An investigation of a fatigue model with two competing failure mechanisms

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Abstract

A new combined fatigue model that considers a defect based and a stress based mechanism is presented. The defect part is based on the weakest link theory, where the integration is performed either over the surface area or the volume of the component. The stress based mechanism is described by the normal distribution, where the stress is either the largest occurring stress value, the point stress, or the point stress adjusted with the stress gradient, denoted the gradient adjusted point stress. Experiments on notched specimens have been performed.

The combined model behaves almost like only a stress based model at high failure probabilities. At low failure probabilities, results are less clear. There, the stress based model is not accurate and the defect based model (both for area and volume) dominates. Using the gradient adjusted point stress in the stress based model and the volumetric weakest link integral in the defect based model being described by gives the best overall fit to the probability of failure. It is noted that for design with respect to high failure probabilities ($> 20\%$), it is enough use only the gradient adjusted stress based model.

1 Introduction

In High Cycle Fatigue (HCF) it is commonly accepted that the failure is initiated at some sort of defects in the material. In a solid mechanics setting, the defects can be modeled using either stress based models like the Weakest Link (WL)-integral, [1, 2], as in [3–5], or defect tolerant approaches for instance [6]. The defects can be either internal, volumetric, defects or surface defects. The types of defects, internal or at the surface, will then lead to two competing fatigue failure modes giving two SN -curves, [7–10]. The competition between the two failure modes (area and volume) has been modeled in HCF using a Poisson process in [7–9]. In [11, 12], the two different failure mechanisms are modeled as two different WL-integrals, one for area and one for volume.

In this paper two mechanisms for fatigue failure are considered. They are, however, not the same as mentioned above. They are described with two models for the probability of fatigue and then, the total probability is found. In order to formulate these models for fatigue, the spatial distribution of stress as well as the time history, the load cycle, need to be considered. The first type of model is called the "D" (Defect) model. It is the well known weakest link integral, in which it is assumed that defects (D) are randomly distributed in size, position and orientation in the material. Thus, in the D-model there is a probability of failure in all volume elements. This probability increases with loading. On the other hand, it is still possible that fatigue is initiated in a region of low stress because a large defect may be found there. The integration is performed either over the specimen volume, as in [3–5, 13–16], or the specimen surface area, as in [4, 5, 17, 18].

The second type of model is called the "S" model, in which the stress (S) is controlling the initiation. The main assumption for this mechanism is that a local stress that is high enough will lead to fatigue. The explanation for this behavior is that the material is quite clean, there is an even density of "defects" and they are small. Hence, it is only the stress that matters in the S-model. It is expected that the scatter of the S-model is less than for the D-model. The stress component to be considered in the S-model is either the largest occurring principal stress value or the largest occurring gradient adjusted stress value. Using the largest occurring stress value, the point stress, gives poor predictive capabilities, [3, 5, 19] whereas adjusting the stress with the stress gradient gives better predictive capabilities, [5, 19].

In the combined probability of fatigue failure, both the D-mechanism and the S-mechanism are possible. In a structure with a very strong stress concentration, a small fillet radius for example, the S-mechanism will dominate. In a smooth structure that have almost uniformly distributed stress, the D-mechanism will dominate. Also, at low loads the D-mechanism dominate and at high loads the S-mechanism will dominate instead. Thus, it should also be possible to study if there is a shift in dominating fatigue mechanism when the load is increased.

The goals of the paper are: *i*) To present a combined model that incorporate two mechanisms of fatigue failures *ii*) Compare the experimental outcome with theory for the combine DS-model as well as for the separate D- and S-models,

respectively (*i.e.* the new model). Finally, it is a goal to *iii*) attain better understanding of the reasons for scatter in fatigue limit test results.

Firstly, the fatigue limit tests are presented. The computations are presented after this section. Secondly, the new combined model, the DS-model, is presented and analyzed. This is followed by a discussion and then summary and conclusions.

2 Experimental investigation

The material used is Uddeholm IMPAX supreme, corresponding to AISI P20 mod, with chemical composition as in Table 1. The low sulphur content, high purity and good homogeneity gives this steel excellent fatigue endurance. The yield strength, $R_{p0.2}$, is 850 MPa, the ultimate tensile strength, R_m , is 1000 MPa and Young's modulus, E , is 205 GPa. This steel is used in moulds for production of plastics, components, for example.

