity distribution \( \Gamma \) (cf Sect.2) that have been used here are:
\[ \phi_0: \phi_m = 50 \text{ kV}; \ r_1 = 15^\circ; \ r_2 = 40^\circ \]

\[ \Gamma: \ \Gamma_0 = 1 \text{ mho}; \ \Gamma_{mo} = 5 \text{ mho}; \ \Gamma_{ma} = 15 \text{ mho}; \]

\[ k = 2; \ L_0 = 5^\circ; \ L_a = 2^\circ; \ r_0 = r_a = 23^\circ. \]

Figs. 2a and 3a show the impressed potential \( \phi_0 \) which is given by Eq. 6. It corresponds to the \( \beta = 0 \) case (i.e. no limitation). In Figs. 2b and 3b a limitation factor of 0.5 has been used which results in a clockwise rotation and slight distortion of the convection pattern. Although the distorted patterns in Figs. 2b and 3b look similar there are some differences of which the following are especially important:

The magnitude of the potential maximum (or minimum) is lower than \( \phi_m \) (\( \phi_m = 50 \text{ kV} \)) for the case where both the Pedersen and Hall current contribution to the secondary Birkeland currents are taken into account (Fig. 2b) but greater than \( \phi_m \) for the case where the Pedersen contribution is neglected (Fig. 3b). With the given boundary conditions this also implies that the magnitude of the electric field is on the average larger for the latter case. In response to a sudden enhancement in the auroral oval conductivity, one should expect the ionosphere to generally modify itself so as to decrease rather than increase the electric field. This is in fact the basic idea behind the approach used here. Thus, the solution presented in Fig. 2b where both the Pedersen and Hall contributions are taken into account appears to be physically more realistic. Another important difference is that the rotation of the equipotential contours within the high conducting auroral oval is larger for the case where the Pedersen contribution is taken into account, causing a westward intrusion of the potential contours (cf. Figs. 2b, 3b). In Figs. 2c and 3c the potential patterns are shown for the two cases for \( \beta = 1 \) (i.e. complete limitation). The patterns are seen to be rotated almost \( 45^\circ \) clockwise having potential maxima of \( \approx 50 \text{ kV} \) and \( \approx 60 \text{ kV} \) respectively. Here, the increase of the potential maximum above \( \phi_m \) for the latter case is even more pronounced. Note also the pronounced westward intrusion of the equipotential contours within the auroral oval in Fig. 2c, a feature not seen in Fig. 3c.
To complete the schema the potential distributions for negative β-values of -0.5 and -1.0 are shown in Figs. 2d, 3d and 2e, 3e respectively. This somewhat artificial situation with β<0 implies that the polarisation electric field no longer opposes the excess meridional ionospheric closure currents, but rather acts to enhance them. This results in large electric fields and potential maxima much larger than Φ_m. The patterns are now instead rotated anticlockwise, the rotation angle being smaller for a given absolute β-value than it is for β>0.

Next, let us concentrate in some detail upon the electrodynamics associated with some of the cases presented above. We consider only the more realistic solutions for which both the Pedersen and Hall current contributions to the secondary Birkeland currents have been taken into account. In Fig.4 the impressed potential distribution, solution to the β = 0 case, is given once more together with the associated Birkeland current distribution. Downward and upward currents are denoted by x and o respectively and the contours represent the 5%, 20% and 80% boundaries of the maximum current density.

For β = 0, Φ_p and Φ_c are identically zero and so these distributions have been omitted in Fig. 4. The j_II-distribution for β = 0 can be divided into: (1) the primary Birkeland currents, j_II^I, located at the polar cap boundary r = r_1 and originating from the divergence of the impressed electric field (first term in Eq.2) and (2) the secondary Birkeland currents, j_II^II, located equatorward and originating from the conductivity gradients across the auroral oval (2nd and 3rd terms in Eq.2). The j_II^II-distribution is seen to be rotated anticlockwise relative to the j_II^I-distribution.

