A Joint Power Control and Resource Allocation Algorithm for D2D Communications

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Abstract—We consider the problem of joint power control, signal-to-noise-and-interference-ratio (SINR) target setting, mode selection and resource allocation for cellular network assisted device-to-device (D2D) communications. This problem is important for fourth generation systems, such as the release under study of the Long Term Evolution Advanced (LTE-A) system standardized by the Third Generation Partnership Project (3GPP). While previous works on radio resource management (RRM) algorithms for D2D communications dealt with mode selection and power control, the problem of resource allocation for the integrated cellular-D2D environment and in particular the joint problem of mode selection, resource allocation and power allocation has not been addressed. We propose a utility function maximization approach that allows to take into account the inherent trade off between maximizing spectrum efficiency and minimizing the required sum transmit power. We implement the proposed RRM algorithms in a realistic system simulator and report numerical results that indicate large gains of D2D communications both in terms of spectrum- and energy efficiency.

I. INTRODUCTION

Device-to-device (D2D) communications in cellular spectrum supported by a cellular infrastructure has the potential of increasing the spectrum and energy efficiency as well as allowing new peer-to-peer services by taking advantage of the so called proximity and reuse gains [1], [2], [3]. In fact, D2D communications in cellular spectrum is currently studied by the 3rd generation partnership project (3GPP) to facilitate proximity aware internetworking services [4], [5].

However, D2D communications utilizing cellular spectrum poses new challenges, because relative to cellular communication scenarios, the system needs to cope with new interference situations. For example, in an orthogonal frequency division multiplexing (OFDM) system in which user equipments (UE) are allowed to use D2D (also called direct mode) communication, D2D communication links may reuse some of the OFDM time-frequency physical resource blocks (RB).

Due to the reuse, intracell orthogonality is lost and intracell interference can become severe due to the random positions of the D2D transmitters and receivers as well as of the cellular UEs communicating with their respective serving base stations.

It is advantageous to use uplink resources for the D2D link, because in some countries regulatory requirements may not allow to use downlink resources by user equipments in the future. Therefore, in this paper we only deal with the case when the D2D links use UL cellular resources, such as the uplink OFDM resource blocks in a cellular Frequency Division Duplexing system or the uplink time slots in a Time Division Duplexing system [6], [7], [3].

To realize the promises of D2D communications and to deal with intra- and intercell interference, the research community has proposed a number of important radio resource management (RRM) algorithms.

Although the objectives of such algorithms may be different (including enhancing the network capacity [10], improving the reliability [11], minimizing the sum transmission power [12], ensuring quality of service [13] or protecting the cellular layer (i.e. the cellular UEs) from harmful interference caused by the D2D layer [14]), there seems to be a consensus that the key RRM techniques include:

1) Mode Selection (MS): MS algorithms determine whether D2D candidates in the proximity of each other should communicate in direct mode using the D2D link or in cellular mode (i.e. via the base station, BS) [15], [16], [17], see Figure 1;
2) Power Control (PC): PC algorithms taking into account the interference situation between the cellular and D2D layer play a key role to achieve various objectives [13], [18];
3) Resource Allocation (RA): Surprisingly, resource allocation in the sense of selecting particular OFDM resource blocks or frequency channels out of a set of available ones for each transmit-receive pair (cellular or D2D) is not been addressed. We propose a utility function maximization approach that allows to take into account the inherent trade off between maximizing spectrum efficiency and minimizing the required sum transmit power. We implement the proposed RRM algorithms in a realistic system simulator and report numerical results that indicate large gains of D2D communications both in terms of spectrum- and energy efficiency.

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seldom addressed in the literature ([8], [19], [20]);

4) Pairing: In the D2D context, pairing refers to selecting the D2D pair(s) and at most one cellular UE that share (reuse) the same OFDM resource block, similarly to multiuser MIMO techniques. Pairing is a key technique to achieve high reuse gains [12];

5) Multiple Input Multiple Output (MIMO) Schemes: Interference avoiding MIMO schemes have been proposed by [21]. Such schemes can be applied, for example for the cellular transmissions to avoid generating interference to a D2D receiver.

In this paper we propose a framework that deals with power control, mode selection and resource allocation in an iterative fashion. In our work we focus on single input single output (SISO) transceivers, although our methodology can be readily applied to the MIMO case as well.

The basic idea of our proposed scheme is to separate the time scales for MS, RA and pairing, from the SINR target setting (rate control) and fast power control loops. The SINR target setting is exercised at the time scale of the tens of milliseconds by the outer-loop, where this outer loop assumes that mode selection and resource allocation have been already done by a heuristic mode selection and resource allocation algorithm. A fast power control inner loop adjusts transmit powers such that the SINR targets set by the outer loop are met. The SINR target setting outer loop and the power control inner loop work in concert and operate on each resource block in isolation maximizing a utility function that balances between spectral and energy efficiency over feasible SINR targets. Outer- and inner loops are executed iteratively until convergence is reached. We test this double loop approach in a realistic multicell system and find that the algorithm provides large gains in terms of spectral and energy efficiency, when compared with traditional cellular communications.

We structure the paper as follows. The next section describes our system model. Section III formulates the SINR target setting and power control problem as an optimization task. Section IV develops a decomposition approach assuming that the specific resources (OFDM resource blocks) have been allocated for the transmitter-receiver pairs in the system. Next, in Section V we formulate the MS problem and in Section VI we address the joint MS and resource allocation problem by allocating the transmitter-receiver pairs to resource blocks such that the intracell interference due to resource reuse between D2D pairs and cellular-UEs is minimized. There are some intimate relationships between outer loop and inner loop that are discussed in Section VII that describes the complete RRM procedure. Numerical results are reported in Section VIII and the conclusions are drawn by Section IX.

II. SYSTEM MODEL

We consider a wireless network with a total of $L$ communicating transmitter-receiver pairs. A transmitter-receiver pair can be a cellular UE transmitting data to its serving base station or a device-to-device (D2D) pair communicating in cellular uplink spectrum. D2D candidates are source-destination pairs in the proximity of each other that may communicate in direct mode, depending on the MS decision that is part of the RRM algorithm developed in this paper. In this paper we assume an orthogonal frequency division multiplexing (OFDM) cellular network, such as the 3GPP LTE-A system, in which the time and frequency resources are organized in physical resource blocks (RB) [7].

