Gas Turbine Plant Modeling for Dynamic Simulation

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October, 2011

Master’s Thesis

KTH Industrial Engineering and Management

Master of Science Thesis
KTH School of Industrial Engineering and Management
Department of Energy Technology EGI-2011-MJ211X
Division of Heat and Power
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Master of Science Thesis EGI 2011:MJ211X

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Approved Date 29/03/2012
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Abstract

Gas turbines have become effective in industrial applications for electric and thermal energy production partly due to their quick response to load variations. A gas turbine power plant is a complex assembly of a variety of components that are designed on the basis of aero thermodynamic laws.

This thesis work presents model development of a single-shaft gas turbine plant cycle that can operate at wide range of load settings in complete dynamic GTP simulator. The modeling and simulation has been done in Dymola 7.3, based on the Modelica programming language. The gas turbine plant model is developed on component-oriented basis. This means that the model is built up by smaller model classes. With this modeling approach, the models become flexible and user-friendly for different plant operational modes.

The component models of the main steady-state compressor and turbine stages have been integrated with gas plenum models for capturing the performance dynamics of the gas turbine power plant. The method of assembly used for gas turbine plant integration is based on models of the components from an engineering process scheme.

In order to obtain an accurate description of the gas-turbine working principle, each component is described by a non-linear set of both algebraic and first-order differential equations. The thesis project provides descriptions of the mathematical equations used for component modeling and simulation. A complete dynamic simulation of a gas-turbine plant has been performed by connecting the complete plant model with PI controllers for both design and off-design operating modes.

Furthermore, turbine blade cooling has been studied to evaluate the changes in power output. This has been done to compare and analyze the blade cooling effect.
Acknowledgements

My utmost gratitude goes to my thesis advisor, Dr Veronica Olesen, whose knowledge and logical way of thinking have been of great value for me. Her understanding, encouragement and personal guidance have provided a good basis for the present thesis.

I believe that one of the main achievements was working with Dr. Veronica and Mr. Andreas J.Johanson and gaining their trust and cooperation.

As a result, my thesis work at Solvina became smooth and rewarding. Joining Solvina for the thesis work was not only a turning point in my life, but also a wonderful experience.

My appreciation goes to examiner Associate Prof. Damian Vogt for the peer review of the thesis project that enables me see detail aspects.

My deepest gratitude goes to my family, my wife and friends in Stockholm for their unflagging love and support; you have always been a constant source of encouragement during my graduate study.

Last but not least, thanks be to the Almighty God, who made everything possible.
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Nomenclature

Abbreviations

GTP   Gas Turbine Power Plant
GT-Unit Gas Turbine Unit
VIGV  Variable Inlet Guide Vane
CSTR  Continuously stirred tank reactor
LHV   The lower heating value of the fuel
DAE   Differential Algebraic Equation
BDF   Backward Differentiation Formulas
P     Proportional
PI    Proportional and Integrative
PID   Proportional, Integrative and Derivative

Symbols

\( A_s \)   Heat surface area [m\(^2\)]
\( A \)   fluid flow area [m\(^2\)]
\( E \)   Internal energy [J]
\( U \)   Internal energy in combustion chamber [J]
\( I \)   Rothalpy [J/Kg]
\( M \)   Moment [Nm]
\( Q \)   Heat energy [J]
\( W \)   Specific energy [J]
\( g \)   Gravitational acceleration [m/s\(^2\)]
\( h \)   Specific Enthalpy [J/kg]
\( \dot{m} \) Mass flow rate [kg/s]
\( \dot{n}_c \) The rated rotational speed
\( \dot{P} \) Pressure [Pa]
\( r \)   Radius [m]
\( t \)   Time [s]
\( c \)   Absolute Velocity [m/s]
\( u \)   Tangential velocity [m/s]
\( w \)   Relative velocity [m/s]
\( Z \)   Height [m]
\( R \)   Molar gas constant [J/kg K]
\( T \)   Temperature [K]
\( h_{mL} \) Mechanical specific enthalpy loss due to profile loss [m]
\( h_{thL} \) Thermal heat loss from GTP
\( W \)   Work done by the system on its surroundings [J/s]
\( D \)   Diameter [m]
\( L \)   Length [m]
\( C_{pg} \) The specific heat capacity of the air flow
\( C_t \) Stodola's turbine constant
\( K \)   the discharge coefficient, depends on turbine design considered
\( P_{b1} \) air bleeding pressure from compressor exit [Pa]
\( V_{cc} \) Volume [m\(^3\)]
\( Q \)   Heat transferred to the system in a given time [J/s]
\( T_{b1} \) air bleeding temperature from compressor exit [deg C]
**Greek Letters**

- The total thermal efficiency of the gas turbine plant
- Nominal efficiency
- Isentropic efficiency
- Specific heat ratio
- Compressor flow coefficient
- Turbine flow coefficient
- Pressure loss coefficient
- Density \([\text{kg/m}^3]\)
- Rotational speed \([\text{rad/s}]\)

**Subscripts**

- Totals
- Inlet Stator
- Outlet Stator/Rotor Inlet
- Outlet rotor
- Nominal
- radial component
- Axial component
- Tangential Component
- Denoting any point in the expansion
- Thermal component
- Combustion chamber
1. Introduction

The thesis project has focused on dynamic modeling of the compressor and turbine components in a constant speed single shaft gas turbine power plant, GTP. The models have been developed thermo dynamically on zero-dimensional basis to predict the plant dynamic behavior both in design and off-design operations. All models have been made in the object oriented simulation software Dymola, based on the Modelica language. Mechanisms of dynamic modeling have been implemented to contribute for more simplified and effective gas turbine plant integration and simulation. The issues of predicting the plant off-design performance have also been considered at a component modeling level.

For dynamic simulation of the complete gas turbine set, an existing fixed step combustor model has been used in combination with the dynamic compressor and turbine models. Fluid-specific functions and routines of the Modelica fluid media package have been used for the GTP modeling and simulation in order to determine the plant physical properties.

The dynamic gas turbine plant model has provided main operational characteristics that ensure the ability to deal with plants having large variations in the operating parameters.

The effects of turbine blade cooling with air injection from compressor exit have also been investigated in order to evaluate its influence on the GTP power output.

1.1. Background

With a dynamic simulator of a power plant, any changes in the power generating system can be tested without disruption and downtime in power production. Virtual performance prediction of a plant system for different operational conditions also helps analyze the performance variation before actual system changes are implemented. Dynamic model simulators can also be used as a tool for operator training.

At Solvina, dynamic simulators are being developed for entire power plants, including the process cycle, electrical systems and control systems. The thesis project has been designed to take part in the development of the next dynamic gas turbine simulator. Accordingly, the project has focused on development of a gas turbine power plant model for dynamic simulation based on dynamic models of compressor and turbine.

1.2. Objective

The focus of this master thesis is on modeling gas turbine process cycle for dynamic GTP simulation. The gas generating process cycle is later connected with power generator in the electrical systems.
The main goal has been developing complete compressor and turbine models that capture dynamic/transient effects in dynamic gas turbine plant simulation.

The models had to be compatible with existing component models in the fluid package with the goal of capturing the essential dynamic behavior at different operating modes. The models also had to be representative for compressors and turbines of different sizes and produced by different manufacturers.

The models should be developed on component oriented basis and be able to be reused in new gas turbine plant model simulations.

The compressor and turbine models should be easily integrated for developing dynamic gas turbine plant simulators that can interact with power generator in electrical system.

1.3. Task Description

The task to develop the gas turbine plant for dynamic simulator can be divided into the following subtasks:

Literature survey on GTP modeling approaches. It is concerning reliable performance correction equations at component modeling level for GTP part load operation.

Development of the quasi-steady compressor and turbine models to express and analyze relevant performance dynamics.

Integrate a complete gas turbine unit using the compressor, combustion chamber, turbine, and input and boundary conditions to simulate and test the model to verify with the existing gas turbine unit at Solvina.

Formulate input and output interfacing signals on the complete gas turbine set for PI controllers for dynamic GTP simulation.

Finally evaluate the performance dynamics in the gas turbine process cycle for dynamic simulation under step in loading perturbation.

1.4. Limitations

The dynamic models of compressor and turbine are developed based on constant rotor speed for part load performance corrections.

The gas turbine plant simulation has involved implementation of simple PI controllers to test and reveal the performance dynamics of the plant gas cycle under different load adjustments.

No advanced control strategies have been investigated or implemented in this project for further integration with power generator in electrical system.
2. Theory

2.1. Gas Turbine Power Plants

The interest in gas turbine has been recognized over a century and a half. Aegidius Elling, a Norwegian researcher who is known as the father of the gas turbine, built the first successful gas turbine with excess power needed to run its own components. His first gas turbine patent was rewarded in 1884. In 1903 he completed the first gas turbine construction that produced net power output of 8 kW using rotary compressors and turbines.

He further developed the concept, and by 1912 he had developed a gas turbine system with separate turbine unit and compressor in series. In the invention, one of the main challenges was to find materials that could withstand the high turbine inlet temperature for high output power generation. His 1903 turbine could withstand inlet temperatures up to 400° Celsius.

Nevertheless, the history of the gas turbine as a feasible energy conversion device began with Frank Whittle’s patent award on the jet engine in 1930 and his static test of a jet engine in 1937. By the time 1930’s, gas turbines were largely the development of Brown Boveri and Company in Switzerland and intended to be an offshoot of their development of the Velox boilers. He further realized that the setup of the compressor, combustor, and turbine for a effective gas turbine could be turned to power production. Later in 1939, the Brown Boveri company introduced a 4 MW gas-turbine-driven electrical power system with a turbine inlet temperature of approximately 1020°F in Neuchatel, Switzerland. In 1942 Brown Boveri installed a 2200 horsepower gas turbine on a locomotive for the Swiss Railway Service. Within 10 years there were 43 manufactures producing gas turbines of a variety of designs and being placed in a variety of services. (Weston, 1992)

The process of fitting gas turbines to stationary power plants was less rapid even though it has same working principle as air birthing jet engines. However, applications of gas turbines have grown at a rapid pace as research and development produces performance and reliability increases and economic benefits (Weston, 1992).

A gas turbine plant is a heat engine that converts chemical energy from the fuel into heat energy, which is converted into mechanical and/or electrical energy. This section focuses on the main parts that integrate the plant cycle.