Fig. 5 shows the results for β = 0.5. The Φ_p-pattern is characterized by two cells rotated clockwise about 110° relative to the initial distribution Φ_0 and by a potential maximum (minimum) of +20 kV (-20 kV) (i.e. about 40% of the initial maximum potential).
The $j_{\parallel}^{I\!I}$-distribution is different from that in Fig. 4 in that it is somewhat more confined latitudinally and rotated clockwise such that the distribution is more symmetric about the noon-midnight magnetic meridian.

The $\varphi_c$-distribution is seen to be anticorrelated with the Birkeland current distribution. A pronounced region of positive space charge dominates on the nightside hemisphere and a similar region of negative space charge dominates on the dayside hemisphere.

In Fig. 6 the corresponding results are shown for $\beta = 1.0$ (i.e. complete limitation). The $\Phi_p$-pattern is here characterized by a similar rotation as for $\beta = 0.5$ (cf. Fig. 5) but a much higher potential maximum as compared to the $\beta = 0.5$ case in Fig. 5. $\Phi_{p_{\text{max}}}$ is now almost 90% of $\Phi_m$. The $j_{\parallel}^{I\!I}$-distribution is rotated even further clockwise, about 70° clockwise of the initial $j_{\parallel}^{I\!I}$-distribution for $\beta = 0$ (cf. Fig. 4). The space charge density is higher than for $\beta = 0.5$ but the distributions are otherwise similar.

In Fig. 7, the corresponding results are shown for an example with a negative $\beta$-value, namely $\beta = -0.5$. The $\Phi_T$-contour is here rotated anticlockwise which is a result of the fact that the positive and negative equipotential contours of $\Phi_p$ have changed place as compared to the pattern for $\beta = 0.5$.

The $j_{\parallel}^{I\!I}$-distribution on the nightside is characterized by downward field-aligned currents in the equatorward part of the oval and upward field-aligned currents in the poleward part. The situation on the dayside is the reverse. Such a distribution of Birkeland currents is not consistent with the satellite observations which have been reported. The $\varphi_c$ distribution is here characterized by regions of predominantly negative space charge on the nightside and positive space charge on the dayside.

To summarize, the ionospheric response to a limitation of the secondary Birkeland currents is to generate a polarisation po-
tential distribution. This results in a modification of the initial impressed potential distribution which here was taken to be a symmetric two-cell pattern. For \( \beta > 0 \), the modification is such that the entire potential pattern rotates clockwise between 0 and 45°. A westward intrusion of the equipotential contours in the nightside part of the oval becomes more pronounced as \( \beta \) increases. The maximum potential decreases slightly or remains roughly the same, and the distribution of the secondary Birkeland currents rotates clockwise towards a more symmetric distribution relative to the noon-midnight magnetic meridian. The associated space-charge distribution is characterized by pronounced space-charge regions being positive on the nightside and negative on the dayside.

For \( \beta < 0 \), the modification is such that the rotation of the potential pattern is anticlockwise, the maximum potential is increased much above \( \phi_m \), the Birkeland current distribution is rotated anticlockwise and finally the space charge regions are now interchanged as compared to the results for \( \beta > 0 \). These results for \( \beta < 0 \), are neither likely physically nor consistent with observations.

4. Discussion

Much effort has been made recently to investigate and understand the ionospheric response during a magnetospheric substorm. Any attempts to treat this problem globally requires that the general assumptions made apply globally. The basic assumption used here, that polarisation electric fields acts so as to oppose or limit the enhanced meridional ionospheric closure currents, which may be Pedersen or Hall currents, or contain components of both, is believed to apply globally. This is also confirmed by a majority of the electric field observations made in the auroral ionosphere (cf. Marklund, 1984) as summarized schematically in Fig. 1.

The ionospheric conductivity model by Kamide et al. (1981) which has been slightly modified here includes the Gaussian conducti-
vity variation in the latitudinal but not the longitudinal direction. The reason for neglecting the local time dependence is that during active conditions the longitudinal conductivity gradients are typically much smaller than the latitudinal gradients. This is clearly demonstrated by e.g. Fig. 8 from Wallis and Budzinski (1981) based on ISIS 2 particle data. Due to this and the fact that the conductivity distribution is not reproducible between various substorms, it is likely that the conductivity model used here is just as accurate as any local-time dependent model. Also, since the results depend as much on the particular form of the impressed potential distribution, it is not necessary to introduce uncertain small-scale variations in the conductivity model while using the global form of the potential distribution.