The network topology is represented by a directed graph with connections labelled $l = 1, \ldots, L$. All (i.e. cellular and D2D) transmitters are assumed to have data to send to their corresponding receivers (saturated buffers) and $s_l$ denotes the transmission rate of Transmitter-$l$. Associated with each link $l$ is a function $u_l(\cdot)$, which describes the utility of communicating at rate $s_l$. The utility function $u_l$ is assumed to be increasing and strictly concave, with $u_l \to -\infty$ as $s_l \to 0^+$. We let $\mathbf{c} = [c_l]$ denote the vector of link capacities, which depend on the communication bandwidth $W$, the achieved SINR of the links ($\gamma_l$) as well as the specific modulation and coding schemes used for the communication. Obviously, the target rate vector $\mathbf{s}$ (which is in one-to-one correspondence with the SINR targets, $\gamma_l^{tg}$) must fulfill the following set of constraints:

$$\mathbf{s} \leq \mathbf{c}(\mathbf{p}), \quad \mathbf{s} \succeq 0.$$  

In this formulation, it is convenient to think of the $\mathbf{s}$ vector as the vector of the rate (translating to SINR) targets, while the $\mathbf{c}$ vector represents the actual capacity achieved by the particular power vector $\mathbf{p}$.

Let $G_{l,m}$ denote the effective link gain between the transmitter of pair $m$ and the receiver of pair $l$ (including path-loss and shadowing) and let $\sigma_l$ be the thermal noise power at the receiver of link $l$, and $P_l$ be the transmission power. The SINR of link $l$ is

$$\gamma_l(\mathbf{p}) = \frac{G_{ll}P_l}{\sigma_l + \sum_{m \neq l} G_{lm}P_m}$$  \hspace{1cm} (1)$$

where $\mathbf{p} = [P_1, \ldots, P_L]$ is the power allocation vector, and $\sum_{m \neq l} G_{lm}P_m$ is the interference experienced at the receiver of link $l$.

Equation (1) can also be written as

$$\gamma_l(P_{l_Rx}^{tot}, P_l, G_{ll}) = \frac{G_{ll}P_l}{\sigma_l + (P_{l_Rx}^{tot} - G_{ll}P_l)}$$  \hspace{1cm} (2)$$

where $P_{l_{Rx}}^{tot}$ represents the total received power measured by the receiver of link $l$. Hence, the SINR in (2) can be computed by Receiver-$l$ without knowing either the power used by other D2D pairs or cellular transmitters or any of the channel gains, except the one related to its corresponding Transmitter-$l$.

For ease of notation, we will use the notation $\gamma_l(\mathbf{p})$ for the SINR at Receiver-$l$. Each link is seen as a Gaussian channel with Shannon-like capacity

$$c_l(\mathbf{p}) = W \log_2 \left(1 + K \gamma_l(\mathbf{p})\right)$$  \hspace{1cm} (3)$$

which actually represents the maximum rate that can be achieved on link $l$. $W$ is the system bandwidth and $K$ models the SINR-gap reflecting a specific modulation and coding.
scheme. In the following we assume \( K = 1 \).

III. THE SINR TARGET SETTING AND POWER CONTROL PROBLEM

In this section we assume that mode has already been selected for the D2D candidates and all (cellular and D2D) links have been assigned a frequency channel or an OFDM RB. From the concept of D2D communications reusing cellular spectrum and the system model of the previous section it follows that the resource blocks may be used by multiple cellular and D2D transmitters. In this section we focus on handling this interference by properly setting the SINR targets and allocating transmit powers, while the MS and resource assignment problems that determine the specific cellular and D2D transmitters that share a given resource block are approached in Sections V and VI.

For the set of interfering links sharing the same resource block and thereby causing interference to one another, we formulate the problem of target rate setting and power control as:

\[
\begin{align*}
\text{maximize}_{\mathbf{p}, \mathbf{s}} & \quad \sum_i u_i(s_i) - \omega \sum_l P_l \\
\text{subject to} & \quad s_i \leq c_i(p), \quad \forall l, \\
& \quad \mathbf{p}, \mathbf{s} \geq 0
\end{align*}
\]

which aims at maximizing the utility while taking into account the transmit powers (through a predefined weight \( \omega \in (0, +\infty) \)) [22] [23]), so as to both increase spectrum efficiency and reduce the sum power consumption over all transmitters sharing a specific RB.

The constraints in Problem (4) formally ensure that the rate (SINR target) allocation of sources does not exceed the link capacities, which is a quantity that is optimized through the power allocation. As it will become clear later, the operation of the so called outer loop is such that SINR targets are always feasible, and so the power allocation by the inner loop ensures that target rate (\( \mathbf{s} \)) and capacity (\( \mathbf{c} \)) vectors coincide at the end of the convergence of the outer and inner loop pair.

A. Convexifying the Problem of Equation (4)

Unfortunately, Problem (4) is not convex, but exploiting the results presented in [22] and [24], we can transform it into the following equivalent form:

\[
\begin{align*}
\text{maximize}_{\mathbf{s}, \mathbf{p}} & \quad \sum_i u_i(e^{s_i}) - \omega \sum_l e^{P_l} \\
\text{subject to} & \quad \log(e^{s_i}) \leq \log(c_i(e^{P_l})), \quad \forall l,
\end{align*}
\]

where \( s_i \leftarrow e^{s_i} \) and \( P_l \leftarrow e^{P_l} \). The transformed Problem (5) is proved to be convex (now in the \( \mathbf{s}_i \)-s and \( \mathbf{P}_l \)-s since the utility functions \( u_i(\cdot) \) are selected to be \( (\log, x) \)-concave over their domains [22]. In this paper we use \( u_i(x) \leftarrow \ln(x), \forall l. \) Under this condition, we can solve Problem (5) to optimality by means of an iterative algorithm where the \( \mathbf{s}_i \)-s (or equivalently the SINR targets) are set by an outer loop. The transmit powers \( \mathbf{P}_l \)-s that meet the particular SINR targets (set in each outer loop cycle) are in turn set by a Zander type iterative SINR target following inner loop [25]. This separation of the setting of the SINR targets and corresponding power levels are detailed in the next Section.

