On the Fundamental Limitations of Timing and Energy Resolution for Silicon Detectors in PET Applications

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Department of Physics, KTH

Supervisor: Mats Danielsson

Martin Sjölin
msjoli@kth.se, 070-3722130

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Abstract

Using silicon based detectors for positron emission tomography (PET) applications has the benefit of being able to use more information about the detected gamma photons than today’s scintillator based PET systems. This includes the angle of incidence which together with the system’s good energy resolution can be used to effectively confirm true coincidences and enable the use of single photons and single-scattered photon pairs in the image reconstruction. Also, the excellent timing of the detections promotes the use of the Time of Flight (TOF) method which together with the fine spatial segmentation of the silicon detectors gives excellent spatial resolution in the image.

The aim of the report is to investigate the fundamental limits of the timing and energy resolution of silicon sensors and the energy and angle resolution of a silicon based PET detector. The detection efficiency of the system is also evaluated under reasonable constraints on the detector size and power consumption in order to estimate suitable detector parameters. The simulations are performed using MATLAB and the Peneloe software.

The report will indicate a timing resolution down to $\sim 50$ ps for 100 keV energy depositions. The system energy resolution will prove to be as good as 3% at 511 keV and the angle resolution $\sim 3^\circ$. The silicon based PET detector provides some challenges concerning the detection efficiency versus power consumption and the amount of produced data, but also many benefits, including a high background rejection ratio, that have the potential of increasing the image quality of PET images.
1 Introduction

Positron emission tomography (PET) is a medical imaging technique that produces two- or three-dimensional images of bodily functions and it is one of the most important tools for detecting tumours and other irregularities in the human body. PET traces activity in the body by finding the position of positron emitting radionuclides that have been attached to metabolically important molecules such as glucose.

The radionuclides are introduced in the body through intravenous injection or, less commonly, inhalation, and will distribute throughout the body in a manner determined by the biochemical properties of the tracer. When the nuclides decay they emit positrons which travel a short distance before annihilating with an electron. The annihilation reaction produces a pair of anti-parallel gamma photons with an energy of 511 keV each.

A photon pair will produce two detections on two opposing sites on the detector and the detected events are used to trace the position of the annihilation by drawing lines of response (LOR) straight between the two detections. If one of the photons scatter inside the scanned object, the method of drawing a LOR will give false information about the annihilation position and therefore it is crucial to be able to distinguish between scattered and non-scattered photon pairs.

Research and clinical studies have been made using the Time of Flight (TOF) method in PET [12], where the small time difference between two photon detections is used to determine a probability distribution along the LOR for the position of the annihilation (Figure 2). The method requires very good timing.

![PET scanner](image1.png)  ![PET image of human brain](image2.png)

Figure 1: (a) PET scanner where the paths of the photon pairs are drawn [24] (b) PET Image of the human brain where the activity in the different parts are indicated by a color scale [25]
of the detected interactions and the greatest benefit of TOF is improved signal
to noise ratio [9]. Todays systems generally have a timing of several hundred
pico seconds [30].

Figure 2: The principle of the Time of Flight method [26]

PET scanners today use detectors with scintillator crystals and photo multi-
plier tubes and the main purpose of this report is to investigate if a silicon
based detector could improve PET performance. Silicon sensors have a much
higher timing resolution than traditional detectors, enabling a performance in-
crease from TOF. Moreover, a silicon based detector can provide a finer spatial
segmentation than a scintillator crystal, which implies a better spatial resolu-
tion. However, due to its low atomic number, the photoelectric cross section
of silicon is low compared to other detector materials, especially in the energy
ranges occurring in PET. Therefore, the typical photon detection in the silicon
based detector consists of multiple Compton scatterings followed by either pho-
toelectric absorption or escape from the detector.

Since the photon does not necessary deposit all of its energy in the detector,
estimating the photon energy by summing up the deposited energies is not ap-
nlicable. In this work it is instead proposed that a Compton camera technique
i used, where the physics of Compton scattering is used to estimate the photon
energy [28]. The Compton camera technique also provides additional advantages
since the angle of incidence can be calculated. It has been proposed that the
angle of incidence can be used to recreate the annihilation position for single-
scattered photon pairs, where one of the photons has been scattered once [1].
Moreover, it is possible to use single photons, where only one of the photons is
detected, by using the cone defined by the angle of incidence (Figure 3) in the
image reconstruction [34].
The major advantage of the silicon based PET detector is the potentially very high background rejection ratio, which refers to the ability to identify which photon pairs to use in the image reconstruction. The silicon based detector has three methods for identifying if a photon-pair is non-scattered:

- The coincidence window is the first method which acts on the fact that if the two detected photons come from the same annihilation the detections must occur close in time. With a good timing resolution it is possible to keep the coincidence window short which reduces the background.