Table 1: Chemical composition of IMPAX supreme, given in wt%.

C	Si	Mn	Cr	Ni	Mo	S
0.37	0.30	1.40	2.0	1.0	0.20	0.008

The specimen used, see Fig. 1, is tailored for the fatigue limit tests. This specimen has been designed for investigating the volume effect, which is why the notches have different size (different diameters). The axi-symmetric specimen is inserted into a rotating bending machine in order to conduct the fatigue tests. The specimen is manufactured using turning from as-received rods. The elastic stress concentration factor for bending, K_t , is 1.48 for notch *A* and 1.32 for notch *B*, according to [20].

Figure 1: Schematic of the specimen used in the experiments.

By changing the location of the applied load, β , the fatigue failure location can be changed. If β is small, failure will occur in notch A and if β is large the failure will occur in notch B .

2.1 Fatigue limit tests

Fatigue limit tests presented in [3] have been analyzed in this paper. The experiments have been performed according to the staircase method, [21]. The experimental outcome is presented in Fig. 2. A run-out is defined as specimen that did not break after $2 \cdot 10^6$ cycles and the failure criterion used is complete failure. The rotational speed in the fatigue tests was 3,000 rpm corresponding to a frequency of 50 Hz.

Figure 2: Experimental outcome from the staircase tests performed, ring indicates a run-out and a cross ($\times$) failure and a) is for notch A and b) is for notch B .

From all the results in Fig. 2, the mean and standard deviation for the fatigue limit can be estimated, see [21]. In notch A they were 404 MPa and 15.8 MPa, respectively. For notch B they were 439 MPa and 17.8 MPa, respectively. The experimental outcome presented in Fig. 2 is also summarized in Table 2.

Table 2: Outcome from the fatigue limit tests using the specimen with $r = 1.5$ mm.

	Notch A ($\beta = 72$ mm)				Notch B ($\beta = 85$ mm)				
Stress level (MPa)	383	397	411	425	411	424	437	450	463
Failures	0	3	3	3	0	1	3	3	2
Run-outs	3	4	3	0	1	4	4	2	0

Another set of fatigue limit tests were performed where the notch root radius, r , was changed from 1.5 mm to 0.4 mm, giving a sharper notch. The

experimental set-up was the same for these experiments, and experiments were performed at three different stress levels. The elastic stress concentration factors for bending are 2.20 for A and 1.88 for B when $r = 0.4$ mm. The experimental outcome and the estimated fatigue limits and standard deviations are presented in Table 3.

Table 3: Outcome from the fatigue limit tests for $r = 0.4$ mm. The number in parenthesis is the number of failures per stress level, a total of five experiments were performed at each stress level.

Notch	Low [MPa]	Medium [MPa]	High [MPa]	Fat. lim. [MPa]	std [MPa]
A	518 (1)	534 (3)	558 (5)	536	17.6
B	501 (1)	519 (2)	537 (5)	524	13.5

It can be noted that the estimated fatigue limits are higher for the specimen with the smaller notch root radius compared to the specimen with the larger notch root radius. Here, the fatigue limit is higher for notch A compared to notch B , which is the contrary to the larger notch root radius. If the nominal stress is computed for both notch root radii it can be seen that the fatigue limit is always higher for notch B . The coefficient of variation ($\text{COV} = \text{std}/\text{mean}$) for notch A is 3.3 % and 3.1 % for notch B . The fatigue limits and standard deviations are determined using linear regression on probability paper.

3 Computations

The specimen presented in Fig. 1 was modeled in Abaqus, in order to analyze the experimental outcome from the fatigue limit tests. The results were post-processed using Matlab. The element types used were C3D8 and C3D6, [22], which are eight noded brick elements and wedge elements, respectively. The wedge elements were used at the center of the specimen in order to simplify the meshing. Different meshes were evaluated and the chosen mesh is denser then required for convergence.