IV. DECOMPOSITION APPROACH

A. Formulating the Decomposed Problem

We now reformulate Problem (5) as a problem in the user rates \( \mathbf{s} \) (Problem-I), which, due to the convexification, can be solved for a given (assumed known) power allocation (\( \mathbf{p} \)). Note that the target rate vector \( \mathbf{s} \) can be uniquely mapped to a target SINR vector \( \gamma^{(\mathbf{s})} \) as it will be shown later. We define Problem-I as:

\[
\begin{align*}
\text{maximize} & \quad \nu(\mathbf{s}) \\
\text{subject to} & \quad \mathbf{s} \in \tilde{\mathbf{S}}
\end{align*}
\]

where \( \tilde{\mathbf{S}} = \{ \mathbf{s} | \log(e^{s_i}) \leq \log(c_i(e^{P_l})), \forall l \} \) represents the set of feasible rate vectors that, for a given power vector \( \tilde{\mathbf{p}} \), fulfill the constraints of Problem (5).

Comparing (5) and (6), it follows that the objective function in (6) is defined as \( \nu(\mathbf{s}) \triangleq \sum_i u_i(e^{s_i}) - \varphi(\mathbf{p}) \), where \( \varphi(\mathbf{p}) \triangleq \omega \sum_l e^{P_l} \) represents the cost in terms of the total transmit power for realizing a given target rate \( \mathbf{s} \). Accordingly, we denote with \( \varphi^*(\tilde{\mathbf{p}}) \triangleq \omega \sum_l e^{P_l} \) the cost of achieving the optimum rates \( \mathbf{s}^* \) that solve the utility maximization Problem (6).

Therefore, Problem-II, for a given \( \mathbf{s} \) vector, can be formulated as

\[
\begin{align*}
\text{minimize} & \quad \omega \sum_l e^{P_l} \\
\text{subject to} & \quad \log(e^{s_i}) \leq \log(c_i(e^{P_l})), \quad \forall l.
\end{align*}
\]

Solution approaches to Problem-I and Problem-II are proposed in the next subsection.

B. Solving the Rate (SINR Target) Allocation Problem

We are now concerned with setting the SINR targets by solving Problem-I. Provided that the objective function \( \nu(\mathbf{s}) \) in (6) is concave and differentiable we can determine the optimal \( \mathbf{s}^* \) by means of projected gradient iterations, with a fixed predefined step \( \epsilon \):

\[
\begin{align*}
\mathbf{s}_l^{(k+1)} = \mathbf{s}_l^{(k)} + \epsilon \nabla_i \nu(\mathbf{s}_l^{(k)}) \\
& \forall i, \quad \mathbf{s}_l^{(k)}
\end{align*}
\]

To compute (9), we first need to find \( \varphi^*(\tilde{\mathbf{p}}) \) by solving the primal Problem-II (7). Since it is convex in \( \tilde{\mathbf{p}} \), it can be conveniently solved by Lagrangian Decomposition as follows. Let \( \lambda \) be the Lagrange multipliers (dual variables) for the constraints in (7) and form the Lagrangian function:

\[
L(\lambda, \tilde{\mathbf{p}}) = \omega \sum_l e^{P_l} + \sum_l \lambda_i \left[ \log(e^{s_i}) - \log(c_i(e^{P_l})) \right].
\]

The Lagrangian dual problem of Problem-II is given by:

\[
\begin{align*}
\text{maximize} & \quad \lambda \\
\text{subject to} & \quad \lambda \geq 0.
\end{align*}
\]

Since the original problem is convex, if regularity conditions hold then the solution of Problem (11) correspond to the
solution of Problem (7), i.e. \( L(\lambda^*) = \varphi^*(\tilde{p}) \). Assuming that \((\lambda^*, \tilde{p}^*)\) represent the optimum solution of Problem-II (7), we are now in the position of calculating \( \varphi^*(\tilde{p}) \) from (10):

\[
\varphi^*(\tilde{p}) = \sum_l \left[ \omega e^{\tilde{p}_l} - \lambda^*_l \log \left( e^{\tilde{p}_l} \right) \right] + \sum_l \lambda^*_l \log(e^{\tilde{s}_l})
\]

and

\[
\frac{\partial}{\partial \tilde{s}_l} \left[ \varphi^*(\tilde{p}) \right] = \lambda^*_l.
\]

Recalling (9), we have:

\[
\nabla_{i\nu}(\tilde{s}) = u_i'(e^{\tilde{s}_l})e^{\tilde{s}_l} - \lambda^*_l = s_i[u_i'(e^{\tilde{s}_l}) - \lambda^*_l],
\]

and so the final target rate update, for all \( i \), is:

\[
s_{i}^{(k+1)} = e^{\tilde{s}_{i}^{(k+1)}} = s_i^{(k)} \exp(\epsilon \nabla_{i\nu}(\tilde{s}(k))).
\]

Combining the above with (9) we can write the SINR target setting outer loop in the form for all \( i \):

\[
s_{i}^{(k+1)} = s_i^{(k)} \exp \left( \epsilon \frac{s_i^{(k)}}{u_i'(s_i^{(k)})} \right) \left( -\lambda^*_l(s_i^{(k)}) \right)
\]

The updating rule of the outer loop given by (12) is useful, because it determines the \((k+1)\)-th rate and SINR that should be targeted by the inner power control. Note that as a natural consequence of the decomposition approach, (12) requires the solution of Problem (7), i.e.

\[
\gamma_{l}^{tg} = \frac{\log(e^{\tilde{s}_l})}{\epsilon} \quad \forall l.
\]

Therefore, Problem (7) can be rewritten as:

\[
\begin{align*}
\text{minimize} & \quad \omega \sum_l e^{\tilde{p}_l} \\
\text{subject to} & \quad \gamma_{l}(\tilde{p}) \geq \gamma_{l}^{tg}(\tilde{s}_l) \quad \forall l, \\
& \quad \tilde{p} \geq 0
\end{align*}
\]

and solved with an iterative SINR target following closed-loop power control (CLPC) scheme [25]:

\[
P_{l}^{(t+1)} = \frac{\gamma_{l}^{tg}(\tilde{s}_l)}{\gamma_{l}(\tilde{p}^{(t)})} P_{l}^{(t)}.
\]

Thus, for a given \( \gamma_{l}^{tg}(\tilde{s}_l) \), the (15) power control inner loop provides an efficient means to set the transmit powers at each transmitter in loop \((t+1)\), provided that the transmitter knows the SINR measured at the receiver in the preceding loop \( \gamma_{l}(\tilde{p}^{(t)}) \).