- The second method is calculating the photon energy in order to confirm that both the photons have an energy of 511 keV.

- The third method is calculating the angle of incidence and comparing it to the geometric angle of incidence which can be created since we know where the second photon was detected. Using the angle of incidence for background rejection is not available for scintillator based detectors and one of the reasons why this system has the potential of having a very high background rejection ratio.

The main effort in this report has been to investigate the fundamental performance limitations of a silicon based detector. The subjects of evaluation are:

- Timing if the interactions, referred to as the timing resolution
- The energy resolution for the individual energy depositions, referred to as the energy resolution of silicon (Si) sensors
- The energy resolution for the photon energy, referred to as the system energy resolution
- The angle resolution for the angle of incidence of the detected photons

Figure 3: Three detected single photons where the first scattering angles are used to construct cones of possible photon paths. The source is assumed to be where the cones intersect [10].
The timing of the interactions is done using a pulse matching method where the signal is compared to a set of reference signals with known starting times and the deposited energy is found by integrating the reference signal with the best match. The system energy resolution is evaluated for both a single-calculation method and a maximum likelihood method.

The simulation of the system has been divided into two major parts: single pixel and full detector.

i) The single pixel model is used to simulate the signal generated when a photon interacts with the detector material. The signal is then used to evaluate the timing of a pulse and the energy resolution of the silicon sensors.

ii) The full detector model simulates the creation of a photon pair, the scattering process inside the scanned object and the scattering process in the detector. The result is a set of interaction positions and deposited energies corresponding to a detected photon pair. This is then used to calculated the energy of the photon and to estimate the accuracy of the calculations as a function of pixel size and energy resolution of the silicon sensors.

An attempt has also been made to suggest reasonable dimensions of a silicon based detector from a power consumption versus detection efficiency point of view. Several practical issues are discussed on the way, such as: radiation damage to electric components in the scanner, charge overflow from very large energy depositions, count rate and amount of generated data.

2 Simulating the Silicon PET Detector

2.1 Geometry of the Detector

The full detector consists of several stacks of 0.5 mm thick silicon wafers (see Figure 4) with a cathode collecting the electrons on one side and a segmented anode collecting the holes on the other side. The readout is done through the anode electrodes where an induced current is measured. The area covered by one anode is referred to as a pixel or a sensor. The silicon wafers are stacked close together and the full detector is therefore modelled as a cylinder of solid silicon (see Figure 5).
2.2 Simulation of the Electric Fields

The electric potential in the detector can be calculated with the Poisson equation

$$\nabla^2 \varphi = -\frac{qN}{\epsilon}$$

where $\epsilon = 11.68 \cdot \epsilon_0$ is the dielectric constant for silicon, $N$ is the net doping concentration of the silicon ($5 \times 10^{11}$ cm$^{-3}$) and $q$ is the elementary charge. The boundary conditions are given by

$$\begin{cases}
\varphi = 1000 \, V & z = 0 \\
\varphi = 0 \, V & \text{for } z = d_z, \text{ on the electrodes} \\
\frac{d\varphi}{dn} = 0 & \text{for } z = d_z, \text{ between the electrodes}
\end{cases}$$

$\hat{n}$ is the normal to the surface, in this case $\hat{e}_z$ and $d_z$ is the depth of the thickness of the silicon wafer. The electric fields are then calculated with

$$\vec{E} = -\nabla \varphi$$
The weighting potential describes the electrostatic coupling between the moving charges and the current induced on the electrode. The field is calculated with the Laplace equation

$$\nabla^2 \varphi_w = 0$$  \hspace{1cm} (4)

Where the potential at the investigated electrode is set to unity and the potential for all other electrodes is set to zero. The equations are solved iteratively using the Successive Over-Relaxation method for a 3D grid.