As can be seen in Fig. 3, the mesh was divided into two different sub meshes (A and B) in accordance with the sub volumes of Fig. 1. The different sub volumes A and B were used for evaluation of local contributions to the total failure probability $p_{f,\text{tot}}$, *i.e.* $p_{f,A}$ and $p_{f,B}$. The total failure probability can be expressed as

$$p_{f,\text{tot}} = 1 - (1 - p_{f,A})(1 - p_{f,B}). \quad (1)$$

In order to determine the failure probability for a sub-volume, say A , the slice-shaped section in Fig. 3 can be used. The following relation holds for the probability

$$p_{f,A} = 1 - (1 - p_{f,A\text{section}})^N. \quad (2)$$

In Eq. (2), $p_{f,A}$ indicates the failure probability for the whole sub-volume and $p_{f,A\text{section}}$ the failure probability for a section and N is the total number of

Figure 3: FE-model of the specimen with the element sets corresponding to the sections of notch sub volumes indicated in Fig. 1.

sections, which here is 20. The failure probability for sub-volume B is found in the same way. The total failure probability is then given by Eq. (1).

The stressing from a dead load on a rotating specimen is quite special. The stress tensor history at a point can be found as the result of a combination of the stresses from two load cases [23], with perpendicular forces, F_1 and F_2 , that are equal to the dead load, $F_1 = F_2$. The resulting stress field from one loading is the 90° -rotation of the other. The total stress history is

$$\boldsymbol{\sigma}(\mathbf{x}, \tau) = \boldsymbol{\sigma}_1(\mathbf{x}) \cos(\tau) + \boldsymbol{\sigma}_2(\mathbf{x}) \sin(\tau). \quad (3)$$

Here $\boldsymbol{\sigma}$ denotes the stress tensor at an arbitrary point $\mathbf{x}$ at pseudo time τ , $\boldsymbol{\sigma}_1(\mathbf{x})$ and $\boldsymbol{\sigma}_2(\mathbf{x})$ denote the stress solutions due to loading 1 and 2, respectively. The pseudo-time τ is here equal to the rotation angle ($0 \leq \tau \leq 2\pi$). Based on this stress history a fatigue post-processing analysis can be performed. An FE-analysis is used to determine $\boldsymbol{\sigma}_1(\mathbf{x})$ and $\boldsymbol{\sigma}_2(\mathbf{x})$.

The largest principal stress amplitude criterion can be easily used in this case with fixed principal stress directions; it is just to determine the largest difference over the stress cycle for the principal stresses. This criterion can be described mathematically as

$$\sigma_{\text{eff}, \Delta\sigma_1} = \frac{1}{2} \max_k \{ \sigma_k(t_1) - \sigma_k(t_2) \}, \quad (4)$$

where $k = 1, 2$ and 3 indicate fixed directions. The right hand side contains

the principal stresses during a stress cycle, $\sigma_k(t)$. They are evaluated at t_1 , at which the largest principal stress is found, $\sigma_k(t_1)$ and at t_2 , at which the lowest principal stress is found, $\sigma_k(t_2)$. For the current stress history, with "alternating" bending type stresses,

$$\sigma_{\text{eff}, \Delta\sigma_1} = \max_t(\sigma_1), \quad \sigma_{\text{eff}, \Delta\sigma_1} \leq \sigma_{\text{eff}, \Delta\sigma_1}^{\text{crit}}. \quad (5)$$

The criterion, Eq. (4), is not applicable in cases when the principal stress directions rotate. The criterion in Eq. (5) is only applicable to alternating stress histories. The largest principal stress criterion will be used here, because the choice of effective stress does not have a large influence at the fatigue limit, see [3, 19].

4 The new failure model based on two mechanisms

The fatigue model presented allows for the failure to be initiated due to the occurrence of defects or due to the local stress directly.