The results presented in Section 3 show a number of interesting features. The orientation of the two-cell equipotential pattern depends critically on the value of the limitation factor $\beta$. The potential pattern is rotated clockwise for $\beta > 0$ and anticlockwise for the less likely case of $\beta < 0$, relative to the undistorted pattern ($\beta = 0$). The rotation increases with $\beta$. As can be seen by comparing Fig. 5 and Fig. 6 this is not because the orientation of the $\Phi_p$-pattern changes but rather that the potential maximum increases with $\beta$. In the extreme case of complete limitation ($\beta = 1.0$) the polarisation potential maximum is almost as high as the potential maximum $\Phi_m$ in the initial potential distribution. This is logical since a complete limitation of a current driven by a given electric field should require an opposing electric field of roughly the same magnitude.

For a given $\beta$, the clockwise rotation will increase with increasing values of the conductivities but also with the Hall to Pedersen conductivity ratio. This ratio was chosen here to be 1.0 for the quiet-time oval and the polar cap region and 2.0 for the disturbed-time oval. Inside a distinct auroral form this value may well exceed 2 but it might be too high to represent an average value in the oval even for disturbed conditions. Typical values of this ratio calculated from Rees' algorithm using rocket
particle observations as input are 1.0 outside and 1.5 inside an arc (cf. Evans et al. 1977, Marklund et al. 1982, 1983). This implies that the rotations of the convection patterns presented here might be somewhat overestimated.

Fig. 2c shows clearly that the equipotential contours are more clockwise rotated inside the highly conductive auroral oval than equatorward or poleward of it where the conductivity is lower. Such a pronounced westward distortion or intrusion of the equipotential contours along the nightside auroral oval cannot be distinguished for the less realistic case in Fig. 3c where the Pedersen contribution is neglected. Kan et al. (1984) in fact neglected the Pedersen current, but obtained nevertheless, a westward distortion of the convection streamlines. The distortion, however, resulted from the large longitudinal gradients that had been assumed in their conductivity model. In Fig. 2c the westward distortion localized to the auroral latitudes is on the other hand a consequence of using an improved and physically more realistic assumption, namely that both the enhanced Pedersen and Hall closure currents may be limited.

The clockwise rotation of the potential pattern seems to be a characteristic feature during a substorm as confirmed by both ground-based magnetometers (Kamide et al. 1981) and incoherent scatter radars (Evans et al. 1980). Yasuhara et al. (1983) obtained a rotation of the convection pattern by imposing a field-aligned current distribution of the form shown in Fig. 9, upon the conducting auroral ionosphere. The $j_{\parallel}^{II}$-distribution for the undisturbed potential distribution shown in Fig. 4 is seen to be rotated anticlockwise relative to this, but as the limitation factor $\beta$ increases to a value between 0.5 and 1.0 (cf. Figs. 5 and 6) the $j_{\parallel}$-distribution becomes roughly similar to that in Fig. 9.

In Fig. 10 the average (or typical) pattern of equipotentials and Birkeland currents are shown for comparison (Wolf and Spiro, 1984). As can be seen the patterns are qualitatively in agreement with those obtained here for $\beta = 0.5$, presented in Fig. 5.
The interplanetary magnetic field (IMF) \( y \)-component is known to control the position of the zero potential line separating the two cells, just as the blockage parameter \( \beta \) does in our study. However, since substorms occur for IMF \( B_y \) being positive as well as negative, and \( \beta \) presumably only attains positive values resulting in the clockwise rotation of the convection pattern the relationship between these two mechanisms is unclear.