Figure 6: Calculated electric fields for the silicon detector for a y-z plane in the center of the voxel

Figure 7: Calculated weighting fields for the silicon detector for a y-z plane in the center of the voxel

The potential difference between the anode and the cathode is usually between 100 and 1000 V. The identification of the starting time of a pulse is easier if the pulse is short and high. This is obtained with a high electric field and therefore we choose to operate at the higher potential difference.
2.3 Simulation of the Signal

A single pixel and its surrounding neighbours are simulated in order to estimate the signal that is generated when a photon interacts with the detector material. For evaluation of the timing resolution an electrode plate size of $0.5 \times 0.5 \text{ mm}^2$ is chosen. The simulations are made with 100 ps steps and the charge carriers will travel through the detector until they are collected by the electrode.

2.3.1 Creation of the Electron Cloud

The energy released in a Compton interaction results in the release of an electron, here called the Compton electron. The initial kinetic energy of the Compton electron is given by

$$E_k = \Delta E - E_b$$  \hspace{1cm} (5)

where $\Delta E$ is the energy deposited by the photon and $E_b$ is the binding energy of the electron. The Compton electron will initially have a large kinetic energy and it deposits its energy to nearby electrons, creating a "cloud" of secondary electron-hole pairs.

The electron cloud is generated using the Monte Carlo based simulation program PENELOPE [5], where the secondary electrons are tracked until their energy is lower than 50 eV (the simulation model is only valid down to this energy). The deposited energies and their respective positions are recorded along the way. The energy tracks are then converted into electron-hole pairs by dividing the deposited energies by the mean energy needed for electron-hole pair production, which is 3.6 eV in silicon [13].

In reality the 50 eV electrons will continue to release their energy while moving and creating small clouds of electron-hole pairs of their own. Using the spherical gaussian cloud approximation [18], the standard deviation $\sigma$ of the charge carrier cloud (in $\mu$m) will depend on the energy of the electron $E_e$ (in keV) like

$$\sigma(E_e) = 0.0044E_e^{1.75}$$  \hspace{1cm} (6)

With $E_e = 50$ eV, the standard deviation of the cloud is $\sigma \sim 10^{-11}$ m and therefore negligible.

2.3.2 Diffusion

The cloud of charge carriers will spread out due to diffusion and the size follows a Gaussian distribution with diffusion coefficient $D_{\text{diff}}$ and a standard deviation $\sigma$ given by

$$D_{\text{diff}} = \frac{kT}{q\mu}$$  \hspace{1cm} (7)

$$\sigma = \sqrt{2D_{\text{diff}}t}$$  \hspace{1cm} (8)

where $k$ is the Boltzmann constant, $T$ is temperature, $q$ is the charge of the particle, $\mu$ is the electron-hole mobility and $t$ is the time measured from the Compton interaction.
2.3.3 Acceleration of Charge Particles

The free electrons will accelerate in the applied electric field and also be subject to a damping force proportional to the velocity of the charge carrier. The total force on the electrons is given by

\[ F = m_e \frac{dv}{dt} = qE - \frac{q}{\mu} v \]  

(9)

Solving the equation for the speed of the charge carriers as a function of time gives

\[ v(t) = E\mu \left( 1 - \exp \left( - \frac{qt}{m_e \mu} \right) \right) \]

(10)

The time it takes to accelerate the electrons and holes to 99% of maximum velocity is 1.26 ps and 3.53 ps respectively. Since the signal length is approximately 4 ns and the sampling period is 100 ps, the acceleration time is negligible and will not be taken into consideration.

2.3.4 Coulomb Forces

The Coulomb forces between the free charged particles will have a relatively small impact on the signal since they are small compared to the forces due to the electric field and are assumed to not greatly effect the pulse shape.

2.3.5 The Induced Current

The electron-hole pairs will travel through the detector under the influence of the applied electric field \( \vec{E} \) and diffusion. The movement of the charge carriers will induce a current in the electrode which is given by the Schockley-Ramo theorem [3]

\[ i(t) = -qE_w \vec{v}_i \]

(11)

\[ \vec{v}_i = \vec{E} + \vec{v}_f \]

(12)

where \( \mu \) is the mobility of the charge carrier, \( \vec{v}_f \) is the velocity caused by the diffusion and \( E_w \) is a weighting field describing the coupling between the induced current and the movement of charge carriers in the detector. An example of an induced current signal is shown in Figure 8.

