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MEASUREMENTS OF ELECTRON ENERGY
DISTRIBUTIONS IN FRONT OF AND
BEHIND A STATIONARY PLASMA SHEATH

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Abstract

Electron energy distributions have been measured in front of and behind a stationary plasma sheath in a low pressure mercury discharge. The sheath appears at a constriction of the discharge tube. The measurements have been done with a spheric probe, using the second-derivative method, and the results show that the energy distribution on the anode side of the sheath is a sum of a maxwellian part and an accelerated distribution. Near the sheath the accelerated electrons suffice to carry the discharge current, but far from it the current is carried by an anisotropy in the thermal part of the distribution function. A simple comparison is made with distribution functions calculated under different assumptions. The cross-sections for electron-neutral and Coulomb collisions are not sufficient to account for the damping of the accelerated population along the tube, suggesting that a plasma instability must be present.

Introduction

Electrostatic sheaths and double-peaked distributions have been discussed in connection with space physics (Block 1972) and also detected in laboratory discharges by e.g. Crawford and Freeston (1963), Wieseemann (1969, 1970), Babić and Torvén (1974), Andersson, Babić, Sandahl and Torvén (1969), Andersson (1970).

The purpose of this paper is to discuss some energy distributions that have been measured in a mercury discharge with a stationary constriction-sheath by Andersson (1970).

The current density in our discharge is two or three magnitudes higher than in the discharge tube of Crawford et al. (1963). It is important that stationary sheaths and normal positive columns are studied at high current densities, as this might cast light on some important phenomena which are more difficult to study experimentally, e.g. non-stationary sheaths and current chopping (discussed by e.g. Carlqvist 1972, Babić et al. 1974 and Torvén and Babić 1975).

Discharge tube

The discharge tube (constructed by M. Babić) is shown in fig. 1. At the constriction of the right side-tube a stationary sheath appears, which can be studied by means of a movable, spherical probe. The sheath is not plane, but more or less hemispherical.

The mercury vapour pressure in the cathode vessel is determined by the water temperature; however, the pressure may vary along the side-tube depending on the current (Sandahl 1971, Andersson et al. 1969).

Method

If a spherical probe is immersed in a (anisotropic) low pressure plasma with the energy distribution $\phi(E)$, and the

sheath around the probe is thin compared to the probe diameter, the current I to the probe is

$$I = \frac{eA}{4} \int_{eV_1}^{\infty} \phi(E) \sqrt{\frac{2E}{m}} \left(1 - \frac{eV_1}{E}\right) dE \quad (1)$$

where e is electron charge (without sign)

m electron mass

A area of probe ($= 4\pi r^2$)

$V_1 = V_0 - V =$ plasma potential - probe potential.

That it is so can be shown by regarding the distribution as a superposition of beams from different angles (fig. 2). The current to the probe will be the same, however, if all these beams are imagined to be parallel (fig. 3). The current in the latter case is easily calculated (if the sheath is thin), and is given by (1).

If (1) is differentiated twice with respect to V_1 , we get

$$\phi(eV_1) = \frac{4}{Ae^2} \sqrt{\frac{mV_1}{2e}} I'' \quad (2)$$

Thus, the electron energy distribution can be derived from the second derivative of the probe characteristic. This method is often called "Druyvesteyn's method", although already Langmuir and Mott-Smith (1926) discussed it for a thick sheath (Suits 1961).

The second derivative I'' is determined by an electronic method. A small AC-signal (1 kHz sine-wave) is superimposed on the probe voltage. Due to the nonlinearity of the probe curve, upper harmonics are present in the probe current. The amplitude of the second harmonic (2 kHz) is proportional to I'' . This is described in greater detail elsewhere (e.g. Andersson 1970, Branner et al. 1963). We have built an apparatus which automatically plots distribution functions according to this method.

Measurements

Distribution functions on both sides of the sheath at different pressures and currents have been measured and are shown in fig. 4-8. In fig. 9 plasma potential V_0 versus x (coordinate along the tube) is shown. As V_0 the potential at which $I''=0$ has been used. V_0 has been put equal to zero at $x = -5$ cm.

