Performance Analysis of Relay-Assisted Cognitive Radio Systems with Superposition Coding

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Performance Analysis of Relay-Assisted Cognitive Radio Systems with Superposition Coding

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Abstract—We study the problem of relay selection in a cooperative cognitive radio system in which a secondary transmitter can act as a relay for the primary transmitter in order to maximize the primary user’s gain. Moreover, we take into account that the secondary users may have certain quality-of-service (QoS) requirements which need to be satisfied. In the cooperative scenario of a cognitive radio system, the primary user should have an incentive to allow the secondary user to transmit in its licensed band. At the same time, the secondary user should be able to transmit its own data with the required QoS in order to be willing to help the primary user’s performance by relaying its data. Hence, we take into account both primary and secondary users’ objectives. We use numerical methods in order to study how different parameters, such as assigned powers in the coding scheme, direct link reliability, and the number of secondary users affect the overall system performance. Our results show that the gain, which the primary user can achieve by cooperation, highly depends on the quality of its direct link as well as the secondary users’ QoS requirements. Furthermore, the percentage of cooperation instances between secondary users and the primary user is dependent on the amount of power the secondary users allocate for relaying the primary’s signal based on the direct link quality.

I. INTRODUCTION

In recent years, emerging wireless applications attract many users while at the same time impose high requirements on spectral efficiency in wireless systems. Satisfying these demands with available resources, such as spectrum and transmit power, brings out new challenges for researchers. There have been extensive studies on the spectrum sharing and utilizing current radio resources in order to increase the spectral efficiency. These studies are mainly in the framework of cognitive radio which was initially introduced by Mitola in his dissertation [1]. The principle of cognitive radio is to increase flexibility in spectrum usage by allowing unlicensed (secondary) users to access the spectrum of licensed (primary) users.

There are three cognitive radio paradigms, namely interweave, underlay, and overlay [2]. In the interweave approach, secondary users can not transmit in the same frequency band as primary users. They have to detect spectrum holes in order to avoid collisions with the primary users. In the underlay scheme, secondary users transmit concurrently with primary users if the interference they create for the primary users is under a certain threshold in order to avoid performance degradation for primary users. Finally, in the overlay network, secondary users overhear primary users and facilitate the transmission of the primary users by cooperation.

We consider the overlay approach with cooperative communication. It is shown in [3] that cooperative communication exploits path diversity in order to achieve higher capacity and communication reliability. Cooperative strategies known in literature are decode-and-forward (DF), coded cooperation, and amplify-and-forward (AF). When the DF scheme is deployed for a three-node system (Source-Relay-Destination), the relay node decodes the message broadcasted by the source node, then, it transmits the decoded message to the destination. If the relay node decodes and re-encodes the source node’s message, the cooperation scheme is called coded cooperation. In the AF method, the relay amplifies the message and forwards it to the destination node. If the relay can not listen and transmit at the same time, i.e., the half-duplex mode, all aforementioned schemes are done in two phases.

It has been shown in [4] that for a three-node system the DF relaying together with the superposition coding improve the spectral efficiency. Therefore, in this work, we employ the superposition coding scheme in a cooperative spectrum sharing system (CSS-SC) with one primary transmitter (PT) and receiver (PR) pair and many secondary transmitter (ST) and receiver (SR) pairs. The objective is to select a ST to act as a relay for the primary user, so as to maximize the primary user’s gain. In addition, we consider the QoS requirement of the secondary users so that the selected ST can communicate to its receiver reliably.
The rest of the paper is organized as follows. In the remainder of this section, we review the related work and present our contribution. The system model is described in Section II. We formulate the relay selection problem in Section III, and present the numerical results in Section IV. Finally, we conclude in Section V.

A. Related work and contributions

Collaborative spectrum sharing with superposition coding is considered in [5] when there exists one PT-PR pair together with one ST-SR pair. It is assumed that the primary user splits its transmission time frame to two equal time slots. The PT transmits in the first time slot while the ST uses the other half of transmission time, and the PR has the capability to perform maximum ratio combining (MRC). The objective of [5] is to quantify the primary system priority in terms of outage probability. This problem is generalized in [6] to one PT-PR pair together with many secondary pairs, and the goal is to select one ST for cooperation which minimizes the outage probability of the primary system. The authors in [6] showed that for a fixed fraction of ST’s transmit power which is allocated for relaying the PT’s signal, the primary user’s outage probability only depends on the distance between the PT and the ST who transmits, and therefore, the nearest ST to the PT gives the minimum outage for the primary system.