4.1 The D-model, defect based mechanisms for fatigue

The Weakest Link (WL) integral, [1, 2] is taken to summarize the defect based model. It has been used previously in the literature [3–5, 19], and has proven to have relatively good predictive capabilities. In order to compute the fatigue failure probability using the WL-integral, a summation of the surface area of the specimen can be performed as

$$p_{f, \text{surface}} = 1 - \exp \left(- \int_{A_{\text{surf}}} \left(\frac{\sigma_{\text{eff}} - \sigma_{\text{th}, \text{surf}}}{\sigma_{\text{u}, \text{surf}}} \right)^n \frac{dA}{A_{\text{ref}}} \right), \quad (6)$$

where σ_{eff} is an effective stress, $\sigma_{\text{th}, \text{surf}}$ is a threshold stress, $\sigma_{\text{u}, \text{surf}}$ is a location parameter, n is material parameter describing the scatter, A_{ref} is an arbitrary reference area and A_{surf} is the surface area over which the integration is performed. Depending on the phenomenon that is important, the fatigue failure probability can also be computed as a volumetric entity, which can be expressed as

$$p_{f, \text{volume}} = 1 - \exp \left(- \int_V \left(\frac{\sigma_{\text{eff}} - \sigma_{\text{th}, \text{vol}}}{\sigma_{\text{u}, \text{vol}}} \right)^m \frac{dV}{V_{\text{ref}}} \right), \quad (7)$$

where V is the specimen volume and the other quantities have the same interpretation as in Eq. (6). In order to compute the fatigue failure probability from FE-results, the integration term in Eq. (7) turns into a summation over the finite elements as

$$p_{f, \text{volume}} = 1 - \exp \left(- \sum_{i=1}^{N_{\text{elem}}} \left(\frac{\sigma_{\text{eff}, i} - \sigma_{\text{th}, \text{vol}}}{\sigma_{\text{u}, \text{vol}}} \right)^m \frac{V_i}{V_{\text{ref}}} \right). \quad (8)$$

In Eq. (8), N_{elem} is the total number of finite elements in the model and all the other entities are the same as above. The same equation can be used for the area-method as well, in this case the summation is performed over the surface elements instead of the element volumes.

4.2 The S-model, stress based mechanisms for fatigue

In the stress based model, the probability of failure depends only on the maximum value of the effective stress in the specimen. The fatigue limit is described by the normal distribution, [5], as

$$p_{f,\text{stress}} = \Phi \left(\frac{\sigma_{\text{eff,max}} - \sigma_{\text{mean}}}{\sigma_{\text{std}}} \right), \quad (9)$$

where Φ is the standard normal cumulative distribution function. Here, σ_{eff} is the largest occurring effective stress, σ_{mean} is the mean strength value and σ_{std} is the standard deviation of the strength. The effective stress value in Eq. (9), σ_{eff} , can be either evaluated directly, called the point stress ($\sigma_{\text{eff,point}}$), or as a gradient adjusted stress

$$\sigma_{\text{eff,grad}} = \sigma_{\text{eff,point}} - \gamma \cdot |\text{grad}(\sigma_{\text{eff,point}})|. \quad (10)$$

In Eq. (10), γ is a material parameter that controls the effect of the stress gradient. The largest occurring gradient adjusted stress has proven to have good predictive capabilities, see [5, 19] for instance. Both the point stress and the gradient adjusted point stress will be investigated as the effective stress in the stress based model.

4.3 The DS-model, a combined model for fatigue

In the DS-model the D- and the S- fatigue models contribute to the total fatigue failure probability in an independent way as

$$p_{f,\text{tot}} = 1 - (1 - p_{f,D})(1 - p_{f,S}) \quad (11)$$

where $p_{f,D}$ is the failure probability using the defect based model, either area Eq. (6) or volume Eq. (7), and $p_{f,S}$ is the failure probability according to the stress based model, Eq. (9). An illustration of the combined model is presented in Fig. 4.

In order to determine the model parameters, the optimization script "fmincon" in MATLAB was used, for a more detailed description of "fmincon", see [24]. The input parameters are; three for the defect based part and two for the stress based part (three if the gradient adjusted stress is used). The following residual is minimized in the optimization scheme

$$Res^2 = \sum_{i=1}^n (p_{f,\text{tot},i} - p_{f,\text{exp},i})^2 \quad (12)$$

Figure 4: Illustration of the combined model.

where $p_{ftot,i}$ is the failure probability according to Eq. (11), $p_{fexp,i}$ is the median rank estimate, [25], of the fatigue failure probability from the experiments and n is the number of load cases. The experimental failure probabilities are computed from the experimental outcome presented in Tables 2 and 3.

