Energy concentration by converging shock waves in gases

by

Malte Kjellander

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Energy concentration by converging shock waves in gases

Malte Kjellander
Department of Mechanics, Royal Institute of Technology (KTH)
SE–100 44 Stockholm, Sweden

Abstract  Converging shock waves have been studied experimentally in a shock tube, and numerically using inviscid calculations and the theory of geometrical shock dynamics. The converging shock waves were created in a shock tube with two modular test sections designed to create cylindrical respectively spherical waves. In the spherical case the shock waves take the shape of spherical cap before propagating into a cone, while the cylindrical shocks converge in a fully circular cylindrical chamber.

The dynamics and symmetry of circular and polygonal cylindrical shock waves with initial Mach numbers ranging from 2 to 4 were studied. The shocked gas at the centre of convergence attains temperatures high enough to emit radiation which is visible to the human eye. The strength and duration of the light pulse due to shock implosion depends on the medium. In this study, shock waves converging in air, argon, nitrogen and propane have been studied. Circular shock waves are very sensitive to disturbances which deform the shock front, decreasing repeatability. Shocks consisting of plane sides making up a symmetrical polygon have a more stable behaviour during focusing, which provides less run-to-run variance in light strength. The radiation from the gas at the implosion centre has been studied photometrically and spectrometrically. The full visible spectrum of the light pulse created by a shock wave in argon has been recorded, showing the gas behaving as a blackbody radiator with apparent temperatures up to 6,000 K. This value is interpreted as a modest estimation of the temperatures actually achieved at the centre as the light has been collected from an area larger than the bright gas core. Circular shock waves attained higher temperatures but the run-to-run variation was significant. The propagation of circular and polygonal shocks was also studied using schlieren photography and compared to the self-similar theory and geometrical shock dynamics, showing good agreement.

Real gas effects must be taken into consideration for calculations at the implosion focal point. Ideal gas numerical and analytical solutions show temperatures and pressures approaching infinity, which is clearly not physical. Real gas effects due to ionisation of the argon atoms have been considered in the numerical work and its effect on the temperature has been calculated.

A second convergent test section was manufactured, designed to smoothly transform a plane shock wave into the shape of a spherical cap. After the convergent transformation the spherical shock propagates through a conical section, where it is aimed to retain the spherical shape and converge in the tip of the truncated cone, which has an end radius of 0.3 mm. Spherical implosion is
more efficient than cylindrical and the target volume is much smaller than that in the cylindrical chamber. The new set-up does not suffer from large losses through reflections. Spectrometric and photometrical measurements of the implosion show significantly stronger radiation of longer duration. Preliminary results show measured apparent blackbody temperatures up to 27,000 K during implosion of shock waves of initial Mach number $M_S = 3.9$.

Descriptors: Shock waves, converging shocks, ionisation, shock dynamics, shock tubes, black body radiation.
Preface

This doctoral thesis in Engineering Mechanics deals with converging shock waves. The work is mainly experimental and complemented with numerical and theoretical work. The advisors for the project have been Dr. Nicholas Apazidis and Dr. Nils Tillmark. An overview is presented in Part I and Part II consists of the papers listed below. The papers published in journals have been adjusted to the thesis format but except minor corrected typographic errors, their content is otherwise unchanged. The respondent’s contributions to each paper are summarised in Part I, Chapter 8.

Paper 1
M. Kjellander, N. Tillmark & N. Apazidis, 2010

Paper 2
M. Kjellander, N. Tillmark & N. Apazidis, 2010

Paper 3
V. Eliasson, M. Kjellander & N. Apazidis, 2007

Paper 4
M. Kjellander, N. Tillmark & N. Apazidis, 2011
Polygonal shock waves: comparison between experiments and geometrical shock dynamics. In proceedings: 28th International Symposium on Shock Waves, 2011, University of Manchester, Manchester, United Kingdom

Paper 5
M. Kjellander, N. Tillmark & N. Apazidis, 2011

Paper 6
M. Kjellander & N. Apazidis, 2012
Numerical assessment of shock tube with inner body designed to create cylindrical shock waves. *Technical report*

Paper 7
M. Kjellander, N. Tillmark & N. Apazidis, 2012

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*Malte Kjellander*
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Part I

Overview and summary
CHAPTER 1

Introduction

Shock waves are essentially waves propagating at velocities higher than the speed of sound. They are very thin and sharply raise the temperature and pressure in the medium they travel through - the stronger the shock, the higher the increase in pressure and temperature. Shock waves can be said to be one of nature’s way of spreading local concentrations of energy and are created by sudden releases of energy, such as lightning strikes or explosions. A shock wave created in a point propagates outwards in all directions, weakening in strength, slowing down, as its front swells. Some energy is dissipated through non-reversible processes within the shock front, which further takes energy away and weakens the shock wave. The shock wave heats the gas it propagates through and in this manner the released energy is spread over a large space. Now reverse the process. By some means, create a shock wave spherical in shape which propagates inwards. Although the dissipative losses within the shock front are still present, the shock wave now accelerates as the available space becomes smaller and gets increasingly stronger. Given perfect symmetry, the shock wave will all but coalesce unto a point, creating a very high concentration of energy.

Converging shocks occur naturally in collapsing spheres, ranging in size from microbubbles to supernovae. Except being of interest from a physicist’s point of view, present and potential applications are found in e.g. medicine and material science. A regular method to deal with troubling kidney or gallstones is by extracorporeal shock wave lithotripsy. Shock waves generated outside of the body are focused on the stones inside the body, which shatters them. The possibility to use similar methods on other types of unwanted intruders, e.g. some types of cancer cells, is being studied. The shock waves generated to break kidney stones are extremely weak, barely stronger than sound waves. Strong converging shock waves are of interest in material synthesis, where the phase, hardness or other characteristics of a material can be changed through shock wave compression: one example is the synthesis of diamonds from carbon. Attempts to initiate fusion reactions which generally required extremely high temperatures have also been made; e.g. gamma-rays have been detected escaping from shock waves converging in deuterium. All these applications have at least one thing in common: it is of importance to be able to create symmetric shock waves converging to a well-defined focus. To create extreme conditions intuition says symmetry is necessary to focus the energy to an as small volume as possible, whereas in the case of lithotripsy, the shock waves must focus on the target stones so that surrounding body tissue is not damaged.

The first work on converging shocks was an analytical study by Guderley (1942), which was followed by experimentally produced shocks about a decade later (Perry
 Already during the first experiments it was found that the amplification of the converging shocks of initially moderate strength heated the gas at focus to such a degree that it became radiating.

The present work is one of basic research. The aim is to study the dynamics of converging shock waves and the light emissions they create in order to determine what level of energy concentrations are achievable. Converging shock waves were produced in a shock tube with two modular test sections: one designed to create cylindrical shock waves and a second designed to shape the plane circular shocks into the shape of a spherical cap. The propagation of the shocks is studied with schlieren photography and the light pulse from the shock implosions investigated by photometry and spectrometry. Figure 1.1 shows a photograph of the light during a run with the cylindrical test section.

![Photograph of radiating argon heated by converging shock wave.](image)

**Figure 1.1:** Photograph of radiating argon heated by converging shock wave.

**1.0.1. Thesis structure**

The main parts of the thesis are the papers presented in Part II. In the introductory Part I the Chapters 2, 3 and 4 are essentially a literature study intended to give an introduction to the topics at hand whereas Chapter 5, 6 and 7 summarise the experimental facility and the results of the present work. The contributions of the individual authors to each paper are stated in Chapter 8. Part II contains seven papers, arranged in the following order: the first three papers appeared in the author’s licenciate thesis whereas the remaining four are added to them in chronological order. Papers 1 and 7 are spectrometric studies on the light emissions created by cylindrical respectively spherical shock waves in argon. Paper 2 is a study concerning the influence of real gas effects on converging shocks using the approximate theory of geometric shock dynamics. Papers 3, 4 and 5 deal with the propagation and dynamics of converging cylindrical shock waves of polygonal and circular forms.
CHAPTER 2

Basic equations

This chapter provides a physical and mathematical description of the gases involved, shock wave jump relations and an introduction to pseudo-steady shock reflections. The gas models used in the thesis are either the standard perfect gas model or the equilibrium model for monatomic gases described in this section.

2.1. Equations of motion

The governing equations of an compressible inviscid fluid are the Euler equations. For an inviscid gas with volumetric mass density $\rho$, temperature $T$, pressure $p$, internal energy per unit mass $e$ and velocity $\mathbf{u} = (u, v, w)$ they are written in the conservation form as follows, neglecting body forces and heat addition (see e.g. Anderson 2003).

The conservation of mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \quad (2.1)$$

The conservation of momentum:

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \mathbf{u} \nabla \cdot (\rho \mathbf{u}) = -\nabla p \quad (2.2)$$

The conservation of energy:

$$\frac{\partial}{\partial t} \left[ \rho \left( e + \frac{u^2}{2} \right) \right] + \nabla \cdot \left[ \rho \left( e + \frac{u^2}{2} \right) \mathbf{u} \right] = -\nabla \cdot (p \mathbf{u}) \quad (2.3)$$

The equations need to be closed with an equation of state. At low pressures and temperatures, most real gases behave as thermodynamically perfect gases and fulfil

$$p = \rho RT = nkT \quad (2.4)$$

where $R$ is the specific gas constant, $n$ the number of atoms per unit volume and $k$ the Boltzmann constant. Departures from the perfect state typically occur when the gas attains very high pressures or temperatures but the simplest definition of a real gas is a gas that does not fulfil the perfect gas law. As a
2. BASIC EQUATIONS

general equation of state does not exist different models for different regimes must be used. For the present study the most important and influential departures are caused by ionisation in argon, which is covered in the following chapter.

2.2. Ionised monatomic gases: equation of state and equilibrium conditions

This section described how the monatomic gases accounting for ionisation are modelled. The model is taken from established theory, see Vincenti & Kruger (1966), Zel’dovich & Raizer (2002) or Cambel & Jennings (1967). Consider a volume of monatomic gas that is heated to high temperatures. As the translational energy of the gas increases, collisions between particles become more frequent and violent. Through the collisions translational energy is transferred to excite electrons to higher levels or transfer them into a free state, ionising the gas. The gas now consists of several components: neutral atoms, electron and ions of different charge. New variables are needed: generally, the subscript $i$ will used for values connected to the ions and $e$ to the electrons. The electron number density is denoted as $n_e$ (dimension $\text{m}^{-3}$) and the number densities of the heavy particles $n_i, i = 0, 1, ..., \ell$ where $i$ is the charge state of the ion and $\ell$ the atomic number. For the neutral atoms, $i = 0$. The total number density of all heavy particles is denoted $n_H$ and is the sum of all $n_i$. The number fraction or degree of ionisation is defined as $\alpha_e = n_e/n_H$. The variable $\alpha_e$ may also be seen as the average number of electrons released by the atoms. The number fractions of heavy particles in ionisation stage $i$ is defined as $\alpha_i = n_i/n_H$. From from these definitions follows that

\[ \sum_{i=0}^{N} \alpha_i = 1 \] (2.5)

Charge is conserved and an ion in stage $i$ has released $i$ free electrons, which translates into

\[ n_e = \sum_{i=1}^{\ell} i n_i = n_H \sum_{i=1}^{\ell} i \alpha_i = n_H \alpha_e \quad \text{and} \]

\[ \alpha_e = \sum_{i=1}^{\ell} i \alpha_i \] (2.6)

The gas consists of a mixture of electron and ionised gases. Assuming that each component individually can be treated as a perfect gas, each has a partial pressure.
2.2. EQUATION OF STATE AND EQUILIBRIUM CONDITIONS

\[ p_e = n_e kT_e \]  \hspace{2cm} (2.8)

\[ p_i = n_i kT_i, \quad i = 0, 1, \ldots \ell \]  \hspace{2cm} (2.9)

for the electrons and ionic components respectively. If the gases are in local thermodynamic equilibrium they may all be described by a single translational temperature, \( T_e = T_i = T \). Using the particle fractions as defined above, the total pressure can then be written according to Dalton’s law as follows, yielding an equation of state for a partially ionised gas,

\[ p = \sum_{i=0}^{\ell} p_i + p_e = kT(\sum_{i=0}^{\ell} n_i + n_e) = n_H kT(1 + \alpha_e) \]  \hspace{2cm} (2.10)

This can be reformulated using \( k = R_i m_i \), where \( R_i \) and \( m_i \) is the specific gas constant and molecular mass for the component \( i \). The weight differences between ions of different stages are negligible and the weights and specific gas constants of all ionic components can be approximated with those of neutral argon, \( m_i \approx m_A \) and \( R_i \approx R_A \). Dropping the index from the gas constant, this leads to \( \rho \approx m_A n_H \) and

\[ p = \rho RT(1 + \alpha_e) \]  \hspace{2cm} (2.11)

The internal energy content at equilibrium of the gas is divided into translational energy, potential energy of the ions and energy bound in excited electronic states. Each atom, ion or electron has three degrees of freedom and each therefore contributes \( 3/2kT \). When a heavy particle is ionised, the energy required to remove the electron becomes bound as potential energy. The energy required to ionise an atom or ion from state \( i-1 \) to state \( i \) is \( I_i \). The total energy required to remove \( N \) electrons from an atom is therefore \( I_{tot} = I_1 + I_2 + \ldots + I_N \). There also exist electrons excited to higher levels within the ions, whose excitation energy is designated \( W_i \). Summarising, the internal energy per unit mass of the ionised gas may be expressed as (remembering that \( k/m_i = R_i \approx R \))

\[ e = \frac{3}{2}(1 + \alpha_e)RT + R \sum_{i=1}^{\ell} \alpha_i \sum_{j=1}^{i} \frac{I_j}{k} + R \sum_{i=0}^{\ell} \alpha_i \frac{W_i}{k} \]  \hspace{2cm} (2.12)

The energy of the excited states may be found from statistical mechanics,

\[ W_i = kT \frac{\partial \ln Q_i^{el}}{\partial T} \]  \hspace{2cm} (2.13)

where \( Q_i^{el} \) is the electronic partition function of component \( i \). The equilibrium values of the ionisation fractions can be determined from the Saha equation, which rewritten using the particle fractions becomes
\[
\frac{\alpha_{i+1}}{\alpha_i} = 1 + \frac{\alpha_e}{\alpha_c} \left( \frac{2\pi m_e}{\hbar^2} \right)^{3/2} \frac{(kT)^{5/2}}{p} \frac{2Q_{el}^{i+1}}{Q_{el}^i} \exp \left( -\frac{I_{i+1}}{kT} \right)
\]  
(2.14)

where \( m_e \) is the electron mass and \( h \) the Planck constant. For a given \( p \) and \( T \) equation 2.14 can be solved, e.g. by the iterative method of Trayner \& Glowacki (1995). Appendix B contains derivations of the energy and Saha equations and for further reading on the topic ionisational equilibrium, see e.g. Drellishak et al. (1963) or Ebeling (1976).

2.3. Shock waves

A shock wave can be briefly described as a wave with finite amplitude, travelling in a medium at velocities higher than the speed of sound in that medium. Over the shock wave the pressure, velocity, temperature and density change abruptly. This change is not reversible; inside the shock wave dissipation of energy occurs, the entropy increases. Shock waves occur in nature when excessive amounts of energy is released rapidly, such as the crack of lightning or during volcano eruptions\(^1\). They also occur when an object is travelling at supersonic speed in a medium – or vice versa, if the medium itself is travelling at supersonic speed compared to its surroundings. The physical shock wave is very thin - of the order of a few mean free paths. The entire width of a shock wave therefore only contains a small number of particles in the longitudinal direction and thus the shock appears as nearly a singularity in the continuum model - yet the existence of shock waves was predicted by considering certain waves travelling in a fluid governed by the Euler equations. Before they were studied in any laboratory, what is now called shock waves were discussed as a mathematical peculiarity by prominent 19th century scientists. It was for a long time an open question whether they existed at all in the physical world. A short summary is provided here – for a more in-depth description on the historical development, see e.g. Salas (2007).