A simple gas turbine power plant consists of three main blocks; a compressor, a combustor, and a gas turbine. The compressor and the gas turbine can be mounted on the same shaft. The compressor unit is either centrifugal or of axial flow type. A gas turbine cycle operates on the principle of the Brayton cycle where compressed air is mixed with fuel, and burned under constant pressure conditions (Weston, 1992). The resulting hot gas is allowed to expand through a turbine to perform work.
The open gas turbine cycle represents a basic gas turbine plant where the working fluid does not circulate through the system as a true cycle.

The turbine inlet temperature is usually limited by thermal resistance of the turbine blade material. For that special metals or ceramics are usually selected for their ability to withstand both high stresses at elevated temperature and effects of erosion and corrosion caused by undesirable components of the fuel. In a typical gas-turbine power plant, approximately two thirds of the work is spent compressing the air; the rest is available for producing mechanical drive or electricity generation (Saravanamuttoo, Cohen, Rogers, & Straznicky, 2009).

In power generation, gas turbine plants have considerable merits with certain limitations over steam turbine power plants. Among the main advantages, cheap plant installation, lesser land space needs, higher turbine inlet temperature at relatively low pressure and faster power generation start up are noticeable. On the other hand, the plant fuel sensitivity and need of high quality fuels as well as its efficiency restriction due to its high exhaust temperature are the considerable limitations that the plant reveals.

2.2. Basic Theory of Turbo-Machines

In general terms, fluid machines that transfer energy between a rotor and a fluid are named as turbo machines. This machine category mainly includes turbines and compressors. Turbines extract energy from a fluid to a rotor whereas compressors absorb energy from a rotor to a fluid. All types of turbo machines obey basic relationships of Newton’s second Law of motion and Euler’s energy equation for fluids.

In turbo machines, thermodynamic and kinetic properties in a stage are defined by their respective velocities to maintain conservation of mass, energy and momentum.
2.2.1. Fluid Flow Phenomena

For brief elaboration of rotor and fluid flow phenomena of a turbo machine, compressor rotor-stator stage velocity diagram denotations and conventions are schematized as shown in Figure 2.

Figure 2 A) Schematic presentation of compressor with inlet (1) and outlet (2); B) Compressor stage denotations illustrated by 1( rotor inlet), 2( rotor outlet) and 3( stator outlet); C) The stage velocity triangles at rotor inlet and outlet

The Figure 2C in particular, illustrates the fluid velocity distributions at stage inlet and outlet which determine the energy transfer rate over the positions. The velocity distributions are plotted in form of velocity triangles. These are constructed by the tangential rotor velocity \( u \), the relative fluid velocity to the rotor \( w \) and the absolute fluid velocity \( c \) to stationary frame of reference. The velocity triangles on the compression and expansion stage lines are dependent on the aerodynamic blade shape, configured by the blade angle, radius, length and thickness. With knowledge of the velocity triangle, extracted or absorbed energy by a turbo machine can be determined. This is done through determination of the total enthalpy change between inlet and outlet of expansion or compression gas path.
To show the inter-dependence of enthalpy and velocity triangles of a turbo machine stage, Figure 2C can be redrawn. In Figure 3 the compressor velocity triangles at rotor inlet and outlet from Figure 2C are redrawn as an enthalpy-entropy chart. The overall phenomena of total and static enthalpy change together with the fluid and rotor velocity are thus illustrated.

Figure 3: Enthalpy versus entropy air compression chart

In the chart, total and static pressures and enthalpies are plotted for conditions at rotor inlet, rotor outlet and stator outlet. The total pressure and enthalpy at each position is the sum of their static and kinetic values.

Where

- "1" determines the static rotor inlet pressure, $P_1$ and enthalpy, $h_1$
- "01" determines the total rotor inlet pressure, $P_{01}$ and enthalpy, $h_{01}$
- "2" determines the static rotor outlet pressure, $P_2$ and enthalpy, $h_2$
- "02" determines the total rotor outlet pressure, $P_{02}$ and enthalpy, $h_{02}$
- "3" determines the static stator outlet pressure, $P_3$ and enthalpy, $h_3$
- "03" determines the total stator outlet pressure, $P_{03}$ and enthalpy, $h_{03}$

The absolute kinetic enthalpies are then determined from the total and static enthalpy differences at each positions on the chart. These are represented by $\frac{c_1^2}{2}$, $\frac{c_2^2}{2}$, $\frac{c_3^2}{2}$ at the...
respective stage positions. The relative kinetic specific enthalpies are represented by $\frac{w_2^2}{2}$, $\frac{w_3^2}{2}$ and the tangential by $\frac{u_2^2}{2}$, $\frac{u_3^2}{2}$ at the rotor inlet and outlet respectively.

Based on the velocity and pressure distribution on a stage, conservation of mass, energy and momentum are combined for determination of the energy rate absorbed or extracted by the stage.

2.2.2. Balance Equations

Conservation of mass is maintained by taking the sum of mass flow rates over all boundaries equal to the change in mass in the control volume

$$\sum_i \dot{m} = \frac{\partial m}{\partial t}$$  \[1\]

In a steady-state process, mass in the turbo machine is constant over time. Thus,

$$\sum_i \dot{m} = 0$$  \[2\]

Conservation of mass over a turbo machine control volume can be written as inflow (1) and outflow (2) in equation [3].

$$\dot{m} = \rho_1 C_{x1} A_1 = \rho_2 C_{x2} A_2$$  \[3\]

The first law of thermodynamics states that energy must be preserved. In equation [4], a change in internal energy is given by the difference between the heat supplied to the system (Q) and the work performed by the system (W).

$$d\dot{U} = dQ - dW$$  \[4\]

This gives:

$$d\dot{U} = \dot{m}(dh_0 + gdZ) = \dot{Q} - \dot{W}$$  \[5\]

Where $dh_0$ is the change in total enthalpy and the term $gdZ$ represents the change in specific potential energy. Except for turbo machines that drive incompressible fluids, the latter can be neglected. Furthermore, for process simplification, turbo machine compression and expansion processes can often be assumed as adiabatic. This gives the conservation of energy for a steady state process as:

$$\dot{W} = \dot{m}(h_{01} - h_{02})$$  \[6\]

In turbo machines, it is worth to note that work producing machines, turbines have $h_{01}>h_{02}$, thus resulting in positive work output $\dot{W}>0$. In contrast, work absorbing machines, compressors have $h_{01}<h_{02}$ resulting in negative work output $\dot{W} < 0$. 
The Euler's turbo machine equation is developed based on conservation of energy and momentum. The mechanical rate of work equals the product of moment and rotational speed.

\[
\dot{W} = M_z\omega
\]  

[7]

Thus the conservation of energy rate can be related to the conservation of momentum as follows,

\[
\dot{m}(h_{o1} - h_{o2}) = \dot{m}(r_1c_{\theta 1} - r_2c_{\theta 2})\omega
\]  

[8]

Substituting \(\omega r_1\) by \(u_1\), \(\omega r_2\) by \(u_2\) and eliminating \(\dot{m}\) yields

\[
(h_{o1} - h_{o2}) = (u_1c_{\theta 1} - u_2c_{\theta 2})
\]  

[9]

Then, equation [9] is referred to as Euler’s turbo-machine equation. It states that any change in total enthalpy is equivalent to a change in tangential flow speed \((C_\theta)\) or tangential rotor speed \((u)\).

Reformulating [9] leads to a fundamental aspect of turbo-machine thermodynamics, called rothalpy.

\[
(h_{o1} - u_1c_{\theta 1}) = (h_{o2} - u_2c_{\theta 2}) = I
\]  

[10]

The rothalpy denoted as \(I\) in [10] is a function that remains constant throughout a rotating machine (rotor) while the total enthalpy \((h + \frac{c^2}{2})\) remains constant in a non-rotating machine (stator).

The general notation of rothalpy is

\[
I = h + \frac{c^2}{2} - uc_\theta
\]  

[11]

Generally, there are three facts of compressor and turbine operation conditions that hold true during compression and expansion on a stage (Dixon, 1998).

These are stated as follows:

- The static conditions at stator outlet and rotor inlet are equal.
- Polytropic compression and expansion line connect static conditions.
- Total enthalpy in a stator and rothalpy in a rotor maintain constant.

2.3. Compressor

A compressor is a main section of a gas turbine cycle and compresses air for the combustion. A compressor consists of rotor and stator assemblies that are mounted between bearings in a casing. The compressor is a multi-stage unit, where the pressure is increased by each stage. Each stage consists of one vertical layer of
rotating blades, and stator vanes. The stator vanes decrease the air velocity and turn it into static pressure. Besides, the stators lead the airflow at a correct angle to the next stage of rotor blades. Sealing between the stages prevent the air from leaking. Owing to the decreasing pattern of the cross section area of the airflow from inlet to outlet, the axial velocity remains constant as the volume decreases during the compression.

At start of the compressor operation, ambient air is taken through an inlet duct and passes a filter before reaching the compressor. The importance of the filter is to prevent objects from entering the compressor and minimize erosion and corrosion. Otherwise, entry of unwanted object creates rotor blade twist that possibly turns the compressor to break down due to minimum air flow intake. Such effects can also occur during compressor start-up and stop operations when the volume air flow is low at high compression pressure ratio. These phenomena are technically named as compressor surge or stall effect that turns the compressor to oscillate.

In principle, surging is when the airflow through the whole compressor is stopped; stall effect is when only some stages are affected by flow break up. As the result, the oscillation due to stalling can damage the compressor since it creates stress on the blades.

To alleviate the consequence of surging, variable guide vanes together with bleed valves are used to optimize for full load operating mode. The variable inlet guide vanes are airflow guiders and adjustable to the rotor inlet blade angle for flow control.

The term variable inlet guide vane (VIGV) refers specifically to the row of variable vanes at the entry of a compressor. The function of such VIGV’s is to improve the aerodynamic stability of the compressor when it is operating at relatively low rotational speed or air mass flow rate at off-design (Saravanamutto, Cohen, Rogers, & Straznicky, 2009).

