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Acknowledgements

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Fredrik Meurling has made a parallel investigation with much the same aim, our discussions and interchange of experience between the two projects have been a big help. (1)

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Last but not least I want to thank Carl-Filip Lindahl, who took from his time to read through this work and gave me very valuable advices to improve the disposition.
1 Introduction

1.1 Work outline
An investigation of the materials deformation history prior to the straightening process will be made. The deformation history and further investigations together will hopefully make it possible to state how much the material properties are changed during the straightening process.

Authentic field studies will be carried out on tubes taken from the pilgering machine and after the 7-roll multi staggered roll straightening machine, RSM. From these tubes specimens will be manufactured and tested to show the different process steps effect on the proof strength, $R_{p0.2}$ value.

Parallel to the above specified procedure there will be a simulated cold working and straightening process. To simulate the process there will be specimens manufactured. The specimens will then be exposed to pre strain and or low cycle fatigue tests, LCF. When the specimens originate from hollows, the specimens will be pre strained to suitable levels depending on the material. In the simulated process it will be easier to isolate different essential parameters and to see the specific effect on the proof strength of those. The simulation process of the straightening will be carried out such as an LCF test with varying strain levels.

An analytical beam model will be helpful during the LCF tests. The analytical model can predict strain levels which will be input into the LCF simulation.

Because of the analytical models limitation to predict only elastic strain there will also be a FEM simulation made. With help from the FEM simulation we hope to get a qualitative estimation of how to interpolate the analytical model to plastic deformation.

1.2 Objective
Traditionally the straightening machines were developed to prevent strain hardening during the process. The major part of tubes that have been straightened has also traditionally been annealed. For some decades some steel manufactures have learned to benefit the austenitic and duplex steels inherent possibilities to strain harden. Cold working induces strain hardening and that lead to considerably higher yield strength. The experience from straightening of heavily cold worked steel grades is therefore limited depending on the above described history. Straightening has mostly been a concern of how to get the tubes straight enough, with little thought of that the process itself may affect the material properties.

In Sandvik Materials Technology, SMT, there are some grades especially situated in oil and gas applications that are of different structures. Two of them are Material A and Material B. Their strength are increased considerably through pilgering at room temperature and followed
by straightening without any annealing in between. SMT need to know more about these two materials behavior depending on straightening.

SMT has therefore decided to make a deeper investigation of the straightening process of the above steel grades which are straightened in the machine 327. The machine 327 is a RSM, just as described above.

Some investigations have already been made, but the results are ambiguous.

With help from an analytical model, LCF tests and a FEM-simulation, the increase of know how should also give better possibilities to set right process parameters in the straightening machine 327. With more specific process parameters one should be able to predict and achieve desired outcome.

Because of the large amount of specimens extracted from just one tube, it will also give better statistics on variations of proof strength throughout the tube.
1.3 **Straightening in a 7-roll multi staggered roll straightening machine (327).**

![Photo of the multi staggered cross roll straightening machine 327.](image)

There are two kinds of continuous straightening processes, reeling and cross roll straightening, see fig.1. The reeling machine is mostly used for making bars straight and the latter for tubes. The cross roll straightener has only concave rolls and the reeling machine has got concave pressure rolls and convex working rolls. (2) The major difference between reeling and cross roll straightening is that in a reeling machine a bending moment is built up even in the specific roll pair, and therefore more of the tube ends can also be straightened. In a cross roll straightener the tube will pass through 5, 6, 7, 9, 10 or as in the most recent machines 12 rolls.

To make possibilities to considerable straightening, bending is used. Straightening by bending is made by three point bending, that is the rolls are displaced from the centre line.

Also ovalization, application of compressive load between the pair of rolls, contributes to straightening of the tubes, see figure 2. In the right picture the arrows to the right show that through the tube wall the stress changes from compression on the inner wall, to tensile on the outer face. Of course the picture is exaggerated to be more descriptive. In the upper part there will be tensile stress induced even on the inner face, but not in magnitudes that will induce plastic strain.
The ovalization will make the tubes more cylindrical in cross section, it will also result in relaxation and equalization of residual stresses, in that sense the ovalization will also make the tube more straight, but not in the magnitude as bending.

Figure 2: Depiction of ovalization. The picture to the right shows that the stress changes from compressive to tensile from the inside to the outside during simultaneous outer point loads.
1.4  **The tensile test curve**

In cold worked strain hardening steels there are some differences in the stress strain diagram compared to an ordinary carbon steel diagram, see figure 3. In a carbon steel there is often a first well defined straight elastic part.

In a cold worked, not annealed, austenitic or duplex stainless steel on the other hand the look of the initial part is often nothing like the previous at all, see fig 4. The initial part does in fact consist of two parts even though that it is hard to recognize. The first elastic part is sometimes very hard to find or determine, because it is so small, for deeper information, see the part about modulus of elasticity below. The part that follows the first small part is then curved due to anelasticity, see passage 1.4.1 below. As the modulus of elasticity should be established in the first small linear elastic part, it is quite obvious that it is not as straightforward as in the earlier mentioned material group.

The very first part is also affected by the tensile testing machines inner friction and could therefore not be used for determination of the modulus of elasticity. To get a reliable result of the modulus of elasticity and later the proof strength another approach to determine the proof strength was therefore developed, described under the passage 1.6

![Figure 3: Depiction of ordinary carbon steel, with a first well defined straight elastic part, followed with Lüders strain and then the strain hardening part before yielding sets in. Source: SS-EN standard 10002-1, pp. 21](image)
Figure 4: Sample of a cold worked material A steel exposed to a few load changes of a total strain of 0.8 %, note that prior to yielding there is a slight change of curvature basically all the way, it is definitely not straight.

In SMT there is also a mathematical algorithm in the evaluation software that works with the tensile testing machine. It should however be mentioned that this algorithm is made mainly for the above described steels with a well defined first straight part.

It should be emphasized that a lower value of the modulus of elasticity will give a higher proof strength, since the slope of the curve will turn the curve to the right. When the modulus of elasticity turns to right the values will then meet the curve further up in the diagram.
1.4.1 Anelasticity

The phenomenon anelasticity itself is divided into two modes, namely the after effect and the internal friction mode. (3) When a constant stress is applied to a specimen, the specimen will instantaneously respond with an elastic strain. The unrelaxed modulus is defined at that zero time. When time elapses the strain will gradually relax and at infinite time the modulus will describe the relaxed modulus. In figure 5 unloading of a plastically strained specimen is depicted. If the unloading is very fast it will describe the part from A to e2 and with time the remaining strain will approach to point e3. If the tensile testing machine is set to one specific strain rate, the unloading part will look like the right side part in figure 6.

In materials that experiences cyclic loading the anelasticity will make the amplitude of vibrations decrease and that will result in energy loss by internal friction. (5)

![Figure 5: Depiction showing the aftereffect.](image)

1.5 Modulus of elasticity

One could certainly have opinions about why a materials scientist should read about modulus of elasticity, ME, in an introduction to a diploma work concerning steel, which should be one of our deepest rooted knowledge.

However there are several different phenomena that can affect the determination and values of the modulus of elasticity in a heavily cold worked strain hardening stainless steel.

Dislocations will rearrange compared to a normal annealed steel structure. That itself may lower the ME, up to in special cases 20 %. We often say that ME could not change because it is a parameter directly connected to inter forces from one atom to the other. What is forgotten in that discussion is that the material properties in commercial steel alloys is set by the dislocation structure. Since ME is a material property that property can change if the dislocation structure changes.

On the other hand other investigations say that anelasticity will affect the determination of ME with traditional tensile testing a lot in cold worked strain hardening steels and should
therefore not be recommended. If the ME is determined with resonance frequency testing instead the ME is still constant. (6)

### 1.6 Determination of proof strength, Rp0.2.

Because of the problems concerning the determination of the modulus of elasticity it is very important to establish a standard procedure, to minimize management errors. Then it is possible to compare results from different tests with a minimum of uncertainty. If the tensile test had been carried out theoretically perfect, the modulus of elasticity would be the derivative of the curve when the stress is zero. But when the stress is zero errors tend to blow up. That is why the tensile test curve often has to be extrapolated backwards in that region, due to loss of data.

In the standard SS-EN 10002-1, 13.1, a special procedure is recommended with a hysteresis loop, see fig. 6. That specific procedure is especially suitable to determine proof strength in these hard evaluated materials showing anelastic behavior.

![Figure 6: Depiction of determination of proof strength with a hysteresis loop. Source: SS-EN standard 10002-1, pp. 21.](image)

### 1.7 Residual stress

Residual stresses are internal stresses that are not necessary to keep a body in equilibrium with its surrounding. (7) Yet they almost always exist in atomic, granular or macro scale. Residual stresses originate from miss fittings between different spatial areas. As indicated there are three types of residual stresses. Type I or macro stresses work over ranges that can be discerned with your eyes. Type II or inter granular stresses work over granular ranges and type III work on atomic scale.

On macro scale, plastic strain bending of a beam is an illustrating example. The outer fibre of the beam has been stretched the most, and the inner fibre the least. After unloading, that will result in that the outer and inner fibres will compete to remain in their respective state. That will give rise to residual stresses on a macro scale.
Type II residual stresses equilibrate typically over ranges of 3-10 granular dimensions. Interphase thermal stresses in a metal matrix composite is an example of type II.

Type III residual stresses exist on atomic scale and equilibrate within a granule. Dislocations and point defects will result in type III residual stresses.

1.8 The Bauschinger effect

The Bauschinger effect was first discovered in 1886 and it is named after the finder. The effect describes what happens in a material, exposed to tensile or compression strain, when it experiences reversed loading, see fig. 7a. The first half cycle A-D is called forward loading, tensile segment in fig. 7b, and it is not necessary that it starts in tensile direction. When the load changes sign in point D the reversed loading starts. D-F is the compression loading portion. The material in reversed loading will yield at a lower stress, tensile or compressive, than in the previous load direction, that is the Bauschinger effect. In fig. 7b a measure of the Bauschinger effect can be read off as the difference between the tensile loading and the amount of the mirrored compression loading portion.

What cause the Bauschinger effect are changes in the dislocation substructure that occur in load reversing and also changes in internal stress systems.

![Figure 7: Depiction of the Bauschinger effect. Source: Fatigue of materials, S.Suresh, p. 98.](image)

The Bauschinger effect can also be used to see what effect different dislocation mechanisms have on strain hardening.
1.9 Process chain

The tubes have gone through several steps in the process chain. The molten steel is first casted in the continuous casting machine. The cast billets are later hot rolled to bars. The bars are then peeled and cut into short billets. Machine turning adds a radius in one of the billets ends, to prevent cracks in the hot extrusion. The prepared billets are heated up to 1200 °C before put into the extrusion press. In the extrusion press the billets are transformed in to a tube that looks a lot like the final result, except for the dimensions. The tubes have again passed a hot step and now they have to be straightened and pickled. The following two steps are the ones that will set the tubes final mechanical properties. In the pilgering the steel is heavily strained plastically, which gives strain hardening and increased strength.