2.3.6 Correction for the Cloud Creation Time

The PENELOPE software simulates the path of the Compton electron and the shape of the created cloud of charge carriers. The induced current on the electrode is then simulated by moving the charge carriers in the applied electric field. This assumes that the time it takes to create the cloud is negligible, but this is not realistic. In this section, the cloud creation time is estimated and corrected for.

The energy loss of the Compton electron as it travels through the material depends on the stopping power \( S_p \), which is defined as

\[ S_p(E_k) = -\frac{dE_k}{dx} \]  

(13)
where $E_k$ is the kinetic energy of the Compton electron. The stopping power in silicon for different electron energies is shown in Figure 9.

The rate at which the energy is transferred from the Compton electron as it travels through the silicon is given by

$$\frac{dE_k}{dt} = -S_p(E_k)v(E_k)$$  \hspace{1cm} (14)
\[ v(E_k) = \frac{c \sqrt{E_k (E_k + 2m_e c^2)}}{E_k + m_e c^2} \]  

(15)

where \( m_e \) is the electron mass and \( c \) is the speed of light. The number of created electron-hole pairs, \( N_e \), after a time \( T \) is given by

\[ N_e(T) = \frac{1}{E_{\text{Si}}} \int_0^T \frac{dE_k}{dt} \, dt \]  

(16)

where \( E_{\text{Si}} = 3.6 \text{ eV} \) is the ionization constant of silicon. The number of created electron-hole pairs as a function of time for 10, 50 and 100 keV electrons travelling in silicon is shown in Figure 10. The stopping power \( S_p \) is calculated using the ESTAR database provided by the NIST physics laboratory [6].

Figure 10: Creation of charge carriers in silicon. It is likely that all charge carriers are created within a single sampling interval (100 ps).

The cloud creation time is corrected for by shifting the signal according to the creation process of the charge carriers. An example of how the shifting is done is shown in Figure 11. For a real signal, the shift is much more subtle since the total cloud creation time for the simulated energies is less than the sampling interval and the total signal is approximately 50 sampling intervals long.

Shifting the signal in this way is a reasonable approximation for small energy depositions where the electric and weighting fields are approximately uniform in the cloud. This is the case here since the largest simulated energy deposition is 100 keV, which in the Gaussian cloud shape approximation 6 creates a cloud with a standard deviation of 0.014 mm. This is very small in comparison to the electrode size of 0.5 mm.

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Figure 11: An example showing the shift of a signal due to the creation time of the cloud. Here 64% of the charge carriers were created between $t = 1$ and $t = 2$, and 36% between $t = 2$ and $t = 3$.

### 2.4 Readout Electronics Modelling

The readout ASIC (Application Specific Integrated Circuit) is modelled as a transimpedance amplifier (TIA) [17] with a sampling frequency of 10 GHz. When a photon interacts in the detector it deposits energy and electron-hole pairs are created. This will create an induced current signal from the diode. The signal is then read out by the TIA and processed by an analogue filter followed by additional digital filters. The transfer from induced current to final filtered signal is modelled here.

To find the transfer function from incoming current $I_i$ to output voltage $V_{out}$ the system is modelled as in Figure 12. Assuming that no current passes over the transimpedance, $G$, we have the current relationship

$$I_i = I_e - I_0$$

$$I_i = V_1 i \omega C_d - \frac{V_{out} - V_1}{R_f} = V_1 \left( i \omega C_d + \frac{1}{R_f} \right) - \frac{V_{out}}{R_f}$$

Now using the relation between $V_1$ and $V_{out}$

$$V_{out} = -(GR_f - 1)V_1 \implies V_1 = \frac{V_{out}}{1 - GR_f}$$

$$I_i = \frac{V_{out}}{1 - GR_f} \left( i \omega C_d + \frac{1}{R_f} \right) - \frac{V_{out}}{R_f} = V_{out} \left( \frac{i \omega C_d + \frac{1}{R_f}}{1 - GR_f} - \frac{1}{R_f} \right)$$