At low speed and mass flow conditions, the variable vanes are in a near closed position, directing and turning the airflow in the direction of rotation of the rotor blades immediately downstream. This reduces the fluid inlet angle at the entry to the blades and the tendency of them to stall as well. As the rotational speed and mass flow of the compressor increases with increasing engine power, the vanes are moved progressively and in harmony towards an optimum open position.
When gas turbine power plants are operating at constant rotor speed, the variable inlet guide vanes are deviating to be positioned at optimum angle of incidence to the rotor inlet. By this, the rotor tangential speed \( (u) \) is maintained constant and the aerodynamic stability of the compressor at different part load operation is preserved. The compressor flow coefficient is described as ratio of axial air flow velocity and the tangential rotor velocity.

\[
\varphi_c = \frac{c_x}{u}
\]  

From the conservation of energy, an equation can be derived that describes the specific work required to achieve a certain pressure ratio over the compressor at a given temperature.

There are some properties of a compressor that cannot be easily calculated analytically, e.g. the isentropic efficiency and the mass flow through the compressor. These must instead come from measured data, given in the form of a compressor map. In the map, the mass flow and efficiency are listed for different values of speed and pressure ratio. In order to reduce the number of variables needed to represent the map, the non-dimensional variables pressure ratio, \( p_{ic} \) corrected speed \( \dot{n}_{corr} \) and corrected mass flow \( \dot{m}_{corr} \) are used. The variables are normalized with the inlet temperature and inlet/outlet pressures during the experiments (Saravanamuttoo, Cohen, Rogers, & Straznicky, 2009).

\[
p_{ic} = \frac{p_{out}}{p_{in}}
\]
For a given pressure ratio and inlet temperature, the compressor model should give a unique mass flow using data from the map. Figure 5 illustrates a typical compressor map that determines the relation of compression pressure ratio and corrected air mass flow at different speed.

\[ m_{\text{corr}} = \frac{m_{\text{flowair}} \sqrt{T_{\text{in}}}}{P_{\text{in}}} \]  

\[ \dot{n}_{\text{corr}} = \frac{\dot{n}}{\sqrt{T_{\text{in}}}} \]  

The curved lines on the main part of the map, in Figure 5 are the constant rotational speed lines. These slightly curved lines are not distributed linearly with flow rate. It is because of the fact that the compressor is fitted with variable stators, which open progressively as speed increases, causing an exaggerated increase in flow in the medium to high speed region. At low speed, the variable guide vanes are locked, causing a more linear relationship between speed and airflow.

Furthermore, beyond design flow, the speed lines close up rapidly due to choking which is the effect of maximum air flow achieved by the compressor. After choke, any further increase in speed will generate no further increase in airflow as it is shown in Figure 5. Thus, the compressor working line lays between the surge and choke line adjusted by optimum positioning of VIGV so as to avoid any outrageous aerodynamic instability on a given constant compressor speed line.

The continuous ellipsoid speed curves can be fitted and parameterized to the different speed curves so that the whole map can be continuously represented.
For each speed curve, one set of these parameters are calculated to make the model continuous in respect to speed based on these discrete sets of parameters. The parameters are fitted as polynomial functions of speed.

Figure 6 Ellipsoid curve for one particular speed of rotation

The curve in Figure 6 describes the curve for one speed and can be approximated as an ellipsoid curve and represented with an ellipsoid equation which relates the mass flow rate and pressure ratio in the constant speed line (Gustafsson, 1998). This is simplified as [16]

\[
\dot{m}_{\text{flowair}} \sqrt{\frac{T_1}{P_1}} = C_c \left(1 - \frac{1}{P_{i,c}^2}\right)^{0.5}
\]

[16]

Where

- \( C_c \) = compressor machine constant which depends on the inlet geometry and flow coefficient.
- \( T_1 \) = Ambient temperature at compressor inlet
- \( P_1 \) = Ambient pressure at compressor inlet
- \( P_{i,c} \) = The compression pressure ratio (\( \frac{P_{\text{out}}}{P_{\text{in}}} \))
- \( \dot{m}_{\text{flowair}} \) = the air inlet mass flow rate

For cases of isentropic compression, the compressor discharge temperature can be calculated as follows

\[
T_{\text{out, is}} = T_{\text{in}} \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right)^{(\gamma_c - 1)/\gamma_c}
\]

[17]
From the steady-state energy flow equation, [6] the compressor specific work \( W_c \) can be written as

\[
W_c = \frac{C_{pg}(T_{in} - T_{out,ls})}{\eta_{is,c}} \tag{18}
\]

Where \( \eta_{is,c} \) = the compressor isentropic compression efficiency
\( C_{pg} \) = the specific heat capacity of the air flow
\( x_c \) = the compression specific heat capacity ratio
\( W_c \) = The specific compressor power absorbed (negative work rate)

The compressor power intake is then determined by multiplying the air mass flow rate through the compressor with \( W_c \).

2.4. Combustion Chamber

The combustion chamber can be considered as a continuously stirred tank reactor (CSTR) and is characterized by a constant volume capacity. It is a pure energy accumulator where no unsteady effects of pressure drop due to mass accumulation are taken into account. With these assumptions the physical and chemical characteristics are the same at each physical location within the chamber. Hence, the same values of pressure and temperature at all points of the combustor correspond to pressure and temperature at the outlet section. The behavior of the combustion chamber is basically described by the following equations under no viscous pressure losses for the gas turbine plant integration.

Mass conservation equation:

\[
V_{cc} \frac{dp}{dt} = \dot{m}_{air} + \dot{m}_{fuel} - \dot{m}_{fluegas} \tag{19}
\]

Energy conservation equation:

\[
\frac{d(\dot{M}_{cc} \dot{U}_{cc})}{dt} = \dot{m}_{air} h_{air} + \dot{m}_{fuel} LHV - \dot{m}_{fluegas} h_{fluegas} \tag{20}
\]

Where \( LHV \) = the lower heating value dependent on fuel composition
\( \dot{M}_{cc} \) = the total mass of gases inside the combustor at each time step
\( \dot{U}_{cc} \) = the internal specific energy in the combustion chamber

The total mass of gases in the combustion chamber depends on outlet pressure, outlet temperature and composition.
2.5. Turbine

The turbine, in the open-cycle engine operates between the pressure at turbine inlet and atmospheric pressure.

One approach of turbine modeling was proposed by (Cooke, 1983). Under the assumption of a constant flow coefficient, the pressure drop on a gas turbine stage can be modeled. The theory is based on a pressure-flow relationship for a turbine where the fluid can expand. This pressure-flow relation in a specific point can be approximated as constant mass flow coefficient as follows:

\[
\varphi = \frac{\dot{m}_i}{\sqrt{P_i \nu_i}} = \text{Constant}
\]  

Where

- \( \varphi \) = mass flow coefficient
- \( \dot{m}_i \) = mass flow to the next stage group [kg/s]
- \( P_i \) = inlet pressure [Pa]
- \( \nu_i \) = specific volume corresponding to state point at \( P_i \) [m\(^3\)/kg]
- \( i \) = subscript denoting any point in the expansion

Under the assumption of the expanding gas behaving like an ideal gas, the relation can be rewritten as:

\[
\varphi = \frac{\dot{m}_i \sqrt{T_i}}{P_i} = \text{Constant}
\]  

Where

- \( T_i \) = Turbine Inlet Temperature

Thus, in 1927, the expansion process on a turbine stage the physicist Aurel Stodola empirically found a substantial relationship to determine the flow behavior in turbine expansion path. This is known as the Stodola’s Ellipse and stated as (Stodola, 1927):

\[
\varphi \propto \sqrt[4]{1 - \left(\frac{P_{\text{in}}}{P_{\text{out}}}\right)^2}
\]

Introducing a constant \( C_t \) in equation \( 23 \) and combining it with equation \( 22 \), the basic Stodola’s turbine equation is found:

\[
\dot{m} = C_t \frac{P_{\text{in}}}{T_{\text{in}}} \sqrt[4]{1 - \left(\frac{P_{\text{out}}}{P_{\text{in}}}\right)^2}
\]
Where \( C_t \) is Stodola's turbine constant, which is a measure of the effective flow area through the turbine and calculated by:

\[
C_t = \frac{\alpha \cdot f_{sp}}{\sqrt{zR}} \tag{25}
\]

Where 
\( \alpha \) = flow coefficient
\( f_{sp} \) = turbine constriction cross-section
\( z \) = number of stages
\( R \) = molar gas constant

If the turbine can be considered isentropic, the discharge temperature can be written as

\[
T_{out,is} = T_{in} \left( \frac{P_{in}}{P_{out}} \right)^{(\gamma_t - 1)/\gamma_t} \tag{26}
\]

From the steady-flow energy equation [6], the turbine specific work \((W_t)\) can be written as

\[
W_t = \eta_{is} (C_{pg,in} T_{in} - C_{pg,out} T_{out,is}) \tag{27}
\]

Where 
\( \eta_{is} \) = the turbine isentropic efficiency
\( C_{pg} \) = the specific heat capacity of the flue gas calculated at state condition of the expansion gas line
\( \gamma_t \) = the turbine expansion specific heat ratio
\( W_c \) = the specific turbine power extracted (positive work rate)

The turbine power output is then determined by multiplying the mass flow rate of combustion gas flowing through the turbine with \( W_t \).

The net power output of the gas turbine set, \( P_{net} \) is influenced by the mass of air processed and can be calculated as

\[
P_{net} = \dot{m}_{air} \left( (1 + f) W_T + W_c \right) \tag{28}
\]

And the thermal efficiency of the engine set will be

\[
\eta_{tt} = \frac{P_{net}}{\dot{m}_{fuel,LHV}} \tag{29}
\]

Where 
\( f \) = the fuel to air ratio
\( \dot{m}_{fuel} \) = the fuel flow rate to combustor
\( LHV \) = the lower heating value of the fuel
\[ P_{net} = \text{the net power output of the gas turbine set} \]
\[ \eta_{tt} = \text{the total thermal of the gas turbine plant} \]

Power turbines are often designed to have repetition stages. This means that stages have the same geometry, a constant flow coefficient and a swirl free inflow condition. Thus, the stage absolute inlet velocity \( C_1 \) and outlet velocity \( C_3 \) remain constant at design condition. However, while at start-up and full load operation conditions, the velocity (kinetic demolition) can create a pressure drop in the expansion path. This creates shock on the turbine stage in form of vibration due to stage profile (mechanical) loss.

### 2.6. Gas Turbine Power Loss Calculations

In complete gas turbine dimensional modeling, the stage profile loss is taken as a sum loss on stator and rotor. The reason for doing so is to be able to relate losses to geometrical properties such as the deviation of the flow in each blade row (Soderberg, 1949).