In oil and gas applications that high strength is particular desirable, when the tubes from the platform down to the bottom can build up to several thousand of meters and have to uphold their own weight.

Unfortunately strain hardened steel is sensitive to altered strain directions, the steel experiences Bauschinger effect and if put to several load changing cycles also cyclic softening. That is why the last step, cross roll straightening, has to be performed with extreme precautions.

1.10 Cyclic stress-strain behaviour

Metals are meta stable when put out to cyclic changing loads. How the material will respond to stress-strain can change drastically when put out to just cyclical strain. If the material has been annealed or cold deformed prior to these cyclic strain changes is of major importance.

“When well annealed FCC single crystal, suitably oriented for single slip, are subjected to cyclic strains under fully reversed loading, rapid hardening is noticed even in the initial few cycles” After that relatively short period hardening ceases to increase and a state called “saturation” sets in. (8)

While a cold deformed material immediately will show cyclic softening after the introductory Bauschinger effect. (9)
2 Test material

There are two grades of material included in this investigation, Material A and Material B.

2.1 Material A

The raw material was manufactured from a round bar with a diameter of 264 mm. The round bar was then extruded to 197 x 20 in kst 927. The hollow was then pilgered to our final dimension 156, 3 x 12, 6 mm.

2.2 Material B

The raw material was manufactured from a round bar with a diameter of 264 mm. The round bar was then extruded to 162, 7x21 in kst 927. The hollow was then pilgered to our final dimension 127, 5 x 15, 8 mm.
3 Means and methods

The analytical beam model is very simplified. There was therefore a FEM-simulation made with the software MSC Marc. The purpose with the FEM-simulation was to verify if the analytical calculations are in the right magnitude. The RSM has got crossed rolls, which makes the analysis quite complex. First the tube is cylindrical, and then the rolls have got a hyperbolic shape. That together will make the stress state between the ovalizing rolls hard to analyze in any other way than with FEM-simulation.

The physical test methods that were carried out in this investigation were tensile test, low cycle fatigue test, hardness test and microstructure evaluation. Also an analytical model was made to decide which strain levels should be used in the low cycle fatigue test, see appendix 1.

3.1 Simulation and calculations of adequate strain

The goal with the simulation was not to make a perfect simulation. It was to verify the old and the new analytical beam models. Therefore the rolls were simplified and the mesh was set quite coarse.

To be able to simulate the deformation with finite element analysis there is need for a true stress-strain curve. From the tensile test, one curve was modified to true stress-strain. The tensile test curve of specimen 4-2-12_1 was used. The starting length of the strain gauge is 12.5 mm in the tensile test machine. From the fracture point of strain 14.6193 %, the force was read from the log of specimen 4-2-12_1, 5255 N. The local fracture neck was measured to 2,025 mm. The fracture stress could then be calculated.

\[ R_{m(true)} = \frac{F_{fracture}}{A_{fracture}} = \frac{F_{fracture}}{\pi \cdot d_{fracture}^2} \cdot 4 = \frac{5255 \cdot 4}{\pi \cdot 2,025^2} = 1632 \left[ \frac{N}{mm^2} \right] \]

The stress-strain curve of specimen 4-2-12_1, see figure 8, was changed with a completing line from the above calculated stress-strain point to the tangent of where the elastic strain stops, see figure 9.
Figure 8: The look of the technological stress-strain curve from the tensile testing machine.

Figure 9: The modified stress-strain curve to true stress-strain curve.
The meaning with the low cycle fatigue test was to be able to use it as a simulation method for the real straightening process in the cross roll straightening machine 327. It is then important to know which strain levels the tubes go through in reality and adopt them in the tests that will be performed. Therefore it has been a little beam FEM model made, see figure 11. The problem has also been analyzed with a traditional beam model, see figure 10. The two separate calculations can then verify the correctness of the other.

Before this investigation started there was a similar beam model to that in figure 10. The major difference between the two is that in the prior model the supports B and C were represented as pinned connections. When that model was developed the focus was to get the answer of which displacement could be needed at the most to get a certain strain. When this investigation first started it was with the purpose to get the answer of what strain a volume element could possibly have experienced. Together they will give some kind of mean value theorem. In between the two calculated values the true value must be found. In a cross roll straightener the contact in between the roll and the tube will give a line contact. That should induce a distributed load instead of a point load and a line contact should also induce more of fixed support than pin connection, that’s why fixed support has been chosen here. If further investigations will be executed it would be preferable to install load cells to get the opportunity to read the force from respective roll. It would then be possible to decide more precisely what character the supports are of.

Figure 10: Load case configuration.

Figure 11: Displacement designations for this specific load case configuration.
Table 1: Scheme of how the separate element matrices will affect the global stiffness matrix. GM means global matrix and EM means element matrix.

<table>
<thead>
<tr>
<th>GM</th>
<th>EM1</th>
<th>EM2</th>
<th>EM3</th>
<th>EM4</th>
<th>EM5</th>
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<table>
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<th>Load vector</th>
<th>Load vector with boundary values</th>
<th>Reduced displacement vector</th>
<th>Reduced load vector</th>
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<td>$\theta_8$</td>
<td>$f_8$</td>
<td>$f_8$</td>
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As shown in table 1, 5 loads are known and 3 displacements. The matrix then has to be rearranged before it can be solved. The 3 unknown forces are moved to the left side, and the displacements were moved to the right side, see appendix 2. The resulting equation system was then solved in MATLAB. To be able to sketch the displacement, transverse force diagram and moment diagram, there was also need for an elementary case analysis. That is because the FEM-model only delivers the resulting forces and moments and to analyze the separate displacements in respective beam part one has to know the separate contributions to the part.
3.2 **Tensile test**

Tensile tests were carried out on both pilgered and straightened material. The tensile tests were performed in pilgering direction, PD, and transverse to PD, TTPD. It was then possible to see if there was any anisotropy in the material, from the tensile tests.

3.3 **Low cycle fatigue test or simulated straightening**

The low cycle fatigue tests have been carried out on material that has only passed through the pilgering process. These tests were only performed in PD. The procedure was much the same as in a low cycle fatigue test, with changing stress states from tensile to compression and tensile again, and so on.

When a tube passes through a cross roll straightener the stress varies from tensile to compressive, just as described above. One of the main purposes with this investigation was to clarify whether the low cycle fatigue test could be used as a simulated straightening process.

3.4 **Authentic tests**

There were specimens collected from one specific tube throughout the last two steps in the process chain. The specimens were collected as described in the pictures 12 to 15. The collected specimens were later put through tensile testing in PD and TTPD.

3.5 **Collection of specimens**

The investigated tube was cut into seven smaller tube sections, see figure 12. After pilgering there were two pieces cut off for further testing, number 1 and 2. The length of tube section 1 and 2 was 500 mm. The remaining tube was cut into five tube sections after straightening. Number 3 and 5 were 700 mm long and number 4 was 500 mm long.

To always be able to trace from where the specific sample was collected, the original tube was arbitrary marked with one original circumferential 12-direction, by analogy with the watch. That original 12-direction was later transferred to the tube sections gradually as they were cut off. The end that was oriented towards the end of the original tube was also marked to make the tube sections direction fully constrained.

Each specimen was marked due to the code X_XX_XXX, where X stands for the lengthwise number 1-5 in figure 12, XX for the lengthwise number 1-7 in figure 13 and XXX for the circumferential number 12, 3, 6 or 9 in figure 13.

The tube sections were later divided into smaller rings. Those smaller rings were then cut into circumferential sectors. From those sectors lengthwise tensile-, transverse tensile- and
Bauschinger specimens could be machine turned. No further machining was executed after turning.

The final machine turned specimens were collected in groups of three pieces, see fig. 13. From the position marked X-1-12 three transverse tensile specimens were collected, from X-2-12 three lengthwise tensile specimens and from X-3-12 three Bauschinger specimens. However in grade Material B specimens were only collected from position 12 and 6, 7 specimens from each position. The tube was too small to allow collection from other positions.

The cross roll straightening machine 327, which is installed in tube mill 63 in Sandviken has a distance of 900 mm between the working rolls. In a cross roll straightener the ends are not properly straightened in about half the distance between the working rolls, that is about 450mm in our case. But they have been adequately ovalized. That is why equal sets of specimens have been collected from both ends of section 3 and 5 in figure 12.

Figure 12: Depiction of the tested tube sections location and numbering along the original tube. The unnumbered sections have been withdrawn from further testing.
Figure 13: Depiction of the location, on the particular tube section, from where the specimens have been collected. The specimens were collected due to this configuration for the sections marked 1 and 2, in figure 12. These tube sections were cut off after pilgering. Also note the lengthwise and circumferential marking code. Position 6 and 7 marks the mirrored positions of position 1 and 2.

Figure 14: Depiction of the location, on the particular tube section, from where the specimens have been collected. The specimens have been collected due to this configuration for the second and the sixth tube section, in figure 1. These tube sections were cut out after straighten.
Figure 15: Depiction of the location, on the particular tube section, from where the specimens have been extracted. The specimens have been collected due to this configuration for the fourth tube section, in figure 2. These tube sections were cut out after straightening.
Table 2: Specimens collected from tubes after pilgering.

<table>
<thead>
<tr>
<th>Kind of specimen</th>
<th>No. of positions (lengthwise)</th>
<th>No. of positions (clockwise)</th>
<th>Number/position</th>
<th>Subtotal</th>
<th>Total</th>
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<td>Tensile test ⊥ ⊥ (front and back)</td>
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<td>12</td>
</tr>
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<td>LCF (front and back)</td>
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<tr>
<td>Bauschinger</td>
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<td>4</td>
<td>3</td>
<td>36</td>
<td>72</td>
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</table>

Total No. of samples 120

Table 3: Specimens collected from tubes after straightening

<table>
<thead>
<tr>
<th>Kind of specimen</th>
<th>No. of positions (lengthwise)</th>
<th>No. of positions (clockwise)</th>
<th>Number/position</th>
<th>Subtotal</th>
<th>Total</th>
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<tr>
<td>Tube pieces/tube</td>
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<td>1</td>
<td>1</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
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<td>4C30</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>24</td>
</tr>
<tr>
<td>Tensile test // (front and back)</td>
<td>4C30</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>Tensile test ⊥ ⊥ (centre piece)</td>
<td>4C30</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>Tensile test // (centre piece)</td>
<td>4C30</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>12</td>
</tr>
</tbody>
</table>

Total No. of samples 120
3.7 Tensile test procedure

The tensile tests were performed in a 100 kN RKM servo hydraulic machine, see figure 16.

