Structural Intelligent Platooning by a Systematic LQR Algorithm

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Abstract

Oil price has rapidly increased in the past centuries and made the fuel consumption a hot topic in the transport business. This was also emphasized by a study made by the heavy duty vehicle manufacturer, Scania CV AB, who concluded that one third of their operational cost could be related to the fuel consumption. An alleviation to this problem might be to form a platoon of vehicles. This will not only increase the capacity of the traffic flow but also decrease the fuel consumption due to the slipstream effect that occurs behind traveling objects. Control has already been developed to handle fixed intermediate distances within a platoon of vehicles and is referred to as Automatic Intelligent Cruise Control in the vocabulary of the manufacturers. However none of these existing regulators are taking the slipstream effect into concern and is therefore not suitable for the purpose of fuel reduction by vehicle platooning. By paying regards to this particular effect the complexity of the dynamics is increased and new more sophisticated regulators are necessary. In this thesis a new distributed regulator is proposed which is able to take the slipstream effect into concern using linear quadratic regulation. This regulator maintains the present informational topology where each vehicle is able to measure the distance as well as the velocity to the vehicle ahead. Through simulations it is shown that the regulator is able to cope with realistic scenarios, a systematic decrease of error throughout the platoon has been observed and robustness has been proven.
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Chapter 1

Introduction

According to a survey, [1], made by the heavy duty vehicle manufacturer, Scania CV AB, the fuel cost constitutes approximately one third of their total operational cost. This together with the fact that the oil price have increased more than 50 %, in the period of 2001-2010, has made the fuel reduction for heavy duty vehicles, a commercial topic.

A terminology often used in sports is the slipstream effect which refers to the atmospheric drag reduction behind a traveling vehicle. This region can therefore be used in advantage by vehicles traveling behind. In [4] the result of an experiment with two trucks traveling on a highway in 70 km/h presents that a fuel reduction of 4.7 – 7.7% could be experimentally obtained with two identical trucks when taking advantage of vehicle platooning.

The fundamental problem in the control of coordinated vehicles is the integration of communication due to the decentralized architectures. The control objectives are often specified in terms of the entire formation, meanwhile the regulator controlling the engine can only be applied on each and every vehicle separately.

1.1 Related Work

During the last decades a large amount of work has been published about stability and control of vehicles in different formations. Heavy duty vehicles, (HDVs) in a vehicle platoon is one of the formations that have been in focus, until now mainly because of the environmental benefits and the safety reasons it result in. The introduction of the air drag reduction implies a new type of coupled dynamics and a new type of regulator than what are proposed in earlier studies, e.g. [6]. Since none of the existing proposed regulators take the air drag reduction into concern, they are neither suitable for the purpose of fuel reduction by vehicle platooning.

1
1.2 Objective

By regulating a vehicle’s velocity, i.e. by acceleration or deceleration, the control strategy produces an increased energy consumption which in turn increases the fuel consumption. The brake-even point is when the energy consumption of the proposed regulator is less than the energy gain obtained by taking advantage of the slipstream effect. Therefore the objective of this thesis is to find a regulator that minimizes its energy consumption meanwhile it takes the slipstream effect into concern, is robust and is able to withstand realistic strenuous scenarios.

1.3 Cruise Control (CC) and Adaptive intelligent Cruise Control (AiCC)

Cruise Control, also referred to as speed control or autocruise control is a system that automatically maintains the speed of a vehicle according to a reference velocity.

In addition to a cruise control an Adaptive Cruise Control takes into account the distance to the vehicle in front. This means that the driver defines a reference velocity as well as a reference distance to the vehicle ahead. The distance is measured by radar or laser. The later is lower in cost but weather sensitive. Exactly as the CC, AiCC holds a reference velocity and in addition to that it also adapts its own velocity to the vehicle ahead by keeping a certain reference distance to it.