The losses in the stator and the rotor respectively are expressed by the outflow velocities such that

\[ h_{SL} = h_2 - h_{2s} = \frac{1}{2} C_2^2 \xi_S \]
\[ h_{RL} = h_3 - h_{3s} = \frac{1}{2} w_3^2 \xi_R \]

Where the stator profile loss factor \( (\xi_S) \) is calculated by:

\[ \xi_S = 0.04 + \left( \frac{\epsilon_S}{100} \right)^2 \]

And rotor profile loss factor \( (\xi_R) \) is calculated by:

\[ \xi_R = 0.04 + \left( \frac{\epsilon_R}{100} \right)^2 \]

The term \( (\epsilon_S) \) is the stator angle deviation and \( (\epsilon_R) \) is the rotor angle deviation that are considered in the loss factor calculations in [32] and [33].

Then, the sum of the two profile loss factors gives the total mechanical loss factor \( (\xi_{ml}) \) that determines the stage mechanical dynamics.

\[ \xi_{ml} = \xi_{SL} + \xi_{RL} \]

However, a complete gas turbine dimensional modeling needs a lot of data which specify thermodynamic and flow quantities along the main flow direction. Thus, most
gas turbine analytical models proposed in the literature are simply design models. Some others, however, are able to predict off-design behavior by means of zero-dimensional models.

Zero-dimensional models are defined as models that consider only thermodynamic transformations across the component without simulating the internal flow field.

For zero dimensional turbine modeling, the lumped effect of the dynamic mechanical losses can be calculated from total pressure drop condition. Thus, the corresponding velocity due to the profile loss in stator and rotor can be captured in a gas plenum. In the gas plenum, the essential aspects of stage dynamic effects are characterized by one main equation which is composed of the physical variables. Accordingly, it has been derived for the transient flow across the gas plenum by which pressure variations due to mass accumulation can be calculated (Liepmann & Roshko, 1957).

\[
\frac{dP_{\text{out}}}{dt} = \left(\frac{\dot{m}_{\text{flow, in}} - \dot{m}_{\text{flow, out}}}{V_p}\right)\gamma R T_m \tag{35}
\]

\[
C_{\text{ml}} = \left(\frac{2\Delta P}{\rho \xi_{\text{ml}}}ight)^{0.5} \tag{36}
\]

And, the mechanical energy loss rate will be:

\[
\dot{Q}_{\text{ml}} = m_{\text{flow}} \frac{C_{\text{ml}}^2}{2} \tag{37}
\]

Where \(\Delta P\) = the total pressure drop due to profile loss
\(C_{\text{ml}}\) = the velocity lost in turbine shock (vibration)
\(\xi_{\text{ml}}\) = average stage profile loss factor
\(\rho\) = the medium gas density
\(m_{\text{flow}}\) = Turbine mass flow
\(\frac{dP_{\text{out}}}{dt}\) = the total pressure drop rate at stage outlet
\(V_p\) = the plenum volume
\(\gamma\) = the gas specific heat capacity ratio
\(R\) = the universal gas constant
\(T_m\) = gas plenum medium temperature

The thermal losses on a stage correspond to the temperature difference between the turbine inter-stage and the surrounding. Besides it depends on the total heat transfer coefficient and area of the gas turbine body. The turbine body construction is basically made with good insulation that allows a minimum heat loss to the
surrounding. This contributes for total cycle efficiency gain. Thus, the thermal loss is calculated by

\[ \dot{Q}_{\text{th,l}} = UA\Delta T \]  

[38]

Where 
- \( U \) = the overall heat transfer coefficient throughout the turbine body
- \( A \) = the heat transfer surface area of the turbine body
- \( \Delta T \) = the internal and external temperature difference

The overall energy transfer phenomenon on a turbine control volume can be illustrated in Figure 7.

![Figure 7 Energy flow phenomena on a turbine stage](image)

The energy flow phenomenon is thus evaluated using first law of thermodynamics for energy conservation as in equation [39]

\[ d\dot{E} = \dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} - \dot{W}_{\text{out}} - \dot{Q}_{\text{th,l}} - \dot{Q}_{\text{ml}} \]  

[39]

Where
- \( d\dot{E} \) = the rate of internal energy stored in the turbine
- \( \dot{Q}_{\text{in}} \) = the rate of input energy to the turbine
- \( \dot{W}_{\text{out}} \) = the rate of energy extracted by the turbine
- \( \dot{Q}_{\text{th,l}} \) = the thermal energy loss to the surrounding
- \( \dot{Q}_{\text{ml}} \) = the mechanical energy loss to the surrounding

2.7. Gas Turbine Design and Off-Design Simulation

Design mode simulation refers to the plant performance at full engine design operating mode. However, real power plants do not always operate at design capacity. Thus, off design
simulations become important in revealing the actual engine performances. Before a new gas turbine can be designed, many alternative thermodynamic cycles are evaluated. Then, a cycle is selected which constitutes the cycle design point (cycle reference point) of the gas turbine. For this design point, all the mass flows, the total pressures and total temperatures at the inlet and outlet of all components of the engine are determined.

Off design mode simulations deal with the behavior of a plant system with given geometry. This geometry is found by running a single cycle design point. To prepare for an off-design simulation, the component design points must be correlated with the component maps. This can be done automatically using standard maps of gas turbines and the standard design point settings in these maps. The maps often need to be scaled before the off-design calculation commences in such a way that they are consistent with the cycle design point.

In recent developments of GTP off design operations, systematic studies have been made to correlate and get typical characteristics equations (Zhang & Cai, 2002). The equations can be analyzed in terms of performance correction using compressor and turbine characteristic curves. However, they cannot represent the general and typical performances of gas turbine plants.

For pressure ratios different from nominal conditions, the isentropic efficiency falls below the nominal isentropic efficiency $\eta_{is}$ (Zhang & Cai, 2002).

For part load compressor operation, the equations have been given as:

\[
\frac{P_{i-c}}{P_{i-cD}} = c_1\hat{n}_c\hat{m}_n^2 + c_2\hat{n}_c\hat{m}_n + c_3\hat{n}_c \tag{40}
\]

\[
\frac{\eta_c}{\eta_{cd}} = [1 - c_4(1 - \hat{n}_c)^2] \left( \frac{\hat{n}_c}{\hat{m}_n} \right) \left[ 2 - \left( \frac{\hat{n}_c}{\hat{m}_n} \right) \right] \tag{41}
\]

Where

- $P_{i-cD}$ = Pressure ratio from outlet to inlet at design operating mode
- $P_{i-c}$ = Pressure ratio from outlet to inlet at off design operating mode
- $\hat{m}_n$ = The air mass flow rate normalized by the design air flow rate
- $\hat{n}_c$ = The rotor speed normalized by the design rotor speed
- $\frac{\eta_c}{\eta_{cd}}$ = The compressor efficiency normalized by the design efficiency

The coefficients $c_1$, $c_2$, and $c_3$ of the equation [40] are calculated by

\[
c_1 = \frac{\hat{n}_c}{p \left( 1 - \frac{m}{\hat{n}_c} \right) + \hat{n}_c(\hat{n}_c - m)^2} \tag{42}
\]

\[
c_2 = \frac{p - 2m\hat{n}_c^2}{p \left( 1 - \frac{m}{\hat{n}_c} \right) + \hat{n}_c(\hat{n}_c - m)^2} \tag{43}
\]

\[
c_3 = -\frac{pm\hat{n}_c - m^2\hat{n}_c^2}{p \left( 1 - \frac{m}{\hat{n}_c} \right) + \hat{n}_c(\hat{n}_c - m)^2} \tag{44}
\]
According to (Zhang & Cai, 2002), the values of m and p should satisfy the equation, $$\frac{2}{p} \geq \frac{2m}{3}$$ to ensure appropriate shape and position of the compressor characteristic curves. Thus, values of, m=1.06, p=0.36, C₄ = 0.3 are also determined and applicable for any compressor performance corrections in constant speed operation.

For turbine part load operation, the correcting equations have been given by:

\[
\hat{\eta}_{gt} = 3.18\hat{W}_n - 4.69\hat{W}_n^2 + 3.69\hat{W}_n^3 - 1.18\hat{W}_n^4
\]

\[
\hat{m}_f = 0.288 + 0.624\hat{W}_n + 0.088\hat{W}_n^2
\]

Where

- \(\hat{W}_n\) = the power normalized by the design power output
- \(\hat{\eta}_{gt}\) = the efficiency normalized by the design efficiency
- \(\hat{m}_f\) = the inlet mass flow rate normalized by the design flow rate

2.8. Gas Turbine Blade Cooling

Turbine blade cooling technology can be categorized in the two major groups, open and closed loop cooling.

Closed loop cooling is a way of blade cooling by which the coolant passes through the blade or vane to absorb heat and reject it outside of the expansion process. Steam cooling injected from the steam cycle in a cogenerating power plant is a typical example of closed Loop Cooling.

Open loop cooling is the most common way of blade cooling performed when the coolant is injected into and mixed with the main flow stream. Air cooling is considered as typical example of open loop cooling technology.

According to (Jordal, 2001), blade cooling is primarily used in large gas turbine power plants which allow the use of higher turbine inlet temperature for better cycle efficiency with low thermal stress compared to the un-cooled. However blade cooling results in significant disadvantages stated as:

- Turbine work output is decreased due to the compressed cooling air by passing one or more stages.
- Turbine work is lost due to the colder cooling air being mixed with hot gases from the main stream, resulting in decreased enthalpy and total pressure.
- Extraction of air from the compressor outlet has a risk of disturbing the flow field into the combustion chamber.
- There is less heat to recover in applications where the exhaust gases are used since cooling effect makes the temperature of the gases leaving the turbine lowered.
• The cost of producing the blades is increased.

2.9. Control Systems

Control systems are used in every process industry today. Besides, the strong market, competition and environmental requirements have forced industries to automate and optimize their systems using different control schemes. The need of an automatic control system in a GTP plant is to handle fast process responses and avoid large disruptions in the power generation

The most common regulator structure is the feedback control. A reference signal \( r(t) \) is sent into the controller and compared to the process output signal \( y(t) \) that needs to be controlled. Depending on the control error calculated as:

\[
e(t) = r(t) - y(t)
\]  

[47]

A control signal is calculated and sent to the process that needs to be controlled. For better understanding of the control mechanism, see Figure 8 as an example of a feedback loop structure.