Figure 16: Photo over the tensile test machine.

Figure 17: Photo over the tensile test specimens called 4C30.

The machine worked test specimens were tested according to ISO standard 6892-1 fig. 6. That specific procedure, with a hysteresis loop, is well suited for materials with considerable anelastic strain, see introduction chapter about determination of proof strength.
The procedure is that the specimen is installed in a tensile test machine and drawn according to practice. When the strain is well over 0.2 %, see figure 18, the specimen is unloaded to about 10 % of current stress. The specimen is drawn again and this time until fracture occurs. In materials showing anelastic strain, the stress-strain curve will now describe a hysteresis loop with a look like a section of a lens. Modulus of elasticity is then evaluated as the slope of the line connecting the peripheries of the lens. A parallel line to the previous one is constructed tangent to the initial part of the stress-strain curve. That is it will sometimes cut the abscissa in negative values. The line is then displaced parallel with a 0.2 strain, in usual manner. It is now possible to read Rp0.2 as the intersection between the constructed sloped line and the stress-strain curve. In this investigation the hysteresis loop was introduced at a total strain of 1.5 %.

![Figure 18: Description of how the evaluation of Rp0.2 with hysteresis loop is made.](image)
3.8 Low cycle fatigue test procedure

The low cycle fatigue, LCF, tests were performed in a 100kN Instron servo hydraulic fatigue machine, see figure 19.

Figure 19: Photo over the machine where the low cycle fatigue tests were performed.
In table 4 a test plan over the LCF tests for Material A is represented.

**Table 4: Low cycle fatigue test plan for Material A.**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Number of specimens</th>
<th>Number of specimens tested</th>
<th>Number of cycles</th>
<th>Strain in %</th>
<th>Number of specimens</th>
<th>Number of cycles</th>
<th>Strain in %</th>
<th>Number of specimens</th>
<th>Number of cycles</th>
<th>Strain in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3-12</td>
<td>3</td>
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<td>15</td>
<td>0,4</td>
<td>1</td>
<td>15</td>
<td>0,8</td>
<td>1</td>
<td>15</td>
<td>1,2</td>
</tr>
<tr>
<td>1-3-3</td>
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<tr>
<td>1-3-9</td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>3</td>
<td></td>
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<td>2-4-12</td>
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</tr>
</tbody>
</table>
In table 5 a test plan over the LCF tests for Material B is represented.

**Table 5: Low cycle fatigue test plan for Material B.**

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Number</th>
<th>Number of specimens tested</th>
<th>Number of cycles</th>
<th>Strain in %</th>
<th>Number of specimens tested</th>
<th>Number of cycles</th>
<th>Strain in %</th>
<th>Number of specimens tested</th>
<th>Number of cycles</th>
<th>Strain in %</th>
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</tr>
<tr>
<td>6-3-12</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>0,8</td>
<td>1</td>
<td>10</td>
<td>0,8</td>
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<td></td>
</tr>
<tr>
<td>6-3-6</td>
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<td>1,2</td>
<td>1</td>
<td>10</td>
<td>1,2</td>
</tr>
</tbody>
</table>

The machine worked Bauschinger test specimens, see figure 20, were tested with a changing load, from tensile to compression and so on until reaching determined number of cycles. The established strain was equal in positive and negative mode. The test data values were collected in one long file, including all cycles from respective specimen, and are not very manageable. That file is of the format *.raw and could be imported to excel, see figure 21. To get a quantitative comparison of the proof strength with or without cyclic loadchanging a method to determine proof strength of each tensile half cycle from the long data collection above was needed. A MATLAB program was therefore used to first create useful outputs, in the shape of one *.raw file for each half cycle.

*Figure 20: Photo over the Bauschinger test specimen.*
To determine proof strength in tensile tests or fatigue tests, a software called Cyclic EDC is used at SMT. Cyclic EDC is programmed according to ISO standard 6892-1, but not the procedure with the hysteresis loop. According to the procedure in ISO 6892-1 the program tries to find the straightest part in the steepest first part of the tensile curve. If strains lower than proof strength should be used it is not possible to use the hysteresis loop, since it should be introduced after the proof strength. Since the first part isn’t especially straight in these materials at all, the determination with Cyclic EDC is not exact. Even though this procedure is more suited to materials that have not been cold worked the errors tend to equalize each other, which have been investigated with comparison between proof strength determined with hysteresis loop method and with Cyclic EDC. Proof strength determined with hysteresis loop in general often lies about 20 MPa higher than determined with Cyclic EDC. Though Cyclic EDC has been used to determine proof strength in the half cycles from the origin long output file in this investigation.

A tensile half cycle represented with a *.raw file, prepared with the MATLAB program, could then be imported to Cyclic EDC, see figure 22. In Cyclic EDC the proof strength for that particular half cycle can be determined.
It has also been noted that the software Cyclic EDC has got an upper limit of ME set to 210 GPa. Therefore all values above 210 GPa should be considered with precautions, they could be higher. A lower value of ME will influence the proof strength value in a positive way that is a lower ME will give a higher proof strength value.

The proof strength values from respective tensile half cycle were then noted in an excel sheet and represented as curves in figure 23. It is called dynamic because the test was running continuous and the proof strength values here could therefore still remain some anelastic rest.
To secure respective dynamic proof strength value also a last tensile test was performed, see figure 24. That proof strength has been called true here.

Figure 23: A diagram from Excel showing the dynamic proof strength values determined with EDC Cyclic.

Figure 24: Diagram over the true proof strength value from the final tensile test, when the specimens were drawn until fracture occurred.
The dynamic proof strength and the true were then brought together in one diagram beginning with the dynamic part and of course ending with the true, see figure 25.

Figure 25: The resulting diagram from the dynamic and true proof strength brought together.
4 Results

4.1 A volume element’s way through the straightener

Before the proof strength results and the LCF results are shown, there is an analysis concerning a volume element’s way through the straightener. The analysis has been made from two perspectives, ovalization and bending. The analysis is only qualitative.

Let us start with the bending analysis, because it is the most critical, regarding strength, see figure 26. The point will enter from right and the first point is determined to be perfectly synchronized with the first maximum of bending moment, see figure 26. The point will alter position with 211° from one roller set to the next. By the ovalization roll pair the point is still quite well in phase with the maximum bending moment, by the third and the fifth roll the point is about as much out of phase for both. By the fourth roll it is completely out of phase. With reference to this knowledge, the point could theoretically speaking change stress sign five times, but it is more likely that it happens twice or three times.

Figure 26: Depiction over a separate point’s way through the cross roll straightener. Here the focus is on bending.
The analysis of the ovalization is easier. Also here the point will enter from right and the point is determined to hit the second roller set perfectly, the ovalizing roll pair, see figure 27. By the second ovalizing roll pair, the point will not experience any ovalization. The depiction shows that while a volume element travels through the straightener it will experience only one ovalization cycle for sure, but it is possible that it won’t be ovalized at all, if the rolls are set at improper angles.

![Figure 27: Depiction over a separate point’s way through the cross roll straightener. Here the focus is on ovalization.](image)

**4.1 Calculation model**

The undetermined beam was solved with a little beam FEM-model and an elementary case analysis, see figure 10 and 11. In figure 28 the plot of the tube deflection is represented. The plot itself is a good check that the calculations are made correctly. The boundary conditions were as said in passage 3.1 fixed in the middle supports and pinned in both ends. The difference in boundary conditions in the outer tube sections will make the maximum deflection occur on the half nearest to the pinned support.
In figure 29 the transverse force diagram is plotted. Because the forces are represented as point loads, the transverse force diagram will show only constant force levels.
In figure 30 the moment diagram is represented. Because the transverse force diagram only shows constant levels the moment diagram will show linear moment changes.
The resulting strain from the beam calculation analysis is represented in figure 31. The maximum strain is achieved by the left ovalizing roll pair, as the deflection was set to the highest value there. A value of about 1.5 % is read from the diagram. According to the other model the strain is 0.6 %. The strain should then lie in between 0.6 and 1.5 % when looking at isolated bending.
Figure 31: Plot of strains from MATLAB.
4.2 FEM-simulation

The FEM-simulation was made parallel to the analytical calculations. As can be seen in figure 32 the sharp edges of the rolls will make considerable indentation in the tube. This is because the model of the rolls was simplified. The simplification was made to generate a more quick computer calculation. Of course this is not acceptable at all and the radius should have been reintroduced. It is not worth to talk about the strain levels in this stage.

*Figure 32: Shows the point contact that comes from the deletion of the outer radius.*

In figure 33 we can follow how the indentation propagates throughout the entire simulation. Also here the lack of the radius makes the most of the strain. Therefore the strain levels are wrong here also.
Figure 33: Shows that the lack of the outer radius will propagate throughout the whole simulation.

The mesh was as mentioned earlier set quite coarse. In figure 34 we can see the effect of the coarse mesh. The deformation runs like a caterpillar around the tube. If the mesh had been perfect it would have rendered in a smooth tape instead of a caterpillar. As previous pictures the strain levels should be ignored.
Figure 34: Shows the caterpillar pattern due to the quite coarse mesh.

On this stage we can’t draw a lot of quantitative conclusions. We can say one thing from these pictures. If the angle of the rolls aren’t set correctly to the present tube diameter we will end up with a tube that consists of very inhomogeneous mechanical properties.

4.3 Modulus of Elasticity

ME had to be evaluated with the same methods as used for determination of proof strength. To get the right proof strength values one also needs correct input from ME. That’s why two diagrams over the ME have been inserted here. In figure 35 a diagram over ME in Material A is represented. It starts on 210 GPa and immediately drops to 192 during the first cycle. At the 10th cycle the ME is down at 179 GPa.
In figure 36 the ME diagram over Material B is represented.
4.4  Tensile tests

4.4.1 Material A

Figure 37 shows the tensile proof strength of Material A TTPD before straightening. Notice the low variance in the sections and from one section to the other. Section 1 is front end and section 2 is rear end. The diagram shows results from 24 specimens in total. The median is about 710 MPa.

Figure 38 shows the tensile strength of Material A in PD before straightening for front and rear end. Also here we see quite uniform properties from front to rear end and in circumferential direction. The median is about 810 MPa, 100 MPa higher than TTPD, that means that the material is showing considerable anisotropic properties.
Figure 38: Tensile strength in pilgering direction before straightening. Tube section 1 is front end and section 2 is the rear end.