5 Predictive capabilities of the DS-model

The DS-model is applied to three different experimental sets; first the specimen with a notch root radius equal to 1.5 mm, second the specimen with a notch root radius $r = 0.4$ mm and thirdly the DS-model is applied to the experimental outcome from all experiments presented in this paper.

5.1 The specimen with notch root radius $r = 1.5$ mm

Table 4: Parameters for the S-model and the D-model, stresses in MPa. When the gradient parameter γ is zero, the point stress is used. The residual, Res , is computed using Eq. (12).

γ [mm]	σ_{mean}	σ_{std}	σ_{th}	σ_u	m [-]	Res [-]	Defect model
0.44	205	9.94	-	-	-	0.156	-
0	441	55.9	-	-	-	0.651	-
-	-	-	302	6.44	2.72	0.491	Volume
-	-	-	0	239	13.6	0.531	Volume
-	-	-	367	0.127	0.868	0.465	Area
-	-	-	0	293	13.9	0.546	Area

In Table 4 the parameters from fitting only the S-model or only the D-model to the experimental data is presented. The parameters were determined by setting $p_{ftot,i}$ in Eq. (12) equal to $p_{f,s}$ for the stress based model or $p_{f,surface}$ or $p_{f,volume}$ for the defect based model. It can be seen that the gradient adjusted

point stress gives the smallest residual and the point stress, γ equal to zero, gives the largest residual.

In Table 5, it can be seen that the residual when combining the point stress and the volume or area stress is lower than just using the defect based model. The parameters are similar when the gradient adjusted stress is used.

Table 5: Parameters for the new combined model, the DS-model, with stresses in MPa. The residual, Res , is computed using Eq. (12).

Comb.	γ [mm]	σ_{mean}	σ_{std}	σ_{th}	σ_u	m [-]	Res [-]	Defect model
Fig. 5a)	0	468	3.71	341	0.10	1.18	0.465	Volume
Fig. 5b)	0.45	200	10.0	371	0.10	1.03	0.153	Volume
Fig. 5c)	0	467	5.78	367	0.11	0.84	0.461	Area
Fig. 5d)	0.45	202	10.1	402	0.39	0.78	0.153	Area

The combined probability functions for the specimen under consideration for the combined models are plotted in Fig. 5. The letters *A* and *B* in this plot indicates fatigue limit tests performed for determination of the fatigue limit distributions in the respective notch. In order to determine the fatigue limits, different values on β and the applied load has been used (see Table 2). The stress on the x -axis is the largest principal stress value in notches *A* and *B* for a given value on the applied load F .

Figure 5: Distribution functions for the new combined model using a) point stress and volume, b) gradient adjusted stress and volume c) point stress and area and d) gradient adjusted stress and area.

In Fig. 5 it can be seen that if the point stress is used in the stress based model, then the defect based model is dominating for low failure probabilities and the stress based for high failure probabilities. There is a shift in dominating failure model at high failure probabilities. The trends are similar for both the volume and the area stress. When using the gradient adjusted stress, the contribution from the defect based model becomes negligible.

It can be concluded that the gradient adjusted stress based model gives the best predictive capabilities. Using a combined model does not improve the predictive capabilities much at all.

5.2 The specimen with notch root radius $r = 0.4$ mm

Fatigue limit models were fitted to the experimental outcome from the fatigue tests performed on the specimen with the smaller notch root radius, see Table 3.

Table 6: Parameters for the S-model and the D-model, stresses in MPa. The residual, Res , is computed using Eq. (12).

γ [mm]	σ_{mean}	σ_{std}	σ_{th}	σ_{u}	m [-]	Res [-]	Defect model
0.302	57.9	1.69	-	-	-	0.126	-
0	530	15.1	-	-	-	0.278	-
-	-	-	350	121	78.7	0.386	Volume
-	-	-	405	113	79.6	0.386	Area

In Table 6 either the S- or the D-model is fitted. It can be observed that the residual is low for all four combinations. The residual from using the point stress is lower than for area and volume.

The result for the DS-model is presented in Table 7. In Table 7 it can be seen that the residual is decreased when the combined model is used.

Table 7: Model parameters for the DS-model where the stresses are in MPa. The residual, Res , is computed using Eq. (12).