In order to build a good controller, apart from knowing the process output signal and reference value, good process knowledge is required.

![Figure 8 Process control using feedback control](image)

A common way of finding the dynamics of the process is disturbing the process with a transient. The most common way of doing this in the process industry is by introducing a step in the control signal and registering the behavior of the output signal (Hägglund, 1997).

In process industries, the most common regulator used is the PID-controller. The PID-controller is a relatively simple controller to tune. Furthermore, it is cheap to use comparing to the more complex controllers. Owing to these factors industries usually prefer to use a lot of PID-controllers instead of a few more advanced controllers (Hägglund, 1997).

Among all these PID controllers, the P and I parts are most commonly used. Thus, in the thesis project, the GTP model is controlled by a set of PI controllers for design and off design simulation calculations. The mathematical formulation of the PI-controller is
2.10. Modelica and Dymola

The component models developed in the flue gas package are all based on the programming language Modelica and are built in the modeling and simulating environment, Dymola.

2.10.1. Modelica

Modelica is an object-oriented, equation-based programming language developed with the main objective to model complex physical systems using small model components in a wide range of engineering fields.

The models are mainly not built by algorithms as in traditional programming, but by differential, algebraic equations (DAE) that describe the physical behavior of the model. These equations are not solved in any predefined order but are solved based on what information the solver has access to. This means that the solver can, and often do, manipulate the equations during the simulation so that it can solve for the desired variables. This is the main reason to why the models are reusable for new plant system integration.

The Non-profit Modelica Association is also responsible for developing the free Modelica Standard Library, which is a library of models, components, equations that are used for the GTP modeling and simulation (Modelica, 2011).

2.10.2. Dymola

Dymola, Dynamic Modeling Laboratory, is a complete object-oriented tool for modeling and simulation of integrated and complex systems for use within automotive, aerospace, robotics, process and other applications. It is a commercial environment for modeling and simulating with Modelica code and is developed by the Swedish company Dassault Systèmes AB (Dymola-Modelica, 2011).

The Dymola environment is divided in two main windows; modeling and simulating. The modeling window is in turn composed of two main areas; text windows for writing Modelica code for models and a graphical interface for connecting models and components by drag and drop. The simulation window is there to simulate the models by numerically solving the differential-algebraic-equations (DAE) that are set up in the models. There are a wide range of numerical solvers to choose from depending on task preferences. In this master thesis the solver Dassl has been used.

In Solvina, power plant models are mostly developed in Dymola, Thus, other programming software preferences have not been considered.
2.10.3. Numeric

Most numerical solvers are produced to solve ordinary differential equations written in the standard form.

\[
\begin{align*}
  \dot{y}(t) &= f(t, y(t)) \\
  y(t_0) &= y_0
\end{align*}
\]  \[49\]

However, they are not made for solving implicit systems of differential algebraic equations (DAE) written in the form, [50] which arises in a lot of physical systems.

\[
\begin{align*}
  F(t, y(t), y'(t)) &= 0 \\
  y(t_0) &= y_0 \\
  y'(t_0) &= y'_0
\end{align*}
\]  \[50\]

For solving these DAE systems, Dassl which is a numerical solver developed by Sandia National Laboratories (Petzold, 1982) are used. It is a non fixed step solver and uses backward differentiation formulas (BDF) to find the solution from one time step to another. Depending on the behavior of the solution, Dassl uses up to five of the previous solved time steps. The length of each step is also dependent on the behavior of the solution.
3. Modeling

The dynamic compressor and turbine modeling have been carried out in Dymola 7.3. The plant components have been separately modeled using the differential-algebraic-equations (DAE) that express each of the component’s characteristics. The modeling has been developed on the bases of zero-dimensional mass and energy conservation equations. This has been performed with the aim to simulate a GTP model for different sets of operations. The component models have been made with respect to dynamic properties at different load settings. Fluid-specific functions and routines of the Modelica fluid media package have been used for all model developments. Thus, the temperature, pressure and gas composition have been used to calculate functions of thermo physical properties. Enthalpy, specific heat capacity, density are the main gas line properties that have been determined for model developments. Continuous testing and evaluation during the modeling phase have been performed to ensure that each component reveals substantial behavior of the GTP model.

3.1. Compressor Modeling

The dynamic compressor model has been developed based on two harmonizing blocks. The first model block has been made for the stage compression on steady-state basis. This has been done in such a way that steady-state pressure ratio and efficiency are calculated from air corrected mass flow at constant rotating speed. To account for the performance correction, curve fitting polynomial equations from compressor map have been implemented in the steady state stage modeling.

The above considerations have been done disregarding any dynamic effects affecting mass and energy conservation. For that a second block, a plenum has been modeled to make up the dynamic effects of compressor stage at the compressor discharge section. The plenum has been modeled to be integrated at the stage exit and represent for passage where the air velocity is nearly diffused to be low enough as to neglect momentum variations. Thus, conservation of energy, and therefore temperature, and pressure across this component has been considered.

3.1.1. Compressor Stage Modeling for Steady State Performance

The compressor stage for steady state performance is normally considered adiabatic. However, the heat loss cannot always be neglected. Thus, quasi-steady-state conditions have been assumed for the adiabatic thermodynamic equation. Besides, the compressor operative characteristics applications in order to consider correction during part load operation have taken part. Thus, the model has been composed of compressor characteristic equations that provide correction on pressure ratio and mass flow rate which in turn corrects the part load efficiency.

In this model, coefficients $c_1$, $c_2$ and $c_3$ of the performance correction equation [40] have been calculated using equations [42], [43] and [44] respectively. Thus, $c_1 = -55.56$, $c_2 = 97.78$ and $c_3 = -41.22$ have been used. At the same time values of m and
p have been given in section 2.7 to satisfy the equation, \( \frac{3}{p} \geq \frac{2m}{3} \) for appropriate shape and position of the curves; as well as c4. Thus, for the compressor modeling, \( m = 1.06, p = 0.36, c4 = 0.3 \) and \( n_c = 1 \) have been assigned to be substituted in equations [40] and [41]. Then, the compressor pressure ratio and efficiency correction equation have been rearranged and implemented in stage modeling as

\[
\frac{p_{1-cd}}{p_{1-c}} = -55.56\dot{m}_n^2 + 97.78\dot{m}_n - 41.22
\]

\[
\frac{\eta}{\eta_{cd}} = \frac{1.1}{\dot{m}_n} (2 - \frac{1}{\dot{m}_n})
\]

The efficiency of the compressor has then been corrected under the quasi-steady assumption at each spot of the compression line.

In the model, air mass flow rate has also been calculated and given by

\[
\dot{m}_{\text{flow air}} = \frac{\sqrt{T_1}}{P_1} \cdot \frac{(\varphi_c \cdot \text{VIGV} \cdot F_m)}{(1 - \frac{1}{P_{1-c}^2})^{0.5}}
\]

\[
\frac{P_{1-c}}{P_{in}} = \frac{P_{out}}{P_{in}}
\]

\[
C_c = \varphi_c F_m
\]

Where \( \dot{m}_{\text{flow air}} \) is the air inlet mass flow rate; \( \varphi_c \) the compressor flow coefficient; \( F_m \), compressor geometry factor and assumed to be 0.03 as Stodola’s turbine machine constant, and \( P_{1-c} \) is outlet-to-inlet pressure ratio. The compressor machine constant, \( C_c \) corresponds to turbine matching.

The power consumption of the compressor has been calculated as the product of air mass flow and enthalpy rise between inlet and outlet sections.

Thus, the negative power output of the compressor can be written as:

\[
W_c = \dot{m}_{\text{air}}(h_{in} - h_{out})
\]

Where \( h_{in} \) and \( h_{out} \) represent the input and output air enthalpy. Enthalpy is a function of temperature, pressure and air composition and can be evaluated by fluid-specific functions and routines of the Modelica fluid media package.

The outlet flow temperature was calculated assuming an adiabatic, non-isentropic transformation.
\[ T_{\text{out}} = T_{\text{in}} \left( 1 + \frac{1}{\eta_{\text{c}}} \left[ \frac{P_{\text{out}}}{P_{\text{in}}} \right]^\frac{\gamma_{\text{c}} - 1}{\gamma_{\text{c}}} - 1 \right) \]  

The specific heat capacity \( C_p \) and specific heat ratio \( \gamma_{\text{c}} \) were calculated at the state medium temperature and pressure at the inlet and outlet sections of the compressor.

The above calculations were made disregarding the effects of mass accumulation. However, these effects must be included in a dynamic simulation and, therefore, a plenum was modeled at the compressor discharge section to account for them.

The steady compressor stage can be represented as in Figure 9 where air flow properties have been considered to depend on temperature, pressure and composition throughout the compressor in the gas turbine set.

![Figure 9 Schematic representation of compressor where 1 represents air inlet; 2 represents air outlet flow and 3 is the power intake extracted from a connected turbine.](image)

The following input data for the model simulation was used

**Inputs:**
- Ambient inlet pressure, temperature, design pressure ratio and air composition
- Air mass flow in the previous time step for initialization to avoid algebraic loops

**Outputs:**
- Power consumption and corrected efficiency.
- Discharge temperature/enthalpy, nominal air mass flow rate

### 3.2. Gas Turbine Modeling

The dynamic turbine modelling has focused on the turbine component in a gas turbine power plant simulator. Thus, the turbine model was built up by smaller model components to predict the turbine dynamic behavior at different operating conditions. Turbine blade cooling was studied using an air stream injecting from compressor outlet to the turbine stator and rotor to avoid high thermal stress on the stage. However, this was made on the expense of a power output loss. This is fully
consistent with the conclusions of (Jordal, 2001) that for low and medium gas turbine power plants, blade cooling can be omitted.

3.2.1. Air-Cooling Passage Modeling

Turbine blade cooling, considered in the thesis project is schematically illustrated in Figure 10. The cooling air implementation was discharged out from the compressor exit and injected in either of the stator or rotor to decrease blade thermal stress for sustainable use of the engine. Besides, the power output has been compared to the un-cooled GTP.