The electron density n_e has been determined in the following way. The shape of the distribution function determines the average velocity $\bar{v}$, as the energy axis is calibrated in volts. The current to the probe at plasma potential determines $n_e e \bar{v} / 4$ according to formula (1). This settles n_e for all values of x as n_e is proportional to the area under the distribution curve. Usually, $n_e e \bar{v} / 4$ has been determined at two or three values of x , and average values of n_e have been calculated.

In addition, the quantities $(n_e e \bar{v})_{\text{total}}$ for the whole distribution, and $(n_e e \bar{v})_{\text{beam}}$ for the accelerated electrons are given. The latter quantity has been obtained by extrapolating the thermal (low-energy) part of the distribution graphically and subtracting it from the whole distribution.

Before these measurements are discussed, some calculated distributions will be presented for comparison.

Calculated accelerated distributions

a. Half-maxwellian distribution, plane sheath.

Let us assume that we have a half-maxwellian velocity distribution $F_1(v)$ before a plane, stationary sheath (fig. 10):

$$F_1(v_x, v_y, v_z) = e^{-(v_x^2 + v_y^2 + v_z^2)} = e^{-v^2} \quad \text{for } v_x \geq 0$$

(in suitable units)

Since current is conserved, the distribution F_2 behind the sheath must fulfill the condition

$$v_x F_2(v_x, v_y, v_z) dv_x = v'_x F_1(v'_x, v_y, v_z) dv'_x$$

$$\text{with } (v'_x)^2 = v_x^2 - \frac{2}{m} E_0 \text{ and } v_x dv_x = v'_x dv'_x$$

($V_0 = \frac{E_0}{e}$ is the potential difference across the sheath).

Thus,

$$F_2(v_x, v_y, v_z) = e^{-v^2 + \frac{2}{m} E_0} \text{ for } v_x \geq \sqrt{\frac{2E_0}{m}}$$

The number of particles with speeds between v and $v + dv$ is then (fig. 10)

$$dN = f_2(v) dv = F_2(v) \cdot 2\pi v(v - \sqrt{\frac{2E_0}{m}}) dv$$

$$\text{Thus } f_2(v) = 2\pi v(v - \sqrt{\frac{2E_0}{m}}) e^{-v^2 + \frac{2}{m} E_0} \text{ for } v \geq \sqrt{\frac{2E_0}{m}}$$

If this is transformed to energy distributions $\phi(E)$, observing that $\phi(E) = \frac{dN}{dv} \frac{dv}{dE} = f(v) \frac{1}{\sqrt{2mE}}$ we get (in suitable units):

$$\begin{cases} \phi_1(E) = \sqrt{E} e^{-E} \\ \phi_{2,a}(E) = (\sqrt{E} - \sqrt{E_0}) e^{E_0 - E} \text{ for } E \geq E_0 \end{cases} \quad (3)$$

where $\phi_1(E)$ is the assumed energy distribution before the sheath and $\phi_{2,a}(E)$ is the resulting energy distribution after the sheath.

This is shown in fig. 11(a) for different values of E_0 .

b. Hemispherical sheath, reflecting walls.

To do a rigorous calculation which takes into account the curvature of the sheath and the reflecting walls seems to be

difficult. However, the importance of these factors can be estimated in the following way.

First, let us assume that we have a beam with the energy distribution $\phi_1(E)$ before a plane sheath (fig. 11, b). To conserve current, the distribution behind the sheath must be

$$\phi_2(E) = \sqrt{\frac{E-E_0}{E}} \phi_1(E-E_0) \text{ for } E \geq E_0$$

If $\phi_1 = \sqrt{E} e^{-E}$ we get $\phi_{2,b} = \frac{E-E_0}{\sqrt{E}} e^{E_0-E}$ (4)

This is shown in the diagram (fig. 11, b).

If we now have the situation in fig. 11, c with $\phi_1 = \sqrt{E} \exp(-E)$ radially directed against a hemispheric sheath and a beam after the sheath (somewhat unrealistic), we must instead have $\phi_{2,c} = 2 \cdot \phi_{2,b}$ to conserve current.