However, it is important to note that the distance-dependent relay selection is only suitable for non-fading environments [7]. Moreover, the rate of the primary user, in addition to the distance between the PT and the ST, is a function of the fraction of time and power that the ST is willing to spend to relay the primary user’s signal along with its own signal. Therefore, relay selection only based on distance may not always result in the optimal system performance. For instance, consider the example in Fig. 1 where each transmitter-receiver pair has a QoS requirement that needs to be satisfied, and the transmission frame is split into two equal time slots. If, only based on the distance to the PT, ST$_1$ is selected for transmission, it should spend a higher fraction of its power for transmitting its own signal to its receiver (to meet its QoS requirement), and consequently, less power for relaying the primary user’s signal. Instead, if ST$_2$ is chosen for transmission, although it is located further away from the PT, it can allocate a much higher fraction of its power for relaying the primary user’s signal, since its own receiver, SR$_2$, is close. So, this choice may be more beneficial in terms of the data rate for the primary user as well as the secondary user with certain QoS requirement.

In [5], [6], the two transmission time slots in one frame are considered to have the same length. However, it has been shown in [4] that the rate improvement in the two-phase relaying with superposition coding is higher when the first time slot is longer than the second one. In this line of thought, [8] considers a scenario with one ST-SR pair and optimizes the time slot duration and the fraction of power that the ST allocates for primary signal’s transmission with the objective of maximizing the secondary user’s rate. However, [8] does not provide any insights on how primary user’s data rate will change corresponding to the secondary user’s rate, and it also does not consider more than one secondary user in the studied scenario.

In this paper, we consider a scenario similar to [6] with one primary user and many secondary users, where our main objective is to maximize the primary user’s data rate. This is due to the fact that the primary user should have an incentive to allow the ST to transmit in its licensed band. On the other hand, the ST should be able to transmit its data with a required QoS in order to be willing to help the primary user’s performance by relaying its data. This requirement, which has not been considered in [6], translates into a rate constraint for the secondary user in the relay selection process. In addition, we also optimize the duration of the transmission time slots for a fixed power fraction of the ST used for relaying. We use numerical methods in order to study how different parameters, such as assigned powers in the coding scheme, direct link reliability, and the number of secondary users affect the overall system performance.

II. SYSTEM MODEL

We consider a cognitive radio network, as shown in Fig. 2, with one PT and one PR along with $M$ secondary transmitters ST$_i$ and receivers SR$_i$, where $i \in \{1, \ldots, M\}$. The PT employs time division multiple access (TDMA) technique and intends to transmit its data to the PR in its licensed band. Secondary users are not allowed to transmit in this frequency band which is licensed to PT. However, PT can share part of its transmission time with secondary users in order to gain a better QoS and link reliability. In this way, STs get the opportunity to transmit their data as well. We assume that among the $M$ secondary users, only one ST is selected in each transmission frame to cooperate with the primary user as a decode-and-forward (DF) relay in half-duplex mode. So, it can not listen and transmit at the same time. As a result, transmission is done in two phases during one unit-time frame (cf. Fig. 3).
Each transmission phase is called a time slot.

\[
T = 1
\]

\[
\begin{array}{c|c|c}
\delta & 1 - \delta \\
\end{array}
\]

Fig. 3. Frame structure.