Poisson (1808) was the first to solve the propagation of a wave in a fluid described by the Euler equations. A problem, or ”a difficulty” as Stokes (1848) called it in his paper treating the subject, appears when considering that different parts of a sinusoidal wave travel at different velocities. Given enough time, the front of the wave will steepen until it becomes vertical: suddenly the solution breaks down. Although he later changed his mind, Stokes suggested that once such a breakdown appears a possible physical result is that the front of the wave continues its motion as a sharp discontinuity.

Riemann (1860) solved the propagation of various initial discontinuities, although he assumed the jumps to be isentropic. He introduced the invariants now bearing his name and the method of characteristics to trace the paths of

\(^1\)If these examples interest the reader he is diverted to e.g. Jones et al. (1968); Saito et al. (2001).
the jumps. Working from a thermodynamic rather than mathematical point of view, Rankine (1870) and Hugoniot (1889) presented the well-known jump conditions over a discontinuity by considering the conservation of mass, momentum and energy. Without the entropy condition, solutions exist for both compression and rarefaction shocks (i.e. where pressure increases and decreases, respectively). The laws of thermodynamics had not yet been firmly established and even though Hugoniot stated that the entropy increases over the compression shocks, it was not until the early 20th century that Lord Rayleigh and Taylor determined that only compression shocks exists in nature, due to the second law of thermodynamics.

The first to actually observe and visualise shock waves was Töpler in 1864. For the purpose he used the schlieren technique that he had recently invented and observed shocks created by electric discharges. Using a precise timing circuit he flashed the schlieren light source after a certain duration from each discharge. By continuously discharging the source and by flashing at the same moment a seemingly stationary schlieren image could be seen in the viewing telescope, which he documented by drawing them by hand. Some of his images and a biography has been published by Krehl & Engemann (1995). Töpler intended to visualise sound and it was not clear what actually had been observed - he designated them as sound waves, travelling at the speed of sound. It would be Mach who, in a set of experiments during 1875-1888 – partly using Töpler’s techniques – would not only visualise shock waves but also conclude that these were not sound but the discontinuities described by Riemann. Among other things, Mach also experimentally showed the steepening of a pressure-pulse into a shock wave, the irregular reflections that bear his name and a host of other contributions to many fields of research. The study of shock waves may have had little practical use in the 19th and early 20th century, but with the advent of supersonic flight and high-speed internal flows the purely mathematical and slightly academic discontinuities of Stokes and Riemann have become a major research field.

2.3.1. Generalised Rankine-Hugoniot relations

Figure 2.1(a) illustrates a standing normal shock wave and particle paths. The frame of reference is chosen so that the shock is stationary. The gas upstream is in a known state (1) and flows into the standing shock with a supersonic Mach number $M_1 = u_1/a_1$. After passing through the shock, which is treated as a sharp discontinuity, the gas continues at a lower and subsonic Mach number $M_2 = u_2/a_2$ being in a new state (2). The unknown state (2) is sought. Assuming that the change of quantities is immediate, consider the states immediately up- and downstream of the shock.
By the laws of conservation,

\[ \rho_1 u_1 = \rho_2 u_2 \]  
\[ \rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2 \]  
\[ h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \]

where \( h = e + pV \) is the enthalpy. The entropy condition demands that dissipation causes the entropy of the gas to increase when it passes the shock, or \( s_2 \geq s_1 \). For calorically perfect gases, where \( p = \rho RT \) and \( h = c_p T \) with a constant \( c_p \), the above set of equations can be solved to give the well-known normal shock relations presented by Rankine and Hugoniot. For many problems a more intuitive approach is from a frame of reference that is at rest with the flow ahead of the shock. The system can be transformed to such a laboratory frame where the shock wave is moving with a velocity \( u'_s = M'_s/a_1 \) into an undisturbed state \( 1 \) by setting \( u'_s = -u_1 \) and \( u'_2 = u_2 - u_1 \). This is illustrated in Fig. 2.1(b): note that the Mach number of the flow behind the shock \( M'_2 \) might now be either sub- or supersonic in that frame. The jump relations solved by Eqs. 2.15-2.17 are identical for both frames.

Consider now a shock so strong that the gas in the post-shock condition \( 2 \) may no longer be considered perfect. Depending on the gas, it might experience vibrational and rotational excitation, chemical reactions, dissociation or ionisation. It is no longer possible to close the system with a simple equation of state, and iterative methods are needed to find a solution, see for example...
2.3. SHOCK WAVES

\[ M < 1 \]

\[ M > 1 \]

\[ p_2, T_2, \rho_2, u_2, h_2, \alpha_{i,2} \]

\[ p_1, T_1, \rho_1, u_1, h_1, \alpha_{i,1} = 0 \]

Figure 2.2: Normal shock wave. The state (1) ahead of the shock is known and the equilibrium state (2) is sought.

Resler et al. (1952), Kozlov & Stupitskii (1968), Nieuwenhuijzen et al. (1992) or Michaut et al. (2004). In this overview a short description of the solution procedure for shocks in monatomic gases is added: a strong shock wave is moving into a gas with known conditions \( p_1, T_1, \rho_1, u_1, h_1 \) and ionisation \( \alpha_{i,1} \). The gas in region (1) is in such a state that \( \alpha_{i,1} = [\alpha_1, \alpha_2, ..., \alpha_\ell]_1 \) can be considered equal to zero. In a frame attached to the shock wave the system is such as illustrated in Fig. 2.2. To find the state (2) the set of Eqs. 2.15-2.17 needs to be solved. The enthalpy for a monatomic gas in the upstream condition is

\[ h_1 = \frac{3}{2}RT + pV = \frac{5}{2}RT. \]

In the post-shock state the gas is ionised; with the energy equation Eq. 2.12 and equation of state Eq. 2.11 the enthalpy in state (2) becomes

\[ h_2 = \frac{5}{2}(1 + \alpha_{e,2})RT_2 + R \sum_{i=1}^{\ell} \alpha_{i,2} \sum_{j=1}^{I} \frac{f_j}{k} + RT_2^2 \sum_{i=0}^{\ell} \alpha_{i,2} \frac{\partial \ln Q_i(T_2)}{\partial T} \quad (2.18) \]

The last term, the electronic excitation energy, is often much smaller than the first two and may then be neglected. To calculate \( \alpha \), local thermodynamic equilibrium is assumed to be established instantaneously and the species distribution is found from the Saha equation, Eq. 2.14. The set of equation is closed, but an iterative method is necessary to find the post-shock conditions:

1. An initial value of \( \rho_1/\rho_2 \) is estimated, based on e.g. the standard Rankine-Hugoniot equations.
2. New values of \( p_2, u_2 \) and \( h_2 \) are calculated using Eqs. 2.15-2.17.
3. With the new values, a temperature that simultaneously fulfils the enthalpy according to Eq. 2.18 and the equilibrium conditions according to Eq. 2.14 is found using a numerical method.
4. A new \( \rho_1/\rho_2 \) can now be found from the equation of state, Eq. 2.11, which is used as a new guess in step 1. The process is repeated until the
error between the guessed and resulting values is as small as acceptable
or machine allows.

Figure 2.3 shows the equilibrium conditions behind a normal shock wave in
argon with initial temperature $T = 293$ K and three different initial pressures
$p_1 = 0.1, 0.01$ and $0.001$ atm. The dashed lines are the Rankine-Hugoniot
relations for a perfect gas without ionisation depending only on Mach number.
The ionisation has a strongly limiting effect on the temperature as energy is
transferred from translational to potential energy. Whereas the compression
approaches an asymptotic value ($p_2/p_1 = 4$ for $\gamma = 5/3$) for the constant-
composition gas this is not the case for the ionising shock. The peak corre-
sponds to the maximum of the first ionisation stage, whereafter the transla-
tional energy increases relative to the potential energy, resulting in a decrease
of density.

![Figure 2.3](image)

Figure 2.3: Effect of ionisation on shock jump conditions (a-c) at three different
initial pressures $p_1 = 0.1, 0.01$ and $0.001$ atm. Dashed lines represent the non-
reacting Rankine-Hugoniot solution. The ionisation is presented in (d).
2.3. Shock Waves

2.3.2. Shock structure

In the previous sections it was stated that the shock front was only a few mean free paths thick, and that the gas assumes equilibrium conditions after shock passage. However, the picture becomes more complicated when the temperature jump is so large that reaction processes start behind the shock. The structure of strong shocks in argon has been studied extensively and a working model for the processes has been developed for shock Mach numbers $M \approx 15 - 30$: see e.g. the theoretical studies (Bond 1954; Gross 1965; Wong & Bershader 1966; Hoffert & Lien 1967; Biberman et al. 1971; Matsuzaki 1974; Kaniel et al. 1986) and experimental (Petschek & Byron 1957; Fomin et al. 2003; Yakovlev 2006). Without going into detail, the basic features are presented here.

Figure 2.4 shows a qualitative sketch of the shock structure in a monatomic gas with the variation of temperatures and ionisation fraction. Region (1) contains the undisturbed gas, while region (4) is the post-shock state in thermodynamic equilibrium. Region (2) is the very front of the shock, where translational equilibrium is reached after only a few collisions. The thickness of this region is thereby a few mean free paths. Immediately behind the front is the so called frozen condition as determined with the standard Rankine-Hugoniot equations. Region (3) is the relaxation zone, in which the gas attains its equilibrium values. In this non-equilibrium area the electron and ion gases have different translational temperatures, $T_e$ and $T$. A commonly used model is the two-step ionisation process: in the first step atoms are excited to the first electronically excited state and in the second step the excited atoms are subsequently

![Figure 2.4: Structure of an ionising shock in a monatomic gas, moving towards the right in the positive $x$-direction: sketch of the temperature and ionisation distribution. Region 1: undisturbed gas, 2: shock front, 3: relaxation zone, 4: equilibrium post-shock state.](image-url)
ionised. Catalysts for both reactions are either heavy particles or electrons. Once a certain number of energetic electrons have been generated the process increases rapidly – giving rise to a so-called electron avalanche – which can be seen in the sudden increase in ionisation in the figure. The relaxation zone can be substantial: e.g. for a shock of strength $M \approx 10$ in initial $p_1 \approx 1$ kPa and $T_1 = 300$ K it is several centimetres. Increasing the shock strength to $M_S \approx 20$ decreases the relaxation zone to less than 0.1 mm (Zel’dovich & Raizer 2002).

2.4. Pseudo-steady shock reflections

The interactions between several shock waves or between shocks and solid boundaries are important in this work, so a brief introduction will be given here. The categorisation and transition of reflections are subject to ongoing research and this section is based on the review of the current state of research by Ben-Dor & Takayama (1992) and Ben-Dor (2006). The reflection pattern appearing when a shock wave collides with an inclined solid surface is dependent on the wall inclination $\theta_w$ and Mach number $M$ of the incoming shock wave (and also on the state of the gas; this introduction only deals with ideal conditions).

The different patterns are categorised in two main groups, regular and irregular reflections. Figure 2.5 illustrates some of the possible shock reflections. A plane shock wave $i$ is moving perpendicularly along a surface from left to right, with velocity $u_S$ and Mach number $M_S$. The shock propagates into a gas at rest. At a certain point it strikes a wall inclination with angle $\theta_w$. If $\theta_w$ is large enough, regular reflection occurs, Fig. 2.5 (a), where the reflected shock $r$ is connected to the incident shock at the surface (point P). Although the shock waves are not stationary in the laboratory frame, the flow is steady in a reference frame attached to point P and such systems are referred to as pseudo-steady.

Irregular reflections occur when the angle $\theta_w$ is so small that a physical flow can not be established by the regular reflection pattern (a). Irregular reflection includes von Neumann reflection and different forms of Mach reflections. Two different Mach reflections are shown in (b) and (c). A shock wave $m$ normal to the surface appears – called a Mach stem after its first observer – inducing a parallel flow close to the surface. The incident and reflected shock waves instead coalesce with the stem at a point away from the wall, called the triple point (T). A slip line divides the gas that has passed the incident and reflected shock from the gas affected by the stem. If the flow immediately behind the triple point between $r$ and $s$ is supersonic relative to $T$, the near part of the reflected shock wave becomes straight. This pattern is designated as a transitional Mach reflection (c). A von Neumann reflection is a weaker form of irregular reflection, where the reflected shock $r$ is a compression wave. Experimental visualisation of many types of reflections can be found in e.g. Takayama & Ben-Dor (1993). The different reflection domains are sketched in Fig. 2.6 for shock waves in
2.4. PSEUDO-STEADY SHOCK REFLECTIONS

perfect air and argon up to $M_S = 5$. It should be noted, that for high Mach numbers non-ideal gas effects have large influences on the regimes (see e.g. Ben-Dor & Glass 1979, 1980).

Figure 2.5: A few shock reflection types: (a) regular reflection; (b) single Mach reflection; (c) transitional Mach reflection. The streamlines in figure (a) are presented as seen from a frame of reference attached to the intersection point P.

Figure 2.6: Approximate reflection regimes for shock waves in perfect air (a, $\gamma=1.40$) and argon (b, $\gamma=1.66$), after Ben-Dor (2006) and Lee & Glass (1984) respectively. The transition lines from (b) are inserted as dashed lines in (a) for comparison.
Shock tubes

Shock tubes are devices used primarily to study high temperature gas kinetics, shock wave interactions and high speed flow. The first shock tube was built slightly more than a century ago: the originator was the French chemist and inventor Paul Vieille (1854-1934) who, working for the French armouries, also invented the smokeless gunpowder. During experiments with detonations he had detected waves in non-reacting gas. For the purpose of investigating if these were the discontinuous waves then recently described by Hugoniot, he constructed the first of the devices which are now called shock tubes: a four meter long tube divided in two sections by a thin diaphragm. One end was filled with air at atmospheric pressure, the other at a high pressure. In a series of experiments, using purely mechanical detectors he registered shock waves travelling at about 600 m/s (Vieille 1899–1900) as well as the expansion wave travelling the opposite direction. A few shock tubes experiments were conducted during the Interwar period (Schardin 1932) but it was not until after the Second World War that a large number of shock tubes appeared in research facilities in many countries, beginning with Payman & Shepherd (1946) in the UK, Bleakney et al. (1949) in the US and Soloukhin (1957) in the USSR. This chapter provides a short introduction to the workings of the simple shock tube. Much of the collected information is based on Oertel (1966) and references therein.

3.1. The simple shock tube

A simple shock tube is a long tube, usually with a rectangular or circular cross section, consisting of two sections separated by a thin membrane. The first is called the driver section and is filled with a gas at high pressure. The other, low pressure section is called the driven section. A shock tube at initial conditions is sketched in Figure 3.1. When the membrane is broken, a shock wave is formed, travelling down the tube. After reflecting on the end wall, the shock wave travels through the previously shock-heated gas compressing and heating it further. The gas is ideally at rest in this hot reflected zone and is used for studying thermodynamics and reactions in hot gases.

3.1.0a. Flowfield. In the ideal situation, a one-dimensional flow without viscosity where a shock wave and expansion is instantly formed at membrane burst,
3.1. THE SIMPLE SHOCK TUBE

![Diagram of shock tube](image)

Figure 3.1: Shock tube before membrane burst. Initially the high and low pressure gases are in states (4) and (1) respectively.