The enthalpy-entropy chart presented in Figure 10 shows the stage pressure and enthalpy variation due to a cooling air application. The chart illustrates that the total inlet pressure for expansion decreases due to mixing. At the same time, total enthalpy also decreases due to heat rejection at cooling. Depending on the proportion of the mixed temperature decrement compared to mass flow gain on the expansion line, the total power output varies significantly compared to the non cooled GTP model.
Figure 11 Enthalpy - Entropy gas expansion chart in a cooled turbine stage

<table>
<thead>
<tr>
<th>0: represents flue gas entry at turbine inlet</th>
<th>0-1: represents cooling air and flue gas mixing on stator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2: represents pressure loss at mixing for stator cooling</td>
<td>2-3: represents adiabatic gas expansion on a stage</td>
</tr>
<tr>
<td>3-4: represents rotor cooling on a stage</td>
<td>4-5: represents pressure loss at mixing for rotor cooling</td>
</tr>
</tbody>
</table>

The cooling air injected into the turbine was modeled using a duct passage to connect the compressor exit with the cooled turbine stage blades. Thus, the cooling air injected into the turbine affects the pressure and temperature of the main gas stream. By that it modifies the expansion line of the gas turbine component. The pressures at different locations where cooling air is injected into the turbine have determined the amount air flow rate bled for respective stage cooling. The mass flow of the cooling air bled for each blade row can be calculated as proposed by (MC Greehan & Schotsch, 1988) and presented in (Chacartegui, Sánchez, Muñoz, & Sánchez, 2011).

\[
\dot{m}_{\text{c, air}} = K \frac{P_{\text{bl}}}{\sqrt{T_{\text{bl}}}} \left(1 - \frac{P_{\text{st}}}{P_{\text{bl}}}\right) \tag{58}
\]

Where
\( \dot{m}_{\text{c, air}} \) = cooling air bled to the turbine
\( K \) = discharge coefficient, depends on turbine design
\( P_{\text{bl}} \) = air bleeding pressure from compressor exit
\( T_{\text{bl}} \) = air bleeding temperature from compressor exit
\( P_{\text{st}} \) = turbine pressure at a specific stage
In the air-cooling injecting duct model development, the above equation was implemented as the main model equation. It was used for maintaining mass and energy conservation when mixing of cooling air and the main gas stream were considered during the dynamic gas turbine simulation.

3.2.2. Turbine Stage Modeling for Steady State Performance

From a thermodynamic point of view, cooled turbine stages cannot be considered as an adiabatic machine. In order to determine the starting point of the expansion line, pressure and temperature are needed at this location. Both parameters are linked to the mass flow of combustion gases through the flow parameter, \( C_t \) whose value is determined at on design operating conditions (Cooke, 1983)

\[
\frac{m_{in}}{P_{in}} \sqrt{T_{in}} = C_t \sqrt{1 - \frac{P_{o-t}}{P_{i-t}}}
\]  

[59]

Where

\( m_{in} \) = is the turbine inlet mass flow rate

\( T_{in} \) = the turbine inlet temperature

\( P_{in} \) = the turbine outlet to inlet pressure ratio

And \( C_t \) is Stodola’s turbine constant, which is a measure of the effective flow area through the turbine.

Starting from these initial conditions, the expansion gas line of the turbine was used to evaluate enthalpy variations for useful work determination. It also helped in calculating the pressures at different locations where cooling air was injected into the turbine.

The model was developed to evaluate the quasi-steady state performance of the gas turbine stage; with assumption of no mass accumulation on the turbine stage. The expansion line of the turbine was constructed with a method proposed by (EL Masri, 1986).

Furthermore, the model was used for simulation of the gas turbine performance with and without stage cooling implementation. The gas inlet temperature has an influence on consideration of stage cooling to protect the turbine blades from thermal stress and erosion.

During the cooled stage modeling, connection of an air nozzle component to the inlet of the gas turbine stage was considered. By that the mixed output temperature was taken as inlet turbine temperature for adiabatic non isentropic expansion.

Then, mass and energy conservation of stage mixing was calculated at constant turbine stage pressure as follows.

\[
m_{tot} = m_{c,air} + m_{flow}
\]

[60]
While mixing at the stator stage before expansion, a certain pressure loss due to mixing was considered at constant total inlet condition. This is usually evaluated from experimental data. Thus, an approximate value of 2% pressure loss from inlet port pressure was taken as default parametric value to calculate the inlet expansion pressure. The same pressure loss was implemented as well for rotor cooling. The pressure loss percentage was also used in the gas turbine model developed by (Chacartegui, Sánchez, Muñoz, & Sánchez, 2011).

The model also considered turbine performance correction using equations [45] and [46] in section 2.7 that provide mass flow rate and the part load efficiency corrections. Thus, the expansion process considered the corrected isentropic efficiency, pressure ratio and mass flow rate as state variables. With these, it was able to determine the thermo physical properties at the inlet and outlet of the expansion path. The adiabatic non isentropic turbine expansion equation implemented was:

\[ T_{out} = T_{in} \left[ 1 - \eta_{is,corr} \left( 1 - \left( \frac{P_{out}}{P_{in}} \right)^{\frac{r-1}{r}} \right) \right] \]  

Where  
\( T_{in} \) = the turbine inlet temperature  
\( T_{out} \) = the turbine outlet temperature  
\( \eta_{is,corr} \) = the corrected isentropic stage efficiency  
\( P_{out} \) = the turbine outlet pressure  
\( P_{in} \) = the turbine inlet pressure  
\( r \) = the specific heat ratio

Here, it is worth noting that the enthalpy of the cooling air is the same for all stages that are considered to be cooled. This is due to the fact that the bleeding point at the compressor discharge is the same for all air cooling nozzles that are connected to cool turbine blades.

3.3. Gas Plenum Modeling

The gas plenum has been modeled to represent a passage at the compressor discharge to combustion chamber and turbine inter-stage free space. The plenum was modeled to capture the dynamic effect of stator-rotor mechanical and thermal losses to the surrounding.

The main model equation is composed of variables that characterize the essential aspects of stage dynamic effects in the plant simulation. Thus, conservation of
energy, and therefore temperature, and pressure across this component is maintained constant. The continuity equation for the unsteady-state flow across the plenum has been used. This can be written as (Liepmann & Roshko, 1957)

$$\left( \frac{dP_{\text{out}}}{dt} \right) = \left( \frac{(\dot{m}_{\text{in}} - \dot{m}_{\text{out}})}{V_p} \right) \gamma R T_{\text{out}}$$  \[63\]

Where

\[ V_p = \text{the volume of the free space (plenum) between each pair of turbine stages} \]

\[ \frac{dP_{\text{out}}}{dt} = \text{rate of pressure variation at outlet port of the gas plenum to the next turbine stage} \]

\[ \dot{m}_{\text{in}} = \text{the inlet flue gas flow rate of the gas the plenum} \]

\[ \dot{m}_{\text{out}} = \text{the outlet flue gas flow rate of the gas plenum} \]

\[ T_{\text{out}} = \text{the outlet flue gas temperature to the next turbine stage} \]

\[ \gamma = \text{the specific heat capacity ratio} \]

\[ R = \text{the universal gas constant calculated at the plenum temperature and pressure and gas composition} \]

Using [63], pressure variations due to mass accumulation in the compressor and turbine passages, transition duct and combustor can be calculated. The performance of the model was tested by connecting two steady-state gas turbine stages with the gas plenum model. It was possible to capture the mechanical and thermal losses to the surrounding using the heat ports. The mechanical loss corresponds to the profile losses that create shock on the stage and result in stage vibration. The thermal loss was evaluated considering the thermal resistance of turbine body and the surrounding temperature.

### 3.4. Model Integrations for GTP Simulation

In this section, the model components of the compressor and turbine have been separately built and simulated to test before they were brought for gas generating unit/plant integration. Then, gas turbine unit has been integrated with the tested compressor, existing verified combustion chamber and turbine models with all the auxiliary component models. Air to fuel ratio calculator to consider stoichiometric combustion and air cooling passage have been the major auxiliary models integrated for the required operation condition. The testing has been carried out by setting inlet and outlet boundary conditions on models. The unsteady/transient effects in the compression and expansion paths have been captured in the plenum model integrated with the steady stage models.
3.4.1. Complete Compressor Model

The complete compressor model consists of the two main model blocks of compressor stage and gas plenum, where the steady and unsteady/transient state effects can simultaneously be captured.

The performance capability of the complete compressor model has been tested by simulating the complete model as shown in Figure 12. The inlet boundary conditions of temperature and pressure and as well as compression ratio have been assigned for simulation so that the physical properties in the compression path have shown varying in line with compressor map. The compressor model has been able to perform well within the operating points of surge and choke line. The compressor operating map has been included in the steady state stage model development expressed in terms of the performance correction equations as stated in equation [51] and [52]. Performance correction equations of compression pressure, temperature, air mass flow rate and isentropic efficiency have been considered and corrected referring to the respective design variables. For that the design air compression ratio has been set as parameter in compressor model so that off design operation refers to it and avoid stall/ surge effect. At the same time, by setting the minimum allowable compression ratio in the model, choking effect is avoided. This formulation has also interrelated with air mass flow and efficiency corrections that the compressor operates accordingly. At the same time, making the design compression ratio as parametric value in the model makes it easily changeable to reuse for different types of compressors.

Figure 12 shows coupled models for a complete compressor model for dynamic GTP integration. The airflow through the system is represented by the deep and light green lines, while the blue lines are scalar signals and the red lines represent heat flows. Air is entering through the deep light green filled rectangle to the bottom left, AirInletFlow going through the compressor stage for compression. The light green line, which is the compressed air, passes through the gas plenum and leaves the system through the slightly green filled rectangle to the upper right, AirFlowOut. Heat and mechanical losses have been considered using the heat port of the gas plenum connected to the turbine wall for heat conduction as well as convection to the ambient air.
3.4.2. Complete Turbine Model

The complete turbine model consists of the two main model blocks turbine stage and gas plenum, where the steady and unsteady state effects can simultaneously be captured.

Complete two stage turbines have been integrated for model comparison test. Exiting static Stodola’s turbine and the newly developed dynamic turbine stages have been simulated for performance comparison. For both simulation test cases, boundary inlet conditions of design expansion ratio and turbine temperature have been assigned. The simulation results have then been plotted to analyze the inter-relatedness of the gas expansion properties with respect to mass flow, power output and efficiency correction. Off design turbine performance has also been tested in order to analyze result variations from the design operating mode. Change in turbine inlet pressure has been done to examine off design operation at constant turbine inlet temperature.

In the simulation, inputs of design turbine pressure ratio and inlet temperature have first been considered for determination of nominal turbine inlet mass flow rate using Stodola’s relation equation [24]. Then, in the model the design inlet mass flow rate has been used as default parameter for cases of performance correction calculations that have gone off design operating mode.