Finally, if we assume that ϕ_2 is half-isotropic (fig. 11, d) after the sheath, we must have $\phi_{2,d} = 2\phi_{2,c}$ to conserve current. A case between c and d, but with continuous angle-dependence seems to be rather realistic in a plasma.

Discussion

A first glance at fig. 4-9 reveals that the distribution looks rather maxwellian on the cathode side of the sheath. On the anode side, however, it is a sum of a thermal population and a population which has been accelerated through the sheath from the cathode side. The accelerated population (the "beam") is further accelerated somewhat in the narrow tube due to the electric field in the plasma and also damped.

In fig. 8 the distribution after the sheath is somewhat surprising. In addition to the expected population at 20 volts there is an extra population at lower energies, which seems to have been accelerated from the anode in the reversed field according to fig. 9.

If the amplitude of the distribution just before the sheath is compared with that of the accelerated electrons after the sheath (at a low x -value), it is clear especially in fig. 5-7 that an intermediate case between $\underline{c}$ and $\underline{d}$ in fig. 11 is a rather good description of reality, whereas case $\underline{a}$ is out of question. It is also interesting to study the quantity $(n_e \bar{v})_{\text{total}}$ just before the sheath. If this quantity equals half the current density i_4 in the narrow tube, which it often does, and the sheath is hemispheric, i.e. has an area which is twice the cross-section of the narrow tube, the distribution before the sheath must be radially directed as in fig. 11 ($\underline{c}$, $\underline{d}$) to be able to carry the current.

In fig. 5 $(n_e \bar{v})_{\text{total}}$ is even less, suggesting that the sheath has expanded further. However, there is no sharp transition between plasma and sheath, and there are inaccuracies in probe measurements, so this should not be pushed to far. This shows, however, that the distribution before the sheath is strongly anisotropic, in contrast to the maxwellian distributions which are applicable at low current densities (Crawford et al. 1963).

Immediately after the sheath, $(n_e \bar{v})_{\text{beam}}$ is equal to i_4 or greater (except in fig. 4 and 8), again suggesting that a case between $\underline{c}$ and $\underline{d}$ in fig. 11 may be applicable. Far from the sheath $(n_e \bar{v})_{\text{beam}}$ is not sufficient to carry the discharge current, however. This implies that there must be an anisotropy in the thermal part of the distribution, and therefore measurements with plane probes to study distribution functions of velocity components are being planned.

In the experiment of Crawford et al. (1963) the damping of the "beam" along the tube is in reasonable agreement (within a factor of two) with the mean free path (m.f.p.) for elastic electron-neutral collisions. This is the case also in our experiment; the m.f.p. for electron-neutral collisions ranges from about 3 cm in fig. 4 to about 30 cm in fig. 8 when the energy-dependence of the cross-sections is taken into account

(Brown 1966). It is, however, difficult to understand how elastic collisions can change the energy distribution. One possibility might be that axially moving electrons are scattered to the wall and, if they have sufficient energy, penetrate the wall sheath and recombine with ions. This requires that a sufficient number of atoms are ionized and impinge on the wall; however, the cross-section for ionization is too small to account for this (Brown 1966). The cross-section for excitation (Hasted 1964) is also too small. The m.f.p. for Coulomb collisions (Alfvén and Fálthammar 1963) is at least one order of magnitude longer than the m.f.p. for electron-neutral collisions.

It seems that some plasma instability must be responsible for the major part of the damping of the "beam" in our experiment. Experiments which, it is hoped, will shed light on these problems are being planned.

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Fig. 1

Discharge tube.

Cross section of narrow tube: 0.95 cm^2

Probe area: 5.8 mm^2

Fig. 2

Fig. 3

x	-5	-1	0	2	7	cm
V_o	0	2.5	5.6	6.8	9.8	volts
n_e	2.4	3.6	7.8	6.6	8.4	$\cdot 10^{16} \text{ m}^{-3}$
$(n_e \bar{v})_{\text{total}}$	0.4	0.6	1.7	1.4	1.5	A/cm^2
$(n_e \bar{v})_{\text{beam}}$	-	-	0.8	0.4	0.15	A/cm^2 (approx.)