The flow field and wave propagation can be solved explicitly from the known initial conditions in states (1) and (4). Fig. 3.2 shows the ideal solution of a shock tube run with air in both sections ("air-air") with \( p_4=14 \) atm and \( p_1=1 \) atm. At \( t=0 \) the membrane bursts and two waves are formed: one shock wave travelling downstream with constant velocity \( u_S \) and Mach number \( M_S = u_S/a_1 \) and one expansion wave propagating with \( u - a \) or in words, propagating upstream relative to the gas with the local speed of sound. When the gas in the driven section is passed by the shock wave it is compressed to a state (2) and a momentarily accelerated to a velocity \( u_2 \). The high pressure gas is expanded in the expansion wave to a state (3) with the same velocity as the shock-compressed gas and is moving into the driven section with the same velocity as the shock-induced flow, \( p_3 = p_2 \) and \( u_3 = u_2 \). The front of the expansion travels upstream with velocity \( -a_4 \) (as \( u_4=0 \)) and the tail with velocity \( u_3 - a_3 \); the tail may move either towards the left or right depending on whether \( u_3 \) is sub- or supersonic. The whole flow can thus be divided into a number of states: (1) the pre-shock initial low pressure state, (2) the shock-compressed state, (3) the expanded cold state, (4) the initial high pressure state. Between states (3) and (4) is the expansion wave wherein the gas conditions change continuously.

The shock Mach number \( M_S \) is dependent on the pressure, the speed of sound and heat capacity ratio in the initial high and low pressure gases. An expression relating the pressure ratio to the Mach number is given in below (for an explicit derivation, see e.g. Resler et al. 1952):

\[
\frac{p_4}{p_1} = \left[ \frac{2\gamma_1 M_S^2 - (\gamma_1 - 1)}{\gamma_1 + 1} \right] \left[ 1 - \frac{\gamma_4 - 1}{\gamma_1 + 1} \frac{a_1}{a_4} (M_S - \frac{1}{M_S}) \right]^{-\frac{2\gamma_1}{\gamma_1 - 1}} \tag{3.1}
\]
Figure 3.2: An example run in air-air: $p_4=14$ atm and $p_1=1$ atm, $T_4 = T_1 = 300$ K, $M_S=1.7$. Flow field (a) and gas conditions at time $t = 0.5$ ms (b) through (e). A wave diagram is shown in (f). The diaphragm is situated at $x=0.33$ m.
3.1. THE SIMPLE SHOCK TUBE

With a known $M_S$, state (2) is determined from the normal shock relations. State (3) can then be determined as the velocity and pressure are the same as in state (2) and as it has been isentropically expanded from state (4), the isentropic relations give the density and temperature. When the expansion front reaches the left wall it will reflect and travel back into the expansion, creating a complex region. The values in the expansion, including the complex region, can be calculated using the method of characteristics. The velocity and Mach number $M_R$ of the reflected shock wave at the right wall are determined by considering that the gas behind it must be at rest. With a known $M_R$ state (5) behind the reflected shock can be acquired from the shock jump conditions.

Returning to Eq. 3.1: the Mach number of the shock wave $M_S$ apparently depends on the initial ratios of the pressures $p_1$ and $p_4$ and the speeds of sound $a_1$ and $a_4$. Considering the limit $p_4/p_1 \rightarrow \infty$ yields and interesting result,

$$M_{max} = \frac{\gamma_1 + 1}{\gamma_4 - 1} \frac{a_4}{2a_1} + \sqrt{1 + \left(\frac{\gamma_1 + 1}{\gamma_4 - 1} \frac{a_4}{2a_1}\right)^2} \quad (3.2)$$

or, for strong Mach numbers,

$$M_{max} \approx \frac{\gamma_1 + 1}{\gamma_4 - 1} \frac{a_4}{a_1} \quad (3.3)$$

An upper limit for the achievable Mach number is set by the ratio of speeds of sound no matter how much the pressure is increased. Lighter driver gases thereby generate stronger shock waves and common drivers are, besides cheap air, H$_2$ and He. Figure 3.3 shows ideal shock Mach numbers $M_S$ for different gas combinations and pressure ratios.

3.1.0b. Measurement times. During studies of e.g. reaction rates in the hot gas in the zone behind the reflected shock, designated here as state (5), it is essential that the measurement time is long enough. The available time for measurements can be determined and optimised by studying the wave propagation. As such wave diagrams as the one presented in Fig. 3.2 are helpful tools. The measurement time is the time from the instant the shock arrives at the end wall until the first disturbance - a reflected shock or expansion - reaches the end wall and changes the gas conditions. By changing the lengths of the low and high pressure sections the wave pattern can be altered in such a way that the measurement time is prolonged. In general the optimal time is when the reflected shock wave, the reflected expansion from the left wall and the contact surface all confluence at the same time.

The measurement time can be increased if the gases and $M_S$ are chosen such that the reflected shock passes the contact surface without reflection. This can only happen if the driver gas after passing of the reflected shock comes to rest and the pressures in the reflected states 1’ and 2’ (the states of 1 and 2 that has been passed by the reflected shock) are equal. The contact
Figure 3.3: Shock Mach number as a function of pressure ratio for different driver-driven gas combinations according to the ideal solution Eq. 3.1. Driven and driver gases are assumed to have the same initial temperature.

As the calculations on measurement times are somewhat lengthy the reader is referred to Oertel (1966).

3.1.0c. Non-ideal effects. Various processes create large or small deviations from the ideal calculations. For the prediction of the shock Mach number, experiments have shown good agreement with the ideal solution for low pressure ratios, but for higher ratios, $p_4/p_1 \gtrsim 10^4$, the ideal solution predicts lower Mach numbers than experiments have shown. Causes for this discrepancy may be heating of the high pressure vessel during filling of the pressurised gas, multi-dimensional effects and finite formation of the shock. The membrane opening is finite and a shock is not instantly formed. When the membrane bursts pressure waves start propagating into the low pressure gas. The speed of sound behind the successive waves increases, resulting in a compression of the waves into a shock wave – a compression shock. White (1958) developed a
one-dimensional theory for a finite opening, which was closer to experimental results but still under-predicted the Mach number. Axisymmetric calculations on multi-dimensional effects were carried out by Petrie-Repar & Jacobs (1998) showing that this too had effect.

Behind the shock front a viscous boundary layer is formed along the walls. For combinations of geometry and shock strength the boundary layers of opposite walls may even unite and the flow is completely turbulent. After reflection, the shock wave will propagate into the boundary layer it had induced and a bifurcation zone is formed as the reflected shock interacts with the boundary layer. Boundary layer effects are large and interactions of boundaries and the contact surface can have great impact on the flow, even decelerating the shock front (Emrich & Wheeler 1958).
CHAPTER 4

Converging shock waves

This chapter recounts past studies of converging shock wave in a brief review. A large number of papers have been published on the topic, and far from all are mentioned here. Instead a selection has been made that connects more closely to the present study.

4.1. Theoretical background

The first study of converging shock waves was made by Guderley (1942). For the implosion of strong cylindrical and spherical shock waves in an inviscid, perfect gas he derived a local self-similar solution to the gas-dynamic equations of the form

\[ \frac{r}{r_0} = \left(1 - \frac{t}{t_0}\right)^\alpha \]  

(4.1)

where \( r_0 \) is the initial radius at the time \( t = 0 \) and \( t_0 \) is the instant of focusing, when \( r = 0 \). The self-similarity exponent \( \alpha \) governs the acceleration of the front, where \( \alpha = 1 \) implies a constant velocity. The solution to the problem is not trivial and values of the exponent, which is dependent on the gas in which the shock is propagating, are determined numerically\(^1\). Solutions to the local and global – taking into consideration the initiation of the shock – problems and determinations of self-similar exponents with an increasing number of significant digits have been made in a great number of studies, e.g. Butler (1954), Stanyukovich (1960), Fujimoto & Mishkin (1978), Lazarus & Richtmyer (1977), Lazarus (1981), Van Dyke & Guttman (1982) and Ponchaut et al. (2006). Table 4.1 shows the history of the exponent for cylindrical and spherical shock waves for \( \gamma = 7/5 \) and \( \gamma = 5/3 \) and corresponding shock trajectories and velocity increase are plotted in Fig. 4.1. Fujimoto & Mishkin (1978) used a different approach and claimed that the problem might be solved in closed form, which yielded quite different values compared to the rest. Other authors challenged the validity of their method (Lazarus 1980; Van Dyke & Guttman 1982; Wang 1982). Nakamura (1983) used the method of characteristics to solve the problem and acquired exponents agreeing well with the self-similar

\(^1\)An overview of the solution process, and of self-similar problems in general, is presented by Zel’dovich & Raizer (2002), pp. 794–806
solution. Chisnell (1998) made an approximate analytical determination of the exponent agreeing very well with those acquired from the exact form. His solution also gave a description of the flow field at all points behind the converging shock front.

Figure 4.1: Spherical and cylindrical solutions to the self-similar solution Eq. 4.1 for two different \( \gamma \) (values for \( \alpha \) taken from Tab. 4.1). Shock trajectories (a) and velocity amplification for shocks with the same initial velocity at \( u(r = r_0) = u_0 \) (b).

Approximate methods neglecting the influence of the flow behind the shock wave were developed independently by Chester (1954), Chisnell (1955, 1957) and Whitham (1958). It is a geometrical approach based on tracking the shock fronts along rays perpendicular to the fronts, analogous to acoustic wave theory. The approach, called the CCW-method after the listed authors above, or geometrical shock dynamics (GSD), works well also with converging shocks and results in good approximations of the similarity solutions. The theory has been expanded by Whitham to allow uniform flow in front of the shock and by Apazidis et al. (2002) to also account for shocks propagating into non-uniform flows. A comparison of the solutions of self-similar theory, geometric shock dynamics as well that of a numerical Euler solver was presented by Hornung et al. (2008), showing good agreement.

### 4.2. Experiments in shock tubes

To create a radially diverging shock wave is relatively uncomplicated. An explosion or electric spark generates an even shock, propagation radially from the point of the charge. To create a converging shock wave is, naturally, more complex.

\[^2\text{Value depending on initial Mach number.}\]
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<table>
<thead>
<tr>
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<td>0.831 ± 0.002</td>
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Table 4.1: Guderley’s self-similarity exponent. Significant digits of numerical values reduced to three. \(M_S\) is the initial shock strength during the experiments.

Problematic. The methods used by researchers have in principle been variations of two different methods. One, to do the opposite of the point explosion and diverging shock: to generate a shock by placing explosives or an array of spark plugs on a spherical (or cylindrical) periphery, and two: to generate a plane or diverging shock wave and shape it into a converging spherical (or cylindrical) shape by shock reflection and diffraction. In this section is a short review and references to past experiments. Fig. 4.2 shows a collection of different means of creating shock waves.

The first experiments on converging shock waves were carried out by Perry & Kantrowitz (1951). They used a standard shock tube with a tear drop-shaped and centrally aligned inner body, sketched in Fig. 4.2 (a). It’s a device that belongs to the second category described above: initially plane shock waves are reflected and diffracted around the tear-drop to form cylindrical shock waves on the downstream side – the shock front at two instants is illustrated in the figure. They found that the shock waves managed to concentrate enough energy to make the gas at the centre of implosion emit light - even more so when argon was used as a test medium. The production of light was believed to be caused by ionised gas and taken as an indicator of high pressures and
temperatures. Two different initial Mach numbers were studied, 1.4 and 1.8. The convergence process was studied with schlieren optics. They observed that symmetrical shapes were more difficult to achieve when the shock waves were initially stronger. For stronger shock waves, reflections appeared on the front breaking up the symmetry.

The design of Perry & Kantrowitz inspired several other works. Kleine (1985) and Takayama et al. (1987) investigated the dynamics and stability of converging circular cylindrical shock waves in air in two different shock tubes with similar construction. The stability and propagation was investigated and they found experimental values for the self-similarity exponent, which agreed well with theory (see Table 4.1). Watanabe & Takayama (1991) continued the stability experiments.

To avoid the need for supports of the inner body that generate the disturbances, a vertical shock tube with an annular membrane was built at the Tohoku University (Watanabe et al. 1995; Hosseini et al. 2000). The resulting converging shocks kept the circular form better than in shock tubes with supports for the inner body. Deformation of the shock shape still occurred however, and reason for this was believed to be small changes in area between the inner and outer body of the coaxial channel. Also in this experiment cylindrical rods were placed in the test section to introduce corresponding disturbances in a controlled way. One conclusion was that when several modes were combined, the lowest dominated the others.

Hosseini & Takayama (2005b) also constructed a hemispherical chamber for focusing of shock waves created by explosives. The final Mach number of the converging shock was between 2.5 and 8. They created a transparent chamber with aspheric outer surface in order to use holographic interferometry. They produced high-speed video recordings of the shock wave propagation and discussed the influence of different methods of shock generation had on shock stability.

4.3. Shocks waves initiated by detonations or explosions

Lee, Lee and Knystautas at the McGill University in Montreal performed experiments with converging detonation and shock waves in different cylindrical chambers filled with acetylene-oxygen mixtures. Lee & Lee (1965) used a cylindrical drum - a "bomb" - divided in two halves by a disc, with an annular opening between the disc and drum wall that allowed the shock to pass from one section to the other. The explosive gas was ignited in the centre of one of the sections, creating a diverging detonation wave entering the second section where it imploded. They found that the detonation strengthened and imploded as a strong shock.

A second implosion chamber (Knystautas & Lee 1967; Knystautas et al. 1969) was created which essentially was a cylindrical disc with a large number of spark plugs positioned in the ends of channels entering the chamber wall.
4. CONVERGING SHOCK WAVES

Figure 4.2: Principle of operation of a few experimental devices designed to create converging shock waves. Devices (a) and (b) create cylindrical shocks, (c) and (d) spherical. Part (a), the first experiments: the shock tube of Perry & Kantrowitz (1951). The plane shock is transformed into a cylindrical shape by the tear drop. A glass window provides optical access. Part (b), the cylindrical implosion device of Knystautas & Lee (1971). The gas is ignited in one end and the detonation wave propagates through the annular section \((t_1 - t_2)\). Before entering the implosion chamber \((t_3)\) it passes through a converging duct to compensate for the attenuation in the bend. Part (c), a hemispherical implosion chamber such as the one employed by Glass (1967). The chamber is filled with an explosive gas which is ignited at the centre. A detonation wave is created (time \(t_1\)), which reflects on the wall and converges as a strong shock \((t_2)\). Part (d), the spherical implosion chamber of Terao (1984). Detonation is initiated by a spark plug at the top of the inlet tube \((t_1)\). The wave enters a cylindrical space \((t_2)\) before it is diverted via a large number of ducts into the implosion chamber into the shape of a spherical segment \((t_3)\).
4.3. SHOCK WAVES INITIATED BY DETONATIONS OR EXPLOSIONS

Arranged in an even array, they were simultaneously discharged to ignite the gas around the periphery. The detonation waves exit the channel and enter the cylindrical chamber. Knystautas et al. (1969) measured the intensity of the light from the implosion focus at two wavelengths and compared to a blackbody radiator, estimating a maximum temperature\(^3\) of \(18.9 \times 10^4\) K. The stability of cylindrical shocks was investigated with a third chamber which was a much improved version of the drum (Knystautas & Lee 1971); it is shown in Fig. 4.2(b). They reported that transverse waves distributed local perturbations thereby attenuating disturbances. Another conclusion was that the energy densities attainable at implosion focus are practically limited by the degree of symmetry.

A considerable amount of work has been made at the University of Toronto by I. I. Glass and co-workers (Flagg & Glass 1968; Roberts & Glass 1971; Glass et al. 1974; Glass & Sharma 1976; Roig & Glass 1977; Glass & Sagie 1982; Saito & Glass 1982). Their research was focused on a hemispherical implosion chamber – a simple sketch of its workings is shown in Fig. 4.2(c) – working on the following principles: in the geometrical centre of the chamber, detonation or shock waves are initiated by explosives or exploding wires. The waves reflect off the periphery and converge as strong shock waves. As a shock wave implodes and reflects from the geometrical centre, a high pressure and temperature region is produced. Roberts & Glass (1971) measured the emission from the light produced during and after implosion. The chamber was filled with oxygen-hydrogen gas at high pressures (6.8-27.2 atm). They found the radiation to be continuous with an apparent blackbody temperature of \(~5000\) K. Saito & Glass (1982) made further spectrometric studies in \(\text{H}_2 - \text{O}_2\). A smaller area was investigated and higher temperatures could be measured as averaging effects with colder regions could be avoided: around \(10 - 13 \times 10^3\) K for regular runs and up to \(17 \times 10^3\) K when the imploding shock was boosted by explosives lined on the periphery. Except for studies on the gas conditions at implosion, the device was also used as a shock tube driver (Glass et al. 1974), to launch projectiles (Flagg & Glass 1968), to synthesise diamonds (Glass & Sharma 1976) and, filled with deuterium-oxygen, in fusion initiation experiments (Glass & Sagie 1982).