Figure 13 shows the connected models in Dymola forming a complete dynamic turbine model with no stage cooling. The combusted flue gas flow through the system is represented by the double deep, medium and light red lines while the blue lines
are scalar signals and the single red lines represent heat flows to the surrounding. Hot flue gas of double deep red line from green filled rectangle at the upper left is entering to another green filled rectangle of the gas plenum to pass through and account for combustion dynamics and expand on turbine stage. The double medium red line, come out from the first stage expansion passes through the intermediate plenum to capture the sage dynamics; and lead to the next turbine stage for further work extraction; and leaving the system in light red line through the green unfilled rectangle to the bottom right. Heat and mechanical losses have been considered by connecting the heat ports of the gas plena to the turbine wall to the ambient air for heat convection.

![Figure 13 Complete Gas Turbine model with no stage cooling](image)

On the other hand, Figure 14 represents the connected models in Dymola forming a complete dynamic turbine model with stage cooling by injecting air from the compressor exit. Air cooling passages from the compressor exit have been modeled. Unsteady-air-Plenum which captures the stage gas dynamics has been connected to AirFlowOut at the second upper left representing the compressor steady-state stage exit. The air bleeding passage models have been connected to the air-plenum outlet and to the green filled rectangle of the turbine stage inlets for cooling. The rest of the model connection is the same as complete dynamic turbine stage with no cooling.
It’s worth noting that air bleeding passage model calculates the amount of air flow rate used for stage cooling depending on the respective cooling stage pressure variation as described by the model equation [58].

3.4.3. Complete GTP Model Integration for Verification

For the newly developed gas turbine plant model verification, the boundary conditions of ambient temperature and pressure at compressor inlet and compression ratio have been assigned. At the turbine outlet, same ambient pressures together with temperature for simulation initialization were assigned. Figure 15 presents the full simulation chart of GTP involving all components necessary for the model verification. All the flow lines have been previously illustrated in the individual compressor and turbine model integration. However, in addition, the GTP model has also included fuel/air calculating model so as to adjust the air flow to compressor with the required air flow for complete combustion. For the model verification, the model simulation chart has followed the same engineering scheme with respect to boundary condition settings as the static GT-Unit at Solvina.

Then, simulation outputs have been plotted and analyzed to compare the power output, flue gas mass flow rate and other main variables together with captured dynamic effects.
The simulation results have confirmed that it has shown persistent performance in capturing the unsteady effects in gas generation unit in line with extracting power. Accordingly, the simulation outputs have been plotted and analyzed in result section.

Figure 15 Complete GTP simulation-chart for Verification

3.5. GTP Model for Control System Setting

The complete gas turbine plant model is built up by the complete compressor and turbine models, together with Solvina’s existing fixed step combustor model. PI controllers were used in this project for control of design and off-design conditions and load adjustments. The complete plant model is also the final test of the compressor and turbine models.

In the thesis project, the compressor and the turbine models have been developed based on constant rotor speed operation condition. Thus, the integrated gas turbine model has been simulated with consideration of no aerodynamic instability. For that optimum variable inlet guide vane (VIGV) position is tuned at the compressor inlet for constant turbine exhaust temperature to maintain the working line unchanged for all part load operation as in (Walsh & Fletcher, 1998). Figure 16 presents the GT-Unit integration with no turbine blade cooling and with sets of input and output interfacing signals. In Figure 16, the inlet and outlet signals represented in blue lines have been extended out and seen connected with PID controllers in complete GTP simulation.
chart presented in Figure 18 and Figure 19 for design and off design operating mode calculation respectively.

On the other hand Figure 17 shows the gas turbine power plant with turbine stage cooling by injecting air from the compressor discharge spot through connecting hoses detailed in Figure 14. This is presented in Figure 17 by the light green double arrow line that connects the compressor discharge and turbine stage inlet of filled green rectangle for turbine cooling by mixing to the main stream before expansion.
3.6. GTP Model with Control System Setting

In this section, the complete gas turbine plant simulation integration is presented. The model has been simulated both at design and off design operation modes.

3.6.1. Design Mode Simulations

In the design mode simulations, the complete gas turbine set has been simulated for determining the design point conditions. Turbine inlet temperature, 1350°C and compression pressure ratio, 20:1 have been assigned as design point conditions of the model. Independent variables of the gas turbine cycle are ambient conditions of 1 bar and 15°C at compressor inlet and 1 bar at turbine outlet.

The design mode simulation of the gas turbine plant has been performed using two PI controllers. The first PI controller has been set to match the air mass flow rate required by the combustion chamber with that of the air mass flow rate to the compressor by tuning the design flow coefficient of the compressor. The second PI controller has been adjusted to control the design turbine inlet temperature by actuating the fuel consumption rate. Thereafter, simulation on the complete GTP model has been performed in order to determine the design compressor flow coefficient that remains constant as parameter for off design simulations. At the same time, the power output has then been used as reference power for part load settings.

Figure 18 shows the gas turbine plant design mode simulation set up with two PID controllers discussed above. VIGV opening to full open position, 1 and air to fuel ratio of 1.25 have also been set in order to determine the design GTP outputs.
3.6.2. Off design Mode Simulations

As all the thermodynamic design properties are known from the design simulations, the off-design calculations can be performed so that comparable performance deviations are observed. Various load adjustment criteria can be adopted in the plant operation. However, in the thesis project, three simple PI controllers have been used to analyze the GTP performance dynamics while operating in a wide range of modes. During the tests, the plant performance dynamics has been revealed due to the interdependent relations among the variables. The controllers’ set up have been implemented in a way that the required set power is directly controlled by the fuel mass flow rate which in turn determine the air mass flow rate for complete combustion. Simultaneously, the air flow rate is affected by VIGV opening positions that are tuned for constant turbine exhaust temperature, TET in constant speed part load operations (Walsh & Fletcher, 1998). This method has been used reveal the dynamic response of the main GTP variables being varied in power generation.

Accordingly, the part load simulations have been performed using the fuel consumption rate to control the power output; the air flow rate to the compressor to be controlled through the effect of fuel flow rate variation using air to fuel ratio meter which senses the oxygen amount in the combustion chamber for complete combustion; and the VIGV openings set to control constant TET of 600°C for all part load simulations.

Figure 19 shows the GTP off-design simulation chart with three PID controllers set accordingly as discussed above.
Figure 19 Control System Setting for regulating part load operating mode
4. Results and Discussion

4.1. Model Verification Results

In this section, simulation results of the model components developed in the thesis project have been verified compared with that of the existing Stodola’s turbine stage and static GT-Unit at Solvina.

4.1.1. Turbine Stage Model Verification Result

Two-turbine stages of the existing static and the newly developed dynamic models have been simulated both at design and off design operation modes for the model comparative verification. For the simulation start up, boundary conditions have been set with design inlet and outlet pressure of 15 and 1 bar as well as inlet temperature of 1400 °C. For the cooled turbine stage simulation test, cooling air bleeding from compressor exit has also been set by boundary condition of a pressure 15.5 bar and a temperature of 400°C. The bleeding temperature was calculated using the adiabatic compression equation with compression pressure ratio of 15 from ambient air condition temperature 15°C and compressor isentropic efficiency, 0.85.

Using the input values, simulations were accomplished and results have been plotted to compare performance variable deviations at the same operation condition. Looking into the results, the model has been put forward to be integrated in the gas turbine plant verification for complete dynamic GTP integration. The aforementioned design input data have been used for design mode simulation.

On the other hand, off design operating condition was performed on the existing verified static and the newly developed complete turbine stage models through varying the turbine inlet pressure. For that turbine inlet pressure of 10 bars has been set on the models to examine the variation of turbine characteristic variables from the design case operation. This has been done to reveal the steady-state properties of the gas expansion as well as capturing the transient/dynamic effects in the intermediate gas plenum.

With this manner, comparison of the existing static and the newly dynamic turbine stage simulation results were plotted to compare the result variations.

Figure 20 has shown that the dynamic gas turbine stage power output has distinctive variation over the static one. This is due to the energy and mass balance on the expansion gas line that has been considerably done with significant amount of pressure, stage efficiency and power correction referring to the respective design values. Besides, the inter-stage power loss in form of mechanical and thermal has been captured by the gas plenum. These had impact for the quasi-steady gas turbine power output decrement compared to the static gas turbine unit at Solvina under design and off-design operating mode.
On the other hand, the inlet and outlet temperature distribution as plotted in Figure 21, shows how the dynamic gas turbine model has revealed a lower inlet temperature and a higher exhaust temperatures at off-design operating mode. This has been resulted from the decrement of the power output which is due to efficiency decrement by the steady stage performance correction and power loss captured in gas plenum for unsteady/ transient operation conditions. This has confirmed the dynamic turbine stage model is capable of capturing the gas path dynamics to put it forward for GTP model integration and verification for complete dynamic GTP simulation.
4.1.2. GTP Model Verification Result

For the developed gas turbine plant model verification, the same boundary conditions have been set as the existing gas turbine unit at Solvina AB. Hence, the set up of boundary conditions have been ambient temperature and pressure of 1 bar and 288K at the compressor inlet and turbine outlet. Besides component parametric values have been assigned together with initial variable values. Hence, the model simulations have set compression ratio of 10 on the compressor model so as to determine the power output by the gas turbine set. Then, input parameters assigned to the total plant in the boundary and component models enable the combustion chamber model to calculate the combustion adiabatic temperature. From the value of the air to fuel ratio, the fuel mass flow rate and composition of the combustion products can be evaluated by the combustion chamber model as well.

Comparable simulation result charts were plotted below so as to see the model performance with respect to the existing GT-Unit.

The plots were plotted in such a way that performances of the static gas turbine unit and the dynamic ones can easily be seen. With that the charts have shown the power output, flue gas mass flow rate and unsteady effects of heat and mechanical losses captured well in line with extracting power. These have been possible owing to energy and mass balance consideration as well as efficiency corrections involved on the air compression and the gas expansion paths of the compressor and turbine models respectively.