In the first time slot with length \( \delta \), where \( 0 \leq \delta \leq 1 \), the PT broadcasts its message \( x_p \), which is a complex-valued signal, with transmit power \( P_p \). This message is received by the PR, STs and SRs. Let \( h_{k,j} = c_i a_{k,j}/d_{k,j}^{\alpha/2} \), with \( k, j \in \{ pt, pr, st_i, sr_i \} \), be the channel coefficients between the transmitter \( k \) and receiver \( j \). In the channel model, the path-loss is defined as \( c_i/d_{k,j}^{\alpha/2} \) where \( c_i \) and \( \alpha \) are the path-loss coefficient and exponent, respectively. \( d_{k,j} \) is the distance between the transmitter \( k \) and receiver \( j \). Moreover, \( a_{k,j} \sim \mathcal{CN}(0, 1) \) is the multi-path fading component, where both its real and imaginary components are i.i.d. Gaussian distributed with \( \mathcal{N}(0, 1/2) \) [9]. We assume a Rayleigh block fading channel where the channel is constant during one frame, but varies over different frames. Then, the received signal \( y_{k,j} \) at the receiver \( j \), in the first time slot, is defined as

\[
\begin{align*}
y_{pt,pr}^{(1)} &= \sqrt{P_p}x_p h_{pt,pr} + \eta_{pr}^{(1)}, \\
y_{pt,si}^{(1)} &= \sqrt{P_p}x_p h_{pt,si} + \eta_{si}^{(1)}, \\
y_{pt,sr}^{(1)} &= \sqrt{P_p}x_p h_{pt,sr} + \eta_{sr}^{(1)},
\end{align*}
\]

where \( \eta_{(t)} \) denotes the zero-mean additive white Gaussian noise with variance \( \sigma^2 \) in the corresponding receivers, and the superscript \( (t) \) denotes the \( t \)-th time slot. The channel knowledge is assumed to be available in both primary and secondary transmitters.

If one of the STs is selected by the PT to act as the DF relay, it transmits in the second time slot, otherwise, the PT continues its transmission (i.e., \( \delta = 1 \)). Let ST\( i \) be the selected secondary transmitter to cooperate with the PT, and \( 1 - \delta_i \) denote its transmission time, corresponding to the duration of the second time slot. We assume ST\( i \) employs the superposition coding scheme and allocates a fraction of its power to transmit the decoded signal received from the PT, and uses the remaining power for its signal. Let \( x_{s,i} \) be the complex-valued signal of ST\( i \), then, the transmitted signal from ST\( i \) is given by

\[
x_{s,i}^{sc} = \sqrt{\nu P_s} x_{p} + \sqrt{1 - \nu} P_s x_{s,i},
\]

where \( P_s \) is the secondary’s transmit power and \( \nu \) is the power fraction assigned for the PT’s signal with \( 0 \leq \nu \leq 1 \). The case \( \nu = 1 \) is equivalent to having a selfless DF relay where the secondary transmitter only relays the primary signal. The received signals by the PR and SR\( i \) in the second time slot are

\[
\begin{align*}
y_{st_i,pr}^{(2)} &= \sqrt{\nu P_s} x_{p} + \sqrt{(1 - \nu)} P_s x_{s,i}) h_{st_i,pr} + \eta_{pr}^{(2)}, \\
y_{st_i,sr}^{(2)} &= \sqrt{\nu P_s} x_{p} + \sqrt{(1 - \nu)} P_s x_{s,i}) h_{st_i,sr} + \eta_{sr}^{(2)}.
\end{align*}
\]

We assume that the secondary receiver is capable of interference cancelation (IC). That is, if a secondary receiver can decode the primary’s signal in the first time slot, it can cancel the primary’s signal from its own signal in the second time slot. Then, what remains at this secondary receiver is given by

\[
y_{st_i,sr} = \sqrt{(1 - \nu)} P_s x_{s,i} h_{st_i,sr} + \eta_{sr}.
\]

Note that we consider the transmitted signals to be zero-mean and uncorrelated with normalized power, i.e., \( E[|x_{p}|] = E[|x_{s,i}|] = 0 \) and \( E[|x_{p}|^2] = E[|x_{s,i}|^2] = 1 \).