$t = 30^\circ\text{C}$, $p = 0.38 \text{ N/m}^2$ in cathode vessel.

$I_4 = 1.0 \text{ A}$ ($i_4 = 1.05 \text{ A/cm}^2$ in narrow tube)

$\bar{v}$ is the average value of the magnitude of velocity.

Fig. 4
Electron energy distributions

x	-3	-1	+1	+4	+7	cm
V_0	1.9	2.5	9.0	10.8	12.0	volts
n_e	5.2	3.9	17	17.7	18.3	$\cdot 10^{16} \text{ m}^{-3}$
$(n_e \bar{v})_{\text{total}}$	0.9	0.7	4.2	3.9	3.9	A/cm^2
$(n_e \bar{v})_{\text{beam}}$	-	-	2.0	1.0	0.6	A/cm^2 (approx.)

$t = 20^\circ\text{C}$, $p = 0.16 \text{ N/m}^2$ in cathode vessel. $I_4 = 2.0 \text{ A}$ ($i_4 = 2.1 \text{ A/cm}^2$ in narrow tube)

$\bar{v}$ is the average value of the magnitude of velocity.

Fig. 5

Electron energy distributions

x	-3	+1	+4	+7	cm
V_0	2.1	11.9	12.9	13.2	volts
n_e	4.8	18.7	21.7	22.4	$\cdot 10^{16} \text{ m}^{-3}$
$(n_e \bar{v})_{\text{total}}$	0.9	4.6	5.2	5.1	A/cm^2
$(n_e \bar{v})_{\text{beam}}$	-	2.4	2.0	1.6	A/cm^2 (approx.)

$t = 15.1 \text{ } ^\circ\text{C}$, $p = 0.11 \text{ N/m}^2$ in cathode vessel. $I_u = 2.0 \text{ A}$ ($i_u = 2.1 \text{ A/cm}^2$ in narrow tube)
 $\bar{v}$ is the average value of the magnitude of velocity.

Fig. 6

x	-2	0	cm
V_0	0	13.5	volts
n_e	11	32	$\cdot 10^{16} \text{ m}^{-3}$
$(n_e \bar{v})_{\text{total}}$	2.2	10.4	A/cm^2
$(n_e \bar{v})_{\text{beam}}$	-	7	A/cm^2 (approx.)

$t = 15.1^\circ\text{C}$, $p = 0.11 \text{ N/m}^2$ in cathode vessel. $I_4 = 4.0 \text{ A}$ ($i_4 = 4.2 \text{ A/cm}^2$ in narrow tube)
 $\bar{v}$ is the average value of the magnitude of velocity.

(This series was discontinued because the probe began to glow)

Fig. 7

x	-2	+2	+4	cm
V_0	1.0	1.5	11.5	volts
n_e	3.6	8.3	7.7	$\cdot 10^{16} \text{ m}^{-3}$
$(n_e \bar{v})_{\text{total}}$	0.8	2.6	2.1	A/cm^2
$(n_e \bar{v})_{\text{beam}}^x$	-	0.9	0.7	A/cm^2 (approx.)

x due to electrons from cathode side of sheath. $\bar{v}$ is the average value of the magnitude of velocity.

$t = 10^\circ\text{C}$, $p = 0.07 \text{ N/m}^2$ in cathode vessel. $I_4 = 1.0 \text{ A}$ ($i_4 = 1.05 \text{ A/cm}^2$ in narrow tube)

Fig. 8

Electron energy distributions

Curve Number	1	2	3	4	5
$P \text{ (N/m}^2\text{)}$	0.38	0.16	0.11	0.11	0.07
$I_4 \text{ (A)}$	1.0	2.0	2.0	4.0	1.0
See fig.	4	5	6	7	8

Fig. 9

Potential variation along the tube

Fig. 10

A half-maxwellian velocity distribution is accelerated through a potential step.

Fig. 11

Results of accelerating an energy distribution $\phi_1 = \sqrt{E} e^{-E}$ through different potential steps, calculated under different assumptions.

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Key words Low pressure arc, Sheath, Double layer, Electrical probes, Druyvesteyn's method, Second-derivative method, Electron energy distribution