D. Determining the \( \lambda^*_l \)-s

We can now determine the \( \lambda^*_l \)-s for the outer loop update (12) by exploiting the intimate relationship between the optimal \( \tilde{p}^* \) and the associated Lagrange multipliers \( \lambda^*_l \)-s. To this end, we rewrite the constraints in (14) as:

\[
\frac{G_{ll} P_{l}}{\sigma_l + \sum_{m \neq l} G_{lm} P_m} - \gamma_l^{tg} \geq 0 \quad \Rightarrow
\]

\[
P_{l} - \gamma_l^{tg} \sum_{m \neq l} G_{lm} P_m - \frac{\gamma_l^{tg} \sigma_l}{G_{ll}} \geq 0 \quad \forall l.
\]

Furthermore, let \( H \in \mathbb{R}^{L \times L} \) and \( \eta \in \mathbb{R}^L \) be defined as follows:

\[
H = [h_{lm}] \triangleq \begin{cases} -1 & \text{if } l = m \\ \frac{\gamma_l^{tg} G_{lm}}{G_{ll}} & \text{if } l \neq m \end{cases}
\]

\[
\eta = [\eta_l] \triangleq \begin{bmatrix} \frac{\gamma_l^{tg} \sigma_l}{G_{ll}} \end{bmatrix}.
\]

Using this notation, we can reformulate Problem (14) as the following Linear Programming (LP) problem:

\[
\begin{align*}
\text{minimize} & \quad \omega \mathbf{1}^T \mathbf{p} \\
\text{subject to} & \quad H \mathbf{p} \preceq -\eta \\
& \quad \mathbf{p} \succeq 0
\end{align*}
\]

with the corresponding Dual Problem

\[
\begin{align*}
\text{maximize} & \quad \mathbf{\eta}^T \lambda_{lp} \\
\text{subject to} & \quad H^T \lambda_{lp} \succeq -\omega \mathbf{1} \\
& \quad \lambda_{lp} \succeq 0
\end{align*}
\]

which is necessary to compute the Lagrange multipliers in Equation (12) for the rate update. The inequality constraints in (18) can be rewritten explicitly as:

\[
\frac{\lambda_{lp}}{\omega} - \sum_{k \neq l} G_{kl} \gamma_{l}^{tg} \frac{\lambda_{lp}}{\omega} \leq 1, \quad \forall l.
\]

Proof:

\[
H^T \lambda_{lp} \succeq -\omega \mathbf{1}
\]

\[
\sum_k h_{kk} \lambda_{lp} \succeq -\omega, \quad \forall l;
\]

\[
-h_{ll} \lambda_{lp} + \sum_{k \neq l} h_{kl} \lambda_{lp} \succeq -\omega, \quad \forall l
\]

\[
\frac{\lambda_{lp}}{\omega} - \sum_{k \neq l} G_{kl} \gamma_{l}^{tg} \frac{\lambda_{lp}}{\omega} \leq 1, \quad \forall l.
\]

By defining

\[
\mu_l \triangleq \frac{\lambda_{lp}}{\omega}, \quad \sigma_l \triangleq \frac{\gamma_l^{tg}}{\omega} \eta_l,
\]

(20)
inequality (19) can be interpreted as an SINR requirement, i.e.

\[
\gamma^{FC}_l(\mu) \triangleq \frac{\mu_l G_{il}}{\sigma_l + \sum_{k \neq l} G_{kl} \mu_k^* / \sigma_k} \leq \gamma^{tgt}_l, \quad \forall l. \tag{21}
\]

**Proof:**

\[
\frac{\lambda^{ap}_l}{\omega} - \sum_{k \neq l} G_{kl} \gamma^{tgt}_k \frac{\lambda^{ap}_k}{\omega} \leq 1
\]

\[
\frac{\lambda^{ap}_l}{\omega} - \sum_{k \neq l} G_{kl} \gamma^{tgt}_k \frac{\lambda^{ap}_k}{\omega} \leq 1
\]

\[
= \frac{\mu_l G_{il}}{\sigma_l} + \sum_{k \neq l} G_{kl} \mu_k^* + 1
\]

\[
\gamma^{FC}_l(\mu) \triangleq \frac{\mu_l G_{il}}{\sigma_l + \sum_{k \neq l} G_{kl} \mu_k^* / \sigma_k} \leq \gamma^{tgt}_l, \quad \forall l. \quad \blacksquare
\]

Therefore, Problem (18) can be reformulated as:

\[
\begin{aligned}
& \text{maximize} & & \omega^T \mu \\
& \text{subject to} & & \lambda^{FC}_l(\mu) \leq \gamma^{tgt}_l, \quad \forall l \\
& & & \mu \succeq 0
\end{aligned} \tag{22}
\]

where the solution \( \mu^* \) can be found through the following distributed iterations

\[
\mu_l^{(t+1)} = \frac{\gamma^{tgt}_l}{\gamma^{FC}_l(\mu^{(t)})} \mu_l^{(t)}, \quad \forall l. \tag{23}
\]

Note that (23) can be interpreted as a reverse link power control problem, in which Receiver-\( l \) becomes a transmitter (transmitting with power \( \mu_l \)) and Transmitter-\( l \) measures the experienced SINR at its position. In this sense Equation (20) represents the SINR requirement of the "forward channel" (FC), that is the SINR requirement related to the transmission from the receiver to the transmitter of link-\( l \).