In figure 39 the tensile strength of Material A TTPD, after straightening, is shown. Compared to figure 37 the level is far from the uniform level before straightening. The median is about 760 MPa. The raise of the proof strength TTPD was therefore about 50 MPa. However the level is far from uniform. It is the ovalization that affects the tensile strength TTPD, see figure 2.
Figure 39: Tensile strength transverse to pilgering direction after straightening. Tube section 1 is front end and section 2 is the rear end.
In figure 40 the result after straightening in PD is represented. In the ends the level from before straightening has not been affected at all. In the passage about how the cross roll straightener works it was mentioned that the ends will not be straightened through bending, just through ovalization and therefore there will be no proof strength loss in PD in the ends. In the other end of the same tube section the specimens were collected so far from the ends that the effect from bending started to become considerable. The lower values represent a proof strength of about 690 MPa. The drop in PD was therefore about 120 MPa at the most during the straightening process.

![Material A, after straightening](chart)

Figure 40: The proof strength after straightening in PD. Note the considerable drop much likely depending on the Bauschinger effect from combined bending and load change.

### 4.4.2 Material B

Unfortunately we could only get tensile tests before straightening for the Material B, see figure 41 and 42. The anisotropy TTPD was considerable in this material, which was not the case in Material A.
Figure 41: Proof strength TTPD for Material B. Note the difference in levels, a difference of 250MPa.

Figure 42: Proof strength in PD for Material B. Smooth level of about 1010MPa.
4.5 Low cycle fatigue test

In figure 43 a plot of a specimen that has only been strained to an elastic level of 0.4% is plotted. Even though the specimen was only elastically loaded the stress-strain diagram is far from a straight line just as discussed in the passage about anelasticity.

![Graph over 15 load change cycles with a strain of 0.4%. Note that the 15 curves almost run in the same trace and therefore show elastic strain, without showing a straight line.](image)

In table 4 the specimens that were tested with low cycle fatigue test are shown. The specimens did not show any plastic strain and could therefore not give any proof strength values for dynamic analysis. One of those specimens is shown in figure 43. In table 6 the proof strength after 15 cycles for 3 of the specimens are shown.

**Table 6:** Proof strength for the 3 specimens that were exposed to cyclic strain that was elastic.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Number</th>
<th>Number of specimens tested</th>
<th>Number of cycles</th>
<th>Strain in %</th>
<th>Proof strength [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4-3</td>
<td>3</td>
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<td>15</td>
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<tr>
<td>1-4-9</td>
<td>3</td>
<td>1</td>
<td>15</td>
<td>0.4</td>
<td>801</td>
</tr>
</tbody>
</table>
The specimens that were strained to a level of 1.2% were not possible to determine the proof strength of, because suddenly the EM showed irrelevant values from the software. It could possibly have been determined by hand.

4.5.1 Material A dynamic & true proof strength curve

The background of the determination of the LCF diagram was derived in the passage 3.4. From the initial level the drop was about the same as in the tensile tests. The proof strength drops about 200 MPa immediately. Since the sign of the load, for sure, has changed this is the Bauschinger effect. The drop is then quite uniform from cycle to cycle and that is much likely due to cyclic softening. In the end the last tensile test pops up a bit. The test has rested for a week after the first cycling before the final tensile test to fracture was performed. It could be because of that the strain velocity was too high in the test and that not all the anelastic effect had disappeared in the dynamic test.

![Figure 44: The LCF proof strength diagram for specimen 1-3-12 in Material A. Note the initial drop due to the Bauschinger effect, the continuous cyclic softening and finally maybe an anelastic effect.](image-url)
Figure 45: The LCF proof strength diagram for specimen 1-4-3 in Material A.

Figure 46: The LCF proof strength diagram for specimen 1-4-6 in Material A.
Figure 47: The LCF proof strength diagram for specimen 1-4-9 in Material A.

Figure 48: The LCF proof strength diagram for the specimens 2-3-12 in Material A. They were tested 1, 2 and 5 cycles.
Figure 49: The LCF proof strength diagram for the specimens 2-4-12 in Material A. They were tested 2 and 7 cycles.

Figure 50: The LCF proof strength diagram for the specimens 2-5-12 in Material A. They were tested 3 and 10 cycles.
Figure 51: The LCF proof strength diagram for the specimens 2-3-12, 2-4-12 and 2-5-12 in Material A. They were tested 5, 7 and 10 cycles.
4.5.2 Material B dynamic & true proof strength curve

The procedure for the LCF-test for Material B was the same as for Material A. In figure 52 a proof strength droop of about 150 MPa. It is not as much as in Material A on a relative basis. The drop is even though of such a high level that there is no process way that could compensate for this kind of proof strength loss.

A difference from Material A was that a more frequent drop was noted in the last fracture tensile test.

Figure 52: The LCF proof strength diagram for the specimens 6-3-6 in Material B. They were tested 1 and 10 cycles. Note that the proof strain is only 0.05 due to low plastic strain.
Figure 53: The LCF proof strength diagram for the specimens 6-3-6 in Material B. They were tested 5 and 10 cycles.

Figure 54: The LCF proof strength diagram for the specimens 6-3-12 in Material B. They were tested 1, 3, 5, 7 and 10 cycles.
Figure 55: The LCF proof strength diagram for the specimens 6-3-6 in Material B. They were tested 1, 5 and 10 cycles.
4.6 Hardness test

In the Material A there were hardness tests preformed TTPD and in PD. As expected the hardness was highest at outer surface, because the strain from rolling was the highest there. The hardness raise a bit at the inner surface, and the strain should have been higher there as well in the pilgering operation. There were no major differences between the ends of the tube, therefore the conclusion can be made that ovalization affects the hardness values the most, since it works along all the tube.

![Hardness transverse to pilgering direction, 0 is at outer surface and 1 is at the inner side of the very ends, 3 is at the end towards 1 and 3M is at the end away from 1 on the same tube section as 3.](image)

There were some differences in PD compared to TTPD. There were no differences in the same graph, only anisotropy. The middle part in PD falls linearly towards the lowest level at inner surface. These properties were very uniform throughout the entire tube, even though showing on some anisotropic behavior.
Figure 57: Hardness parallel to pilgering direction, 0 is at outer surface and 1 is at the inner side of the very ends, 3 is at the end towards 1 and 3M is at the end away from 1 on the same tube section as 3.

Figure 58: Hardness transverse to and parallel to pilgering direction. 0 is at outer surface.
In the Material B the hardness TTPD and in PD has been plotted in the same diagram. There are no major differences between the two directions, although more specimens should be evaluated before any big conclusions can be made.

4.7 Micro structure

Prior to this study there was a microstructure analysis made on the extruded material on this specific working order. The microstructure was approved. The microstructures in fig. 59-62 show the outer and inner surfaces, in pilgering direction and transverse to pilgering direction, of pilgered and pilgered and straightened material. No foreign phases were observed. In fig. 59-61 structures of material A are shown. In fig. 62 structures of material B are shown. A great lot of working twins can be observed in material A structures. In the upper figs, taken in pilgering direction, it can be recognized that the grains have been stretched during the working operation. It is even easier to notice the strain in pilgering direction in the material B structures, see the upper pictures in fig. 62.

![Microstructure](image)

*Figure 59: Microstructure, from position 1-12, Material A.*
Figure 60: Microstructure, from position 3-12, Material A.

Figure 61: Microstructure, from position 3-12M, Material A
Figure 62: Microstructure, from position 6-12, Material B.
5 Discussion

5.1 Tensile testing procedure

In tensile testing with hysteresis loop, the loop was introduced at a total strain of 1.6%. It would have been possible to introduce it as early as 0.65%. The consequence of that has not been further analysed. The residual stresses are more normalised the more strain that is induced. That would plead for a late loop introduction as above. On the other hand higher strain level will strengthen specific texture components. Though 1.6% is a fairly low strain level. The strain hardening will also continue, but the slope in that region is almost zero. In the standard the recommendation is to unload to only 10% of stress when the hysteresis loop is introduced. In this investigation the unload level is about 30% of the hysteresis loops introduction level. That itself will make the value of ME to increase. A higher value of ME in its turn will result in a lower proof strength value.

This entire investigation rendered in a quite thorough sub investigation about determination of proof strength in materials showing considerable anelastic behaviour. It has already been mentioned that the proof strength earlier was determined only with the EDC Cyclic software. On an early stage it was clear that the ME could vary considerably from one specimen to another. Further the specimens also showed ME a lot below 170GPa. Something was odd with this behaviour. It is recognized that ME could vary as declared in the introduction, but when it does it almost always do it in a continuous way as what is recognized in the cycles after the first. However from the first cycle to the other there was a big drop of the ME from sometimes 210GPa to 190GPa, see figures 35 and 36. As also described in the introduction, ME can look like it has changed a lot even though what sometimes really has happened is that ME has been evaluated with tensile test procedure that is not appropriate for these steels with anelastic behaviour. A determination with resonance frequency will then give more correct values (6). With the background with the above discussion one could definitely wonder what is the proof strength then, if the ME is not correct determined and the proof strength must be determined with an offset from the ME. A way to evaluate if the hysteresis loop procedure presents the right proof strength could be to first determine the proof strength with the Cyclic software and then know a preliminary value. Then prepare some specimens and make a new tensile test up to that preliminary level and then unload and look if the load drop will meet the abscissa at a strain of 0.2%. If that is not the case the procedure could be adjusted and redone until the load drop hits 0.2 remaining strain, then the unloading was introduced at the proof strength. This could also be valuable in another question. In the hysteresis loop procedure the hysteresis loop ME is translated to make the tangent to the initial part of the tensile curve. The tangent is then extrapolated backwards to the abscissa. That point often lies on the negative side of the ordinate, even though the tensile test started from the origin. The above procedure could show if that is to conservative thinking.
Even though a new procedure with a hysteresis loop was introduced to the tensile testing it could unfortunately not be used in the LCF testing and the Cyclic software had to be used. This is exemplified with the figures 35 and 36.

5.2 Tensile test results

5.2.1 In pilgering direction

As described in passage 4.4.1 the proof strength drops about 120 MPa in pilgering direction at the most. One could almost see the pitch from the simulation in figure 34. The middle section shows the same behavior of the values as the inner ends which is just as one could predict with the background of how the machine works. This is particularly serious because it means that the final mechanical properties will be set in the straightening process, not in the pilgering process. Compensation for this major drop in the pilgering means that the reduction must increase. With increased reduction comes a problem to fulfill requirements of fracture elongation. The Bauschinger effect might affect the elongation in a positive sense. These thoughts have to be investigated further to get a more quantitative model.