So far we have merely assumed that a limitation of secondary Birkeland currents occur whenever ionospheric currents are driven across sharp conductivity gradients without questioning why the limitation occurs. If the number of charged particles out in the magnetosphere are not enough to carry these secondary Birkeland currents along a given fluxtube, there will be a current limitation causing polarisation electric fields to form. However, it is not likely that such a limitation will occur with an equal probability at any local time sector. In other words, \( \beta \) presumably varies with local time, implying that the degree of distortion and rotation of the convection pattern differs at various portions of the auroral oval.

As the magnetic field topology changes from tail-like to dipolar during the expansive phase of a substorm part of the neutral sheet current will be disrupted and has to close in the ionosphere. As discussed by Boström (1974) it is likely that a limited slot region depleted with plasma will form in the disrupted current region (cf. Fig.11). Since the cross-tail current is essentially driven by protons and the field-aligned currents by electrons, a depletion of plasma will occur at the western edge of the slot and a plasma enhancement at the eastern edge, resulting in a westward propagation of the depleted plasma region. The typical propagation speed 1 km/s as calculated by Boström (1976) is comparable to a typical speed of the Westward Travelling Surge. Thus it appears likely that the Westward Travelling Surge is closely associated with the westward distortion of the convection streamlines resulting from a limitation in the flow of secondary Birkeland currents in a limited local time sector on the nightside. However, polarisation electric fields exist
not only during the expansive phase but during the entire course of a substorm at any local time. Thus, the limitation mechanism appears to be a relatively persistent and general feature at auroral latitudes. Superposed on this a more effective current limitation associated with the Westward Travelling Surge may temporarily be responsible for a more localized distortion of the convection pattern on the nightside.

The extent of ionospheric polarisation here represented by the limitation factor $\beta$, has by some authors been suggested to be determined by the ratio of the characteristic Alfvén wave conductance, $\Sigma_{||} = (\mu_0 V_A)^{-1}$, to the height-integrated Pedersen conductivity $\Gamma_P$ (cf. Carlson and Kelley, 1977; Mallinckrodt and Carlson, 1976). Whether $\beta$ simply could be replaced by a function of this ratio is however not so clear. The high field-aligned impedance is according to this theory an inductive effect caused by time-varying currents. A consequence of this is that $\beta$ ought to vanish in a stationary situation. Since polarisation electric fields ($\beta \neq 0$) are observed both during relatively stable conditions and during the substorm expansive phases, this might not always be the case. Since $\Sigma_{||}$ depends on the uncertain value of the plasma density in the magnetospheric source region, it was found advantageous in this study to keep $\beta$ as a free parameter.

5. Summary and conclusions

This study presents numerical examples of how the auroral ionosphere responds to a substorm-enhanced magnetospheric convection, represented by a two-cell equipotential distribution. A modification of this occurs due to the superposition of a polarisation potential distribution, that results from a limitation of secondary Birkeland currents (limitation factor $\beta$) caused by ionospheric conductivity gradients. The main findings are summarized below.

1. As the secondary Birkeland currents are fed by the enhanced ionospheric closure currents, which may be Pedersen or Hall cur-
rents depending on the local time sector, a physically realistic solution that applies globally requires necessarily that both of these contributions are taken into account.

2. The potential pattern will be rotated clockwise for $\beta > 0$ and anticlockwise for the less realistic case of $\beta < 0$. Both the potential pattern and the associated global Birkeland current distribution is found to be in good agreement with observations for $\beta > 0$ but not for $\beta < 0$.

3. The clockwise rotation of the convection pattern will increase both with increasing conductivities and with the Hall to Pedersen conductivity ratio. There will, therefore, also be a distortion of the potential pattern since the degree of rotation varies with the latitude-dependent conductivity.

4. In the centre of the nightside auroral oval the westward intrusion of the equipotential contours, as $\beta$ increases, will therefore be faster than poleward or equatorward of the oval. This intrusion may be associated with the Westward Travelling Surge, as also suggested by Kan et al. (1984). It should be noted that the westward intrusion is much more pronounced for the case where both the Pedersen and Hall current contributions to the secondary Birkeland currents are taken into account. This result does not require any assumed longitudinal conductivity gradients, as were required in the model by Kan et al. (1984).