Matsuo et al. at the Kumamoto University have conducted a series of investigations using a cylindrical implosion chamber, in which converging shocks are generated by explosives lined on the circular periphery. The light emission at the focus, produced by shocks in air, was measured and compared to the blackbody function. Time-resolved intensity was measured with photomultiplier tubes at a number of separate wavelengths between 400 and 500 nm and temperatures in the range of 13,000-34,000 K were found (Matsuo & Nakamura

\(^3\)It was later pointed out by Ref. Roberts & Glass (1971) that the temperature analysis was flawed due to erroneous use of Wien’s law.
4. CONVERGING SHOCK WAVES


Terao (1983) constructed a cylindrical and a hemispherical chamber to study converging cylindrical and spherical shock waves, carrying out pressure and propagation measurements. In a number of papers measurements on spherical converging detonation waves in a propane-oxygen gas were presented (Terao 1984; Terao & Wagner 1991; Terao et al. 1995). The propagation and pressure evolution of the shocks were studied and compared with theory. Spectrometric measurements on the light emissions were made and high gas and electron temperatures at the implosion focus (Terao et al. 1995) were reported. One of his constructions is sketched in Fig. 4.2 (d).

4.4. Dynamic instability

The question whether converging shocks are dynamically stable is of great importance. Perry and Kantrowitz observed how “shock-shocks”, appeared on a circular shock front breaking up the symmetry. The disturbances had been introduced by the supporting struts of the inner body. Butler (1954) conducted perturbation calculations showing that strong cylindrical shocks are unstable and Whitham (1973) used his ray-shock formulation to come to the same conclusion. Neemeh & Ahmad (1986) studied the stability of cylindrical shock waves, experimentally and theoretically. Perturbations were introduced externally, by placing cylindrical rods in the path of the shocks. They made a number of conclusions: the region of collapse was shifted due to the disturbance and depending on whether the shock was strong or weak, the shift was either on the disturbed side of the centre or beyond. Perturbations were found to grow exponentially, in good agreement with Butler’s theoretical work, indicating that cylindrical shocks are unstable.

Stability was investigated by Takayama et al. (1987) in two tubes (in Sendai, Japan and Aachen, Germany) of similar design: one tube had three supporting struts for the inner body and the other four. In the tube with three struts the deformations became triangular, while square deformations appeared in the second tube. The deformations were designated as three- and four-mode instabilities. The stability of cylindrical shock waves was further studied in the Sendai tube by Watanabe & Takayama (1991). Using holographic interferometry the shock waves and density variations in the whole flow field behind them were studied. They showed how initial disturbances in the density and pressure fields behind a shock that initially looked completely circular grew as the shock converged. The shock shape slowly deformed until the gradients behind the front became so large Mach reflections occurred.

The tube at KTH, which works on similar principles, also exhibits the four-mode instability due to the struts. The multiple-exposure schlieren image in Fig. 4.3(a) shows how the initially circular shock is progressively deformed and eventually collapses as Mach reflections occur. Fig. 4.3(b) shows a reflected
shock wave and how the circular shape is retained. Close-ups of initially circular shock waves are presented in Fig 4.4, at radii equal and less than that of the last central exposure in Fig. 4.3(a). Fig 4.4(d) shows a reflected shock that regains the circular shape almost instantly.

![Figure 4.3: Multiply exposed schlieren images of circular cylindrical shock wave: (a) converging shock; (b) diverging shock after focus. The gradual collapse of the circular shape that culminates in the appearance of reflections may be seen in (a). Each exposure is 0.3 µs and the delays between them are 2.2 µs.](image)

4.5. Previous work at KTH

Experiments on converging shock wave were initiated at KTH Fluids Physics Laboratory in 1996. Apazidis & Lesser (1996) conducted a theoretical study using Whitham’s geometrical shock dynamics to design a chamber aimed to produce converging polygonal shock waves. The background to this was the numerical work by Henshaw et al. (1986) and Schwendeman & Whitham (1987) who found that a symmetric polygonal shock is dynamically stable in the sense that the shock front will undergo a periodic transformation between \( n \) and \( 2n \) sided polygonal form while retaining the symmetry of the shock structure. (Johansson et al. 1999; Apazidis et al. 2002) proceeded to build a confined cylindrical chamber with smooth exchangeable boundaries. A shock wave was generated in the centre of the chamber by electric discharges or exploding wires. The shock wave diverged, reflected on the smooth polygonal boundary and converged. Schlieren photography was used for visualisation. The experimental results agreed well with the modified geometrical shock dynamics for shocks
Figure 4.4: Schlieren photographs of the collapse of cylindrical shock waves. The scale applies for all images. The photographs are from separate runs. Image (d) shows a reflected diverging shock wave (DS). The shock wave is clearly seen framing the turbulent region. The eight reflected shocks (RS) are those created from the Mach reflections seen in Figs. (a)–(c). Small arrows indicate wave direction.

moving into a non-uniform flow. More information can be found in the licentiate thesis by Johansson (2000).

However, the method of initiating the shock in the chamber created a disturbance zone in the centre. To avoid these disturbances a horizontal shock tube was constructed. The tube works on similar principles as that of Perry & Kantrowitz (1951) and Takayama et al. (1987) and is described in Chapter 5.
The shock tube has exchangeable reflector boundaries akin to those used in the confined chamber. Polygonal shocks with different number of sides were generated in the tube and studied with schlieren optics (Eliasson et al. 2006). The four-mode instability reported by Takayama et al. (1987) was also observed in the KTH tube. Another way of reshaping the shocks was used, previously employed by Wu et al. (1977), Neemeh & Ahmad (1986): small cylinders were inserted in the tube to deform the shocks into polygonal shapes (Eliasson et al. 2007a). The light production was also studied with a photomultiplier tube (Eliasson et al. 2007b). The total intensity of the light pulse was measured for polygonal and circular shock shapes. It was shown that the light intensity between different shock tube runs was more consistent when the shocks had polygonal shapes, albeit not as strong. More information can be found in the doctoral thesis by Eliasson (2007).

Summary converging shock waves
Table 4.2 lists a number of experimental studies, covering the past six decades. The Method column specifies how the shock waves are initiated and their geometry: C for cylindrical and S for spherical (including hemispherical etc.). In the detonation-driven experiments the test gas itself is ignited, while in those categorised as "explosive" shock waves in non-combustible gas are created with explosive charges. The Measurement column briefly lists which methods of diagnostics were used and/or what was the focus of the study: the temperature measurements were made by spectroscopy while visualisation was made with various methods and therefore written out explicitly. Although the author makes no pretence that the list is complete, it gives an overview of the past and present experiments and groups. The possibility or promise to generate extremely high temperatures and pressures continues to drive interest in the field. New methods of shock shaping are investigated to overcome the problems with asymmetric or unstable shock waves (e.g. Dimotakis & Santaney 2006; Zhai et al. 2010; Vandenboomgaerde & Aymard. 2011) and new experiments continues to appear.

\^Especially a large number of Russian works are left out (see e.g. Sokolov 1990, and references therein).
<table>
<thead>
<tr>
<th>Author(s) (year)</th>
<th>Test gas</th>
<th>Method (geometry)</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perry &amp; Kantrowitz (1951)</td>
<td>air, Ar</td>
<td>shock tube (C)</td>
<td>schlieren, photometry</td>
</tr>
<tr>
<td>Belokon et al. (1965)</td>
<td></td>
<td>(C)</td>
<td>temperature</td>
</tr>
<tr>
<td>Lee &amp; Lee (1965)</td>
<td>C₂H₂-O₂</td>
<td>detonation (C)</td>
<td>streak photography, pressure</td>
</tr>
<tr>
<td>Knystautas &amp; Lee (1967)</td>
<td>C₂H₂-O₂</td>
<td>detonation (C)</td>
<td>schlieren photography</td>
</tr>
<tr>
<td>Flagg &amp; Glass (1968)</td>
<td>H₂-O₂</td>
<td>detonation (S)</td>
<td>applied: hyper-velocity launcher</td>
</tr>
<tr>
<td>Knystautas et al. (1969)</td>
<td>C₂H₂-O₂</td>
<td>detonation (C)</td>
<td>streak photography, temperature</td>
</tr>
<tr>
<td>Lee &amp; Knystautas (1971)</td>
<td>C₂H₂-O₂</td>
<td>detonation (C)</td>
<td>stability</td>
</tr>
<tr>
<td>Knystautas &amp; Lee (1971)</td>
<td>C₂H₂-O₂</td>
<td>detonation (C)</td>
<td>schlieren photography</td>
</tr>
<tr>
<td>Roberts &amp; Glass (1971)</td>
<td>H₂-O₂-He</td>
<td>detonation (S)</td>
<td>temperature</td>
</tr>
<tr>
<td>Fujiwara et al. (1971)</td>
<td>H₂-O₂</td>
<td>detonation (C)</td>
<td>smoke film</td>
</tr>
<tr>
<td>Glass (1972)</td>
<td></td>
<td></td>
<td>comprehensive report on project</td>
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<tr>
<td>Setchell et al. (1972)</td>
<td>Ar</td>
<td>shock tube (conical)</td>
<td>velocity (piezo-electric probe), schlieren photography</td>
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<tr>
<td>Fujiwara &amp; Taki (1974)</td>
<td>C₂H₂-O₂</td>
<td>detonation (C)</td>
<td>temperature</td>
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<tr>
<td>Glass &amp; Sharma (1976)</td>
<td>H₂-O₂</td>
<td>detonation (S)</td>
<td>applied: diamond synthesis</td>
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<tr>
<td>Wu et al. (1977)</td>
<td>air</td>
<td>shock tube (C)</td>
<td></td>
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<tr>
<td>Matsuo &amp; Nakamura (1980)</td>
<td>air</td>
<td>explosives (C)</td>
<td>photography, streak photography</td>
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<tr>
<td>Wu et al. (1980)</td>
<td>air</td>
<td>shock tube (C)</td>
<td>schlieren photography, pressure</td>
</tr>
<tr>
<td>Baronets (1981)</td>
<td>Ar</td>
<td>induction-discharge (C)</td>
<td>photography, schlieren</td>
</tr>
<tr>
<td>Glass &amp; Sagie (1982)</td>
<td>D₂-O₂</td>
<td>detonation (S)</td>
<td>scintillator, applied: fusion initiation</td>
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<tr>
<td>Saito &amp; Glass (1982)</td>
<td>H₂-O₂</td>
<td>detonation (S)</td>
<td>temperature</td>
</tr>
<tr>
<td>Baronets (1984)</td>
<td>Ar</td>
<td>induction-discharge (C)</td>
<td>photography, schlieren, temperature</td>
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<tr>
<td>Matsuo (1983)</td>
<td>air</td>
<td>explosives (C)</td>
<td>spectrometry, photometry</td>
</tr>
<tr>
<td>Terao (1984)</td>
<td>C₃H₃-O₂</td>
<td>detonation (S)</td>
<td>gas and electron temperature</td>
</tr>
</tbody>
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Table 4.2: Sixty years of experiments with converging shock and detonation waves in gases. Continues on the next page.
<table>
<thead>
<tr>
<th>Author(s) (year)</th>
<th>Test gas</th>
<th>Method (geometry)</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berezhetskaya et al. (1984)</td>
<td>air</td>
<td>spark discharge (C)</td>
<td>shadowgraphy, pressure</td>
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<tr>
<td>Kleine (1985)</td>
<td>air</td>
<td>shock tube (C)</td>
<td>schlieren</td>
</tr>
<tr>
<td>Takayama et al. (1987)</td>
<td>air</td>
<td>shock tubes (C)</td>
<td>stability, holographic interferometry, pressure</td>
</tr>
<tr>
<td>Matsuo et al. (1985)</td>
<td>air</td>
<td>explosives (C)</td>
<td>spectrometry, photometry, shadowgraphy</td>
</tr>
<tr>
<td>Barkhudarov et al. (1988)</td>
<td>air</td>
<td>spark discharge (C)</td>
<td>shadowgraphy</td>
</tr>
<tr>
<td>de Rosa et al. (1991)</td>
<td>air</td>
<td>electric discharge (S)</td>
<td>interferometry, shadowgraphy</td>
</tr>
<tr>
<td>Terao &amp; Wagner (1991)</td>
<td>C3H8-O2</td>
<td>detonation (S)</td>
<td>pressure, temperature</td>
</tr>
<tr>
<td>Baronets (1994)</td>
<td>Ar</td>
<td>induction-discharge (C)</td>
<td>wave propagation (shadowgraphy)</td>
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<tr>
<td>Fujiwara et al. (1992)</td>
<td>air</td>
<td>detonation, flyer disc (C)</td>
<td>propagation (photography)</td>
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<tr>
<td>Watanabe &amp; Takayama (1991)</td>
<td>air</td>
<td>shock tube (C)</td>
<td>stability, interferometry, pressure</td>
</tr>
<tr>
<td>Terao et al. (1995)</td>
<td>C3H8-O2</td>
<td>detonation (S)</td>
<td>temperature</td>
</tr>
<tr>
<td>Watanabe et al. (1995)</td>
<td>air</td>
<td>vertical shock tube (C)</td>
<td>proof of concept</td>
</tr>
<tr>
<td>Johansson et al. (1999)</td>
<td>air</td>
<td>electric discharge (C)</td>
<td>schlieren</td>
</tr>
<tr>
<td>Hosseini et al. (2000)</td>
<td>air</td>
<td>vertical shock tube (C)</td>
<td>interferometry</td>
</tr>
<tr>
<td>Hosseini &amp; Takayama (2005a)</td>
<td>various</td>
<td>vertical shock tube (C)</td>
<td>Richtmyer-Meshkov instability, interferometry, pressure</td>
</tr>
<tr>
<td>Hosseini &amp; Takayama (2005b)</td>
<td>air</td>
<td>explosives (S)</td>
<td>propagation, stability, shadowgraphy</td>
</tr>
<tr>
<td>Eliasson et al. (2006, 2007a)</td>
<td>air</td>
<td>shock tube (C)</td>
<td>schlieren</td>
</tr>
<tr>
<td>Eliasson et al. (2007b)</td>
<td>air, Ar</td>
<td>shock tube (C)</td>
<td>photometry, schlieren</td>
</tr>
<tr>
<td>Bond et al. (2009)</td>
<td>CO2, N2</td>
<td>shock tube (wedge)</td>
<td>schlieren, pressure</td>
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<tr>
<td>Hosseini &amp; Takayama (2009)</td>
<td>air</td>
<td>vertical shock tube (C)</td>
<td>interferometry, pressure</td>
</tr>
<tr>
<td>Zhai et al. (2010)</td>
<td>air</td>
<td>shock tube (C)</td>
<td>schlieren</td>
</tr>
<tr>
<td>Kjellander et al. (2010, 2011)</td>
<td>various</td>
<td>shock tube (C)</td>
<td>schlieren, photometry (this work)</td>
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CHAPTER 5

Experimental equipment

The experiments were performed at the shock tube facility of the Fluid Physics Laboratory at KTH Mechanics. The facility consists of a shock tube with circular cross section and equipment for detection and measurement of shock propagation and light emissions. Two end sections ("test sections") on the tube have been used; one designed to create cylindrical converging shock waves and a second to obtain spherical shocks. This chapter describes the experimental setup and serves as an introduction to future users of the facility.