This gives the output voltage

$$V_{out} = I_i \left( \frac{i \omega C_d + \frac{1}{R_f}}{1 - GR_f} - \frac{1}{R_f} \right)^{-1} = I_i \left( \frac{1 - GR_f}{i \omega C_d + \frac{1}{R_f} - \frac{1 - GR_f}{R_f}} \right)$$

$$\implies V_{out} = I_i \left( \frac{1 - GR_f}{i \omega C_d + G} \right)$$
The noise is modelled as filtered white noise which can be expressed as the convolution of white noise and the impulse response of the filter, which according to the convolution theorem implies multiplication in the frequency domain. The output noise voltage spectral density, $S_{\text{von}}$, is given in the frequency domain by the squared-magnitude of the frequency response scaled by the variance of the driving white noise. It is given by Eq.(25) and Eq.(61) in [17] as

$$S_{\text{von}}^2 = \frac{4kT\gamma}{g_{m1}} \left| \frac{1}{1 + j\omega R_f C_d} \right|^2$$

(23)

The filtered noise is therefore modelled as white noise with a variance of $\sigma^2 = 4kT\gamma/g_{m1}$ which is multiplied by the bandwidth, Fourier transformed and multiplied with the frequency response of the filter (the expression within the squared magnitude in Eq.(23)).

The constant $g_{m1}$ determines the noise level and depends on the allowed level of power consumption in the input transistor of the amplifier. The analogue power consumption is given by

$$P_{\text{analogue}} = g_{m1} V_{\text{eff}} V_{dd}$$

(24)

with notation from [17]. If nothing else is stated, $g_{m1} \simeq 30 \text{ mA/V}$, $V_{\text{eff}} \simeq 75 \text{ mV}$ and $V_{dd} \simeq 1 \text{ V}$ giving $P_{\text{analogue}} = 2.25 \text{ mW}$. The used values of the other parameters are $\gamma = 1$, $C_d = 1 \text{ pF}$, $G = 30 \text{ mA/V}$ and $R_f = 3k\Omega$.

The rest of the analogue process is assumed to consume 30% of $P_{\text{analogue}}$ and the digital part of the power consumption is assumed to be 2 mW, giving a total power consumption per pixel of $P_{\text{pixel}} = 1.3P_{\text{analogue}} + 2 \text{ mW}$.

The filtered signal and the filtered noise voltage are added at the exit of the amplifier and filtered with an analogue low-pass filter with the transfer function

$$f_{\text{analog}} = \frac{1}{(1 + j\omega t_1)^2}$$

(25)
The highest frequency that the sampling can detect without risk for aliasing is according to the sampling theorem equal to half the sampling frequency (the Nyquist frequency). The bandwidth is therefore chosen in order to reproduce frequencies up to the Nyquist frequency correctly, i.e \( f_1 = 5 \text{ GHz} \).

If the noise level is not sufficiently low for identifying the pulse after the analogue filter, it is possible to apply a digital low-pass filter with a lower bandwidth. The drawback of filtering with lower bandwidth is that information about the pulse shape will be lost.

### 2.5 Simulation of the Scattering Process

While interacting with the detector material, the photon can undergo Compton scattering, Rayleigh scattering or photoelectric absorption. The probability of the interaction being of type \( i \) is given by

\[
p_i(E) = \frac{\sigma_i(E)}{\sigma_{\text{photo}}(E) + \sigma_{\text{compton}}(E) + \sigma_{\text{rayleigh}}(E)}
\]

where \( \sigma \) represents the respective interaction cross section and \( E \) is the energy of the photon. In the case of Rayleigh scattering the photon changes its direction, but does not deposit any energy and therefore the interaction is not detected. In the case of photoelectric absorption the photon deposits all its energy. In a Compton scattering, the photon interacts with an electron in the silicon, depositing energy to the electron and changing its direction of propagation with a scattering angle \( \theta \). The distribution of scattering angles, \( \theta \), for a certain photon energy, \( E \), is given by the Klein-Nishina formula

\[
\frac{d\sigma}{d\theta} \propto \sin \theta \left(1 + \cos^2 \theta + \frac{k^2(1 - \cos \theta)^2}{1 + k(1 - \cos \theta)}\right)(1 + k(1 - \cos \theta))^2
\]

\[
k = \frac{E}{m_e c^2}
\]

where \( m_e \) is the electron mass and \( c \) is the speed of light.

![Compton scattering](image)

Figure 13: Compton scattering of a photon of wavelength \( \lambda \) with the scattering angle \( \theta \). The deposited energy results in a change of wavelength (energy) of the photon.
The probability of the photon scattering within an angle $\Delta \theta$ is obtained by integrating the Klein Nishina formula with respect to $\theta$.