III. PROBLEM FORMULATION

We define the achievable rate as a baseline for primary user’s network performance metric. Therefore, the secondary user that provides the maximum end-to-end achievable rate for the primary system is allowed to transmit and act as a cooperative DF relay for the primary user, provided that this rate is higher than the achievable rate of the primary user without cooperation. Otherwise, the primary receiver decodes what it received from the direct link. Let \( R_\text{dir} \) be the achievable rate when the PT transmits to its receiver on the direct link without any cooperation with the secondary users. We have

\[
R_\text{dir} = \log_2 (1 + \Gamma_\text{dir}),
\]

where \( \Gamma_\text{dir} \) is the received SNR at the PR, and is defined as

\[
\Gamma_\text{dir} = \frac{|h_{pt,pr}|^2 P_p}{\sigma^2}.
\]

Let \( R_{p,i}^{(1)} \) be the rate between the primary transmitter and the \( i \)-th secondary transmitter in the first time slot with \( i \in \{1, \ldots, M\} \), and \( R_{p,i}^{(2)} \) be the rate between the \( i \)-th secondary transmitter and the primary’s receiver. Then,

\[
\begin{align*}
R_{p,i}^{(1)} &= \delta_i \log_2 (1 + \Gamma_{pt,si}), \\
R_{p,i}^{(2)} &= (1 - \delta_i) \log_2 \left( 1 + \frac{\nu}{1 - \nu + \Gamma_{st_i,pr}} \right),
\end{align*}
\]

where \( \Gamma_{pt,si} \) is the received SNR at ST\( i \) from the PT, and \( \Gamma_{st_i,pr} \) is the received SNR at the PR from ST\( i \) with

\[
\begin{align*}
\Gamma_{pt,si} &= \frac{|h_{pt,si}|^2 P_p}{\sigma^2}, \\
\Gamma_{st_i,pr} &= \frac{|h_{st_i,pr}|^2 P_s}{\sigma^2}.
\end{align*}
\]

\( \delta_i \) and \( 1 - \delta_i \) account for the time each transmission takes place with normalized bandwidth. Note that the PR receives the signal from the PT in the first time slot, however, when there exists a ST to cooperate, it can not combine this signal with the one it receives in the second time slot due to the different lengths of two time slots. In other words, the PR can not perform maximum ratio combining, and it ignores what has been received from the direct link in the first time slot.

The end-to-end achievable data rate for the primary user when it cooperates with the ST\( i \) is limited by the minimum of data rates in the two transmission slots [3], i.e.,

\[
R_{p,i} = \min\{R_{p,i}^{(1)}, R_{p,i}^{(2)}\}, \quad i \in \{1, \ldots, M\}.
\]
Since the PT should benefit when allowing a ST to access the spectrum, the candidate STs should satisfy
\[ R_{p,i} \geq R_{\text{dir}}. \] (4)

This condition gives the feasible set \( A \) of all the secondary users who can decode the PT’s signal and also provide the PT with a rate more than the direct link transmission rate. The PT’s objective is to find a ST, among the possible candidates in \( A \), who maximizes \( R_{p,i} \), i.e.,
\[ \max_i \min\{R_{p,i}^{(1)}, R_{p,i}^{(2)}\}. \] (5)

However, \( R_{p,i} \) is limited to either \( R_{p,i}^{(1)} \) or \( R_{p,i}^{(2)} \), which in turn are related to the values of \( \nu \) and \( \delta_i \). Therefore, the primary rate is optimized based on the values of \( \nu \) and \( \delta_i \).

It has been shown in [4] that the rate improvement in the two-phase relaying with the superposition coding is higher when the first time slot is longer than the second one. Therefore, we optimize the time slot duration \( \delta_i \) for a fix power fraction \( \nu \). From (1)–(2), we observe that for a given \( \nu \), \( R_{p,i}^{(1)} \) is an increasing function in \( \delta_i \) while \( R_{p,i}^{(2)} \) is a decreasing function. Thus, the optimal value in (5) is achieved at the equality of these two terms [8], [10], i.e.,
\[ R_{p,i}^{(1)} = R_{p,i}^{(2)}. \] (6)

Solving (6) with a given \( \nu \), we obtain the optimal value of the time slot duration as
\[ \delta_i^*(\nu) = \frac{\log_2\left(1 + \frac{\nu}{1-\nu \Gamma_{p,i} R_{p,i}}\right)}{\log_2\left(1 + \Gamma_{p,i} R_{p,i}\right)} + \frac{\log_2\left(1 + \frac{\nu}{1-\nu \Gamma_{s,i} R_{s,i}}\right)}{\log_2\left(1 + \Gamma_{s,i} R_{s,i}\right)}. \] (7)