5.1. Shock tube

This section describes the shock tube and equipment common to both test sections whereas the test sections are described in separate sections. The common section of the shock tube is circular with an inner diameter of 80 mm and a length of 1830 mm including the driver section. An outline of the facility is given in Figure 5.1, where the capital letters designate different parts of the tube: (A) is the driver section, (B) and (C) the driven section. The diaphragm is located at the intersection of parts (A) and (B). The driven section consists of an inlet tube (B), its purpose is to allow the shock wave to attain a plane form before entering the test section (C). The low pressure section is evacuated by a two-stage rotary vacuum pump connected to the tube at (6.). Test gas is introduced into the tube through the valve at (5.). When a test gas other than air is used, the section is repeatedly evacuated and filled with the gas twice to ensure pure test gas in the test section. After the final evacuation the gas is allowed to retain room temperature, a process that takes about two minutes. The argon gas used in the present experiments had a purity rate of 99.99%.
Figure 5.1: Schematic of the shock tube facility. The two working sections (C) are described in separate parts. DPI 150: the Druck pressure indicator recording $p_4$ and $p_1$, the high and low pressures before membrane rupture.
5. EXPERIMENTAL EQUIPMENT

<table>
<thead>
<tr>
<th>Membrane</th>
<th>Breaking pressure</th>
<th>Typical $M_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al 0.3 mm</td>
<td>0.8 MPa</td>
<td>1.7 – 2.5</td>
</tr>
<tr>
<td>Al 0.5 mm</td>
<td>1.6 MPa</td>
<td>2 – 6</td>
</tr>
<tr>
<td>Al 0.7 mm</td>
<td>2.3 MPa</td>
<td>4 – 8</td>
</tr>
<tr>
<td>Mylar 0.05 mm</td>
<td>0.3 MPa</td>
<td>1.3 – 2</td>
</tr>
</tbody>
</table>

Table 5.1: Breaking pressures ($p_4 - p_1$) for membranes of different material and thickness and typical Mach number ranges for which the membranes were used. Depending on driver filling time and knife sharpness the measured pressures could be varied with about 200 kPa for the thicker Al membranes and 100 kPa for the thinner. The plastic mylar membranes were used only occasionally and no such variation has been tested. Run-to-run variations for all diaphragms were around 10-20 kPa.

Regular air or commercial helium were used as driver gas and introduced at connection (1.). To get full control over the driver gas composition when only helium was used the driver section was evacuated to about 2.5 kPa prior to filling. Filling the driver section must be done slowly. It is essential that the high pressure gas is in thermal equilibrium with its surrounding to obtain a correct gas temperature. Moreover the pressure transducer is located some distance from the membrane and connected through a small tube, which in the case of a rapid gas filling would give a false pressure reading.

The membranes are inserted in a flange between sections (A) and (B). The driver section is mounted on slides attached to the driven section. A cross-shaped knife that ensures consistency in the mechanical opening process and membrane ruptures at a set pressure difference is located behind the diaphragm at the low-pressure side. The pressure difference required forcing the diaphragm towards the knife is determined by the thickness and strength of the membrane. During present experiments mostly aluminium diaphragms were used but plastic mylar-film was also used to obtain weaker shocks. Table 5.1 shows the bursting pressures and Fig. 5.2 shows a photograph of ruptured membranes. The 0.3 mm aluminium diaphragms had a tendency to be ripped apart and clutter the tube with debris, and were for that reason used sparingly. The burst pressure and its variation between runs are dependent on the handling of the filling of the gas and may thus differ from the tabulated values. The sharpness of the knife-edge also influences the bursting pressure. If care is taken to fill the tube in the same manner each run, experience shows that the variation between shots is as low as 10 kPa.

The pressures in the sections are measured with a pressure transducer and indicator (GE Druck DPI 150), connected to the driver section at (2.) and to the driven section at (4.). An external module (GE Druck IDOS) connected to
the indicator is used to measure the high pressure. The pressure at the instant of membrane rupture is registered and assumed to be the high pressure \( p_4 \). The shock speed is measured by means of temperature sensors sensing the shock passing the sensor. The sensor element is a strip of platinum film painted on the flat surface at the end of a 10 mm diameter glass cylinder inserted through the shock tube wall and aligned flush with the inner surface of the tube. They are mounted on the test sections but are described here as they are common to both sections. The platinum strip is connected to a high frequency amplifier through a high pass filter and the resistance change in the strip is instant when the shock passes its surface. The circuit diagram is shown in Fig. 5.3. Fig. 5.4 shows a graph over measured Mach numbers compared with the ideal solution Eq. 3.1. The measured Mach numbers are smaller than the ideal values except for the smallest pressure ratios. The plotted Mach numbers have been measured in either of the two test sections; they have not been distinguished in the graph as no different trend could be seen.

5.2. Cylindrical test section

One working section is designed to create cylindrical converging shock waves. It is similar in principle to the shock tubes of Perry & Kantrowitz (1951) and Takayama et al. (1987). A coaxially aligned conical inner body transforms the plane tube cross-section to an annular channel, which opens into an open compartment where the shock wave converges cylindrically. The total cross-section is kept constant through the plane tube, transformation section and

Figure 5.2: Two used membranes; one 0.05 mm plastic film and one 0.5 mm aluminium membrane. The Al membrane shows the imprint of the burst indicator on the tip of one leaf.
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Figure 5.3: Circuit diagram of shock sensor amplifier.

Figure 5.4: Measured Mach numbers $M_S$ for different pressure ratios compared with the ideal solution Eq. 3.1. Typical measurement errors for a few Mach numbers and pressure ratios have been added; they are similar for all gas combinations.

The height of the channel is 10 mm and ends into the open chamber with a 90° bend. The test section is made up of this open chamber and is 5 mm wide and has a radius of 70 mm. The shock wave reflects off the end wall of the annular channel and diffracts down into the test section. A high pressure and temperature region created by the reflection off the end wall
5.2. CYLINDRICAL TEST SECTION

Figure 5.5: Sketch of the shock wave propagation through the tube. The plane shock enters the transformation section from the inlet tube, becomes annular before going through the bend into the cylindrical test section. The part of the shock wave that reflects on the test section wall and returns up the annular channel is omitted for clarity.

drives the flow towards the centre of the cylindrical test section. A drawing of the construction is shown in Figure 5.6; note that the second set of supports is rotated by 45° around the central axis from the first set. The shock propagation is illustrated in Figure 5.5. The test section is framed on both sides by glass windows for easy visualisation: the inner body is hollow to allow equipment for illumination of the test section. The observation window is a 15 mm thick thermal-resistant borosilicate glass (Borofloat 33) disc with optical transmission down to the UV wavelengths.

The shock sensors are inserted flush with the outer wall of the annular channel and separated by 25.0 cm.

5.2.1. Triggering and Synchronisation

Capturing images or spectra of a very rapid phenomenon requires an accurate triggering system to turn on cameras or other equipment. Several triggering methods have been used: using the shock sensors, a shock-deflected laser system and photo-multiplier tubes.

5.2.1a. Shock sensor triggering. The temperature sensors positioned on the outer surface of the annular tube may be used for triggering. It is a robust system and apt for all triggering purposes except when rapid events very close to the focal region are studied, when a more precise system is necessary. The shock wave propagation time from passing the downstream shock sensor to implosion in the test section is several hundred µs and the time variation between runs
5. EXPERIMENTAL EQUIPMENT

![Diagram of experimental equipment]

Figure 5.6: Drawing of the annular-cylindrical unit. The second set of supporting struts is rotated by 45° from the first around the axis. The first set is hollow to allow a laser beam to illuminate the test section. Measures in mm.

amounts to several microseconds. Typically, the signal output is connected to a time delay unit (DG525, Stanford Research Systems), which in turn triggers a laser, camera, oscilloscope or spectrometer.

5.2.1b. Laser triggering. A system for detecting the shock waves close to the implosion point was designed for the cylindrical convergence section as the precision using the shock sensors is too low. A non-intrusive method was therefore developed to detect the shock closer to the focus. A continuous laser beam is directed through the test section, about 15 mm from the centre. Deflections of the laser beam caused by the passing shock wave are detected by a photodiode.

Figure 5.7 shows a photograph and a principal sketch of the set-up. A HeNe laser beam is directed through the test section at a small angle, passes through both glass windows and is reflected back by a mirror inside the inner body. The beam path is in the horizontal plane of the tube centreline. The beam exits the test section on the opposite side of the centre. A lens (f=+80 cm) focuses the beam on an optical fibre that leads the light to a fast photodiode (Hamamatsu S5973). The circuit that amplifies the photodiode current is given in Fig. 5.8. It consists of a primary current amplifier with very fast response and a secondary amplifier to increase the output voltage to the triggering levels of the time delay unit. Characteristics were determined with a pulse laser: the rise time of the primary amplifier is about 20 ns and the rise time of the combined circuit including the secondary amplifier is 100 ns.
5.2. CYLINDRICAL TEST SECTION

Figure 5.7: Photograph and sketch of the laser triggering set-up. Description for both: (1.) HeNe laser; (2.) mirror (inside tube); (3.) 80 mm lens, focusing the laser beam on (4.) optical fibre mounted in a traverse, which is connected to a (5.) photodiode and amplifier. The path of the laser beam is sketched in the photo. Two fibres collecting light from the implosion to the photo-multiplier tube and spectrometer (lower left corner) can also be seen in the photo. A damping filter and a knife edge may be used additionally.

The response to a passing shock wave consists of four peaks: the first two correspond to the converging shock wave passing the laser beam going into and out from the test section respectively, while the second pair corresponds to the outgoing reflected shock wave. The peaks have a certain slope depending on the angle between the laser beam and test section. The angle of the laser varied between experiments, depending on the other measurement equipment placed in front of the test section. For triggering, the photodiode signal is set to the trigger input of the delay unit. The system is very sensitive to the position
5. EXPERIMENTAL EQUIPMENT

Figure 5.8: Circuit diagram of the photodiode amplifier. $V_s$ may be set to $5 - 15$ V.

of the receiving optical fibre. The fibre is fastened on a traversing system and before each run it is moved to its most sensitive position where it generates the maximum signal. The triggering level on the DG535 is set to just below the value of the maximum: when the shock passes the generated signal drops.

5.2.1c. Photo-multiplier signal. A simple way to trigger the spectrometer is to use the photomultiplier tube detecting the light created by the shock wave itself. It is particularly useful for measuring the light spectrum after shock focusing, during the relaxation phase of the gas in the centre, but the small rise in emission just prior to the collapse enables this method to be used to detect the beginning of the implosion pulse as well. The problem that arises in the latter case is that this initial light increase is relatively slow and shows large variations in terms of amplitude, which reduces the likelihood of triggering at the same instant relative to implosion each run.

5.2.1d. Burst indicator. Immediately downstream of the membrane in the driven section, an electric conductor runs through the tube wall at connection (2.) in Figure 5.1. The conductor is electrically insulated from the metal tube and the conductor terminal is aligned flush with the inner side of the tube. When the membrane bursts it hits the conductor tip, connecting it with the tube. The change in potential may be used as an indicator for the membrane burst event or as a trigger.

5.2.1e. Pressure triggering. To take schlieren or normal photographs with the Nikon camera the system is in enclosed in a blacked-out enclosure, the camera shutter opened before membrane rupture and left open for 30 seconds exposing during the whole experiment (see Section 5.3.1 below). To do this an automatic system was built that monitors the pressure in the driver section and triggers an infrared remote control (a modified Nikon ML-L3) to turn on the camera once the pressure reaches a predetermined value, slightly below what is required to
break the membrane. A pressure transducer (Keller Series 21R) was connected on a T-joint inserted on the tube between the Druck pressure indicator and the driver section (not shown in the drawing). The sensor signal is connected to a comparator circuit, drawn in Fig. 5.9, which toggles a relay when the pressure signal exceeds 2/3 of the comparator supply voltage. A potentiometer is used to tweak the triggering voltage/pressure and a LED indicates when the relay is activated. The circuit connects the two conductors in the output cable to the infrared remote that triggers the camera.

5.3. Flow visualisation: Schlieren optics

Flow visualisation is provided with schlieren optics. A schlieren system makes use of the density-dependence of the refractive index of light. Here follows a short introduction and description of the present set-up. Three main components are necessary: a collimated light source, a light blocker - called a schlieren stop - and a camera. The principle is simple: the parallel light rays illuminate the test section of interest and are afterwards focused on the stop, which partially or completely blocks the light. Density gradients in the test section make the parallel rays deflect. Light that would otherwise have been blocked by the stop will now pass it (or vice versa - light that would have passed may instead be blocked). The density changes will thereby appear as darker or brighter areas on the image plane after the schlieren stop.

Two principal optical set-ups were used. One that had all optical elements arranged on the centre line of the shock tube and one that made use of the Schlieren Optical Unit (SOU) seen in Figure 5.10 and 5.11 where the optical axis was twice folded. A schematic drawing of the latter system is presented in figure 5.10. Light was provided with laser. The laser head is mounted outside of the shock tube, perpendicular to it. The beam enters the inner body of the annular section through one of the support struts and expands, thereby
5. EXPERIMENTAL EQUIPMENT

Figure 5.10: Schematic drawing of schlieren set-up. The laser beam enters the tube through a hollow support and is directed through the test section via a mirror and two lenses. M: mirrors, L: lenses, S: schlieren stop.

illuminating the test section through the glass window. On the receiving side the collimated light was focused with lens $L_1$ ($f=1350$ mm) on a schlieren stop. To be able to detect density gradients in all radial directions a circular stop was used. Either a small micro-sphere blocking most of the beam - typically a 0.67 mm ball bearing - or a pinhole were used, the former shown in Fig. 5.12. The pinhole blocks light from areas of large gradients, generating bright images with dark shock waves, and the other way around for the sphere.

A second lens $L_2$ creates an image for the camera. Magnification is decided through choices of lenses $L_1$ and $L_2$ and the distances between them and the object plane (the test section). When the SOU is used, the lens $L_1$ is an in-built 1350 mm lens. The system that does not use the SOU is in principle no different, except in that the optical axis is not mirrored.

5.3.1. Cameras

Two CCD cameras are used: a PCO SensiCam and a Nikon D80 system camera. The SensiCam (12 bits, 1280x1024 pixels, pixel size: 6.7x6.7 µm) is equipped with a 80 mm Canon lens and can take either singly or multiply exposed images. It is controlled by a computer that receives an external TTL-level trigger signal via a PCI-board. For single exposures, an Nd:YAG laser (New Wave Orion) is used with both cameras. The pulse length of the laser is about $4 - 5$ ns. The timing of the pictures is determined with the laser: the camera was left open for a longer interval ($5$ µs for the SensiCam and $30$ s for the Nikon) and the laser fired at the desired instant for photographing.

The Nikon D80 is a regular digital system camera equipped with a Micro Nikkor 60 mm macro lens. The shutter of the Nikon D80 could not be satisfactorily triggered without internal modification of the camera, so the shutter was simply left open for 30 seconds. It was triggered using the pressure comparator.
described above. At a pressure just below membrane burst pressure, the output triggered a commercial infra-red remote (Nikon ML-L3) which opened the camera shutter. The remote was modified to be triggered by the relay circuit. The delay between the given IR signal and the shutter opening was very long - longer than the propagation time of the shock wave from membrane rupture and focusing, which made it necessary to trigger the camera before the actual membrane opening. Although the optical set-up is shielded from stray light, an exposure problem occurs with the Nikon camera with the shutter left open for long periods. This causes the camera to not only capture the schlieren image enlightened by the laser beam, but also to be exposed to the implosion light pulse. The implosion pulse may be very bright and over-expose the photograph. The unwanted exposure due to the implosion light flash is damped by placing neutral filters in front of the camera and compensating with increased laser power.