The deposited energy $\Delta E$ corresponding to a scattering angle $\theta$ is calculated using the Compton scattering formula

$$\Delta E(E, \theta) = E \left( 1 - \frac{1}{1 + k(1 - \cos \theta)} \right)$$  \hspace{1cm} (29)

The energies deposited by a 511 keV photon in silicon will range from 0 to 340 keV, where the later corresponds to a scattering angle or $180^0$. The Klein-Nishina distribution of deposited energies for 511 keV photons is shown in Figure 27.

After one interaction has occurred, the location of the next interaction is found by generating a new travelled distance. The probability density function for the distance $r$ travelled by the photon is given by an exponential distribution

$$f(r|\mu) = \frac{1}{\mu} \exp \left( -\frac{r}{\mu} \right)$$  \hspace{1cm} (30)

where $\mu$ is the linear attenuation coefficient for the material that the photon propagates in and depends on the energy of the photon\(^1\).

The position for the next interaction is given by

$$\mathbf{r}_{\text{next}} = \mathbf{r} + R \cos \theta \hat{z} + R \sin \theta \cos \phi \hat{x} + R \sin \theta \sin \phi \hat{y}$$  \hspace{1cm} (31)

where $R$ is the distance travelled, $\theta$ is the scattering angle, $\phi$ is the azimuthal angle, $\hat{z}$ is the direction of propagation before the scattering at $\mathbf{r}$ and $\hat{x}$, $\hat{y}$ and

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\(^1\)http://physics.nist.gov/PhysRefData/Star/Text/ESTAR.html, the ESTAR program, National Institute of Standards and Technology
\( \hat{y} \) are orthonormal Cartesian basis vectors. The azimuthal angle is uniformly distributed between 0 and \( 2\pi \). The angles are shown in Figure 15.

![Figure 15: Definition of scattering angles. The photon initially travels in the z-direction.](image)

### 2.6 Random Processes

There are four random processes; the distance travelled between interactions, the type of interaction, the scattering angle in the Compton scattering and the scattering angle in a Rayleigh scattering. The outcomes of the random processes are simulated by generating a random number \( a \) between 0 and 1 and choosing the first value that corresponds to the value \( a \) of the cumulative distribution function for the process.

The photons do not always travel exactly anti-parallel since the positron and the electron do not always have zero speed when they annihilate. This effect has not been taken into account.

### 3 Discussion

#### 3.1 Detection Efficiency

Designing the detector is an optimization problem with many parameters including: total power consumption, power consumption per pixel, pixel size, and
the relationship between the height and thickness of the detector. These parameters will in turn define the energy resolution of the silicon sensor, timing, system energy resolution, detection efficiency and spatial resolution of the detector. In the end these parameters will determine the image quality. Solving the full optimization problem is not attempted here, instead a typical case is investigated in order to estimate if the setup is reasonable from a detection efficiency point of view.

A reasonable total power consumption could be 20 kW and the possible dimensions of the detector for three different detector volumes are overlaid in Figure 16 for different pixel sizes and power consumptions per pixel.

The detection efficiency is investigated as a function of the thickness and height of the detector. The detection efficiency is here defined as number of good detections/number of annihilation events. For photon pairs, a good detection is defined as one where both the photons are detected with 2 interactions or more. For single photons, a good detection is defined as one where the photon is detected with 3 interactions or more. The imaging case is a point source in the center of the detector and the resulting efficiency contours are shown in Figure 16. The detection efficiency can be compared to the detector volume and translated to power consumption per pixel and pixel size using Figure 17.

It can be seen that it might be beneficial to be able to reconstruct images using the single photons, i.e. using cones defined by the first scattering angle in the detector. The cones are of course worse than the LORs concerning precision, but it will be possible to create many more cones than LORs since the detection efficiency is much higher (∼3×) for single photons.

![Figure 16](image.png)

Figure 16: The detection efficiency for photon pairs and single photons. The contours for three different volumes of the detector is overlaid.

The total efficiency of the system is the product of the detection efficiency and how good the system is at using the detected photons for image reconstruction. This includes the how good the system is at distinguishing between scattered
Figure 17: The pixel size as a function of power consumption per pixel for three different detector volumes assuming a total power consumption of 20 kW

and non-scattered photons, the ability to find the right order of the interactions, the ability to identify an incomplete set of data, etc. The total efficiency of the system is left for further research.