Comb.	γ [mm]	σ_{mean}	σ_{std}	σ_{th}	σ_{u}	m [-]	Res [-]	Defect model
Fig. 6a)	0	535	2.62	226	1.01	1.00	0.167	Volume
Fig. 6b)	0.30	60.0	1.40	200	10	1.37	0.112	Volume
Fig. 6c)	0	535	2.58	76.0	161	1.00	0.188	Area
Fig. 6d)	0.30	60.0	1.42	104	245	1.03	0.111	Area

The DS-model when using the parameters in Table 7 is plotted in Fig. 6.

In Fig. 6 it can be seen that when the gradient adjusted stress is combined with either the area or the volume stress, Fig. 6b) and 6d), the D-model is dominating for low failure probabilities and S-model for high failure probabilities. When the point stress ($\gamma = 0$ mm) is used as stress based model, Fig. 6a) and 6c), the combined model is "equal" to the defect based model for failure probabilities lower than 20 % and for failure probabilities higher than that the DS-model is "equal" to the stress based model.

Figure 6: Distribution functions for the DS-model applied to the specimen with the notch root radius $r = 0.4$ mm. a) is point stress and volume, b) is gradient adjusted stress and volume c) is point stress and area and d) is gradient adjusted stress and area.

5.3 Fitting the model to both specimen

As a final analysis the DS-model was fitted to the experimental outcome from both sets of fatigue tests presented in Table 2 ($r = 1.5$ mm) and Table 3 ($r = 0.4$ mm). In order to combine the two different sets of fatigue tests, weighing was applied as

$$R^2 = \sum_{i=1}^{15} \frac{n_i}{N} (p_{f_{tot},i} - p_{f_{exp},i})^2, \quad (13)$$

where the total number of load cases is 15, n_i is the number of experiments performed at load case i , $p_{f_{tot},i}$ is the failure probability according to the model, Eq. (11), $p_{f_{exp},i}$ is the experimental failure probability computed using median ranks [25] and N is the total number of fatigue tests, which is 68.

The parameters from fitting only the D- or the S-model to the both sets of fatigue tests are presented in Table 8. The parameters are chosen so that the residual in Eq. (13) is minimized. However, the residual presented in Table 8 is determined using Eq. (12) in order to be able to compare the different fits.

Table 8: Parameters for the S-model and the D-model when fitted to all fatigue tests. Stresses in MPa. The residual, Res , is computed using Eq. (12).

γ [mm]	σ_{mean}	σ_{std}	σ_{th}	σ_u	m [-]	Res [-]	Defect model
0.09	386	33.8	-	-	-	0.783	-
0	480	100	-	-	-	1.03	-
-	-	-	1.01	216	10.7	0.760	Volume
-	-	-	1.10	237	7.15	0.861	Area

If the DS-model is fitted to all experiments using Eq. (13) the parameters presented in Table 9 are obtained.

Table 9: Model parameters for the DS-model when fitted to all experiments and stresses are in MPa. The residual, Res , is computed using Eq. (12).

Comb.	γ [mm]	σ_{mean}	σ_{std}	σ_{th}	σ_{u}	m [-]	Res [-]	Defect model
Fig. 7a)	0	535	2.66	217	1.86	1.00	0.574	Volume
Fig. 7b)	0.44	205	10.6	1.38	414	64.6	0.454	Volume
Fig. 7c)	0	536	2.26	219	8.93	1.00	0.660	Area
Fig. 7d)	0.44	205	10.6	2.91	468	63.7	0.469	Area

The smallest residuals in Table 9 are obtained if the gradient adjusted point stress is taken to be the stress based model. It can be seen that the residual when combining the two models is lower compared just fitting one model to the experimental data, see Table 8. The combined probability functions for the specimen under consideration are plotted in Fig. 7.

Figure 7: Distribution functions for the DS-model when fitted to all experimental outcome, plotted as function of applied force. Filled lines are the DS-model, dashed lines the stress based model and dash dotted lines the defect based model. a) is point and volume, b) is gradient and volume, c) is point and area and d) is gradient and area.