Once the iterative procedure (23) converges to the optimum \( \mu^* \), the optimal dual variables \( \lambda^{ap*}_l \) can be retrieved from Equation (20) as

\[
\lambda^{ap*}_l = \omega \mu_l^* \eta_l^{* -1}, \quad \forall l. \tag{24}
\]

The original nonlinear power control problem (7) and its (LP) formulation (17) are equivalent in the sense that there is the following specific relation between their optimal solutions \((\bar{p}^*, \lambda^*)\) and \((\mu^*, \lambda^{ap*})\):

\[
\begin{aligned}
P_l^* &= e^{P_l^*} \quad \forall l \\
\lambda_l^* &= \log(1 + \gamma^{tgt}_l)^{1+\gamma^{tgt}_l} P_l^* \log(2) \omega \mu_l^* \gamma^{tgt}_l \quad \forall l.
\end{aligned} \tag{25}
\]

Hence, when we achieve both \( P_l^* \) and \( \mu_l^* \), by means of Equations (24) and (25), we are able to compute \( \lambda^* \) as

\[
\lambda^*_l = \log(1 + \gamma^{tgt}_l)^{1+\gamma^{tgt}_l} P_l^* \log(2) \omega \mu_l^* \frac{G_{il}}{\sigma_l^* \gamma^{tgt}_l} \quad \forall l. \tag{26}
\]

and use it to update the user rates of Equation (12).

**E. Summary**

This section developed a dual loop iterative solution approach to the convex optimization problem (5). The basic idea has been to decompose the problem to separate subproblems in \( \bar{s} \) (Problem-I) and \( \bar{p} \) (Problem-II). Problem-I can be solved by gradient iterations and using Lagrangian duality to obtain the SINR targets, while Problem-II can be solved by an iterative SINR target following inner loop. We exploited the relationship between Problem-I and Problem-II such that the necessary Lagrange multipliers in the iterations of Problem-I are provided by solving Problem-II. In a practical setting, the outer and inner loop can be started off by setting a low SINR target vector and running the inner loop to determine the transmit power levels and the corresponding \( \lambda^*_l \)'s. The updated power levels and Lagrange multipliers are then used as the input values to the outer loop update rule (12), see Figure 2.

Figure 2 shows the implementation of the Inner- and Outer-Loop mechanism in the network, clarifying which information must be exchanged between the transmitter and receiver of each pair and which computations must be performed by both nodes in order to achieve the optimal transmit power.

**V. Mode Selection (MS)**

While cellular users (UEs) communicate with their respective serving base stations, D2D users may communicate both directly or in cellular mode. In the former case, D2D transmitters are allowed to reuse cellular resource blocks (D2D reuse mode). Alternatively, when D2D candidates use the direct mode, they can be allocated orthogonal, i.e., dedicated resource blocks (D2D dedicated mode), in which case the reuse gain of D2D communications is not harvested. However, even in this dedicated resource assignment case, D2D communications can realize the proximity and hop gains [3].
With this notation and terminology, formulating the resource paper, in forced D2D mode, all D2D candidates operate using cases for the adaptive MS algorithm proposed later in the index UEs always transmit in mode and 

\[ \sum_{n \in N} x_{n,j} + \sum_{m \in M} x_{m,j}(q) \leq 1, \quad \forall j; \]

- Adaptive mode selection:

\[ \sum_{n \in N} x_{n,j} + \sum_{m \in M} x_{m,j}(q = 0) \leq 1, \quad \forall j; \]

where the last inequality expresses that a specific D2D pair \( m \) can only be either in D2D or cellular mode over Resource-\( j \). Note that formally a specific D2D pair \( m \) is allowed to use a resource in D2D mode and another resource in cellular mode.

VI. MODE SELECTION AND RESOURCE ALLOCATION PROBLEM

A. Problem Formulation

With the notation introduced in the previous section, we are now interested in formulating and solving the resource allocation problem that is concerned with selecting the mode (\( q \)) for D2D candidates and allocating resource blocks to cellular UEs and D2D candidates. We formulate the resource allocation task as a cell-based optimization problem, in which we wish to maximize the overall spectral efficiency assuming fixed transmit powers \( P \) for each user and thermal noise \( \sigma \) for each link. Recalling the definition of spectral efficiency for user-\( l \) on a given resource-\( j \):

\[ \eta_{l,j} = \log_2(1 + \frac{G_{k,l,j}P}{\sigma + I_{l,j}}) \]

we notice that it actually depends on the path gain \( G_{k,l,j} \) between Transmitter-\( k \) and Receiver-\( l \) on the RB-\( j \), and on the intracell interference \( I_{l,j} = \sum_{k \neq l} P \cdot G_{k,l,j} \), due to the possible in-cell resource sharing between D2D pairs and cellular-UEs. Hence, our target to maximize the spectral efficiency can be interpreted as our wish to both minimize the intracell interference and, when there are enough orthogonal resources, to select for D2D candidates the transmission mode (\( q \)) that takes advantage of their potential proximity (i.e. higher path gain).

Let \( L = N + M \) and \( J \) denote the number of users i.e. both cellular UE and D2D transmitters and resource blocks (RB) respectively. To formulate the resource allocation problem we assume that each transmitter can only be assigned a single RB. Thus, the user assignment task becomes (Problem-III):

\[
\begin{align*}
\text{maximize} & \quad \sum_{j} \sum_{l} \log_2(1 + \frac{G_{k,l,j}P}{\sigma + I_{l,j}}) \\
\text{subject to} & \quad \sum_{j} x_{l,j}(q = 0) \leq 1, \quad \forall j, \forall q \in \{0, 1\} \quad \text{(C1)} \\
& \quad \sum_{j} x_{l,j}(q = 1) = 1, \quad \forall l \quad \text{(C2)} \\
& \quad x_{l,j}(q = 1) + x_{l,j}(q = 0) \leq 1, \quad \forall j, \forall l \quad \text{(C3)}
\end{align*}
\]

The constraints (C1) express that each RB can be allocated to at most one user in cellular mode due to the orthogonality constraint. Constraints (C2) express that each user must get allocated exactly one RB and constraints (C3) ensure that to each user is assigned only one of the two possible modes. Obviously, cellular-UEs are always assigned to mode \( q = 0 \).
B. A Heuristic Algorithm to Solve the User Assignment Problem