3.2 Count Rate and Amount of Generated Data

There will be a significant amount of data produced when sampling and storing the signals from all the detector pixels during a scan. This might possibly be a limitation of the system and therefore an investigation is made to confirm that the data flow is reasonable.

In order to make an estimation of the count rate, i.e. the number of detected photon interactions per second, a lot of parameters have to be taken into account. First we have the biological process of distributing the injected radio nuclide in the body and second we have the geometrical and intrinsic properties of the detector.

We make an estimation of the count rate for the case of brain imaging with $^{18}F - FDG$ (fluorine-18 fluorodeoxyglucose) where the radio nuclides are injected intravenously.

The activity at a time $t$ is given by the radioactive decay law

$$A = A_0 \exp \left( -\frac{\ln 2 \cdot t}{t_{1/2}} \right)$$

(32)

where $A_0$ is the initial activity and $t_{1/2}$ is the half life of the nuclide. The injected dose of FDG prior to a PET scan is typically $A_0 = 370 - 555$ MBq [22]. The fraction of the injected activity that will be distributed to the brain under normal circumstances is $\sim 7\%$ [21]. A PET scan typically starts after 45-60 min after the injection and the half life of the nuclide $^{18}F$ is 110 min. This implies
that the activity in the brain when the scanning begins has a maximum value of

\[ A_{\text{brain}} \simeq 0.07 \cdot 555 \cdot \exp \left( -\frac{-\ln(2) \cdot 45}{110} \right) = 29 \text{ MBq} \quad (33) \]

It is reasonable to assume that all decays release a positron and that all positrons will annihilate and create two 511 keV photons.

Now we want to find the average number of interactions per annihilation event. The detector is chosen to be 4 cm thick and 13.3 cm high and the imaging case is a sphere of soft tissue with a 9 cm radius and uniform distribution of radionuclides. The propagation of the photons are Monte-Carlo simulated to include scattering in the sphere and the interaction with a detector with the given dimensions. The simulation predicts an average number of interactions per annihilation of \( N_{\text{int}} = 0.86 \). It is assumed that every interaction will give pulses in approximately 5 pixels due to induced currents in neighbouring pixels, but the actual number depends on the pixel size and the size of the energy deposition. Assuming that the interactions are uniformly distributed in the detector we have a mean pulse frequency in each pixel given by

\[ \frac{5 \cdot N_{\text{int}} \cdot A_{\text{brain}}}{N_{\text{pixels}}} = 12.6 \text{ Hz} \quad (34) \]

We record pixel data continuously and keep the 100 latest samples with a sampling rate of 10 GHz. When a pulse is detected we save 100 values from some time before the pulse to some time after. Then we perform an AD-conversion of these 100 values with a resolution of 8 bits, thus producing 800 bits. The pixel will then create a message including these 800 bits, plus a pixel address of 32 bit and a time stamp of 48 bits (covers 8h with the resolution of 100 ps). This will create a total message size of 880 bits. This will give us a total data flow of

\[ 880 \cdot 5 \cdot N_{\text{int}} \cdot A_{\text{brain}} = 111 \text{ Gb/s} \quad (35) \]

This is a large amount of data which shows that this is an issue that has to be considered in detail before attempting a construction of the system.

### 3.3 Absorbed Dose in the ASIC

The ASIC is integrated with the detector material and it consists of inactive silicon. Since the photons come from all directions, the ASIC is difficult to protect and there is a risk that the radiation damages the ASIC. The ASIC has a maximum dosage tolerance of approximately \( 0.1 \) – \( 1 \) Mrad (Joule of energy per kilogram of matter) before there are errors in the measurement. An estimation of the dose that the ASIC receives during one scan in order to estimate the life time of the ASICs is made.

The number of detected annihilation pairs that are used to reconstruct images in traditional (scintillator) PET is typically \( \sim 10^6 \) – \( 10^8 \). The number of detections needed for image reconstruction using this detector model will be less than for a traditional system so these figures can be used as a limiting values.
Events that can be used for the image reconstruction is here defined as events where both photons are non-scattered when they enter the detector and both are detected with two interactions or more. Using the same imaging case and detector dimensions in Section 3.2, the ratio of usable events and all simulated annihilation events was found to be \( \sim 2.6 \cdot 10^{-3} \). The total number of events needed to produce a suitable number of good detected events is therefore \( N_{\text{decays}} = 3.8 \cdot 10^{10} \). The volume of the detector is given by

\[
V_{\text{detector}} = h\pi(r_{\text{outer}}^2 - r_{\text{inner}}^2) = 5 \cdot 10^3 \text{ cm}^3
\]

