TRITA-EPP-81-03

AN APPENDIX TO THE PAPER "$T_e$ DETERMINATION IN LOW-DENSITY PLASMAS FROM THE HE I 3889 A AND 5016 A LINE INTENSITIES"

Nils Brenning

October 1981

Department of Plasma Physics
Royal Institute of Technology
S-100 44 Stockholm, Sweden
AN APPENDIX TO THE PAPER "$T_e$ DETERMINATION IN LOW-DENSITY PLASMAS
FROM THE HE I 3889 A AND 5016 A LINE INTENSITIES"

N. Brenning
Royal Institute of Technology, Department of Plasma Physics,
S-100 44 Stockholm, Sweden

Abstract
The use of the HeI 3889 A line for measurements of plasma electron temperature
is discussed. The reduction in line intensity due to absorption is calculated,
and a "useful range" of parameters is determined, within which the $T_e$ deter-
mination is uninfluenced by the effects of absorption.
An appendix to the paper "Te determination in low-density plasmas from the He I 3889 A and 5016 A line intensities"

Nils Brenning
Department of Plasma Physics, Royal Institute of Technology, Stockholm, Sweden

In a recent paper by the author (Brenning 1980, here referred to as paper I) the determination of plasma electron temperature from the absolute intensity of the He I 3889 A line was discussed. The calculations were restricted to the case when the plasma is optically thin against the 3889 A line radiation.

The requirement of optical thinness is here discussed further. A "useful range" of parameters is determined, within which the Te determination is uninfluenced by reabsorption of the 3889 A line. When one operates in the outskirts of this useful range, the temperature measurements become uncertain; this uncertainty is also calculated. The results are given in graphical form in Figure 1.

It is not possible to make a general calculation of reabsorption. One has to choose a geometry where the equation of radiative transfer can be solved, and also make some assumptions about the density of absorbers. The 3889 A (3P-2S) line is absorbed by helium atoms in the metastable 2S level. The population density of this level can vary considerably from experiment to experiment.

We consider the case when the absorption is the strongest possible with given values of helium density and electron temperature. This corresponds to the following assumptions about the metastable density n2S and the radiative transfer process: (a) n2S is the equilibrium population with respect to the ground state, (b) de-excitation of metastables due to collisions with neutrals can be disregarded, (c) diffusion of metastables out of the plasma is unimportant, (d) no absorbed radiation is re-emitted into the line of sight, and (e) there is no scattering of radiation into the line of sight. The absorption calculated from these assumptions represents an upper limit to the absorption in a real experiment.

The calculation of the intensity decrease in the escaping radiation is essentially straightforward. Details are given in the appendix. The density of the metastables can be calculated from the data in paper I; it can reach up to 1% of the total neutral helium density. The treatment of the imprisonment of a doppler-broadened line can be found in textbooks (e.g. McWhirter 1965). The end result of the calculations is that the reduction in line intensity can be written as a function of Te, THe and the integrated
helium density $\int n_{\text{He}} \, dL$ through the plasma. The consequences for the $T_e$ determination are then found by combining these results with the data in paper I, where the line strength as a function of $T_e$ is calculated. The result is given in Figure 1, which shows how the useful range for the diagnostic method is restricted by the effect of reabsorption.

Acknowledgements

I thank Dr I. Axnäs for stimulating discussions. This research has been financed by the Swedish Natural Science Research Council.
References


Griem H R 1964 Plasma Spectroscopy (McGraw-Hill)

Appendix: Calculations

The population density of the metastable level is related to the ground-state population by the rate equation

\[
\frac{\partial n_{2^3S}}{\partial t} = n_e n_{1^1S} S_{1^1S+2^3S} - n_e n_{2^3S} S_{2^3S+1^1S}
\]  
(1)

where \( S_{1^1S+2^3S} \) and \( S_{2^3S+1^1S} \) denote the sums of the rate coefficients for excitation by electron impact to and from the \( 2^3S \) level. They are given by Figure 5 of paper I. In equilibrium, the relative population density \( n_{2^3S}/n_{1^1S} \) is found from Eq. (1) to be

\[
\frac{n_{2^3S}}{n_{1^1S}} = \frac{S_{1^1S+2^3S}}{S_{2^3S+1^1S}}
\]  
(2)

This is a function only of the electron temperature. Numerical values are given in Table I.

<table>
<thead>
<tr>
<th>( kT_e (\text{eV}) )</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{2^3S}/n_{1^1S} )</td>
<td>5.49 \times 10^{-6}</td>
<td>1.30 \times 10^{-4}</td>
<td>5.56 \times 10^{-4}</td>
<td>1.22 \times 10^{-3}</td>
<td>2.78 \times 10^{-3}</td>
<td>5.88 \times 10^{-3}</td>
<td>7.41 \times 10^{-3}</td>
<td>8.7 \times 10^{-3}</td>
<td>9.1 \times 10^{-3}</td>
</tr>
</tbody>
</table>

Table I. The equilibrium population of metastables

The dominating broadening mechanism at the plasma densities considered here \( (n_e < 10^{19} \text{m}^{-3}) \) is doppler broadening. This gives the line profile, normalized to unity

\[
P(x) = \frac{1}{\sqrt{\pi}} \exp(-x^2).
\]  
(3)