To solve Problem (27) we propose a new straightforward procedure based on the shadowed path loss measurements. This scheme, that we call MinInterf, exploits the proximity between D2D candidates for the Mode Selection, and performs Resource Allocation by minimizing the intracell interference. It involves two steps. Firstly, orthogonal resources are allocated to cellular UEs. Since MinInterf disregards frequency selective fading, to each UE it randomly picks and assigns an available RB. Next, for each D2D candidate in the cell, MinInterf considers two possible cases:

- **D2D transmission with dedicated resource.** If there are orthogonal resources left, they can be assigned to the D2D candidate so that the D2D transmission does not affect others within the same cell. In this case, the D2D transmitter can also choose which is the best mode to communicate with his corresponding receiver (i.e. Cellular Mode or D2D Mode) on the basis of the channel gains it experiences both towards the D2D receiver \( G_{d2dMode} \) and towards the Base Station \( G_{CellularMode} \). Specifically, if \( G_{CellularMode} \leq G_{d2dMode} \), then the direct mode is preferred.

- **D2D transmission with resource reuse.** When there are no unused RBs in the cell, the D2D pair must communicate in direct mode (D2D Mode) and reuse RBs. Sharing resources with other users within the same cell produces intra-cell interference. To reduce this intracell interference, for each resource-\( j \) we consider the sum

\[
S(j) = [G_{2Tx_1Rx,j} + G_{1Tx_2Rx,j}]
\]

as a measure of the potential interference, that assigning the D2D-pair to resource-\( j \) causes. Here \( G_{2Tx_1Rx,j} \) represents the channel gain between the D2D transmitter and the receiver of link(s) already allocated to resource-\( j \), which may be the cellular base station and/or another D2D receiver. \( G_{2Tx_1Rx,j} \) takes into account the interference that the D2D pair produces transmitting on RB-\( j \). \( G_{1Tx_2Rx,j} \), on the other hand, is the channel gain between the transmitter(s) already allocated to resource-\( j \) (which can be both a cellular-UE and/or other D2D transmitters) and the receiver of the new D2D pair to be allocated. \( G_{1Tx_2Rx,j} \) is therefore related to the interference that the D2D pair might perceive due to the reuse. Once expression (28) is computed for each available resource-\( j \), the D2D pair is assigned to that resource corresponding to the minimum value.

\[
[0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5] \leq [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]
\]

Figure 5. An example of a D2D transmission with resource reuse. D2D Tx node communicates directly to its D2D Rx node sharing a resource block (RB) with the cellular user UE. The shared RB is selected in such a way to minimize an estimate (Eq. (28)) of the intracell interference that D2D communication might perceive (related to the gain \( G_{1Tx_2Rx,j} \) between the UE and the D2D Rx node) and produce (related to the gain \( G_{2Tx_1Rx,j} \) between the D2D Tx node and the BS).

It is worth noting that the final Resource Allocation achieved with the presented MinInterf scheme represents a suboptimal solution of Problem (27), nevertheless numerical results show that its interplay with the iterative Power Control procedure, which takes into account also the intercell interference, allows to attain good performance in terms of spectrum and energy efficiency. Algorithm 1 summarizes the main steps of the MinInterf scheme.

VII. THE COMPLETE JOINT POWER CONTROL, MODE SELECTION AND RESOURCE ALLOCATION ALGORITHM

With the solution to Problem-III in hand, we are now in the position to propose the solution to the joint mode selection, resource allocation and power control problem (based on the boxed equations in the previous sections). We assume that all transmitter-receiver pairs in the system have been assigned exactly one resource block. Note that multiple transmitter-receiver pairs may be assigned to a single RB in order to...
Algorithm 1 MinInterf

Allocate orthogonal resources to cellular-UEs (Randomly)

for Each D2D candidate do
    if there is an orthogonal resource-\(l\) left then
        if \(G_{Cellular \ Mode} \leq G_{D2D \ Mode}\) then
            D2D candidate transmits in \(D2D-\text{Mode}\) on resource-\(l\)
        else
            D2D candidate transmits in \(Cellular-\text{Mode}\) on resource-\(l\)
    end if
    else
        for Each available resource-\(j\) do
            \(S(j) = [G_{2Tx,1Rx,j} + G_{1Tx,2Rx,j}]\)
        end for
        D2D candidate transmits in \(D2D-\text{Mode}\) on resource-\(j\)
        corresponding to the minimum value of \(S\)
    end if
end for

accommodate for cases in which resources must be overallocated due to high load. Once performed the resource allocation/scheduling for all users, we let the outer (Eq. (12), (13), (26)) and inner loops (Eq. (1), (15), (21), (23)) determine the optimal transmit powers that maximize the utility associated with each RB.

VIII. NUMERICAL RESULTS

A. Simulation Setup and Parameter Setting

In this section we consider the uplink (UL) of a 7-cell system, in which the number of UL physical resource blocks (RB) is 4 (per cell). We perform Monte Carlo experiments to build some statistics over the performance measure of interests when employing the joint resource allocation and power control (outer-inner loop) described in the previous section. In each cell we drop two users ("cellular UEs") that communicate with their respective serving base station (BS), that is transmit data to their serving BS. In addition, two or four D2D candidate pairs are also dropped in the coverage area of each cell. When two D2D pairs are dropped, the D2D pairs must use orthogonal resources with the cellular users, but the system may select D2D (also called "direct") mode or cellular mode for them to communicate. When the D2D pair uses the cellular mode, the D2D transmitter transmits data to the BS in the UL band, and the BS sends this data to the D2D receiver in the DL band. In our study, we do not model the DL transmission, essentially assuming that the DL resources are in abundance so that we can focus on the UL performance. When the D2D pair communicates in the direct mode, the D2D transmitter sends data to the D2D receiver using UL resources. This case is referred to "MS" to emphasize the role of the mode selection for the D2D candidates.