(36)

The average deposited energy per annihilation event in the field of view is found to be \( \sim 36 \text{ keV} \) which gives a total deposited dose per cm\(^3\) of

\[
D = \frac{E_{\text{dep}} \cdot N_{\text{decays}}}{V_{\text{detector}}} = 2.7 \cdot 10^{11} \text{ eV/cm}^3 = 4.4 \cdot 10^{-8} \text{ J/cm}^3
\]

(37)

The density of silicon is \( \rho = 2.329 \cdot 10^{-3} \text{ kg/cm}^3 \) and this gives a deposited energy per kilogram of \( D/\rho = 1.9 \cdot 10^{-5} \text{ J/kg per scan} \). This gives a very long average lifetime of the ASIC.

### 3.4 Large Energy Depositions

The distribution of deposited energies range from 0 to 511 keV, where all above 340 keV must be photo electric absorptions since a 511 keV photon can release at most 340 keV in a Compton scattering. If the electron cloud is approximated with a spherical Gaussian distribution [18] we can see that some clouds will be larger than the detector pixels. This implies that there will be a lot of charge sharing between the pixels and many high energy electrons will hit the electrodes.

A typical electrode could be a \( \sim 0.7 \mu\text{m} \) thick aluminium plate and the distance between the detector elements could be in the range of a few hundred \( \mu\text{m} \). A brief investigation of the transmission of electrons through aluminium and air ([19], [20]) suggests that a large amount of the high energy electrons (\( > 10 \text{ keV} \)) will pass right through the electrode and the air and into the neighbouring detector element and continue to create electron hole pairs there. This is positive in the sense that we can trace most of the energy deposited by the photon.
Figure 18: The normalized radial profile of the electron density in Gaussian distributed electron clouds created by three different energy depositions in silicon. The pixel boundary in the case of 0.5×0.5 mm$^2$ pixels is marked with the dashed line.

4 Conclusions

The timing of the interactions through pulse matching has a potential to be very accurate, especially for high deposited energies ($\sigma = 50$ ps for 100 keV energy deposition). This will increase the efficiency since it makes it easier to find the right order of interactions and the spatial resolution will increase due to TOF. Also, the timing will enable the use of a shorter coincidence window which will decrease the number of random coincidences and increase the number of correctly identified photon pairs.

The system energy resolution is good for 511 keV photons ($FWHM/E \approx 3\%$) and will be useful for identifying non-scattered photon pairs. The maximum likelihood estimation of the energy has shown to improve the energy resolution. The angle resolution has also shown to be good ($FWHM \approx 3^\circ$) which will lead to an increase of the number of correctly identified non-scattered photon pairs and enable the use of single photons and single-scattered photon pairs.

The spatial resolution in the image will largely depend on the pixel size, although it might be possible to obtain sub-pixel resolution and in that case it will be favourable to have larger pixels. The spatial resolution will in any case be a large benefit for the silicon based detector.

The amount of produced data will have to be considered when attempting a construction of the system, but since the information technology is developing in a phenomenal rate this will most likely not be a problem. The power consumption versus the detection efficiency will also be a very important issue.

Some suggestions for further investigation of the system are:
- A major simulation comparing the system with a ‘state of the art’ scintillator detector system in order to estimate the relative efficiency and signal to noise ratio
- Sub-pixel spatial resolution using the induced currents on the neighbouring pixels and pulse shape analysis.
- Image reconstruction using the weighting of the images where one uses straight LORs from photon pairs and the other uses cones from single detected photons and parts of cones from single scattered photon pairs.
- Estimate the dead time of the detector elements and the coincidence window that should be used and compare this to the expected count rate. The count rate should be low enough to avoid detections in neighbouring areas of the detector during the same coincidence window.

In summary, the silicon based PET detector has several challenges to deal with but if these are solved in a sufficient way then there might be a large enhancement of the image quality.

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6 List of References

References


[34] Garry Chinn, Angela M. K. Foudray, and Craig S. Levin, "A Method to Include Single Photon Events in Image Reconstruction for a 1 mm Resolution PET System Built with Advanced 3-D Positioning Detectors", IEEE Nuclear Science Symposium Conference Record, 2006