5.3.2. Lasers

And Orion Nd:YAG pulse laser was used as light source for the schlieren photography. The laser Q-switch can be triggered internally or externally. In each mode, it first receives a TTL signal to start the flash lamp ("Fire lamp"). In internal QS mode, the laser pulse follows the "Fire lamp" signal after 328 µs. In the external mode, the laser is fired after receiving a second triggering signal ("Fire QS"), typically around 200 µs after the "Fire lamp" signal. The external handling of the Q-switch generates much stronger light than in internal mode. The output laser beam strength is controlled manually. Two energy modes, called High and Low, are available and is supplemented with an energy scale ranging from 0-99. Typical used values were Low 4 with the external Q-switch mode, and Low 15 with internal Q-switch mode. The effective exposure time for the schlieren photographs when using the Orion is determined by the laser pulse length, which is 4-5 ns.

The Orion laser could not be used for multi-exposed images since its maximum pulse frequency is 1 Hz. Instead a continuous laser was used as light source: an argon-ion (Spectra-Physics BeamLok 2060) and a HeNe laser were used interchangeably.

5.3.3. Arrangement procedure

To arrange the system the following procedure is followed.

The first step is to find the optical axis. The axis follows the centre line of the shock tube and is relatively easy to find if the optics are to be aligned in a straight line. However, when the SOU is used the optical axis is twice folded by 90° and SOU must first be aligned. Starting in the camera end might be the simplest course of action. A HeNe laser can be used as an alignment assistant. First, the laser is placed at the position of the camera and the beam
Figure 5.11: Set-up for schlieren photography: (a) laser light source; (b) 1350 mm lens (inside tube); (c) schlieren stop; (d) focusing lens; (e) camera.

Figure 5.12: The etched 0.67 mm micro-sphere used as a schlieren stop.

is aligned straight with the the optical rail on top of the SOU and directed through the centre of the light hole on the tower. A mirror should be used to control that the beam is aligned along the optical axis of the system. With the laser beam coming out of the SOU along its optical axis, the whole unit can be positioned by moving it until the HeNe beam enters the centre of the shock tube perpendicularly. When properly aligned, the laser beam should go through the whole system and hit the schlieren laser orifice.
The illumination laser should be aligned through the hollow support perpendicular to the tube. The alignment of the mirror $M$ inside the tube can be made from outside. A remote-controlled electric motor controls the motion of the mirror around the vertical axis. The motion around the horizontal axis is handled with a manual screw going through the hollow strut on the opposite side of the light-entering strut.

With the SOU and schlieren laser aligned, the optical instruments - lenses, stop and camera - can be placed. A clear camera image is obtained by putting a semitransparent paper grid in the centre of the test section and focusing the camera on the grid. When all optical elements are aligned, the schlieren stop is positioned at the focus point of the laser light.

5.3.4. Shock wave shaping

Two different methods have been employed to shape the shock waves from cylindrical into primarily polygonal forms: by cylindrical obstacles creating a reflection and diffraction pattern or by wings dividing the test section into radial channels where plane sides are created. The annular-cylindrical tube was designed to create shock waves shaped as polygons by using replaceable reflector plates around the periphery of the test section. This method has not been used in the work but is mentioned for completeness (for details see Eliasson et al. 2006).

5.3.4a. Cylindrical obstacles. By placing small cylindrical objects in the test section, the diffraction of the converging cylindrical shock wave around the obstacles changes the overall form. If the size and position of the obstacles are arranged in certain way, symmetrical polygonal forms may be achieved. The diameters of the cylindrical objects ranges from 7.5 to 15 mm. They are positioned between the glass windows using guides. During mounting they are temporarily kept in position with a small amount of glue: equipped with o-rings they are afterwards kept in place mechanically by the pressure from the glass windows.

5.3.4b. Biconvex wing profiles. Another method is to place biconvex wings in the test section with their chords aligned radially. The incoming shock wave reflects on the wings and if arranged properly, the shock wave attains polygonal structure with almost plane sides when leaving the channels. Since the wings have sharp leading and trailing edges, less pressure is lost compared to the case when circular objects are used, in which case reflected waves travel upstream. Figure 5.13(a) shows the test section with the wing dividers. The leading edges are aligned flush with the inner surface of the annular channel and the trailing edges end 20 mm from the centre of the test section. Calculations were made to find the appropriate lengths and widths to ensure plane shock wave exiting the channels into the open centre of the chamber. One purpose of the wings is
5. EXPERIMENTAL EQUIPMENT

Figure 5.13: Wing matrix mounted in the test section (a) and before assembly (b).

to improve the control of shock shaping and to allow the same blockage ratio no matter the number of wings - and consequently the number of sides of the polygonal shock wave - by altering the thickness and length of the wings from case to case. The measurements in the present study however, only feature a configuration with eight such dividers, creating a cylindrical octagonal shock wave.

5.3.5. Inner body alignment

The eccentricity of the inner body inside the shock tube has major impact on the symmetry of the shock waves. Referring to Figure 5.1, the inner body is supported by two sets of struts. The downstream set is located close to the test section and may be aligned with the help of a mechanical guide. The upstream set is situated too far upstream for this method to be useful. It was found that the wings provided a good tool for tuning the position of the body. The wing matrix divided the test section in eight radial channels and the velocity of each segment of the otherwise connected circular shock wave could be seen in detail.
5.3. FLOW VISUALISATION: SCHLIEREN OPTICS

Figure 5.14: Schlieren images of shocks arriving at the open center of the test section to illustrate the effect of non-aligned inner body: (a) before and (b) after alignment.

Figure 5.15: Interaction of two shock waves close to the trailing edge of a wing (outlined with dashed lines) at slightly different times. In (a) the lower shock wave has already arrived and diffracted around the tip and reflected with the upper shock wave. A vortex can be seen forming due the shear flow. A Mach stem is formed as the shocks reflect. It is shifted upwards due to the asymmetric reflection (b). At the bottom of the image the Mach reflection coming from the lower wing tip can be seen. Arrows indicate wave direction. Each image is from a separate run.
Figure 5.14 shows schlieren photographs of shock waves exiting the channels. The shocks have clearly different velocities as they arrive at different instants. This suggests that the inner body is eccentric with the annular channel being slightly narrower at the part of the channel corresponding to the position of the faster shocks and vice versa. To ensure that asymmetric construction of the matrix did not give rise to the irregularities, the matrix was gradually rotated between several runs. The shock pattern was unchanged with rotation and it was concluded that the matrix construction was good. The struts were adjusted accordingly to the schlieren photographs. By trial and error the arrival of the shock fronts at the end of the matrix could be improved. Figure 5.14 (a) shows the shock pattern before alignment: the whole lower half of the shock is faster than the upper. Figure 5.14(b) shows the pattern after alignment: the general shape is much improved. Photomultiplier records measuring the light of the implosion pulse showed a large increase in strength after the alignment indicating a more symmetric implosion. Figure 5.15 shows reflections around the right encircled wing-tip in Fig. 5.14(a).

5.4. Converging test section

A second test section was designed to create shocks with spherical symmetry. The section is joined to the main shock tube and consists of a converging pipe with a smoothly changing cross section. The idea is to form the wall in such a way that the shock wave foot remains normal to the wall without reflection (or with minimal reflection) and that the shock front has a spherical shape as it leaves the section. If the pressure is evenly distributed behind the shock and all parts of the shock front propagates at the same speed, the front will be close to spherical. The existence of such a solution was discussed and proved by Dumitrescu (1983, 1992); Saillard et al. (1985). Fig. 5.16 shows a sketch of the principle: a plane shock wave enters the tube and where the cross section changes disturbances move along the curved shock progressively increasing the curvature and gradually accelerating it. In order for the shock front not only to have a circular symmetry in the propagation axis plane but also to have the same speed at all parts, to ensure a continued spherical shock front, a series of calculations were made to find the wall shape. The final design is shown in Fig. 5.17. It consists of an extended inlet tube (with shock-sensors) followed by a smooth contraction and a small conical end. Eq. 5.1 provides the shape of the contracting surface in parameter form. The cone half angle is 21°.

\[
\begin{align*}
  x &= A \sin \theta \\
  y &= B - R(1 - \cos \theta)
\end{align*}
\]

for \(0 \leq \theta \leq 0.35\pi\)  \hspace{1cm} (5.1)

where \(A=300.7\) mm, \(B = 40\) mm and \(R = 57.3\) mm. The transformation part was constructed by casting a plastic material around a steel mold. The cast part is housed in a steel tube with a flange and fastened to the shock tube. It is
291 mm in total length and terminates in a straight cone of 21 mm. The cone is made of steel truncated 0.4 mm from its tip leaving a circular opening with a radius of 0.3 mm. The opening is covered with a 1.5 mm thick quartz window fixed to its position by a threaded brass sleeve. The conical part is shown in section in Fig. 5.18. The sleeve contains optical fibre mounts. One fibre is mounted coaxially, viewing straight into the tube. A second fibre views the opening at an oblique angle covering a volume stretching no more than 0.5 mm into the tube. The surface inside the contraction and cone is smooth in order to avoid disturbances introduced into the very sensitive converging process.

Figure 5.17: Convergence section for three-dimensional implosion. Inlet tube with shock sensors $S_1$, $S_2$ and $S_3$, a smoothly changing contraction and 21 mm conical end cone. The end diameter of the conical section is 0.6 mm. A quartz window closes the tube. Measures in millimetres.
5.5. Spectroscopic instrumentation and its calibration

Two spectrometers have been used in the experiments, both on loan from the KTH Physics Department. They are both of echelle type and the diffracted light is recorded on an intensified charge-coupled device (ICCD). The final spectrum is analysed though computer software. The first spectrometer, which was used for the measurements on the cylindrical implosions (Paper 1) was a Mechelle 7500 (Multichannel Systems, Sweden) equipped with an Andor Istar ICCD. Details about the specific and echelle spectrometers in general can be found in Lindblom (1998). The spectrometer was able to record spectra in the wavelength interval $180 - 880$ nm. The sensitivity of the CCD is dependent on wavelength, which needs to be accounted for when analysing the data. Figure 5.19(a) shows the ICCD sensitivity. The test section glass window also limits the light transmission, to between roughly 350 and 880 nm. The transmittance of the 15 mm thick borosilicate glass is presented in Figure 5.19(b).

During the measurements the spectrometer exposure was started by the laser triggering system with a precision of 10 ns relative the emission peak, which for synchronisation purposes was measured separately with a photomultiplier tube.

The second spectrometer (Aryelle 200, Lasertechnik Berlin) used an improved version of the Andor Istar ICCD. A wavelength calibration was made with a mercury lamp with the aid of the supplied software. In order to reconstruct the spectrum a radiometric calibration against a calibrated deuterium lamp and, for the longer wavelengths, against a tungsten lamp with known filament temperature ($3000 \pm 50$ K) was carried out. The temperature of the tungsten filament was measured using two pyrometers and calculated by measuring the resistance $R_{ref}$ at room temperature of the lamp and using tabulated values of $R/R_{ref}$ vs $T$. The value of $T$ is very sensitive to $R_{ref}$, and great care was taken to measure it. The lamp tension was measured at thes
5.5. SPECTROSCOPIC INSTRUMENTATION AND ITS CALIBRATION

Figure 5.19: (a) Relative CCD wavelength sensitivity of the Mechelle 7500 system and (b) transmittance of the test section borosilicate window.

The bulb socket and the zero resistance was determined to \( R_0 = 0.28 \, \text{kΩ} \) by extrapolation of \( R = U/I \) to \( U = 0 \), see Fig. 5.20.

The measured spectrum of the deuterium lamp is shown in Fig. 5.21 together with the given calibrated emission. The quickly decreasing sensitivity in the deep UV is due to the optical fibre as the manufacturer gives the ICCD sensitivity in the ultraviolet region as fairly constant. The red calibration using the tungsten lamp was patched with the ultraviolet calibration to create a single calibration file. The good agreement between the overlapping regions of the separate calibrations shown in Fig. 5.21 is taken as an indication of a successful calibration.

Figure 5.20: Radiometric calibration: (a) Determination of zero resistance \( R_{ref} \) and filament temperature. \( R_{ref} \) is determined through extrapolation to \( U = 0 \) (red dashed line). The temperature can subsequently be estimated from \( R/R_{ref} \). (b) Patching region of ratio between measured and known spectra for tungsten filament (dashed, red) and deuterium lamp (full line).
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Figure 5.21: Raw spectrum of deuterium lamp compared to the given calibrated emission (blue dashed line).

Figure 5.22: Relative sensitivity for the Aryelle system, including optical fibre (FC400 solarisation-resistant fibre) and observation window (quartz). The decrease per order is visible: the orders are denser at lower wavelengths.

The resulting sensitivity function is shown in Fig. 5.22. The periodicity is due to the varying sensitivity of each diffraction order. The sensitivity drastically drops below 250 nm and above 950 nm, reducing the accuracy in those parts of the spectrum.
Numerical calculations were made with Euler solvers unstructured triangular grids with or without adaptive mesh refinement. Three different solvers were used: a single-component and a multi-component solver and a single-component solver taking into account equilibrium ionisation. First- and second order finite volume discretisation schemes were used. The convective flux was in all cases calculated using the artificially upstream flux vector splitting (AUFS) scheme introduced by Sun & Takayama (2003), whose fundamental idea is to overcome the disadvantages of up-winding schemes by introducing artificial wave speeds into the flow which simplifies the discretisation. The single-component solvers are briefly described here while the multi-component solver is described in paper 6 including more details on the discretisation and mesh adaption also relevant for the single-component case. The 2D Euler equations for compressible inviscid flow:

\[ U_t + F_x + G_y = 0 \] (6.1)

where the vector \( U \) contains the conserved variables while \( F \) and \( G \) are the fluxes in \( x \)- and \( y \)-directions:

\[
U = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{pmatrix}, \quad F = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho v u \\ \rho E u + p u \end{pmatrix}, \quad G = \begin{pmatrix} \rho v \\ \rho w \\ \rho v^2 + p \\ \rho E v + p v \end{pmatrix}
\] (6.2)

where \( \rho \) is the mass density, \( \rho u \) and \( \rho v \) and \( \rho E \) the energy per unit volume. The energy per unit mass is the sum of the specific internal energy and the kinetic energy \( E = e + \|u\|^2/2 \). The equations were discretised on an unstructured triangular mesh. Referring to the grid cell illustrated in Figure 6.1, Eq. 6.2 may be formulated and calculated over the normal interfaces between the cells. Variables denoted L refer to the states inside the cell and R to those in the neighbouring cells. We have:
Figure 6.1: A grid cell $i$. $U_i^L$ is the state in the cell while $U_j^R$, $j = 1, 2, 3$, are the states in neighbouring cells.