5.2.2 Transverse to pilgering direction

The ovalization is more predictable than bending, because of no load changes, just as described in the introduction. Therefore it is possible to affect the anisotropy with various ovalization levels. In the tensile testing TTPD it was showed that the proof strength increased from about 710MPa to about 760MPa. It was clear that the proof strength didn’t drop in a single point. When the proof strength in PD was determined after straightening the ovalization could be adjusted to get a more corresponding value of  \( \text{Rp0.2} \) in the transverse direction and therefore minimize anisotropy. However the pitch must be adjustable in order to get in to phase with the diameter of the tube. In short time period it might be possible to run a tube twice and put the second helical in between the first one. A double straightening together with a possibility to change the angle of the working rolls slightly could give a tube with quite homogeneous properties.

5.3 Modulus of elasticity

As told in passage 4.3 ME started on 210 GPa and immediately fell to 192 during the first cycle. At the 10th cycle the ME was down at 179 GPa. In a heavily cold worked strain hardening steel showing anelastic behavior that is not strange. There are reported cases were ME drops up to 20 %, in our case that was 42 units and means a lower value of 168 GPa, from that point of view these values could be possible. The fact should though be dealt with caution because in these materials determination of ME with tensile testing is not recommended and there are other investigations that say that ME is fairly constant (6)(12).

It should be mentioned that even though the software for evaluation of proof strength produces an answer, there is no guaranties that the answer is correct. There is a great possibility that there is an anelastic contribution that affects the result from the software.
There is also an indication from the fluctuations of the ME that strengthens that theory. It should also be mentioned that there is a built in, in the software, upper limit by 210 GPa.

When the Rp0.2 value turns up in the LCF tests it is a sign that ME in the previous cycle might have been wrong evaluated.

In the material that was only strained 0.4% ME was kept in quite reasonable levels compared to the specimens that were only strained 0.8 and 1.2%. The levels of values below 170 GPa should be considered with great scepticism.

To read out from the discontinuous first drop, something happens when load direction changes, and that happens immediately and only once. So far despite level of plastic strain, only if there are some plastic strain that is enough. Afterwards the stress drop is continuous, just like in investigations about anelasticity and change of ME says. Three mechanisms might be involved. The most possible explanation is that the first drop is due to Bauschinger effect and the continuous drop thereafter is due to cyclic softening.

5.4 Analytical calculation and FEM-simulation

As described in passage 3 the problem is quite complex from an analytical calculation view. The analysis was therefore simplified and a complementary FEM-model would be encouraging to have. The deflections were set to 7.5, 1.3 and 3.2 mm. Both the analytical beam model and the little FEM-model gave strain levels of about 1.5%. The old beam model gave about 0.6%. The true strain level should be somewhere in between depending on boundary conditions. With thought of the LCF-tests it seems quite right. The drop in the LCF-test for Material A was between 120-200 MPa, in the first cycle. In the authentic tests the drop was at the most about 120 MPa. In the LCF-test the whole specimen was strained too the same level. In the authentic test the outer fiber of the tube will have the strain level printed above. That means that the strain level will decrease towards the middle of the tube. To this discussion the very coarse adjustment of the straightening machine should be added. With this background the correlation with the analytical model plus the LCF-test and the authentic tests should be considered very good.

When it comes to the FEM- and analytical model the correlation is very bad. On this level the FEM-model can’t be used for any quantitative analysis. Compressive stresses were exaggerated, especially on the outer surface and right beneath it. The rolls were simplified, see passage 4.2. The outer fillets were removed in order to make the simulation to run quicker. One run lasted for three days anyway. Also the mesh was quite coarse. The mesh is always made from straight lines even though they are connected into a circle. A coarse mesh will then result in sharp edges. The sharp edges will then give stress concentrations depending on the coarse mesh. That can be seen as caterpillars in the helical around the tube.

At last I want to refer to a line in the passage 1.2, objective. “The main purpose of this investigation is to clarify when specific grades soften or harden during the straightening
process.” The mission was to answer the question whether the proof strength increased or decreased during the straightening process. That must be considered as fulfilled.

6 Conclusions

The strength decreases drastically, about 120 MPa, and immediately in pilgering direction when the load changes sign and the material is deformed plastically.

The strength increases, about 50 MPa, transverse to pilgering direction when the material is deformed plastically in circumferential direction by ovalization.

The strength varies a lot more after straightening then after pilgering, because of not proper rolls for the tube diameter. That also gives a tube with strongly inhomogene properties, compared to after pilgering.

The absolute ends of the straightened tube are not affected concerning straightening by bending, therefore it is not likely to find strength values representative for the whole tube there.

The hardness is not affected much under normal straightening parameters.

The hardness is higher at outer surface.

The correlation between the analytical beam model and the FEM-simulation is at the moment not god at all; therefore more job remains on the FEM-model to see if the load case on the whole is possible to break down with this approach.

Some kind of adjustment of the straightener is desired to be able to adjust the angle from the tube to the rolls that gives the right pitch, and therefore gives more concise material properties.

7 Proposed measures for short time quality improvement

Let the tubes pass the straightener two times with lengthwise adjustment, so that the bending moments will overlap from the first lap to the second, and then give more homogeneous properties.

The displacement should be considered as a process parameter, therefore the displacement sensors should be fool proof and the displacement configuration should be noted on respective working order.
8 Proposed further investigations

Evaluate modulus of elasticity with resonance frequency equipment.

Compare the pressurized and the tensile effected material during bending and straightening press straightening to cross roll straightened material.

Map a tube more seriously to check against the route through the straightening machine.

Investigation about how the stress states vary from pilgering to straightening.

Evaluate what ovalization is needed to make a particular increase of the transverse tensile strength, whit the thought of creating better anisotropic properties.
9 References

1. **Meurling, Fredrik.** Results of the 4 1/2” and 5 1/2” trial of the Sandvik Material C. 2011.


5. **Youngs modulus measured in different ways on different steels subjected to various pretretments.** Engberg, G. 1995.


Appendix 1 - Derivation of analytical beam model

A1 Elementary case analysis

A1.1 Forces of reaction, displacement and support angles in beam part 1:

![Figure 63: Depiction of beam part 1, point load elementary case.](image)

Force balance in figure 64 and symmetry induces:

\[ R_{A1} = R_{B1,1} = \frac{P_1}{2} \quad (1) \]

![Figure 64: Depiction of beam part 1, moment elementary case.](image)

Force and momentum balance in figure 65 induces:

\[ \uparrow: R_{A2} + R_{B1,2} = 0 \quad (2) \]

\[ \sim \text{ around } B: M_{B1} + l \cdot R_{A2} = 0 \quad (3) \]

(3) induces:

\[ l \cdot R_{A2} = -M_{B1} \]
\[ R_{A2} = -\frac{M_{B1}}{l} \] (4)

(1) and (4) induces:
\[ R_A = R_{A1} + R_{A2} = \frac{P_1}{2} - \frac{M_{B1}}{l} \] (5)

(4) in (2) induces:
\[ R_{B1,2} = -R_{A2} = \frac{M_{B1}}{l} \] (6)

(1) and (6) induces:
\[ R_{B1} = R_{B1,1} + R_{B1,2} = \frac{P_1}{2} + \frac{M_{B1}}{l} \] (7)

Displacement and support angles:

Originating from point load:

\[ \delta \left( \xi = \frac{l}{2}, \frac{1}{l} = \frac{1}{2} = \beta \right) = \frac{p_l^3}{6EI} \cdot \beta \left[ (1 - \beta^2)\xi - \xi^3 \right] \]

\[ = \frac{p_l^3}{6EI} \cdot \frac{1}{2} \cdot \left[ \left( 1 - \frac{4}{4} - \frac{1}{4} \right) \frac{1}{2} - \frac{1}{6} \right] \]

\[ = \frac{p_l^3}{6EI} \cdot \frac{1}{2} \cdot \frac{2}{8} \]

\[ = \frac{1}{48} \cdot \frac{l^3}{EI} \cdot P_1 \] (8)

\[ \theta_{B1,1} \left( \alpha = \beta = \frac{1}{2} \right) = \frac{p_l^2}{6EI} \cdot \alpha \beta (1 + \alpha) \]

\[ = \frac{p_l^2}{6EI} \cdot \frac{1}{4} \left( 1 \cdot \frac{2}{2} + \frac{1}{2} \right) \]

\[ = \frac{p_l^2}{6EI} \cdot \frac{1}{4} \cdot \frac{3}{2} \]
\[
= \frac{3}{48} \cdot \frac{l^2}{EI} P_1 \quad (9)
\]

**Originating from bending moment:**

\[
\delta \left( \xi = \frac{1}{2}, M_A = 0 \right) = \frac{l^2}{6EI} \cdot [M_A(2\xi - 3\xi^2 + \xi^3) + M_B(\xi - \xi^3)]
\]

\[
= \frac{l^2}{6EI} \cdot M_B \left( \frac{1}{2} \cdot \frac{4}{4} - \frac{1}{8} \right)
\]

\[
= \frac{l^2}{6EI} \cdot \frac{M_B}{8} = \frac{3}{48} \cdot \frac{l^2}{EI} M_B \quad (10)
\]

\[
\theta_{B1,2}(M_A = 0) = \frac{M_A \cdot l}{6EI} + \frac{M_B \cdot l}{3EI} = \frac{1}{3} \cdot \frac{l}{EI} M_B \quad (11)
\]

(8) - (11) induces the following equation system:

\[
\begin{align*}
\frac{1}{48} \cdot \frac{l^3}{EI} P_1 - \frac{3}{48} \cdot \frac{l^2}{EI} M_B &= 7.4 \cdot 10^{-3} \\
\frac{3}{48} \cdot \frac{l^2}{EI} P_1 - \frac{l}{3EI} M_B &= 0
\end{align*} \quad (12)
\]

**A1.2** Forces of reaction, displacement and support angles in beam part 2:

Figure 65: Depiction of beam part 2, point load elementary case.