So far, we have discussed the requirements of the primary user, but what is the achievable rate of the secondary user? We have already mentioned that the secondary receiver can perform interference cancelation (IC) if it can decode the primary’s information during the first time slot, i.e., if it satisfies
\[ \delta_i \log_2\left(1 + \Gamma_{p,i} R_{p,i}\right) \geq \log_2\left(1 + (1-\nu)\Gamma_{s,i} R_{s,i}\right). \] (8)

Therefore, the rate of the secondary user is given by
\[ R_{s,i} = \begin{cases} (1-\delta_i) \log_2\left(1 + (1-\nu)\Gamma_{s,i} R_{s,i}\right) & \text{with IC,} \\ (1-\delta_i) \log_2\left(1 + \frac{1-\nu}{\nu + \Gamma_{s,i} R_{s,i}}\right) & \text{without IC,} \end{cases} \] (9)

where,
\[ \Gamma_{s,i} = \frac{|h_{s,i} R_{s,i}|^2 P_s}{\sigma^2}. \] (10)

From (9), we observe that if the secondary transmitter can cancel the interference, it can achieve higher data rates. Moreover, the achievable rate of the secondary users is a decreasing function of \( \nu \). If the secondary users also have specific rate (or QoS) requirements, these requirements must also be fulfilled so that they accept to cooperate with the PT and relay its signal, otherwise, they choose not to transmit at all. That is, the relay candidates should as well satisfy
\[ R_{s,i} \geq R_{s,i}^{\text{th}}. \] (11)

The secondary users that satisfy the constraint in (11) as well as (4), form the final feasible set \( K \subset A \) of possible relay candidates that the PT can choose from.

The data rate gain resulted from collaboration of the selected secondary user with the primary user, for a given \( \nu \), is defined as
\[ G_p = \frac{\max_{i \in K} R_{p,i} - R_{\text{dir}}}{R_{\text{dir}}}. \] (12)

In the following section, we study how different parameters, including the rate requirements of the secondary users, affect the achievable gain of the primary user.

**IV. NUMERICAL STUDY**

We investigate the effects of different system parameters on the cooperation between the primary and secondary users and the achievable gain of the primary user by means of Monte-carlo simulations. The steps of the simulation for numerical optimization of the system is depicted in Alg. 1.

**Algorithm 1:** Performance evaluation process.

- **Data:** \( A \): set of secondary users allowed to transmit, \( K \): set of allowed secondary users which also fulfill their rate thresholds (\( K \subset A \)).
- **Input:** \( M \): number of secondary users, \( S \): set of average SNR levels for the direct link, \( N \): set of possible power fraction values \( \nu \), \( MC \): number of Monte-Carlo simulation runs.

\begin{algorithm*}
\begin{algorithmic}
\For {\( s \in S \)}
\For {\( m = 1 : MC \)}
\State Generate \( M \) secondary users uniformly distributed in the area with radius \( 0.1R \leq d \leq 0.5R \)
\State Calculate all channel gains \( |h_{k,j}|^2 \)
\State For \( \nu \in N \) do
\State \hspace{1em} \Calculate the optimal time slot duration \( \delta_i^* \) from (7)
\State \hspace{1em} \Calculate \( R_{p,i} \) from (1), (2), and (3)
\State \hspace{1em} Form \( A \) using (4)
\State \hspace{1em} Calculate secondary user rates \( R_{s,i} \) from (9)
\State \hspace{1em} Form \( K \) from \( A \) and the condition (11)
\State \hspace{1em} Find the relay \( k \in K \) based on the objective in (5)
\EndFor
\EndFor
\end{algorithmic}
\end{algorithm*}

In our performance analysis, we try to answer the following questions:

- How does the primary user’s gain relate to the direct link quality?
- What are the parameters that affect the percentage of cooperation between the secondary users and the primary user?
• How does the number of available secondary users affect the primary user’s gain?
• How does the rate requirements of the secondary users as well as the fraction of the power they allocate for relaying limit the primary user’s gain?