Accordingly, Figure 22 presents the consistency of the model simulation result in flue gas mass flow rate with respect to the existing GT-unit model under cooled and un-cooled turbine stage conditions. However, Figure 23 presents the new gas turbine stages power outputs which have explicit decrement from the existing static gas turbine by a significant amount. This has indeed been due compression and expansion performance correction in the by the steady stage and power loss captured in gas plenum for unsteady/ transient operation conditions. Corresponding differences have been resulted in stage efficiencies, temperature and pressure as well.
Figure 22 Simulation results the static, the new cooled and un-cooled gas turbine mass flow rate

Figure 23 Simulation results the Existing static GT-Unit Turbine Power Output and the new cooled and un-cooled gas turbine unit models
4.2. GTP Model Dynamic Simulation Result

Primarily, design mode simulation have been carried out in order to attain all the thermodynamic quantities calculated before performing the off-design calculations. Thus, the off-design simulation have been carried out using power output from the design mode simulation, 242MW as reference design power to adjust part load settings. In the simulation, the same independent ambient inlet and outlet conditions have been used as for the design mode simulation. Hence, simulation results have been investigated for the two part load settings at 25% and 90% of the design power output. Then, the plant output variables that determine performance characteristics have been plotted and presented below.

A simulation period of 250 seconds has been set for all operation modes. A step up in power demand of 10MW has been done after 150 seconds of simulation. This has been done in order to observe the performance dynamics of the GTP model after reaching steady state simulation as test to load perturbation.

Figure 24 illustrates the GTP response in power output for the required power setting in the gas turbine control system. The corresponding fuel flow for the set power has been tuned and plotted in Figure 25. Besides, the plots reveal the GTP model performance dynamics regulated well after step in load perturbation on steady state simulation at 150 sec.

![Figure 24 The measured power by the GTP cycle represented by red curve line and set power for off-design simulation, shown by the blue line converge to the same value by implementing the PI controller scheme.](image-url)
The Power PI controller delivers a signal output of fuel flow rate to respond for the power regulation and tune the fuel flow as control variable. It further reveals the effect of the step in loading being actuated correspondingly.

At the same time, Figure 26 shows the air flow rate to the compressor leaned with the amount of combustion air required for a given load setting. The air to fuel ratio has been tuned by the PI controller in response to achieve the required air flow rate that the fuel flow to the combustion chamber requires for complete combustion. Simultaneously, constant exhaust temperature has been set for all part load simulation controlled by the VIGV openings that in turn vary the air flow in constant rotor speed operation.
Figure 27 plots the air flow PI controller tuning reaction on the air to fuel ratio meter for complete combustion.

![Air to fuel ratio response for air flow control](image)

Figure 27 The PI controller of the airflow delivers signal output of air to fuel ratio for airflow regulation at the 90% part load setting.

Besides, VIGV opening has been tuned by PI controller of constant turbine exhaust temperature (TET). Figure 28 shows the GTP model response to the step in loading perturbation at regulating the TET using VIGV opening as control variable to optimize air flow rate to the compressor in line with the fuel flow rate that controls the power setting.
Figure 28 The plot illustrates the measured turbine exhaust temperature (TET), blue curve line and set TET by the red line in the off-design simulation. They converge to the same value by implementing the PI controller scheme. The TET controller has revealed the effect of the step in loading after the steady operation reached at 150 seconds.

Figure 29 The PI controller of the turbine exhaust temperature delivers signal output of VIGV opening position for the respective airflow adjustment for constant TET off design simulations. The model controller reveals the dynamic effect of step in loading after reaching steady state operation at 150 seconds that confirms the GTP model robust for load perturbations.
For further analysis and observation of the fuel flow rate variation for different load adjustments, additional off design simulation at 25% of design power setting has been carried out. Hence, Figure 30 presents the measured power outputs of the off design simulations performed at 25 and 90% of the GTP design power. The corresponding fuel flow rate at the respective loadings has been plotted in Figure 31.

Each of the simulation output variables has revealed the effect of load perturbation after steady state operation is reached. With these, the GTP model is confirmed to be robust for any off design simulation and can be put to replace the gas turbine process cycle in producing electricity in GTP simulator.

![Figure 30](image1.png)

**Figure 30** Part load power output distribution at the respective load setting of 25% (red line) and 90% (blue line) of the design power.

![Figure 31](image2.png)

**Figure 31** Part load fuel consumption rate for the respective power load measured at 25%(blue line) and 90% (red line) of design power set for off-design simulations.
At the same time, the result outputs of the air flow rate together with the tuned air to fuel ratio are plotted in Figure 32 and Figure 33 respectively. Simultaneously, the VIGV opening variations for constant TET control have been revealed in the respective off design simulations in Figure 35. In this figure, the VIGV opening has been shown limited to opening gap that ranges between minimum, 58% and maximum, 73%. On the other hand, the compression pressure ratio outputs in Figure 37 at both off design conditions has revealed the operating range stays between 18.6 and 13.5. Hence, the maximum compression pressure ratio and the VIGV opening at 90% design operation have confirmed safe operation with respect to the compressor model developed to operate at maximum pressure ratio of 20 assigned as parameter to avoid surging/stall effect. Thus, the GTP model operates dynamically safe up to full load operation.

Figure 32 Part load air flow rate variation for the respective load settings has shown distinctive variation by the control system strategies implemented for dynamic response verification of the part load settings.
Figure 33 Part load air to fuel ratio tuning variation at the respective air flow rate control for part load settings.

Figure 34 Measured constant turbine exhaust temperatures, TET for the respective load settings of 25 and 90% have shown convergence by the control system strategies implemented in the GTP model off-design simulation. The plot has revealed the GTP model robustness at the step in loading after steady state simulation achieved at 150 seconds.
Figure 35 Part load VIGV tuning variation for maintaining constant TET through air flow rate variation in constant speed off-design simulation. The plot reveals its dynamic response for the step in load perturbation implemented in the off-design simulation.

In the simulation, results of the main GTP model defining variables have also been analyzed and plotted to show their dynamic reaction over the step in loading. Thus, turbine inlet temperature (TIT) and compression pressure ratio/combustion pressure distribution have been plotted in Figure 36 and Figure 37 respectively during the same test. The plots show realistic simulation results of the main GTP model performance variables under the implementation of the control system strategy for the off-design simulation.

Figure 36 Part load Turbine Inlet Temperature variation at the respective part load settings.
4.3. Simulation Result of Cooled and Un-Cooled GTP

The performance of a cooled gas turbine plant model have been analyzed in design mode simulation and compared to the non-cooled GTP model. The simulation results are shown in Figure 38. The figure reveals that the power output has decreased as result of the cooling effect. The same results have been found by (Jordal, 2001), that except in cases of large gas turbine power plant, disadvantages with blade cooling overshadows the advantages from the energy extraction point of view.
4.4. Discussion

The GTP model for dynamic GTP simulation has first been integrated and tested its performance with no control system but with boundary input values to compare with the existing static GT-Unit at Solvina. Thus, results have shown that the GTP model can play a significant role in substituting the gas generating cycle for dynamic GTP integration with control system setting.

Afterwards, the GTP model simulations at design and off design mode have been performed. In the off design mode simulation, step in load perturbation have been introduced after reaching the steady-state simulation period so as to analyze and observe the model dynamic responses in its main defining variables. For that, the GTP model has been set with PI controllers that make the variables vary at different power load adjustment and show the physical dynamics correspondingly.

Design mode simulation has been done with set of VIGV opening at the full position, design compression pressure ratio and design turbine inlet temperature TIT.

The design mode simulation results have been used to set design parameters and variables to be referred within off-design mode simulations.

Each of the off-design simulations have been done for 250 seconds. The GTP measured values and the set values to PID controllers with PI controlling scheme have converged to each other agreeably after dynamically responding to step in load perturbation. The effect of the step in part load perturbation has also been revealed in GTP variables comparatively at the same instant. These effects have been revealed in the plots analyzed in terms of the dynamic increase in the fuel flow, turbine inlet temperatures and compression pressure ratio.

Apparently, fuel mass flow increases uniformly by increase of the load while VIGVs is tuned for constant TET by decreasing the air flow to the compressor in constant rotor speed for all part load settings.

Gas turbine with blade cooling has been simulated to compare the power output. The design mode simulation has shown that power output of the cooled turbine decreased compared to the un-cooled GTP model. This is due to turbine work output decrement from compressed cooling air extraction and turbine work lost due to the main stream enthalpy and total pressure decrement at mixing. Thus, in this case, turbine blade cooling is advised to be considered only from the material point of view.
5. Conclusions

The GTP model has been developed thermodynamically based on zero-dimensional conservation equations applied to each component of the engine for steady state performance calculations and intermediate plena to account for dynamic effects.

Dymola Multi-Engineering Modeling and Simulation have been used for the model development and simulation. Thus, high flexibility in the choice of the adjustment criteria and model reuse is allowed by changing parameter values at the model component according to the required operation.

Performance correction of gas turbine plant at part load operating mode has also been considered at compressor and turbine modeling level.

Design and off-design dynamic simulations of single-shaft constant speed GTP model have been done after the GTP model has been verified with the existing static GT-Unit model at Solvina.

PI control systems have been used for tuning parameters in design mode and for plant control in off-design mode.

Design point calculations on the GTP model have resulted in obtaining the gas turbine plant general design point characteristics.

Tuning of both compressor and turbine models together in design mode simulation has been performed so that design parameters are achieved for off design operation reference.

GTP off-design mode simulations have been made to determine approximate plant working line characteristics that reveal operational variations.

A gas turbine plant model has been developed and tested for its dynamic output response at step in load perturbation after reaching steady-state simulation period.

The GTP has also found to be robust to represent the gas generating cycle to integrate with generator as input shaft power for further inertia, mechanical and generator efficiency consideration.

In summary, a model for dynamic simulation and analysis of gas turbine cycle is presented in this paper. The model has proved to be good at predicting part load analysis of the GTP. The application of the model to different power plant analyses at different operational scenarios will lay basis for the model validation as well as calibration for specific purpose in future works.
6. Future Works

Implementations of simple PI controllers have been done to show the dynamic performance variations of the gas turbine process cycle. Thus, in the future works,

The GTP model in this thesis project should be integrated with power generator model of GTP simulator where moment of rotor inertia, mechanical and generator efficiency are considered.

Implementation of advanced multivariable dynamic control strategies should be done for internal and external GTP regulation. This is in order to optimize the desired power output dependent on the main plant system variables in the gas turbine cycle, generator and transmission grid.

Test scenarios for frequency perturbation and other abnormal effects needs to done to validate and calibrate the plant model for specific operation.
References