**Force balance in figure 66 and symmetry induces:**

\[
R_{B2,1} = R_{C1,1} = \frac{P_2}{2} \quad (13)
\]
Force and momentum balance in figure 67 induces:

\[ \sim \text{around } B: -M_{B2} - l \cdot R_{C1,2} = 0 \quad (14) \]
\[ \sim \text{around } C: M_{C1} + l \cdot R_{B2,2} = 0 \quad (15) \]

(14) induces:

\[ R_{C1,2} = -\frac{M_{B2}}{l} \quad (16) \]

(15) induces:

\[ R_{B2,2} = -\frac{M_{C1}}{l} \quad (17) \]

(13) and (17) induces:

\[ R_{B2} = R_{B2,1} + R_{B2,2} = \frac{p_2}{2} - \frac{M_{C1}}{l} \quad (18) \]

(13) and (16) induces:

\[ R_{C1} = R_{C1,1} + R_{C1,2} = \frac{p_2}{2} - \frac{M_{B2}}{l} \quad (19) \]

(7) and (18) induces:

\[ R_B = R_{B1} + R_{B2} \]
\[ = \frac{p_1}{2} + \frac{M_{B1}}{l} + \frac{p_2}{2} - \frac{M_{C1}}{l} \]
\[ = \frac{1}{l} (P_1 + P_2) - \frac{1}{l} (M_{C1} - M_{B1}) \quad (20) \]
Displacement and support angles:

Originating from point load:

\[ \delta \left( \xi = \frac{1}{2} = \beta \right) = \frac{1}{48} \cdot \frac{P_2 l^3}{EI} \]

(21)

\[ \theta_{B2,1} = \theta_{C1,1} \left( \alpha = \beta = \frac{1}{2} \right) = \frac{3}{48} \cdot \frac{P_2 l^2}{EI} \]

(22)

Originating from bending moment:

\[ \delta \left( \xi = \frac{1}{2} \right) = \frac{l^2}{6EI} \cdot \left[ M_A \left( 2 \cdot \frac{1}{2} - 3 \cdot \left( \frac{1}{2} \right)^2 + \left( \frac{1}{2} \right)^3 \right) + M_B \left( \frac{1}{2} - \left( \frac{1}{2} \right)^3 \right) \right] \]

\[ = \frac{l^2}{6EI} \left[ M_A \left( \frac{2}{4} \cdot \frac{4}{4} - \frac{3}{4} \cdot \frac{2}{2} + \frac{1}{8} \right) + M_B \left( \frac{1}{4} \cdot \frac{4}{4} - \frac{1}{8} \right) \right] \]

\[ = \frac{l^2}{6EI} \cdot \left[ \frac{3}{8} M_A + \frac{3}{8} M_B \right] \]

(23)

\[ \theta_{B2,2} = \frac{M_A l}{6EI} + \frac{M_B l}{6EI} \]

(24)

\[ \theta_{C1,2} = \frac{M_A l}{6EI} + \frac{M_B l}{6EI} \]

(25)

(21) to (22) induces the following equation system:

\[
\begin{align*}
\frac{1}{48} \cdot \frac{l^3}{EI} P_2 - \frac{3}{48} \cdot \frac{l^2}{EI} M_A - \frac{3}{48} \cdot \frac{l^2}{EI} M_B &= 7.4 \cdot 10^{-3} \\
\frac{3}{48} \cdot \frac{l^2}{EI} P_2 - \frac{1}{6} \cdot \frac{l}{EI} M_A - \frac{1}{3} \cdot \frac{l}{EI} M_B &= 0
\end{align*}
\]

(26)
A1.3 Forces of reaction, displacement and support angles in beam part 3:

Figure 67: Depiction of beam part 3, point load elementary case.

Force balance in figure 68 and symmetry induces:

\[ R_{C2,1} = R_{D1} = \frac{P}{2} \quad (27) \]

Figure 68: Depiction of beam part 2, moment elementary case.

Force and momentum balance in figure 69 induces:

\[ \uparrow: R_{C2,2} + R_{D2} = 0 \quad (28) \]

\( \sim \text{arr}ound C: -M_{C2} - l \cdot R_{D2} = 0 \quad (29) \)

(29) induces:

\[ R_{D2} = -\frac{M_{C2}}{l} \quad (30) \]

(30) in (28) induces:

\[ R_{C2,2} = -R_{D2} = \frac{M_{C2}}{l} \quad (31) \]

(27) and (31) induces:
\[ R_{C2} = R_{C2,1} + R_{C2,2} = \frac{P_3}{2} + \frac{M_{C2}}{l} \]  
(32)

(19) and (32) induces:

\[ R_C = R_{C1} + R_{C2} \]
\[ = \frac{P_2}{2} - \frac{M_{B2}}{l} + \frac{P_3}{2} + \frac{M_{C2}}{l} \]
\[ = \frac{1}{2} (P_2 + P_3) - \frac{1}{l} (M_{B2} - M_{C2}) \]  
(33)

(27) and (30) induces:

\[ R_D = R_{D1} + R_{D2} = \frac{P_3}{2} - \frac{M_{C2}}{l} \]  
(34)

Displacement and support angles:

Originating from point load:

\[ \delta \left( \xi = \frac{1}{2} = \beta \right) = \frac{1}{48} \cdot \frac{P_3 l^3}{EI} \]  
(35)

\[ \theta_{C2,1} = \theta_{D1} \left( \alpha = \beta = \frac{1}{2} \right) = \frac{3}{48} \cdot \frac{P_3 l^2}{EI} \]  
(36)

Originating from bending moment:

\[ \delta \left( \xi = \frac{1}{2}, M_B = 0 \right) = \frac{3}{48} \cdot \frac{l^2}{EI} M_{C,2} \]  
(37)

\[ \theta_{C2,2} = \frac{M_B l}{3EI} \]  
(38)

(35) to (38) induces the following equation system:

\[
\begin{aligned}
\frac{1}{48} \cdot \frac{P_3 l^3}{EI} - \frac{3}{48} \cdot \frac{l^2}{EI} M_{C,2} &= 3.2 \cdot 10^{-3} \\
\frac{3}{48} \cdot \frac{P_3 l^2}{EI} - \frac{1}{3} \cdot \frac{l}{EI} \cdot M_B &= 0
\end{aligned}
\]  
(39)
\( \sigma = \frac{M}{I} z \) \hspace{1cm} (40)

\( \sigma = E \cdot \varepsilon \) \hspace{1cm} (41)

(40) and (41) induces:

\[ E \cdot \varepsilon = \frac{M}{I} \cdot z \]

\( \varepsilon = \frac{M}{EI} \cdot z \) \hspace{1cm} (42)
Appendix 2 - Stiffness matrix

\[
[k_e] = [k_2] - [k_3] = \frac{E I_z}{L^3} \begin{bmatrix}
12 & 6L & -12 & 6L \\
4L^2 & -6L & 2L^2 \\
& & & \\
& & & 
\end{bmatrix}
\]

\[
[k_1] = [k_e] = \frac{E I_z}{L^3} \begin{bmatrix}
\frac{12}{a^3} & \frac{6L}{a^2} & -\frac{12}{a^2} & \frac{6L}{a} \\
\frac{4L}{a^2} & -\frac{6L}{a} & \frac{2L^2}{a^2} & \frac{6L}{a} \\
& & & \frac{4L}{a^2} \\
& & & \frac{6L}{a}
\end{bmatrix}
\]

\[
K = \begin{bmatrix}
\sum k^{(1)}_{22} + k^{(2)}_{22} & k^{(1)}_{23} + k^{(2)}_{23} & k^{(1)}_{24} + k^{(2)}_{24} & 0 & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{bmatrix}
\]

- 77 -
\[ k_{11} = k_{11}^{(1)} = \left\{ C = \frac{E I z}{L^3} \right\} = C \cdot \frac{12}{\alpha^3} \]

\[ k_{12} = k_{12}^{(1)} = C \cdot \frac{6L}{\alpha^2} \]

\[ k_{13} = k_{13}^{(1)} = -C \cdot \frac{12}{\alpha^3} \]

\[ k_{14} = k_{14}^{(1)} = -C \cdot \frac{6L}{\alpha^2} \]

\[ k_{22} = k_{22}^{(1)} = C \cdot \frac{4L^2}{\alpha} \]

\[ k_{23} = k_{23}^{(1)} = -C \cdot \frac{6L}{\alpha^2} \]

\[ k_{24} = k_{24}^{(1)} = C \cdot \frac{2L^2}{\alpha} \]

\[ k_{33} = k_{33}^{(1)} + k_{11}^{(2)} = 12C \cdot \left(1 + \frac{1}{\alpha^3}\right) \]

\[ k_{34} = k_{34}^{(1)} + k_{12}^{(2)} = 6C \cdot L \left(1 - \frac{1}{\alpha^2}\right) \]

\[ k_{35} = k_{57} = k_{79} = k_{911} = k_{13}^{(1)} = -12C \]

\[ k_{36} = k_{58} = k_{710} = k_{912} = k_{14}^{(2)} = 6C \cdot L \]

\[ k_{44} = k_{44}^{(1)} + k_{22}^{(2)} = 4C \cdot L^2 \left(1 + \frac{1}{\alpha}\right) \]

\[ k_{45} = k_{67} = k_{89} = k_{1011} = k_{23}^{(2)} = -6C \cdot L \]

\[ k_{46} = k_{68} = k_{810} = k_{1012} = k_{24}^{(2)} = 2C \cdot L^2 \]

\[ k_{55} = k_{77} = k_{99} = k_{33}^{(2)} + k_{11}^{(3)} = 12C \cdot \left(1 + \frac{1}{\alpha^3}\right) \]

\[ k_{56} = k_{78} = k_{910} = k_{34}^{(2)} + k_{12}^{(3)} = -6C \cdot L \left(1 + \frac{1}{\alpha^2}\right) \]

\[ k_{66} = k_{88} = k_{1010} = k_{44}^{(2)} + k_{22}^{(3)} = 4C \cdot L^2 \left(1 + \frac{1}{\alpha}\right) \]

\[ k_{1111} = k_{33}^{(5)} + k_{11}^{(6)} = 12C \cdot \left(1 - \frac{1}{\alpha^4}\right) \]

\[ k_{1112} = k_{34}^{(5)} + k_{12}^{(6)} = -6C \cdot L \left(1 - \frac{1}{\alpha^3}\right) \]

\[ k_{1113} = k_{13}^{(6)} = -C \cdot \frac{12}{\alpha^3} \]

\[ k_{1114} = k_{14}^{(6)} = C \cdot \frac{6L}{\alpha^2} \]

\[ k_{1212} = k_{44}^{(5)} + k_{22}^{(6)} = 4C \cdot L^2 \left(1 + \frac{1}{\alpha}\right) \]

\[ k_{1213} = k_{23}^{(6)} = C \cdot \frac{6L}{\alpha^2} \]

\[ k_{1214} = k_{24}^{(6)} = C \cdot \frac{2L^2}{\alpha} \]

\[ k_{1313} = k_{33}^{(6)} = C \cdot \frac{12}{\alpha^3} \]

\[ k_{1314} = k_{34}^{(6)} = -C \cdot \frac{6L}{\alpha^2} \]

\[ k_{1414} = k_{44}^{(6)} = C \cdot \frac{4L^2}{\alpha} \]
Appendix 3-Matlab code for analytical beam model