\[
U_t + F_n = 0 \iff U_t + AU_n = 0 \quad (6.3)
\]

where $A$ is the Jacobian matrix $A_{ij} = \frac{\partial F_i}{\partial U_j}$ and $n$ refers to the normal direction. The matrix $A$ has four real eigenvalues corresponding to the four wave speeds of the system, $(u_n - a, u_n, u_n, u_n + a)$, where $u_n = u_n x + u_n y$ is the normal velocity across the interface. Provided that $u_n < c$ somewhere, the system therefore contains waves going both upstream and downstream which makes up-winding difficult. However, observed from some frame of reference all waves propagate in the same directions. By introducing the artificial wave speeds $s_1$ and $s_2$ corresponding to such a moving frame of reference the flux can be rewritten to simplify the discretisation. After some manipulation the flux becomes:

\[
F = (1 - S)F_1 + SF_2 = (1 - S) \left[ \frac{1}{2}(P_L + P_R) + \delta U \right] + S \left[ U^d(u_n^d - s_2) + P^d \right] \quad (6.4)
\]

where $S = s_1/(s_1 - s_2)$, $\delta U$ artificial viscosity, $P = (0, pn_x, pn_y, pu_n)$ and $d$ is either $L$ (if $s_1 > 0$) or $R$ (if $s_1 \leq 0$) , depending on whether the corresponding wave goes into or out of the cell. The pressure is acquired from the internal energy: $e = p/(\gamma - 1) = E - u^2/2$ while the temperature is attained from the perfect gas law.
The artificial viscosity term is:
\[
\delta U = \frac{1}{2a} \begin{pmatrix}
(p^L - p^R) \\
(pu)^L - (pu)^R \\
(pu)^L - (pv)^R
\end{pmatrix}
\frac{a^2}{\gamma - 1}(p^L - p^R) + \frac{1}{2}((pU^2)^L - (pU^2)^R)
\]
(6.5)

where \( U^2 = u^2 + v^2 \) and \( \bar{a} = (a^L + a^R)/2 \) is the average speed of sound of domains \( L \) and \( R \). The artificial wave speeds were chosen as

\[
s_1 = \frac{u^L_n + u^R_n}{2}
\]
(6.6)

\[
s_2 = \begin{cases} 
\min(0, u^L_n - a^L, u^R_n - c^*) & s_1 > 0 \\
\max(0, u^*_n + c^*, u^R + u^R) & s_1 \leq 0
\end{cases}
\]
(6.7)

\[
u^* = \frac{1}{2}(u^L_n + u^R_n) + \frac{a^L - a^R}{\gamma - 1}
\]
(6.8)

\[
c^* = \frac{1}{2}(a^L + a^R) + \frac{1}{4}(\gamma - 1)(u^L_n - u^R_n)
\]
(6.9)

For details of the derivation the reader is to directed to Sun & Takayama (2003). The solution update for the grid \( i \) between time-steps \( n \) and \( n + 1 \) is then calculated by a first-order method:

\[
U_{i}^{n+1} = U_{i}^{n} - \sum_{k=1}^{3} \frac{\Delta t}{h_k} F_{k}
\]
(6.10)
\[ e = \frac{3}{2}(1 + \alpha_e)RT + R \sum_{i=1}^{\ell} \left( \alpha_i \sum_{j=1}^{i} \frac{I_j}{k} \right) \]  \hspace{1cm} (6.12)

where \( \alpha_e \) is the average ionisation fraction and \( I_j \) are the ionisation potentials as defined in chapter 2. With the additional variables \( \alpha_i \), the system is closed with the Saha equations which can be expressed as functions of \( T \) and \( \rho \) and solved in the same manner as presented by Trayner & Glowacki (1995):

\[ \frac{\alpha_{i+1}}{\alpha_i} = \frac{1}{\alpha_e} \left( \frac{2\pi m_e}{h^2} \right)^{3/2} \frac{m_H(kT)^{3/2}}{\rho} \frac{2Q_{i+1}^e}{Q_i^e} \exp \left( -\frac{I_{i+1}}{kT} \right) \]  \hspace{1cm} (6.13)

where \( m_H \) is the mass of the neutral atom. Solving the equation in terms of a given primitive variable \( \rho \) instead of \( p \) is preferred, since the latter is a derived variable. The temperature and ionisation fractions for each grid are carefully balanced and calculated from the given primitive variables during each time-step. An iterative method is used to find the ionisation and temperature that fulfills the energy requirement 6.12 as well as the set of Saha equations 6.13. This implies finding the root of the transcendental equation \( T - f(\alpha_e(T, \rho)) = 0 \), where the numerically evaluated function \( f \) is determined from the known energy 6.12. Explicitly written out this becomes

\[ T - \left[ e - R \sum_{i=1}^{\ell} \left( \alpha_i(T, \rho) \sum_{j=1}^{i} \frac{I_j}{k} \right) \right] \left[ \frac{3}{2}(1 + \alpha_e(T, \rho))R \right]^{-1} = 0 \]  \hspace{1cm} (6.14)

Equation 6.14 may be solved by a bi-section method with initial lower bound \( T = T_0 \) and upper bound set to the ideal non-ionising temperature. Once the temperature and ionisation fractions are found, the pressure is extracted from the equation of state 6.11.

The artificial wave speeds \( s_1 \) and \( s_2 \) are chosen in the same manner as previously, but the speed of sound now becomes the equilibrium speed of sound, \( a_e = (\partial p/\partial \rho)_s \), which can be calculated from derivatives of \( \alpha \) (see Appendix A).

6.0.0b. Single ionisation. When only one stage of ionisation is likely to be present, Eqs. 6.13-6.14 can be simplified to significantly reduce calculation time. Such a scheme has been presented in Aslan & Mond (2005). Only one Saha equation remains, for \( i = 1 \) where \( \alpha_0 = 1 - \alpha_1 \), which does not need iteration. In this temperature range, the partition function ratio can be adequately approximated by a constant \((2Q_1/Q_0 \approx g_0 \approx 11)\) and Eq 6.13 reduces to
\[
\frac{\alpha_1^2}{1 - \alpha_1} = g_0 \left( \frac{2\pi m_e}{h^2} \right)^{\frac{3}{2}} \frac{m_H(kT)^{3/2}}{\rho} \exp \left( -\frac{I_1}{kT} \right) = g_0 C \frac{T^{3/2}}{\rho} \exp \left( -\frac{I_1}{kT} \right)
\]

(6.15)

where the constant \( C \approx 1.603 \times 10^{-4} \text{ kg m}^{-3} \text{ K}^{-3/2} \) for argon.

The approximation of the partition function ratio as a constant carries a certain error, which is exemplified in Fig. 6.2. The post-shock conditions resulting from the approximation are compared to those where the partition functions included a summation over the first few terms. As evident, the error is reasonably small until \( M \approx 30 \). Above that Mach number second stage ionisation becomes significant (compare with Fig. 2.3) and the model validity is in any case becoming questionable. The calculations are made with initial pressure 0.1 atm and \( T = 300 \text{ K} \).
Figure 6.2: Error caused on shock relations by the assumption $Q_1/Q_0 = 11$ (dashed line) compared to $Q_1/Q_0 = f(T)$: (a) post-shock temperature; (b) pressure; (c) degree of ionisation and (d) shock compression.
CHAPTER 7

Conclusions and contributions

This is a short summary of the results presented in the papers in Part II.

7.1. Cylindrical shock waves

- A new system of shaping the circular shock wave was introduced: the convergence chamber was divided into channels using wing profiles which split the incoming shock wave. For some wing lengths and widths, the shock waves exit the channels as straight sections and a polygonal shape is created.

- Spectrometric studies on shock convergence were made using polygonal shock waves. The shape had shown to produce more repeatable results in terms of the light emitted by the implosion than circular. Blackbody radiation was measured during the beginning of the implosion light pulse, showing a peak value of 6,000 K, which is lower than expected. Light was collected from an unnecessarily large area, which is significantly colder than the hot central core.

- The dynamics of symmetric polygonal shapes were studied in order to compare with theoretical studies on the peculiar behaviour of polygonal shock waves. The repeating and alternating formation of the initial polygon due to Mach reflections in the corners was seen and found to match the theory.

- Calculations using geometrical shock dynamics for converging cylindrical and spherical shock waves were performed taking in account real gas effects during the convergent process. Ionisation, electronic excitation and coulombic forces were taken into account.

- The design of the apparatus creating cylindrical shock waves was studied numerically by axi-symmetric Euler calculations: it was found that the 90° bend and contraction works to create strengthened converging cylindrical shock waves. The initially diffracted shock at the bend is weak and attenuated, but the shock reflected at the end wall turns into the flow field created by the diffraction, overtakes and merges with the diffracted shock. This strong shock converges efficiently, the flow driven by the high pressure created by the reflected shock in the annular channel. Three-dimensional effects due to the bend are initially large, but quickly diminish although they are not completely damped.
7. CONCLUSIONS AND CONTRIBUTIONS

- The convergence of circular cylindrical shock waves was studied to determine the self-similarity exponent for shocks in three different gases, argon, nitrogen and propane. According to established theory, the exponent depends on the ratio of specific heats, $\gamma$. The experiments confirmed the variation and the acquired values agreed well with theory. The Mach numbers in these experiments were kept low so $\gamma$ of the gas would not change due to real-gas effects (before implosion).

7.2. Spherical shock waves

- A new experimental section was designed and constructed for the shock tube. It consists of a transformation section with smoothly convergent cross-section ending with a straight cone. The wall curve was designed to slowly change the shape of the plane shock wave into the shape of a spherical disc (imagine a part of a spherical shell cut out by a cone with its apex in the sphere centre) when exiting the transformation section. Numerical calculations were made to test the shape and found to work well for a range of Mach numbers. In the first version of the experiment no sensors were inserted to avoid disturbing the flow.

- The radiation from the imploding shock in the conical section was measured for shock waves in air. Significantly stronger radiation was recorded than in the cylindrical case for the same initial Mach number. In the first part of the implosion light pulse a strong continuum was seen, while bound-bound line radiation of argon appeared in the cooling phase. Compared to the cylindrical case, more radiation from unwanted sources appeared: e.g. iron and aluminium from the shock tube and diaphragm. Preliminary results show a highest blackbody temperature of about $2.7 \times 10^4$ K.

- The work on the spherical test section opens possibilities and raises several questions. Further experiments should be made to more accurately clarify what level of shock strengthening is achieved, which could be done by e.g. measuring the propagation. The end of the cone could be reconstructed to enable clearer measurements. Instead of an abrupt wall at the end of the cone, a small cavity or tube could be attached, into which the strengthened shock could propagate. The light emission could then be studied by placing the collecting fibres perpendicular to the axis instead of along it. On the numerical side, the addition of viscosity and/or a collisional-radiative model to the Euler calculations could provide better understanding of the fast processes around focus. With more accurate calculations the shape of the convergent section could be optimised.
CHAPTER 8

Papers and authors contributions

Paper 1

*Thermal radiation from a converging shock implosion.*

M. Kjellander (MK), N. Tillmark (NT) & N. Apazidis (NA).


This paper is a spectrometric and photometric study of the light emission produced by converging shock waves in argon. For repeatability purposes, polygonal shape shocks were created. The experiment was set up by MK and NT with assistance from Olli Launila and Lars-Erik Berg, KTH Applied Physics and performed by MK. Numerical calculations complemented the study, performed by NA and MK. The paper was written by MK and NA, with feedback from NT. Parts of this work has been presented at:

**27th International Symposium on Shock Waves,**

19 – 24 July 2009, St Petersburg, Russia

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Paper 2

*Shock dynamics of imploding spherical and cylindrical shock waves with real gas effects.*

M. Kjellander, N. Tillmark & N. Apazidis.


This paper is a study on the high temperature gas processes close to the centre of convergence of cylindrical and spherical shock waves in monatomic gases. The method of characteristics was used, with a gas model accounting for ionization and Coulomb effects. The initial idea was proposed by NA and the calculations were performed mostly by MK. Theoretical derivations were made by MK and NT. The paper was written by MK with feedback from the co-authors.
Paper 3

*Regular versus Mach reflection for converging polygonal shocks.*

V. Eliasson (VE), M. Kjellander & N. Apazidis.


Different reflection patterns in polygonal shock waves were investigated. Square and triangular shocks were created by cylindrical rods placed in the path of the shocks. The experimental setup and work was mainly done by VE, but also by MK: MK set up and performed the experiments with the cylinders placed at 61.5 mm from the centre with higher optical magnification. The paper was written by VE with feedback from NA. Parts of this work was presented at:

60th Annual Meeting of the American Physical Society - Division of Fluid Dynamics,
18 – 20 November 2007, Salt Lake City, Utah, United States

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Paper 4

*Polygonal shock waves: comparison between experiments and geometrical shock dynamics.*

M. Kjellander, N. Tillmark & N. Apazidis.

*In proceedings: 28th International Symposium on Shock Waves, 2011, University of Manchester, Manchester, United Kingdom*

Schlieren photography is used to compare the dynamics of polygonal shocks with theory. Symmetric shock waves with 6, 8 and 12 sides were studied. The experimental set-up was made by MK and NT, the experiments were performed by MK. The writing was made by MK, with feedback from NA. This work has been presented at:

28th International Symposium on Shock Waves,
17 – 22 July 2011, Manchester, United Kingdom

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Paper 5

*Experimental determination of the self-similarity constant for converging cylindrical shocks.*

M. Kjellander, N. Tillmark & N. Apazidis.


This is a continuation of previous experiments concentrating on the dynamics of converging cylindrical shock waves. The self-similarity exponent for the motion of cylindrical imploding shock waves was measured for different gases.
The setup and experiments were made by MK with support from NT. MK wrote the paper with feedback from NT and NA.

**Paper 6**

*Numerical assessment of shock tube with inner body designed to create cylindrical shock waves.*  
M. Kjellander & N. Apazidis  
*Technical report*

This is a numerical study on the performance of the cylindrical convergence chamber. The numerical code was written by MK and NA; MK performed the calculations and wrote the report.

**Paper 7**

*Generation of spherical converging shocks in a shock tube by wall shaping.*  
M. Kjellander, N. Tillmark & N. Apazidis.  
*Manuscript.*

A study on the convergence of shock waves in a smoothly convergent shock tube designed to create a spherical shape of the shock during the last stage of implosion. The design of the setup was made by the authors jointly. MK performed the experiments and wrote the paper, with feedback from NT and NA. Olli Launila and Lars-Erik Berg, KTH Applied Physics, provided invaluable contributions to the spectrometric setup and interpretation of the results.
Acknowledgements

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May 2nd, 2012 Stockholm

Malte
APPENDIX A

Specific heat and speed of sound from $\alpha$

In the case of no Coulomb interactions the specific heats and equilibrium speed of sound can be rewritten in terms of derivatives of $\alpha_i$, which simplifies the numerical work in some cases where these are practically already calculated. The equilibrium speed of sound $a_e$,

$$a_e^2 = \left( \frac{\partial p}{\partial \rho} \right)_s = \gamma \left( \frac{\partial p}{\partial \rho} \right)_T$$  \hspace{1cm} (A.1)

where $\gamma = c_p/c_v$. We aim to express $a_e$ in terms of known quantities and derivatives. The heat capacities are found from the enthalpy and energy. Neglecting electronic excitation, these are

$$h = \frac{5}{2} (1 + \alpha_e)RT + R \sum_{i=1}^{\ell} \alpha_i \sum_{j=1}^{I} \frac{I_j}{k}$$  \hspace{1cm} (A.2)

$$e = \frac{3}{2} (1 + \alpha_e)RT + R \sum_{i=1}^{\ell} \alpha_i \sum_{j=1}^{I} \frac{I_j}{k}$$  \hspace{1cm} (A.3)

so that

$$c_p = \left( \frac{\partial h}{\partial T} \right)_p = \frac{5}{2} (1 + \alpha_e)R + \frac{5}{2} TR \left( \frac{\partial \alpha_e}{\partial T} \right)_p + R \sum_{i=1}^{\ell} \left( \frac{\partial \alpha_i}{\partial T} \right)_p \sum_{j=1}^{I} \frac{I_j}{k}$$  \hspace{1cm} (A.4)

and

$$c_v = \left( \frac{\partial e}{\partial T} \right)_v = \frac{3}{2} (1 + \alpha_e)R + \frac{3}{2} TR \left( \frac{\partial \alpha_e}{\partial T} \right)_v + R \sum_{i=1}^{\ell} \left( \frac{\partial \alpha_i}{\partial T} \right)_v \sum_{j=1}^{I} \frac{I_j}{k}$$  \hspace{1cm} (A.5)

where

$$\left( \frac{\partial \alpha_e}{\partial T} \right)_F = \sum_{i=1}^{\ell} \left( \frac{\partial \alpha_i}{\partial T} \right)_F$$  \hspace{1cm} (A.6)
With the equation of state Eq. 2.11 the speed of sound can then be written

\[ a^2_c = \gamma \left( \frac{\partial p}{\partial \rho} \right)_T = \frac{c_p}{c_v} \left( \frac{\partial p}{\partial \rho} \right)_T = \frac{c_p}{c_v} \left[ \frac{\partial \rho}{\partial \rho} (\rho(1 + \alpha_e)RT) \right]_T = \]

\[ = \frac{c_p}{c_v} \left[ (1 + \alpha_e)RT + \rho RT \left( \frac{\partial \alpha_e}{\partial \rho} \right)_T \right] \tag{A.7} \]