We assume a circular cell with radius $R = 200$ m, where the primary transmitter is located in the center of the cell and the primary receiver on the cell edge. In each simulation run, $M$ secondary transmitters are generated randomly with uniform distribution over the cell area with radius $0.1R \leq d_{pt, st} \leq 0.5R$ from the PT. The lower bound on the distance of the STs are enforced so that the path-loss model is valid. Similarly, for each secondary transmitter, a secondary receiver is generated uniformly over the area with radius $0.1R \leq d_{st, sr} \leq 0.5R$. The path-loss exponent is considered to be $\alpha = 4$, and the fading component of the channel is generated randomly in each realization. We assume normalized transmit power for all transmitters in the cell, and receiver noise power of $10^{-9}$ W. The set $S$ of fixed average SNR levels for the direct link is considered with $S = \{-2, 0, 2, 4\}$, from which the path-loss coefficient $c_t$ and the channel gains are obtained. Furthermore, secondary users are assumed to have the same rate requirements, i.e., $R_{th}^s = R_{th}$. The number of Monte-Carlo simulation runs is set to 10000.

Fig. 4 shows the average primary user’s gain in (12) versus different values of direct link average SNR, for given values of $\nu = \{0.85, 0.9, 0.95\}$, $M = 15$, and $R_{th}^s = 1$ bps. It can be observed that relaying is only beneficial for the primary user when the direct link quality is poor. Also as expected, the more the fraction of secondary user’s power that is assigned to the primary user’s signal, the more the primary user gains.
In Fig. 5, the average number of instances that secondary users cooperate with the primary user is shown for different direct link conditions. It can be seen that when the direct link quality is low, such as the case with $\Gamma_{\text{dir}} = -2\,\text{dB}$, the number of secondary users which satisfy the constraint in (4) is high, and therefore, the size of the feasible set $A$ is large. If secondary users in this set can assign more transmit power to their own signal (which corresponds to a small value of $\nu$), the number of secondary users who can fulfill their rate requirements in (11) increases. Consequently, there are more instances that the final feasible set $K$ is not empty, and the primary user can find a secondary user from this set to cooperate with. As the value of $\nu$ increases, fewer secondary users can satisfy their rate requirements which results in less cooperation.

However, when the quality of the direct link is better, such as the case with $\Gamma_{\text{dir}} = 4\,\text{dB}$, and the value of $\nu$ is small, the number of secondary users that can fulfill the constraint in (4) is small. Therefore, the feasible set $A$, and consequently $K$, shrinks, resulting in fewer number of cooperation instances. On the other hand, with a larger value of $\nu$, the final feasible set becomes larger, resulting in higher number of cooperation instances.

Now, if we fix the direct link SNR to $\Gamma_{\text{dir}} = 2\,\text{dB}$ and vary the number of secondary users in the cell, we can see in Fig. 6 that the average gain of the primary user increases. That is, with higher number of secondary users, there is a higher probability that the primary user can find a helpful relay which can also satisfy its own rate requirement.

Fig. 7 depicts how the average primary user’s gain behaves as a function of the rate requirement of the secondary users. It can be seen that if secondary users have higher rate requirements, the primary user’s gain decreases noticeably. We can also observe that if the rate requirement of the secondary user is high, increasing the value of $\nu$ does not provide a higher average gain for the primary user.

V. CONCLUSIONS

In this paper, we considered the problem of relay selection in a cooperative cognitive radio system in which a secondary transmitter can act as a relay for the primary transmitter in order to maximize the primary user’s gain. Moreover, we assumed that secondary users have certain quality-of-service requirements that need to be satisfied. We showed that the primary user’s data rate gain is a function of both time slot duration and the fraction of power that the secondary user allocates for relaying the primary’s signal. Fixing the power coefficient, we derived the optimal time slot duration for every possible relay candidate based on which, the primary user’s rate is obtained. The secondary user which maximizes the primary user’s gain while satisfying its own QoS requirements is selected as the relay. We studied how different parameters, such as the assigned powers in the coding scheme, direct link reliability, and the number of secondary users affect the overall system performance. We have assumed that both primary and secondary users have perfect knowledge about the link qualities of each other and it will be interesting to investigate the case when this is not the case.

REFERENCES