\( x \) is the normalized distance from the wavelength center,

\[
x = (\nu - \nu_0)/\Delta \nu_D ,
\]  
(4)

where \( \Delta \nu_D \) is the doppler width

\[
\Delta \nu_D = \nu_0 \left( \frac{2kT}{mc^2} \right)^{3/2}.
\]  
(5)
When the velocity distribution of the absorbers and the emitters is the same, the optical depth at a frequency \( \nu \) (or distance \( x \) from the line center) is a function of the optical depth \( \tau \) at the line center:

\[
\tau(x) = \tau_0 \exp(-x^2). \tag{6}
\]

We here treat the case when scattering and re-emission of absorbed radiation can be neglected. The escaping fraction \( g(\nu) \) of the emitted radiation is then given by the solution to the equation of radiative transfer at the frequency \( \nu \):

\[
g(\nu) = (1 - \exp(-\tau))/\tau. \tag{7}
\]

The average decrease in intensity for the whole line is found by integrating \( g(\nu) \) across the line profile. Equations (3), (6) and (7) then give

\[
\bar{g} = \frac{1}{\sqrt{\pi} \tau_0} \int_0^\infty (1 - \exp(-\tau_0 \exp(-x^2))) \, dx. \tag{8}
\]

We call \( \bar{g} \) the escape factor for the line. It is the fraction of the radiation emitted in a certain direction that escapes absorption. \( \bar{g} \) is a function only of the optical depth, \( \tau_0 \), across the plasma at the center of the line. Some values are given in Table II.

<table>
<thead>
<tr>
<th>( \tau_0 )</th>
<th>0</th>
<th>0.1</th>
<th>0.25</th>
<th>0.50</th>
<th>1.0</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>30</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(\tau_0) )</td>
<td>1.0</td>
<td>0.966</td>
<td>0.917</td>
<td>0.845</td>
<td>0.725</td>
<td>0.631</td>
<td>0.556</td>
<td>0.447</td>
<td>0.372</td>
<td>0.319</td>
<td>0.256</td>
<td>0.256</td>
<td>0.256</td>
<td>0.256</td>
<td>0.256</td>
<td>0.256</td>
</tr>
</tbody>
</table>

Table II. The escape factor as a function of the optical depth \( \tau_0 \) at the center of the line

The optical depth at the center of the line is determined by the metastable density. The absorption coefficient \( K_A \) is (Griem, p.181):

\[
K_A(\omega) = 2\pi^2 r_0 c f_{mn} n_{2\Delta_S} P(\omega), \tag{9}
\]

where \( r_0 \) is the classical electron radius \( r_0 = 2.82 \cdot 10^{-15} \text{m} \), and \( f_{mn} \) is the absorption oscillator strength, 0.05705 for the 3889 \text{A} line. \( P(\omega) \) is the line
profile as a function of angular frequency, which can be found from equation (3). The optical depth at the center of the line is given by the integral along the line of sight through the plasma

$$\tau_0 = \int K_A(\omega) dL.$$  \hspace{1cm} (10)

With numerical values inserted, equations (1) and (10) yield

$$\tau_0 = 5.152 \times 10^{-16} (T_{He})^{-\frac{1}{2}} \int n_{2^{3}S} dL.$$ \hspace{1cm} (11)

This equation can be combined with the data in Tables I and II to give the escape factor as a function of $T_e$, $T_{He}$ and $\int n_{He} dL$, the integrated helium density (not metastable density) along the line of sight.

The last step in the calculations is to examine how this reduction in line intensity influences the $T_e$ determination. This calculation is best illustrated by an example. We consider the case when the line intensity, evaluated as if absorption could be neglected, yields a temperature of 10 eV. We want to determine the value of $\int n_{He} dL$, for which our measurement is uncertain with 10%. The true value of the temperature would then be 11 eV. From the data in paper I we find that a true temperature of 11 eV will be evaluated as 10 eV if the line intensity is reduced by a factor 0.89. Interpolation in Table II yields the corresponding optical depth $\tau_0 = 0.34$, and equation (11) gives the integrated metastable density $\int n_{2^{3}S} dL$. The integrated helium density, finally, is found by interpolation in Table I.
Figure 1. The useful range for the 3889 Å line.

The figure shows the combinations of $T_e$ and $\int n_{He} \, dL$, for which the 3889Å line can be used for $T_e$ determination. $T_e$ is here the measured temperature obtained from the line intensity with the effects of reabsorption disregarded. $\int n_{He} \, dL$ is the integrated helium density through the plasma along the line of sight to the spectrograph. The figure corresponds to a neutral helium temperature $T_{He} = 300^0K$. It can be used for other temperatures if $\int n_{He} \, dL$ is replaced by $(300/T_{He})^{3/2} \int n_{He} \, dL$. 
Royal Institute of Technology, Department of Plasma Physics,  
S-100 44 Stockholm, Sweden

AN APPENDIX TO THE PAPER "$T_e$ DETERMINATION IN LOW-DENSITY PLASMAS  
FROM THE HE I 3889 Å AND 5016 Å LINE INTENSITIES"

N. Brenning,  
October 1981, 8 pp. incl. ill., in English

The use of the He I 3889 Å line for measurements of plasma electron temperature  
is discussed. The reduction in line intensity due to absorption is calculated,  
and a "useful range" of parameters is determined, within which the $T_e$ deter-  
mination is uninfluenced by the effects of absorption.

Key words: Helium spectroscopy, Plasma diagnostics, Plasma spectroscopy