Letting the S-model be described by the gradient adjusted point stress gives the best fits, see Fig. 7b) and 7d). When the point stress is used in the S-model, Fig. 7a) and 7c), it can be seen that the D-model is dominating at low failure probabilities and the S-model for high failure probabilities. In Fig. 7b) and d) the defect based model describes the experimental outcome for $r = 0.4$ mm and the contribution from the S-model becomes negligible. For $r = 1.5$ mm the stress based model describes the experimental outcome and part from the defect based model becomes negligible.

5.4 Comparison between the different investigated sets of fatigue tests

In order to make a comparison between the different investigated sets of fatigue tests, the residuals are divided with the number of data points per set of fatigue tests. A data point is a load case, combination of F and β , used in the fatigue limit tests.

The residuals in Table 4, 6 and 8 are divided with by the number of data points used in the fits. These residuals are obtained from fitting either only the D- or the S-model to the experimental outcome and are presented in Fig. 8.

Figure 8: Residual/data point for either the D- or the S-model for all the investigated sets of fatigue tests.

In Fig. 8 it can be seen that the gradient adjusted point stress gives the smallest residual. For area and volume, the residuals are almost the same for all three investigated configurations. The residual/data point for the combined DS-model is presented in Fig. 9. The residuals are obtained by dividing the residuals presented in Tables 5, 7 and 9 with the number of data points used in the different fits.

The best fits in Fig. 9 are for using the gradient adjusted point stress as the stress based model. If the point stress is used, the residual is higher. For area and volume, the residual is almost the same for $r = 1.5$ mm and $r = 0.4$ mm.

6 Discussion

The results in Table 4 are consistent with the results presented in [3–5, 19]. Best fit, *i.e.* lowest residual, is from using the gradient adjusted point stress and worst fit is when using the largest occurring stress value, the point stress. For the D-model the residual is similar for the area and volume formulation. If a two-parameter model is used, *i.e.* $\sigma_{th} = 0$ MPa, the residual is larger. These results show that the gradient adjusted point stress have good predictive capability. For the specimen with the smaller notch root radius, Table 6 ($r = 0.4$ mm), the gradient adjusted point stress shows the smallest residual followed by

Figure 9: Residual/data point for all combinations of the DS-model fitted to the three experimental setups.

the point stress and then area and volume. This shows that the S-model should work well for the smaller notch root radius. When either only the D- or the S-model is fitted to both notch root radii, Table 8, the smallest residual is obtained for the volume method followed by the gradient adjusted point stress. The largest residual is once again for the point stress. It can be concluded that the gradient adjusted point stress shows the smallest residual and thus have the best predictive capabilities.

If the D- or S-model parameters for the smaller notch root radius, Table 6 ($r = 0.4$ mm), are applied to the specimen with the larger notch root radius ($r = 1.5$ mm) the experimental outcome can not be predicted at all. The failure probability is then either zero or one at the stress levels in the experiments. If the contrary is done, *i.e.* the D- or S-model parameters from the larger notch, Table 4, are applied on the specimen with the smaller notch root radius the failure probability is overestimated using point, area and volume. The D-model based on volume has the best transferability. It is concluded that the gradient adjusted point stress has no transferability at all between the notches.

Also, the transferability of the DS-model is studied, *i.e.* applying the parameters from fitting the models to one notch root radius to the other notch root radius. Firstly, when the parameters from Table 5 ($r = 1.5$ mm) are applied to the specimen with $r = 0.4$ mm, it is noted that when the point stress is used as the S-model, the failure probability is overestimated (it is always equal to one). If the gradient adjusted point stress is used and the D-part of the model is based on the area, then the DS-model works rather well for low failure probabilities, but very bad for high failure probabilities. When using the volume based D-part of the model, the failure probability becomes zero. If the D-part of the model is based on area, the prediction is good for high failure probabilities and if it is based on volume it is good for low failure probabilities. When applying the parameters in Table 7 ($r = 0.4$ mm) to the specimen with the larger notch root radius, $r = 1.5$ mm, the transferability is good when the point stress is used in combination with the volume based D-part of the model. If the point

stress is combined with the area based D-part of the model, the DS-model gives a good prediction for low failure probabilities. The DS-model is then almost equal to only the D-model, and the predictive capabilities are rather good. For the gradient adjusted point stress the failure probability is always one.