Below the MATLAB code for the elementary case analysis is found.

```matlab
% Balkböjning av rör i riktmaskin 327, med elementarfallsmetoden

clc;
format long

% Konstanter
E=2.1e11; % elasticitetsmodul [GPa][N/m²]

% Variabler
D=156.3e-3; % [m] rörets ytterdiameter
t=12.6e-3; % [m] rörets godstjocklek
l=900e-3; % [m] elementets längd
v2=7.4e-3; % [m] rörets utböjning
v4=1.4e-3; % [m] rörets utböjning
v6=3.2e-3; % [m] rörets utböjning
a=1; % korrigering för annan elementlängd

% Beräknade variablar
D=d=2*t; % [m]
I=pi*(D^4-d^4)/64 % [m⁴]
C=E*I

% Balkdel 1

xi=x1/l;

E1=[(1/48)*(l^3/C) -(3/48)*(l^2/C)
    (3/48)*(l^2/C) -(1/3)*(l/C)]
D1=[v2
    0];
L1=inv(E1)*D1

L1(1,1)
delta_P1=(L1(1,1)*l^3)/(12*C)*(3/4*xi-xi.^3);

delta_P1M=0;
for i=1:length(delta_P1)-1
    delta_P1M(1,i)=delta_P1(1,length(delta_P1)-i);
end

delta_P1=[delta_P1 delta_P1M];

x=0:1e-3:900e-3;
xi=x/l;

L1(2,1)
delta_M1=l^2/(6*C)*(L1(2,1))*(xi-xi.^3);
```
\[
\delta_1 = -\delta_P_1 + \delta_M_1;
\]
% Balkdel 2

\[
x_1 = 0:1e^{-3}:450e^{-3};
\]

\[
\xi = x_1/l;
\]

\[
E_2 = \begin{bmatrix}
\frac{1}{48}l^3/C & -\frac{3}{48}l^2/C & -\frac{3}{48}l^2/C & \frac{3}{48}l^2/C & -\frac{1}{3}l/C & -\frac{1}{6}l/C \\
\frac{3}{48}l^2/C & -\frac{1}{3}l/C & -\frac{1}{6}l/C & -\frac{1}{3}l/C & -\frac{1}{6}l/C & -\frac{1}{3}l/C
\end{bmatrix};
\]

\[
D_2 = \begin{bmatrix}
v_4 \\
0 \\
0
\end{bmatrix};
\]

\[
L_2 = \text{inv}(E_2) * D_2;
\]

\[
\delta_P_2 = \frac{(L_2(1,1) * l^3)}{(12C)} * (3/4 * \xi - \xi^3);
\]

\[
\text{delta}_P_{2M} = 0;
\]

\[
\text{for } i = 1: \text{length(}\delta_P_2\text{)} - 1
\]
\[
\text{delta}_P_{2M}(1,i) = \text{delta}_P_2(1, \text{length(}\delta_P_2\text{)} - i);
\]

\[
\text{end}
\]

\[
\text{delta}_P_2 = [\text{delta}_P_2 \text{ delta}_P_{2M}];
\]

\[
x = 0:1e^{-3}:900e^{-3};
\]

\[
\xi = x/l;
\]

\[
\delta_M_2 = \frac{l^2}{6C} * (L_2(2,1) * (2 * \xi - 3 * \xi^2 + \xi^3) + L_2(3,1) * (\xi - \xi^3));
\]

\[
\delta_2 = -\delta_P_2 + \delta_M_2;
\]

% Balkdel 3

\[
x_1 = 0:1e^{-3}:450e^{-3};
\]

\[
\xi = x_1/l;
\]

\[
E_3 = \begin{bmatrix}
\frac{1}{48}l^3/C & -\frac{3}{48}l^2/C & -\frac{3}{48}l^2/C & -\frac{1}{3}l/C \\
\frac{3}{48}l^2/C & -\frac{1}{3}l/C & -\frac{1}{6}l/C & -\frac{1}{3}l/C
\end{bmatrix};
\]

\[
D_3 = \begin{bmatrix}
v_6 \\
0
\end{bmatrix};
\]

\[
L_3 = \text{inv}(E_3) * D_3;
\]

\[
\delta_P_3 = \frac{(L_3(1,1) * l^3)}{(12C)} * (3/4 * \xi - \xi^3);
\]

\[
\text{delta}_P_{3M} = 0;
\]

\[
\text{for } i = 1: \text{length(}\delta_P_3\text{)} - 1
\]
\[
\text{delta}_P_{3M}(1,i) = \text{delta}_P_3(1, \text{length(}\delta_P_3\text{)} - i);
\]

\[
\text{end}
\]

\[
\text{delta}_P_3 = [\text{delta}_P_3 \text{ delta}_P_{3M}];
\]

\[
x = 0:1e^{-3}:900e^{-3};
\]
\[
\delta_{M3} = \frac{l^2}{6C} \left( \frac{L3(2,1)}{2} \cdot (2\alpha - 3\alpha^2 + \alpha^3) \right);
\]
\[
\delta_3 = -\delta_{P3} + \delta_{M3};
\]
\[
\text{plot}(x,-\delta_{P3})
\]
\[
\text{hold on}
\]
\[
\text{plot}(x,-\delta_{M3})
\]
\[
\text{hold on}
\]
\[
\text{plot}(x,\delta_3)
\]
\[
\% \text{Plotta hela balken}
\]
\[
x = 0:1e-3:2700e-3;
\]
\[
\text{delta} = [\text{delta}_1(:,1:900) \text{ delta}_2(:,1:900) \text{ delta}_3];
\]
\[
\% \text{Beräkna reaktionskrafterna}
\]
\[
\text{Ra} = L1(1,1)/2 - L1(2,1)/1 \quad \% \text{Balkdel 1}
\]
\[
\text{Rb}_1 = L1(1,1)/2
\]
\[
\text{Rb}_1 = L1(2,1)/1
\]
\[
\text{Rb}_1 = \text{Rb}_1_1 + \text{Rb}_1_2
\]
\[
\text{Rb}_2 = L2(1,1)/2 \quad \% \text{Balkdel 2}
\]
\[
\text{Rb}_2 = L2(3,1)/1
\]
\[
\text{Rb}_2 = \text{Rb}_2_1
\]
\[
\text{Rc}_1 = L2(1,1) + L2(2,1)/1
\]
\[
\text{Rc}_2 = L3(1,1)/2 - L3(2,1)/1;
\]
\[
\text{Rb} = \text{Rb}_1 + \text{Rb}_2
\]
\[
\text{Rc} = \text{Rc}_1 + \text{Rc}_2
\]
\[
\text{Rd}
\]
\[
\% \text{Beräkna reaktionsmomenten}
\]
\[
\text{Mb} = -L1(2,1) + L2(2,1)
\]
\[
\text{Mc} = -L2(3,1) + L3(2,1)
\]
\[
\text{plot}(x,\text{delta})
\]
\[
\text{title('Plot of tube deflection')}
\]
\[
\text{xlabel('Position along tube [m]')}
\]
\[
\text{ylabel('Deflection [m]')}
\]
\[
\% \text{axis([0 2.7 -10e-3 0])}
\]
\[
\text{set(gca,'XTick',0:0.45:2.7)}
\]
\[
\text{set(gca,'YTick',-20e-3:1e-3:0)}
\]
\[
\text{grid on}
\]
\[
\% \text{Kontroll av reaktionskrafterna}
\]
\[
\text{Rest} = \text{Ra} - L1(1,1) + \text{Rb} - L2(1,1) + \text{Rc} - L3(1,1) + \text{Rd}
\]
Appendix 4-Matlab code for the limited FEM-model

Below the MATLAB code that solves the little FEM-model is found.

```matlab
clc;
format long

%Konstanter
E=2.1e11;            % elasticitetsmodul [GPa][N/m^2]

%Variabler
D=156.3e-3;         % [m] rörets ytterdiameter
D=156.3e-3;         % [m] rörets godstjocklek
l=450e-3;           % [m] elementets längd
v2=7.4e-3;          % [m] rörets utbökning vid krafter P1
krafter P2
v6=3.2e-3;          % [m] rörets utbökning vid krafter P3

a=1;                % korrigeren för annan elementlängd
r=14;               % antalet randvillkor

x1=0:1e-3:450e-3;    % Definierar balkens olika element
x2=450e-3:1e-3:900e-3;
x3=900e-3:1e-3:1350e-3;
x4=1350e-3:1e-3:1800e-3;
x5=1800e-3:1e-3:2250e-3;
x6=2250e-3:1e-3:2700e-3;

%Beräknade variablen
D=156.3e-3;         % [m] rörets ytterdiameter
D=156.3e-3;         % [m] rörets godstjocklek
l=450e-3;           % [m] elementets längd
v2=7.4e-3;          % [m] rörets utbökning vid krafter P1
v6=3.2e-3;          % [m] rörets utbökning vid krafter P3

a=1;                % korrigeren för annan elementlängd
r=14;               % antalet randvillkor

%Korrigerat elements styvhetsmatris
k1=C*[12/a^3 6*l/a^2 -12/a^3 6*l/a^2
     0 4*l^2/a -6*l/a^2 2*l^2/a
     0 0 12/a^3 -6*l/a^2
     0 0 0 4*l^2/a];

%Elementets styvhetsmatris
k2=C*[12 6*l -12 6*l
     0 4*l^2 -6*l 2*l^2
     0 0 -6*l 4*l^2
     0 0 0 4*l^2];

K=zeros(14);       % Ursprunglig global styvhetsmatris
F=[0;1;1;0;0;1;1;0;1;0;1;1;0;1;1]; % Den ursprungliga förskjutningsvektorns positioner
```
for i=1:4
  for j=1:4
    for m=0:1
      K(i+10*m,j+10*m)=k1(i,j); % Läser in elementen från elementstyvhetsmatrisen k1 i den globala matrisen
    end
    % i är rad och j kolumn, den plockar över första raden från elementstyvhetsmatrisen till globala styvhetsmatrisen
    end
    % m tar hänsyn till att det är den första och den 6:e elementstyvhetsmatrisen som kommer från k1, dvs det skiljer 10 elementpositioner dem emellan
  end
K;

for i=1:4
  for j=1:4
    for m=1:4
      K(i+2*m,j+2*m)=K(i+2*m,j+2*m)+k2(i,j); % Läser in elementen från elementstyvhetsmatrisen k2 i den globala matrisen
    end
    % m tar hänsyn till att det är 2, 3, 4 och den 5:e elementstyvhetsmatrisen som kommer från k2, dvs det skiljer 2 elementpositioner dem emellan
  end
end

K;

for i=1:r
  for j=1:r
    if i~=l
      K(j,i)=K(i,j); % Speglar den globala matrisen i diagonalen och gör den symmetrisk
    end
  end
end

j=0; % Räknar igenom förskjutningsvektorn, registrerar RV som ger bidrag, dess plats och ser hur många obekanta det finns.
for i=1:r
  if F(i,1)~=0
    j=j+1;
    m(j)=i;
  end
end