When four D2D pairs are dropped in addition to the two cellular users, two of them must use direct mode and overlapping resources with either other D2D direct mode users or with cellular users. This is because we assume only four resources per cell accommodating 6 transmitters and we assume that cellular users and D2D candidates in cellular mode (that is transmissions to the cellular BS) must remain orthogonal within a cell. We refer to this case as the "MS Reuse" to highlight that there is a degree of mode selection freedom for two D2D candidates but cellular resources now must be reused by multiple transmitters in each cell. Intuitively, we expect some SINR degradation on the reused resources, but an increase in the total rate (and spectrum efficiency) due to more transmissions per cell.

Distinguishing the two D2D pairs case and the four D2D pairs case allows us to study the proximity gain (there is no reuse in the first case) and the reuse gain (expected in the second case). The main simulation parameters are given in Table VIII-A.

B. Operation of the Outer and Inner Loops

Figures 6-10 illustrate the interplay of the outer and inner loops for the case of a single Monte Carlo drop, now assuming 7x4 transmitters in the 7-cell system. Figure 6 shows the SINR target(s) set at each iteration of the outer-loop, while Figure 7 illustrates the users’ SINR measured at each receiver during the inner loop (power control) iterations. The initial SINR is determined by the initial power levels and the geometry of the whole system, but the final SINR is set by the SINR target evolution, together with the power control procedure. The outer loop adjusts the SINR target for each link from an initial low value to a user specific optimal (utility maximizing) value, taking into account the current interference situation in the network and trying to push the SINR target of each user as

\begin{table}[h]
\centering
\caption{Parameters of the 7-cell system under study}
\begin{tabular}{|l|c|}
\hline
Parameter & Value \\
\hline
System Bandwidth & 5MHz \\
Carrier Frequency & 2GHz \\
Gain at 1 meter distance & -37dB \\
Thermal noise \(N_0\) (\(\text{in MHz}\)) & -107 dBm \\
Path Loss coefficient & 3.5 \\
Lognormal shadow fading & 6dB \\
Cell Radius & 500m \\
Number of cells & 7 \\
Max Tx Power & 200mW \\
Min Tx Power & 5e-6W \\
Number of RB’s requested by users & 1 \\
Max. Number of Outer-Loop iterations & 200 \\
Max. Number of Inner-Loop iterations & 10 \\
Number of Monte Carlo simulations & 100 \\
Initial power & 0.01 W \\
Initial \(\text{SNR}_{\text{tgt}}\) & 0.2 \\
Initial \(\mu\) & 0.01 \\
\(\epsilon\) & 0.05 \\
\(\omega\) & 0.1 \\
Distance between cellular UE and the BS & 100-400m \\
Distance between D2D pairs & 50-250m \\
\hline
\end{tabular}
\end{table}
Recall from Section III, the SINR targets correspond to the rate vectors $\bar{s}$ of the iterative procedure used to solve Problem (6). Since vectors $\bar{s}$ always belong to a set of feasible rate vectors, the beauty of our scheme is that due to the iterative interplay between the outer and inner loops, the final SINR targets not only utility maximizing, but also remain feasible. This is the second (major) advantage over fix SINR target schemes. As the initial SINR target is set to the same value for all users, also their initial rate is the same (Figure 8). Clearly the maximum rate achievable by each user depends on its specific link capacity, which is a quantity that the outer- inner loop procedure aims to maximize. As the outer

Figure 6. The evolution of the SINR target for the 7x4=28 users during the 200 iterations of the outer loop. Initially, all SINR targets are set to a very low value (around -8dB), and the outer loop gradually adjusts all SINR targets to the optimal (overall utility maximizing) value.

Figure 7. The evolution of the individual SINR values during the course of the outer loop. Initially, the SINR is determined by the low initial transmit power and the geometry of the system. The outer loop successively adjusts the individual SINR targets and these SINR targets are reached by each user.

Figure 8. As the outer loop evolves, each user increases its rate ($s$) from the initial low value until the individual rates that are overall optimal are reached. The operation of the outer and inner loops ensure that the individual SINR targets remain feasible.

Figure 9. The Shannon capacity of each link (transmit-receive pair) as dictated by the outer loop. This Shannon capacity is then realized by the operation of the inner loop power control for both the cellular and D2D links.
loop evolves, in fact, the Shannon capacity of each link is optimized (Figure 9) and, at the same time, each users rate increases until convergence to its maximum achievable value, represented precisely by the link capacity. Figures 8-9 show how the final values of rates and capacities coincide at the end of the convergence of the outer and inner loop. Finally, Figure 10 illustrates the evolution of the individual transmit powers of the transmitters in the system. At each iteration of the outer loop, power levels are set in such a way to fulfill the required values of SINR (i.e. to achieve the rate defined by the outer loop), and those new power levels are obtained through the inner loop iterations (see zoomed section in Figure 10). The merit of the inner loop is that it works in a distributed fashion and typically reaches the convergence within a very few iterations (2-3).

C. Resource Allocation and Mode Selection (MS)

Figures 11-18 examine the performance of the proposed scheme when D2D pairs are forced to use cellular mode (in the Figures: "UE-Mode") or may choose between cellular and direct D2D modes without/with resource reuse ("MS" and "MS-Reuse" respectively). In each of these figures there are 6 curves showing the performance for the 3 modes (UE-Mode, MS, MS-Reuse) and 2 predetermined D2D distances (50m and 100m). The cellular users are dropped around their serving base stations at a predetermined distance (either 200m or 400m). The MC experiments are executed such that when the D2D distance and the cellular UE-base station distance are set to some nominal value, there is an allowed ± 10m zone within which the actual drop may vary.

Figure 11. The distribution of the SINR of the cellular users (that is the users transmitting to their respective serving base stations), when the cellular users are approximately 200m from their respective serving base stations. The figure shows this SINR distribution in 6 different cases, depending on the D2D distance (50m or 100m) and the D2D mode (D2D pair in cellular mode, D2D pair in D2D (direct) mode using dedicated resources and D2D pair in D2D mode reusing cellular resources). There is a ca. 1 dB loss in the SINR in the reuse mode when the D2D distance is large.

Figures 11 and 12 compare the SINR distribution of the cellular users when they are kept at a predetermined distance (200 ± 10 and 400 ± 10 meters respectively) from their respective serving base stations. The D2D candidate pairs are dropped according to a surface uniform distribution, but the
D2D distances are constrained to 50 and 100 meters. We can see that the impact of D2D communication on the cellular layer is basically negligible, since only a small SINR degradation can be observed when the D2D distance is high and reuse mode is enforced.