In some literature the Adaptive intelligent Cruise Control is simply known as ACC. The introduction of the word intelligent to this abbreviation is simply adapted by some automobile manufacturers such as Scania AB to keep apart the AiCC from Air Climate Control, which is a totally different system.
Chapter 2

Vehicle Modeling

2.1 Chapter outline

The purpose of this chapter is to derive a continuous linear vehicle model which describes the dynamics of a heavy duty truck and which is based on a specified platoon configuration. This is necessary in order to be able to produce a regulator as well as simulating the performance of a platoon utilizing the implemented regulator. There are many ways of developing a linear vehicle model of this kind, in this thesis the model developed in [2] is used. The derivation of the model is done by first considering the forces acting upon a truck, thereafter expressing these forces as functions of vehicle parameters and finally linearizing the system in order to express the system as a regular linear time invariant, (LTI), system.

2.2 Platoon configuration

Figure 2.1 shows the configuration of the platoon. The platoon consists of \( i = \{1, 2, ..., N\} \) vehicles, where \( i = 1 \) denotes the leading vehicle. Each vehicle is able to measure the distance and the velocity of the vehicle in front of it, as well as its own velocity.

![Figure 2.1. Platoon of N vehicles traveling on a flat road, each vehicle is able to measure the distance to and the velocity of the vehicle ahead.](image)

For simplicity, the movement of the platoon is assumed to be longitudinal which means that these measurements are scalars. The three dimensional heading will instead implicitly be taken into concern through the forces acting on each vehicle.
CHAPTER 2. VEHICLE MODELING

Furthermore, the reference distance between the vehicles is defined by a minimum distance, based on a time gap, $\tau$. This ensures that there is enough space between the vehicles in case of an emergency braking. The minimum distance is defined as:

$$d_{i,i-1}^\tau = \tau v_{i-1}, \ v_{i-1} > 0$$

(2.1)

$$\Delta d_{i,i-1}^\tau = \tau \Delta v_{i-1}, \ v_{i-1} > 0$$

(2.2)

2.3 External forces acting on vehicles

Figure 2.2 shows the external longitudinal forces acting on a heavy vehicle in motion. These forces can be decomposed through Newton’s second law:

$$m \dot{v} = F_w - F_{ad} + F_{adr} - F_{roll} - F_{gravity}$$

(2.3)

The aerodynamic force produced by the air drag is described by

$$F_{ad} = \frac{1}{2} c_d A_o \rho \alpha v^2$$

(2.4)

where $A_o$ is the maximal cross sectional area of the vehicle, $c_d$ is the air drag coefficient and $\rho \alpha$ is the air density. The reduction of the air drag produced by the vehicle in front is defined as

$$F_{adr} = \frac{1}{2} f(d) \frac{100}{100} c_d A_o \rho \alpha v^2$$

(2.5)
where $d$ is the intermediate distance between the vehicles and $f(d)$ is a non-linear function for the air drag reduction due to one heavy duty vehicle in front, depicted in Figure 2.3. Since vehicles cannot be closer than 5 meters due to safety reasons this relationship between the air drag coefficient and the intermediate distance can be fairly described by a least square approximation.

$$f(d)_{lsq} = -0.414d + 41.29$$

$$0 \geq d \leq 99$$

![Mapping of $c_D$ reduction](image)

**Figure 2.3.** The change in the air drag coefficient and the distance vehicles in between, [4].

The rolling resistance is given by

$$F_{roll} = c_r mg \cos(\alpha),$$  \hspace{1cm} (2.6)
where $c_r$ is the roll coefficient. In turn the gravitational force affecting the vehicle is described by

$$F_{\text{gravity}} = mg \sin(\alpha), \quad (2.7)$$

where $g$ is the gravitational constant. The remaining force, $F_w$ will be derived in the next section.

### 2.4 System dynamics

It is natural to use the engine torque as the input to the system. In order to involve this variable in the dynamics the connections between the vehicle and the engine torque is considered. This series of connections is called the powertrain, Figure 2.4 shows a basic model defined in [2].

**Engine:** An engine is on its own a very complicated system, in this model the clutch given by Newton’s second law is taken into account

$$J_e \dot{\omega}_e = T_u - T_c, \quad (2.8)$$
where $\omega_c$ is the angular velocity of the shaft between the engine and the clutch.