The above expression together with A.4 and A.5 is used to calculate \( a_e \). All thermodynamic variables are known but the derivatives of \( \alpha_i \) has to be evaluated numerically around the current state of the gas \( p, \rho, T, \alpha_i \). A simple evaluation can be made as

\[ \left( \frac{\partial \alpha_i}{\partial F} \right)_G = \frac{\alpha_i(F + dF_1, G) - \alpha_i(F - dF_2, G)}{dF_1 + dF_2} \tag{A.8} \]
APPENDIX B

Coulomb effects on thermodynamic variables

In a partly ionised gas Coulomb forces between the charged particles lead to departures from the ideal state. When the effect is weak, consideration to the Coulomb interactions may be taken in form of correction terms to the thermodynamic variables. Different models exist for different gas conditions: here is a derivation using the Debye-Hückel model for the ion charges for weak Coulomb interactions. To derive the corrections due to the Coulomb forces on the thermodynamic state and the species distribution, the electrostatic energy contribution to the free energy is found, which in turn gives the desired corrections. The electrostatic potential around a point charge is found by considering the other particles not as individual charges but as a uniform charge cloud and solving the Poisson equation. The derivation of the potential may be found in e.g. Griem (1962), Ebeling (1976) or Salzmann (1998). The electrostatic energy of a gas in a volume $V$ resulting from this first approximation is given as

$$E_c = -\frac{kT V}{8\pi r_D^3}$$  \hspace{1cm} (B.1)

The parameter $r_D$ is the Debye radius which is a characteristic of the surrounding charge cloud and determines the sphere of influence of the ion charge, which for a single-temperature plasma may be written

$$r_D = \left[ \frac{q^2}{\epsilon_0 kT} (n_e + \sum_i n_i z_i^2) \right]^{-1/2} = \left[ \frac{q^2}{\epsilon_0 kT} (N_e + \sum_i N_i z_i^2) \right]^{-1/2}$$  \hspace{1cm} (B.2)

where $q$ is the elementary charge, $\epsilon_0$ is the vacuum permittivity, $z_i = i$ is the charge state of the ion $i$. Note that several of the cited authors have used other unit systems, while SI units are used here. The number of particles of ionic and electronic components $N_i$ and $N_e$ in the volume $V$ and number densities $n_i = N_i V$ and $n_e = N_e V$ are defined as usual. Outside the Debye sphere, which is the sphere around the ion with a radius $r_D$, the ion is effectively screened by the cloud. A typical validity requirement for the statistical Debye-Hückel model is that several ions must be present within a Debye sphere.
The influence of the Coulomb forces on the free energy is expressed as a correction term to the ideal gas energy, \( F = F_{ig} + F_C \), which derives from the electrostatic energy. Using \( E = -T^2 \partial / \partial T (F/T) \), the correction to the free energy becomes

\[
F_C = -\frac{kTV}{12\pi \rho_D^3}
\]  

(B.3)

**B.0.1. Equation of state**

The pressure follows from the free energy as \( p = (\partial F / \partial V)_{N_i,T} \). The ideal translational contribution to the pressure is given in Eq. 2.11. The correction term is then found from Eq. B.3,

\[
\delta p_C = -\left( \frac{\partial F_C}{\partial V} \right)_{N_i,T} = \frac{kT}{12\pi \rho_D^3} - \frac{kTV}{12\pi \rho_D^3} \frac{3}{2V} = -\frac{kT}{24\pi \rho_D^3}
\]  

(B.4)

For completeness the total pressure including the Coulomb correction is then written

\[
p = p_{ig} + p_C = \rho(1 + \alpha_e)RT + \delta p_C
\]  

(B.5)

**B.0.2. Saha equation**

The Saha equation may be derived from minimising the free energy considering the ionisation reaction where the \((i+1)\)th electron is removed from the atomic species \( A \),

\[
A_i \rightleftharpoons A_{i+1} + e^-, \quad i = 0, 1, 2, \ldots, \ell - 1
\]  

(B.6)

where \( \ell \) denotes the atomic number of \( A \). The free energy of the ideal gas \( F_{ig} \) is given by statistical mechanics. With the Coulombic correction the free energy of a partially ionised gas in local thermodynamic equilibrium becomes

\[
F = F_{ig} + F_C = -\sum_{i=1}^{\ell} N_i kT \ln \frac{Z_i e}{N_i} - N_e kT \ln \frac{Z_e e}{N_e} + F_C
\]  

(B.7)

where \( Z_i \) and \( Z_e \) are the partition functions of the ions and free electrons. Differentiating and setting \((\delta F)_{V,T} = 0\) gives

\[
\delta F = \sum_j \frac{\partial (F_{ig} + F_e)}{\partial N_j} \delta N_j = \sum_j \left( \frac{\partial F_{ig}}{\partial N_j} + \mu_{j,C} \right) \delta N_j = 0
\]  

(B.8)

where \( \mu_{C,j} = \partial F_C / \partial N_j \) and the summation \( j \) is made for \( j = i, j = i + 1 \) and \( j = e \).
According to the reaction in Eq. B.6 \( \delta N_i = -\delta N_{i+1} = -\delta N_e \) and Eq. B.8 becomes

\[
\left( \frac{\partial F_{ig}}{\partial N_i} - \frac{\partial F_{ig}}{\partial N_{i+1}} - \frac{\partial F_{ig}}{\partial N_e} + \mu_{C,i} - \mu_{C,i+1} - \mu_{C,e} \right) \delta N_i = 0 \rightarrow \\
-kT \ln \frac{Z_i}{N_i} + N_i kT \frac{1}{N_i} + \mu_{C,i} + \\
-(-kT \ln \frac{Z_{i+1}}{N_{i+1}} + N_{i+1} kT \frac{1}{N_{i+1}} + \mu_{C,i+1}) + \\
-(-kT \ln \frac{Z_e}{N_e} + N_e kT \frac{1}{N_e} + \mu_{C,e}) = 0 \rightarrow \\
\ln \left( \frac{Z_{i+1} Z_e N_i}{N_{i+1} N_e Z_i} \right) - 1 = -\frac{\mu_{C,i} - \mu_{C,i+1} - \mu_{C,e}}{kT}
\]

(B.9)

Defining the reduction in ionisation potential due to the Coulomb interactions as

\[
\Delta I_{i+1} \equiv \mu_{C,i} - \mu_{C,i+1} - \mu_{C,e}
\]

(B.10)

Eq. B.9 becomes

\[
\frac{N_{i+1} N_e}{N_i} = \frac{Z_{i+1} Z_e}{Z_i} \exp \left( \frac{\Delta I_{i+1}}{kT} \right)
\]

(B.11)

This can be written in terms of the particles densities \( n_j \) by dividing with the volume \( V \) and using that \( N_j = n_j V \):

\[
\frac{n_{i+1} n_e}{n_i} = \frac{1}{V} \left( \frac{Z_{i+1} Z_e}{Z_i} \right) \exp \left( \frac{\Delta I_{i+1}}{kT} \right)
\]

(B.12)

The partition functions for a monatomic ion consist of one translational and one internal part, \( Z_i = Z_{tr}^i Z_{el}^i \), the latter accounting for the excited electrons within the ion. The translational contributions is

\[
Z_{tr}^i = V \left( \frac{2 \pi m_i kT}{\hbar^2} \right)^{3/2}
\]

(B.13)

where \( m_i \) is the molecular weight of the \( i \)th ion, \( k \) the Boltzmann constant and \( \hbar \) the Planck constant. Since the weight difference of the successive ions are negligible the translational part of the partition functions cancel in Eq. B.11 and B.11. The electronic contribution can be written (Zel’dovich & Raizer 2002) as

\[
Z_{el}^i = \sum_l e^{-\varepsilon_{l,i}/kT} = e^{-\varepsilon_{0,i}/kT} \sum_l e^{-(\varepsilon_{l,i}-\varepsilon_{0,i})/kT} = e^{-\varepsilon_{0,i}/kT} Q_{el}^i
\]

(B.14)
B. COULOMB EFFECTS ON THERMODYNAMIC VARIABLES

where \( \varepsilon_0 \) is the ground state of ion \( i \) and the summation is taken over all energy states. In other words the transformed partition function \( Q^e \) relates the energy of each electronic level to the ground state of the individual ions instead of to the ground state of the atom. The energy differences of the successive ionic ground states are equal to the ionisation potentials, \( \varepsilon_{0,i+1} - \varepsilon_{0,i} = I_{i+1} \).

The partition function of the free electrons has one temperature-dependent contribution from the translational energy and one constant contribution related to the spin, \( Z^{\text{spin}} = 2 \). The total electron partition function is then

\[
Z_e = 2V \left( \frac{2\pi m_e kT}{\hbar^2} \right)^{3/2} \tag{B.15}
\]

where \( m_e \) is the electron weight. Inserting Eqs. B.13, B.14 and B.15 into Eq. B.12 yields the Saha equation:

\[
\frac{n_{i+1}n_e}{n_i} = 2 \left( \frac{2\pi m_e kT}{\hbar^2} \right)^{3/2} \frac{Q^e_{i+1}}{Q^e_i} \exp \left( \frac{-I_{i+1} - \Delta I_{i+1}}{kT} \right) \tag{B.16}
\]

Using the particle fractions \( \alpha_i = n_i/n_H \) and \( \alpha_e = n_e/n_H \) this is rewritten as

\[
\frac{\alpha_{i+1}\alpha_e}{\alpha_i} = 2 \frac{n_H}{n_H} \left( \frac{2\pi m_e kT}{\hbar^2} \right)^{3/2} \frac{Q^e_{i+1}}{Q^e_i} \exp \left( -\frac{I_{i+1} - \Delta I_{i+1}}{kT} \right) \tag{B.17}
\]

Noting that \( \rho \approx n_H m_H \), Eq. B.17 has the same form as Eq. 6.13. The equation of state B.5 can be used to rewrite the equation as a function of temperature and pressure,

\[
\frac{\alpha_{i+1}}{\alpha_i} = 1 + \alpha_e \left( \frac{2\pi m_e}{\hbar^2} \right)^{3/2} \left( \frac{kT}{p - \delta pC} \right)^{5/2} \frac{2Q^e_{i+1}}{Q^e_i} \exp \left( -\frac{I_{i+1} - \Delta I_{i+1}}{kT} \right) \tag{B.18}
\]

The potential reduction according to the Debye-Hückel method is received by taking the derivative of Eq. B.3:

\[
\Delta I_{i+1} = \frac{\partial F_C}{\partial N_i} - \frac{\partial F_C}{\partial N_{i+1}} \frac{\partial F_C}{\partial N_e} = \frac{(i + 1)q^2}{4\pi \varepsilon_0 r_D} \tag{B.19}
\]

B.0.3. Energy and enthalpy

The expression for the energy may be calculated from \( E = -T^2 \partial \partial T (F/T) \) using the partition functions. Again \( F = F_{ig} + F_C \), with \( F_{ig} \) for a partially ionised monatomic gas in local thermodynamic equilibrium given in Eq. B.7. The Coulombic correction from the elecrostatic potential is given in Eq. B.1. Dividing the partition functions in their translational and electronic parts, \( Z = Z^{tr} Z^e \) yields
is acquired directly from Eq. B.5 and Eq. B.23:

\[ \frac{\partial}{\partial T} \frac{F_{\text{tot}}}{T} \big|_{V,N} = k \sum_{i=0}^{\ell} \left[ N_i \left( \frac{\partial \ln Z_{\text{e}}^\ell}{\partial T} \right)_{V,N} + N_i \left( \frac{\partial \ln Z_{\text{p}}^\ell}{\partial T} \right)_{V,N} \right] - kN_e \left( \frac{\partial \ln Z_{\text{el}}^\ell}{\partial T} \right)_{V,N} \]

\[ = -k \sum_{i=0}^{\ell} \left[ N_i \frac{3}{2} \frac{1}{T} + N_i \left( \frac{\partial \ln Z_{\text{e}}^\ell}{\partial T} \right)_{V,N} \right] - kN_e \frac{3}{2} \frac{1}{T} \]  \hspace{1cm} (B.20)

Inserting the total number of heavy particles \( N_H = \sum_{i=0}^{\ell} N_i \) the above yields

\[ E_{\text{ig}} = -T^2 \left( \frac{\partial F_{\text{tot}}}{\partial T} \right)_{V,N} = \frac{3}{2} (N_H + N_e) kT + kT^2 \sum_{i=0}^{\ell} N_i \left( \frac{\partial \ln Z_{\text{e}}^\ell}{\partial T} \right)_{V,N} \]

\[ = \frac{3}{2} (1 + \alpha_e) N_H kT + kT^2 N_H \sum_{i=0}^{\ell} \alpha_i \left( \frac{\partial \ln Z_{\text{e}}^\ell}{\partial T} \right)_{V,N} \]  \hspace{1cm} (B.21)

The last term is rewritten using Eq. B.14,

\[ \sum_{i=0}^{\ell} \alpha_i \left( \frac{\partial \ln Z_{\text{e}}^\ell}{\partial T} \right)_{V,N} = \sum_{i=0}^{\ell} \alpha_i \left( \frac{\partial \ln Q_i^\ell}{\partial T} \right)_{V,N} + \sum_{i=0}^{\ell} \alpha_i \left( \frac{\partial \ln e^{-\epsilon_{0,i}/kT}}{\partial T} \right)_{V,N} = \]

\[ = \sum_{i=0}^{\ell} \alpha_i \left( \frac{\partial \ln Q_i^\ell}{\partial T} \right)_{V,N} + \sum_{i=1}^{\ell} \sum_{j=1}^{i} \frac{I_j}{kT} \]  \hspace{1cm} (B.22)

Using that the masses of the ionic species are approximately equal to the atomic mass \( M_i \approx M_A \to N_H k \approx m R_A \) and the energy is rewritten

\[ E_{\text{ig}} = \frac{3}{2} (1 + \alpha_e) m R_A T + m R_A \sum_{i=0}^{\ell} \alpha_i \sum_{j=1}^{i} \frac{I_j}{k} + m R_A T^2 \sum_{i=0}^{\ell} \alpha_i \left( \frac{\partial \ln Q_i^\ell}{\partial T} \right)_{V,N} \]  \hspace{1cm} (B.23)

The enthalpy per unit mass \( h = e + p/\rho \) is similarly divided into an ideal and Coulombic part: \( h = e_{\text{ig}} + e_C + (p_{\text{ig}} + p_C)/\rho \). The ideal contribution to the enthalpy is acquired directly from Eq. B.5 and Eq. B.23:

\[ h_{\text{ig}} = \frac{5}{2} (1 + \alpha_e) R_A T + R_A \sum_{i=1}^{\ell} \alpha_i \sum_{j=1}^{i} \frac{I_j}{k} + R_A \sum_{i=0}^{\ell} \alpha_i \frac{W_i}{k} \]  \hspace{1cm} (B.24)

where the energy of the electronic excitation is

\[ W_i = kT^2 \left( \frac{\partial \ln Q_i^\ell}{\partial T} \right)_{V,N} \]  \hspace{1cm} (B.25)

The two Coulomb corrections have been derived above and we have

\[ h_C = e_C + p_C/\rho = \frac{1}{mV} E_C + p_C/\rho = -\frac{kT}{8\pi \rho r_B^3} - \frac{kT}{24\pi \rho r_B^3} = -\frac{kT}{6\pi \rho r_B^3} \]  \hspace{1cm} (B.26)
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