The D2D SINR distributions are shown in Figures 13 and 14 (for cellular user distances 200m and 400m respectively). It is obvious that the D2D SINRs are radically improved when mode selection is allowed as compared to the forced cellular communication mode: this improvement can be as high as 20-25 dB. What is noteworthy is that even though in reuse mode the D2D SINR drops a bit as compared to using cellular mode (remember that in reuse mode the system resources are overloaded to allow for the reuse gain), this SINR degradation as compared with using orthogonal resources is small as compared with the large proximity gain that is achieved by mode selection (MS). It is, in fact the large proximity gain that helps to harvest the reuse gain by moving the D2D SINR into a high (+20 dB) regime and allowing D2D pairs reuse cellular resources. Our smart resource allocation ensures minimal interference between transmitters using the same resources and so the proposed scheme realizes the expected gains of D2D communications.

Figure 13. The distribution of the SINR of the D2D users in the same 6 cases as in Figure 11. The mode selection (MS) with dedicated resources (dashed lines) show a 10-25 dB proximity gain. MS with resource reuse (2 extra D2D transmitters accommodated in each cell) yields a bit lower SINR (dotted lines), but due to the large proximity gain, these SINR values are still much higher than when using the cellular mode for the D2D pairs (solid line).

Figure 14. This figure is similar to Figure 13, but now the cellular users are 400m far from their respective serving base stations. Also in this case, the proximity gain on the D2D SINR is large, thanks to the operation of the resource allocation and the outer and inner loop operation.

Finally, the distribution of the D2D transmit powers are

![CDF of D2D SINR](image1)

![CDF of D2D Power](image2)

The distribution of the power levels used by the cellular users that are 200m away from their serving base stations. When the D2D pairs reuse cellular resources (dotted lines), the UE power levels are higher to ensure high utilities, especially for cell edge users. However, when D2D pairs are in D2D mode using orthogonal resources the UE power levels are lower (dashed line) than when the D2D pairs are in cellular mode (solid line), because the intercell interference is somewhat lower.

The distribution of the power levels used by the cellular users are shown in Figure 15 and 16. Again, we note that the impact of the D2D users on the cellular layer is basically negligible, except when the D2D distance is relatively large (100m) and the system is overloaded by D2D users. In this case, the cellular users tend to raise their transmit powers to keep the utility at the optimal level, but still using feasible power levels. As we have seen in the SINR graphs, this power control ensures an essentially unaffected SINR level for the cellular users.
This figure is similar to Figure 15, but here the distance between the cellular users and the serving base station is 400m. When D2D pairs use dedicated resources and D2D direct mode, they disturb the (basically cell edge) cellular users more than when they are in cellular mode (compare the solid line with the dashed lines). D2D pairs in reuse mode cause the cellular users to use even higher powers (dotted lines).

Figure 17. The distribution of the D2D pair transmit power levels when the cellular users are 200m far from their serving base stations. When the D2D distance is 100m (dashed and dotted lines to the right from the solid lines) the transmit power levels are higher than in cellular mode (solid lines). In contrast, when the D2D distance is 50m, the D2D transmit power levels are lower than when the D2D pairs are in cellular mode.

Figure 18. This figure is similar to Figure 17, but now the cellular users are 400m far from their serving base stations.

cellular users are close to the BS, the D2D transmit power in cellular mode is quite low (thanks to the low intercell interference) and when the D2D pairs use D2D mode (at large D2D distance), they need to increase their power from this quite low level to maintain a high utility.

D. The Impact of $\omega$

Recall from the problem formulation of 4 that the parameter $\omega$ sets the weight of the overall power consumption in the utility. In this section we set $\omega$ to 0.1 (rendering the power consumption less important than the achieved sum rate) and to 10 (stimulating low sum power operation of the system) and we are interested in the overall (average) achieved rates and power levels as the cellular user-base station and D2D distances are varied over the coverage area of a cell.

Figures 19-20 compare the average rate when $\omega$ is set to 0.1 and 10 respectively and we again emphasize that the shown average rates are the averages of the optimum sum rates that can be achieved in a given Monte Carlo drop.

As expected, the utility maximizing (optimum) average rate is much (with around 50%) higher when $\omega$ is small. However, this rate increase comes at the price of a large power increase, especially when the transmitter-receiver distances (both for cellular and D2D pairs) increase, see Figures 21-22.

IX. CONCLUSIONS

In this work, we addressed the problem of selecting communication mode, allocating resources and setting SINR targets and transmission power in an integrated multicell cellular and D2D communication system. Our solution approach hinges upon a utility function that can easily takes into account the inherent trade off between spectrum and energy efficiency and an iterative algorithm that solves the mode selection
(MS), resource allocation (RA) and power control problem at different time scales. The MS and RA problem is solved at a coarse time scale, which is indeed in line with recent proposals on cellular network assisted D2D communications [4], while the SINR target adjustment and power control loops are executed on a finer time scale. The strength of this approach is not only that it finds the utility maximizing joint allocation but also that it operates in a distributed fashion and remains feasible. This is very important for D2D communications, where the SINR dynamics of the system is necessarily large and in practice would render fix SINR target setting approaches inappropriate.

The numerical results provide a number of important and interesting insights. Due to the utility optimal allocation, the impact of the D2D layer on the cellular layer is small, which is a key requirement in any system that allows licensed spectrum resources to be used by D2D traffic. Also, the proximity gain of D2D communications is so large when the D2D lie in the proximity of one another that there is a potential for overloading (reusing) cellular resources by multiple D2D pairs. Finally, our proposed scheme can readily accommodate a single parameter ($\omega$) that tunes the spectral efficiency - energy efficiency trade off by setting the system operational point into, e.g. a lower average rate regime and drastically reducing the overall energy consumption. A future work, triggered by the insight of the large SINR dynamics (and potential fairness problems) among the population within the coverage area is to study the setting of the $\omega$ parameter on an individual basis, rather than using a single common $\omega$.

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