**Clutch:** The clutch is considered to be stiff which implies that the clutch does not yield any changes of torque and angular velocity, namely:

\[
T_t = T_c \tag{2.9}
\]

\[
\omega_t = \omega_c \tag{2.10}
\]

**Transmission:** In this model the transmission is characteristic by two properties, namely the conversion ratio $i_t$ between the input torque and output torque, as well as the efficiency of the gearbox $\eta_t$. These two properties affect the propeller shaft according to:

\[
T_p = i_t \eta_t T_t \tag{2.11}
\]

\[
\omega_p = i_t \omega_c \tag{2.12}
\]

**Propeller shaft:** The connection between the propeller shaft and the final drive is considered to be stiff, hence:

\[
T_p = T_f \tag{2.13}
\]

\[
\omega_p = \omega_f \tag{2.14}
\]

**Final drive:** Similar to the transmission, a conversion ratio and an efficiency constant characterizes the final drive. In the same manner these are taken into account as:

\[
T_d = i_f \eta_f T_f \tag{2.15}
\]

\[
\omega_d = i_f \omega_f \tag{2.16}
\]

**Drive shaft:** The connection between the wheels and the final drive is approximated to be stiff.

\[
T_w = T_d \tag{2.17}
\]

\[
\omega_w = \omega_d \tag{2.18}
\]
Wheel: In this model the slip of the wheels will be neglected, hence the Newton’s second law provides

\[ J_w \omega_v = T_w - r_w F_w, \]  

where

\[ r_w = \frac{v}{\omega_w} = \frac{v_i i_f f_i}{\omega_e}. \]  

Finally, equation (2.8) - (2.20) provides the expression of the force inflicted upon the wheels:

\[ F_w = \{ \dot{w} = \frac{\dot{v}}{r} \} = -\frac{J_w + i_t i_f^2 \eta_f J_e}{r_w} \dot{v} + \frac{i_t^2 i_f^2 \eta_f^2 J_e}{r_w} T_u \]  

By applying the specified forces, (2.4) - (2.7) together with (2.21) to equation (2.3)

\[ (\frac{J_w}{r_w^2} + m + \frac{i_t^2 i_f^2 \eta_f J_e}{r_w^2}) \ddot{v} = (\frac{i_t i_f \eta_f J_e}{r_w} T_u - \frac{1}{2} c_d A_o \rho_o v^2 + \frac{1}{2} f(d) c_d A_o \rho_o v^2 - c_r m g \cos(\alpha) - m g \sin(\alpha)), \]  

where the expression in the parenthesis on the left hand is known as the accelerated mass.

\[ \Rightarrow \ddot{v} = \frac{r_w^2}{J_w + m r_w^2 + i_t^2 i_f^2 \eta_f J_e} \left( \frac{i_t i_f \eta_f J_e}{r_w} T_u - \frac{1}{2} c_d A_o \rho_o v^2 + \frac{1}{2} f(d) c_d A_o \rho_o v^2 - c_r m g \cos(\alpha) - m g \sin(\alpha) \right) \]  

By utilizing the following four notations:

\[ c_e = \frac{r_w^2}{J_w + m r_w^2 + i_t^2 i_f^2 \eta_f J_e} \frac{i_t i_f \eta_f J_e}{r_w} \]  

\[ c_o = \frac{r_w^2}{J_w + m r_w^2 + i_t^2 i_f^2 \eta_f J_e} \frac{1}{2} A_o \rho_o c_d \]  

\[ c_f = \frac{r_w^2}{J_w + m r_w^2 + i_t^2 i_f^2 \eta_f J_e} c_r m g \]  

\[ c_g = \frac{r_w^2}{J_w + m r_w^2 + i_t^2 i_f^2 \eta_f J_e} m g \]  

the dynamic of the vehicle can be simplified to:

\[ \dot{v} = c_e T_u - c_o \phi(d) v^2 - c_f \cos(\alpha) - c_g \sin(\alpha) \]  

\[ \phi(d) = (1 - \frac{f(d)}{100}). \]
2.5 Linearized system dynamics

2.5.1 General linearization technique

Linearization of a function \( g : \mathbb{R}^n \rightarrow \mathbb{R} \) around the point of interest \( p \) is performed by a first order Taylor expansion:

\[
g(x) \approx g(p) + \nabla g|_p (x - p)
\]

assumed that \( g \) is continuous and first order differentiable in the vicinity of the point \( p \).