5. The current limitation, occurring if there is a high impedance for secondary Birkeland current flow, surely varies both with time and local time. Polarisation electric fields, are however observed during all the different phases of a substorm and for almost any local time, which suggests that the limitation mechanism is a relatively global and persistent feature during a substorm. During the substorm expansive phase, an effective limitation may occur locally, due to a plasma depletion caused by the disruption of the cross-tail current. This depleted plasma region will propagate westward (as discussed in Section 4) and
possibly at the ionospheric end, in the form of the Westward Travelling Surge, as suggested by Boström (1976).

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Figure Captions

Fig.1. Typical orientation of the ambient electric field ($\mathbf{E}_0$), the substorm-associated polarisation electric field ($\mathbf{E}_p$), and the ionospheric closure current ($\mathbf{J}_P$ or $\mathbf{J}_H$) at different local times in the auroral oval.

Fig.2. High-latitude equipotential patterns for different cases of secondary Birkeland current limitation (limitation factor $\beta$, see text for details). The thick line indicates zero potential, and the closed equipotential contours are separated by 5 kV with signs as indicated.

Fig.3. Same as Fig.2 except that only the Hall current contribution to the secondary Birkeland currents is limited by the factor $\beta$.

Fig.4. Equipotential pattern and associated Birkeland current distribution for the case $\beta = 0$ (no polarisation). Downward and upward currents are denoted by $\mathbf{x}$ and $\mathbf{O}$ respectively and the contours represent the 5%, 20% and 80% boundaries of the maximum current density.

Fig.5. Total equipotential pattern ($\Phi_T$), polarisation potential pattern ($\Phi_P$), Birkeland current ($j_B$) and polarisation charge density ($\sigma_C$) for the case $\beta = 0.5$. Positive and negative space charge is indicated by the open and shaded regions, respectively. The contours represent the 5%, 20% and 80% boundaries of the maximum space charge density. The Birkeland current representation is the same as in Fig.4.

Fig.6. Same as Fig.5 but for the case $\beta = 1.0$.

Fig.7. Same as Fig.5 but for the case $\beta = -0.5$. 
Fig. 8. Height-integrated Pedersen ($I_p$) and Hall ($I_H$) isoconductivity contours ($\Omega^{-1}$) produced by average particle precipitation and background ionization sources, for $K_p$ less than 3$_0$ (left) and greater than 3$_0$ (right). From Wallis and Budzinski (1981).

Fig. 9. Birkeland current distribution used by Yasuhara et al. (1983) to produce a clockwise rotation of the convection pattern.

Fig. 10. Equipotential pattern and Birkeland current distribution during moderate substorm activity (from Wolf and Spiro, 1984).

Fig. 11. Illustration of a mechanism suggested by Boström (1974) producing a westward propagation of an initial slot region formed in the magnetospheric tail by disruption of the cross-tail current. PD and PE indicate regions of plasma depletion and plasma enhancement, respectively.
Fig. 1
Fig. 2
Fig. 4
Fig. 5
Fig. 9
It is shown how the high-latitude convection pattern may be modified by substorm-enhanced polarisation electric fields. These are generated whenever the flow of those Birkeland currents which are associated with ionospheric conductivity gradients is limited. Such Birkeland currents are fed mainly by the enhanced Pedersen current in the evening and morning sectors of the auroral oval and by the enhanced Hall current around local midnight. As the current limitation increases, the ionospheric potential, represented here by a symmetric two-cell pattern, will rotate clockwise and deform, just as the associated Birkeland current distribution. The resulting patterns are shown to agree well with observations. A pronounced westward intrusion of the equipotential contours occurs in the auroral oval, and may be associated with the Westward Travelling Surge. This feature does not however require any assumed longitudinal conductivity gradients. Rather it falls out naturally when the limitation of the enhanced Pedersen current is taken into account.

Key words: Electric field, Polarisation, Birkeland current, Convection pattern, Current limitation, Polar cap potential drop, Westward Travelling Surge, Auroral substorm.