K_temp_rad=zeros(j,r); % Skapar en matris för att lagra bidragande kolumner
for i=1:j
  K_temp_rad(i,:)=K(m(i),:);
end
K_temp_rad;

K_red=zeros(j,j); % Definierar den reducerade matrisen
for i=1:j
K_red(:,i)=K_temp_rad(:,m(i));
end

K_red;
L=zeros(j,1);

L=-v2*K_red(:,2); %Beräknar lastvektorn, genom att flytta över de känsa förskjutningarna till lastvektorn
L=L-v4*K_red(:,4);
L=L-v6*K_red(:,6);

L2=[0 -1 0 0 0 0 0 0]; %Radmatriser för att kunna flytta de okända lasterna till förskjutningsvektorn
L4=[0 0 0 -1 0 0 0 0];
L6=[0 0 0 0 0 -1 0 0];

K_red(:,2)=L2'; %Byte ut kolonnerna som flyttats till lastvektorn emot de transponerade radmatriserna ovan
K_red(:,4)=L4';
K_red(:,6)=L6';

%K_red

F_red=inv(K_red)*L; %Beräknar förskjutningsvektorn, som också innehåller de okända lasterna

F_global=zeros(r,1); %Definierar en global förskjutningsvektor

for i=1:j %Flyttar över de beräknade förskjutningarna från den reducerade förskjutningsvektorn till den globala sanna förskjutningsvektorn
    F_global(m(i),1)=F_red(i,1);
end

F_global(3,1)=v2; %Läser in de ursprungliga förskjutningarna i den globala förskjutningsvektorn
F_global(7,1)=v4;
F_global(11,1)=v6

%F_global

%Beräknar reaktionskrafter och moment

for i=1:r
    L_global(i,1)=K(i,:)*F_global;
end

L_global

for i=1:r/2
    if i==1
        L_res=L_global(i,1);
    else
        L_res=L_res+L_global(2*i-1,1);
    end
end
L_res;%Kontrollerar resultantkraften efter summering av reaktionskrafterna 
och de yttre krafterna

%Balkdel 1

x1=0:1e-3:3450e-3;%Definierar halva intervallet, för att formeln bara gäller 
upp till halva intervallet
l=900e-3;

xi=x1/l;

delta_P1=(L_global(3,1)*l^3)/(12*(E*I))*(3/4*xi-xi.^3);%Beräknar 
utböjningen av punktlasten

delta_PIM=0;%Speglar utböjningen, eftersom den är helt symmetrisk

for i=1:length(delta_P1)-1
    delta_PIM(1,i)=delta_P1(1,length(delta_P1)-i);
end

delta_P1=[delta_P1 delta_PIM];%Lägger ihop de två delarna

x=0:1e-3:900e-3;

E1=[(1/48)*(l^3/(E*I)) -(3/48)*(l^2/(E*I))]
    -(3/48)*(l^2/(E*I)) -(1/3)*(l/(E*I))];

D1=[v2
    0]

L1=inv(E1)*D1;%Förskjutningsvektorn

delta_M1=l^2*(L1(2,1)/(6*E*I))*(xi-xi.^3);%Beräknar utböjningen av det 
enskilda momentet, OBS! Den globala förskjutningsvektorn innehåller det 
resulterande momenten ifrån de olika balkelementens respektive moment

delta_1=-delta_P1+delta_M1;%Beräknar den superponerade utböjningen

Mb_1=L1(2,1)%Beräknar momentbidrag från balkdel 1

%Balkdel 2

x1=0:1e-3:3450e-3;

xi=x1/l;

E2=[(1/48)*(l^3/(E*I)) -(3/48)*(l^2/(E*I))]
    -(3/48)*(l^2/(E*I)) (-1/3)*(l/(E*I)) (-1/6)*(l/(E*I))
    (3/48)*(l^2/(E*I)) (-1/6)*(l/(E*I)) (-1/3)*(l/(E*I))];

D2=[v4
    0
    0];
\begin{verbatim}
L2=inv(E2)*D2;

delta_P2=(L_global(7,1)*l^3)/(12*(E*I))*(3/4*xi-xi.^3);

delta_P2M=0;
for i=1:length(delta_P2)-1
    delta_P2M(1,i)=delta_P2(1,length(delta_P2)-i);
end

delta_P2=[delta_P2 delta_P2M];

x=0:1e-3:900e-3;
xi=x/l;

delta_M2=l^2/(6*(E*I))*(L2(2,1)*(2*xi-3*xi.^2+xi.^3)+L2(3,1)*(xi-xi.^3));

delta_2=-delta_P2+delta_M2;

Mb_2=-L2(2,1)
Mc_1=-L2(3,1)

% Balkdel 3

xi=x/l;

E3=[(1/48)*(l^3/(E*I)) -(3/48)*(l^2/(E*I))
    (3/48)*(l^2/(E*I)) -(1/3)*(l/(E*I))];

D3=[v6
    0];

L3=inv(E3)*D3;

for i=1:length(delta_P3)-1
    delta_P3M(1,i)=delta_P3(1,length(delta_P3)-i);
end

delta_P3=[delta_P3 delta_P3M];

x=0:1e-3:900e-3;

end

delta_M3=l^2/(6*(E*I))*(L3(2,1)*(2*xi-3*xi.^2+xi.^3));

end

delta_3=-delta_P3+delta_M3;

Mc_2=-L3(2,1)

% Plotta hela balken

x=0:1e-3:2700e-3;

\end{verbatim}
delta=[delta_1(:,1:900) delta_2(:,1:900) delta_3];

plot(x,delta)

%delta

title('Plot of tube deflection')
xlabel('Position along tube [m]')
ylabel('Deflection [m]')

axis([0 2.7 -10e-3 0])
set(gca,'XTick',0:0.45:2.7)
set(gca,'YTick',-20e-3:1e-3:0)
grid on

figure (2)

%Plot av tvärkraftsdiagram

R1=L_global(1,1);
R_1=[0 R1];
X_1=[0 0];

R_2=[R1 R1];
X_2=[0 1/2];

R_3=R1+L_global(3,1);
R_3=[R1 R3];
X_3=[1/2 1/2];

R_4=[R3 R3];
X_4=[1/2 1];

R_5=R3+L_global(5,1);
R_5=[R3 R5];
X_5=[1 1];

R_6=[R5 R5];
X_6=[1 1*3/2];

R_7=R5+L_global(7,1);
R_7=[R5 R7];
X_7=[1*3/2 1*3/2];

R_8=[R7 R7];
X_8=[1*3/2 1*2];

R_9=R7+L_global(9,1)
R_9=[R7 R9];
X_9=[1*2 1*2];

R_10=[R9 R9];
X_10=[1*2 1*5/2];

R_11=R9+L_global(11,1)
R_11=[R9 R11];
X_11=[1*5/2 1*5/2];
R_{12}=[R_{11} R_{11}];
X_{12}=[l*5/2 l*3];

R_{13}=R_{11}+L_{global}(13,1)
R_{13}=[R_{11} R_{13}];
X_{13}=[l*3 l*3];

R=[R_{1} R_{2} R_{3} R_{4} R_{5} R_{6} R_{7} R_{8} R_{9} R_{10} R_{11} R_{12} R_{13}];
X=[X_{1} X_{2} X_{3} X_{4} X_{5} X_{6} X_{7} X_{8} X_{9} X_{10} X_{11} X_{12} X_{13}];

plot(X,R)
hold on

X_{led}=[0 3*l]
Y_{led}=[0 0]

plot(X_{led},Y_{led},'k')

title('Transverse force diagram')
xlabel('Position along tube [m]')
ylabel('Force [N]')

figure (3)

\% Plot av momentdiagram

\%Balkdel 1
M_{1}=-x_{1}*(L_{global}(3,1)/2)+x_{1}*(M_{b_{1}}/l);
e_{1}=M_{1}/(E*I)*(D/2);

M_{2}=-(1/2-x_{1})*(L_{global}(3,1)/2)+x_{2}*(M_{b_{1}}/l);
e_{2}=M_{2}/(E*I)*(D/2);

\%Balkdel 2
M_{3}=-x_{1}*(L_{global}(7,1)/2)-x_{1}*(M_{b_{2}}/l)-(l-x_{1})*(M_{c_{1}}/l); \%Sammanlagda momentet
e_{3}=M_{3}/(E*I)*(D/2);

M_{stod_{1}}=[M_{2}(\text{length}(M_{2})) M_{3}(1)];
e_{stod_{1}}=M_{stod_{1}}/(E*I)*(D/2);

M_{4}=-(1/2-x_{1})*(L_{global}(7,1)/2)-x_{2}*(M_{c_{1}}/l)-(1/2-x_{1})*(M_{c_{1}}/l);
e_{4}=M_{4}/(E*I)*(D/2);

\%Balkdel 3
M_{5}=-x_{1}*(L_{global}(11,1)/2)-(l-x_{1})*(M_{c_{2}}/l);
e_{5}=M_{5}/(E*I)*(D/2);
\[M_{\text{stod}_2} = [M_4(\text{length}(M_4)) \ M_5(1)]\]
\[e_{\text{stod}_2} = \frac{M_{\text{stod}_2}}{(E*I)}*(D/2)\];

\[M_6 = -(l/2-x1)*(L_{\text{global}}(11,1)/2)-(l/2-x1)*(M_{c_2}/l)\]
\[e_6 = \frac{M_6}{(E*I)}*(D/2)\];

\[\text{XP}_3 = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]\]
\[\text{MP}_3 = [M_1 \ M_2 \ M_3 \ M_4 \ M_5 \ M_6]\]

plot(\text{XP}_3,\text{MP}_3)
hold on
plot(X_{\text{led}}, Y_{\text{led}}, 'k')

set(gca, 'XTick', 0:0.45:2.7)
set(gca, 'YTick', -10e5:1e5:10e5)

grid on

figure (4)
\[\text{e}_3p = [e_2(1,1451) \ e_4(1,1)]\]
\[x_3p = [x_3(1,1) \ x_3(1,451)]\]

\[\text{XP}_4 = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6];\]
\[\text{ep}_4 = [e_1 \ e_2 \ e_3 \ e_4 \ e_5 \ e_6];\]

plot(\text{XP}_4,\text{ep}_4)
hold on
plot(X_{\text{led}}, Y_{\text{led}}, 'k')

set(gca, 'XTick', 0:0.45:2.7)
set(gca, 'YTick', -15e-3:2e-3:15e-3)

grid on
Appendix 5-Additional lcf proof strength diagrams

Material B, strain 0.8%
Rp0.2 dynamic & true

Material B, strain 0.8%
Rp0.2 dynamic & true