In Fig. 5a) and 5c) ($r = 1.5$ mm) and Fig. 6a) and 6c) ($r = 0.4$ mm), the point stress is used in the S-model. It can be seen that there is a shift between the dominating model, the D-model is dominating for low failure probabilities and the S-model for high failure probabilities. This is particularly clear in Fig. 6a) and 6c), where the results of the combined model is close to the pure D-model for low failure probabilities and the S-model for high failure probabilities. In Fig. 7a) and 7c) there is shift from the D-model, either area or volume based, to the S-model, point, for the smaller notch root radius. Thus the D-model is dominating for low failure probabilities and the S-model for high failure probabilities. The D-model is completely dominating for the specimen with the larger notch root radius, $r = 1.5$ mm.

If the gradient adjusted point stress is used in S-model, the results shown in Fig. 5b) and 5d) ($r = 1.5$ mm) are obtained. Here it is seen that the S-model is completely dominating. In Fig. 6b) and 6d) ($r = 0.4$ mm) the D-model is dominating for failure probabilities lower than 10 % and above that the S-model is dominating completely. The contribution from the D-model in Fig. 5b) and 5d) and Fig. 6b) and 6d) is very small. When the DS-model is applied to both specimen, the results in Fig. 7b) and 7d) are obtained. It is seen that only the D-model describes the fatigue limit results for $r = 0.4$ mm and only the S-model for $r = 1.5$ mm. This gives the best fit, which is a bit contradictory to the results presented above.

As mentioned, the transferability of the S-model when based on gradient adjusted stress was not good. But when the DS-model is fitted to the experimental outcome for both notch radii, the result is promising in a curve fitting sense, Fig. 7b) and 7d. However, the transferability of the combined DS-model is not satisfactory on physical grounds. The reason for the agreement between the model and the experimental outcome is that the DS-model is "split" in two parts during the search for the best parameters. The result is that the D-part of the model fits to one notch while the S-part fits to the other. This behavior could have been anticipated from the fact that the stress ranges in the fatigue limit tests for the two radii, Table 2 and 3, do not overlap. This result shows the limitations of the DS-model.

In Fig. 8 and 9 the residuals/data points for the different fits are shown. It can be seen that the gradient adjusted point stress gives the best fit. Further, it can be seen that the improvement in fit when going from either just the D- or the S-model to the combined DS-model is small for the specimen with the notch root radius equal to 1.5 mm. The decrease in residual is less than 6 %. For the specimen with $r = 0.4$ mm, the decrease in residual is about 32 % when going from just one model to the combined model. There is an 11 % decrease in residual when the gradient adjusted point stress is used.

It is seen in Fig. 5, 6 and 7 that the D-model becomes important for low failure probabilities and the S-model for high failure probabilities. Thus, if the

failure probability used in design is high, then the S-model should be used. Design with respect to the fatigue limit, *i.e.* at a failure probability of 50 %, is in the range that is dominated by the S-model with the gradient adjusted point stress. In this paper, the S-model is formulated in a general way, which gave poor transferability in-between geometrise (notches with different radii). It is possible to augment it in order to increase transferability. If the failure probability used in design is lower it is better that the D-model, *i.e.* the WL-integral, is used. The accuracy of this latter statement is not fully justified by the experiments, but the shift from the S- to the D-model is clear.

7 Summary and Conclusions

The following conclusions can be drawn

- For the specimen with the larger notch root radius ($r = 1.5$ mm) the best fit for only one model is obtained when using the S-model with gradient adjusted point stress.
- Using the gradient adjusted point stress in the S-part of the model gives the best fit for the DS-model.
- At high failure probabilities (>20 %) the S-model is dominating and for low failure probabilities, the D-model is more important.
- When the specimen with the smaller notch root radius ($r = 0.4$ mm) is analyzed, the residuals become lower than for $r = 1.5$ mm.
- For the DS-model, the experimental outcome for $r = 0.4$ mm is described by the D-model and for $r = 1.5$ mm it is best described by the S-model with gradient adjusted point stress.
- If the fatigue limit is to be modeled ($p_f = 50$ %), it is sufficient to use the S-model with the gradient adjusted point stress.

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