For this specific case, \( g = \dot{v} \), represents the change of velocity, or in other words acceleration of each vehicle in the platoon. Since the objective is to find a control law where each and every vehicle should act as smoothly as possible in the sense of velocity deviation, \( g|_p \) is chosen to be \( g|_p = 0 \).

2.5.2 Velocity and relative distance as states

The linearization points are chosen around a constant reference distance \( d_i = d_0 \), a constant reference velocity \( v_o = v_{ref} \) and the corresponding reference engine torque \( T_{u} = T_{ref} \). The slope is considered to be a constant parameter \( \alpha_0 = \alpha_{ref} \), which implies that the states of the system is reduced to:

\[
\begin{bmatrix}
\Delta T_{iu} \\
\Delta v_i \\
\Delta d_i
\end{bmatrix}
\]

Applying the Nabla operator

\[
\nabla g|_p = \left( \frac{\partial g}{\partial T_{iu}}, \frac{\partial g}{\partial d_i}, \frac{\partial g}{\partial v_i} \right)|_p = (c_e, -c_\omega \tilde{\phi} v_0^2, -2c_\omega \phi(d_i)v_i)|_p =
\]

\[
= (c_e, -c_\omega \tilde{\phi} v_0^2, -2c_\omega \phi(d_0)v_0),
\]

where \( \tilde{\phi} = \frac{\partial \phi}{\partial d} = 0.0414 \).

\[
\Rightarrow \dot{v}_i \approx c_e \Delta T_{iu} - c_\omega \tilde{\phi} v_0^2 \Delta d_i - 2c_\omega \phi(d_0)v_0 \Delta v_i
\]

Notice that the leading vehicle in the platoon will obtain slightly different dynamics, therefore the following definitions are made:

- \( \gamma_1 = -2c_\omega v_0 \)
- \( \gamma_i = -2c_\omega \phi(d_0)v_0 \) for \( i \in \{2, \ldots, N\} \)
- \( \mu_i = -\tilde{\phi}c_\omega v_0^2 \) for \( i \in \{2, \ldots, N\} \)
- \( b_i = c_e \) for \( i \in \{1, 2, \ldots, N\} \)
In turn the linearize system dynamics becomes:

\[
\begin{bmatrix}
\Delta v_1 \\
\Delta d_{2,1} \\
\Delta v_2 \\
\Delta d_{3,2} \\
\Delta v_3 \\
\vdots \\
\Delta v_{N-1} \\
\Delta d_{N-1,N} \\
\Delta v_N
\end{bmatrix}
\begin{bmatrix}
\gamma_1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & \mu_2 & \gamma_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \mu_3 & \gamma_3 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & \gamma_{N-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & \mu_N & \gamma_N
\end{bmatrix}
\begin{bmatrix}
\Delta v_1 \\
\Delta d_{2,1} \\
\Delta v_2 \\
\Delta d_{3,2} \\
\Delta v_3 \\
\vdots \\
\Delta v_{N-1} \\
\Delta d_{N-1,N} \\
\Delta v_N
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
b_1 & 0 & 0 & \cdots & 0 \\
0 & b_2 & 0 & \cdots & 0 \\
0 & 0 & b_3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & \cdots & b_N
\end{bmatrix}
\begin{bmatrix}
\Delta T^1_u \\
\Delta T^2_u \\
\Delta T^3_u \\
\vdots \\
\Delta T^N_u
\end{bmatrix}
\]
Chapter 3

Optimal control and Robustness

3.1 Chapter outline

This chapter provides the most necessary theory needed to be able to follow the reasoning in the sequent chapters. The theory of Optimal Control is the core in the derivation of the regulator while the section of Robustness provides the most vital theory of the analysis of the derived regulator.

3.2 Optimal control

Optimal control is the theory of how to solve dynamic optimization problems. More precise, it gives an answer to the question: How to find a control law for a certain system such that some optimality criteria are fulfilled? Due to the linearity of the vehicle model, the by far most adopted linear control theory is presented, namely the theory of linear quadratic control.

3.2.1 Linear quadratic control

A typical setup when dealing with LQ problems is to minimize a quadratic cost function:

\[ J(u) = \int_{t_0}^{t_f} x(t)^T Q(t)x(t) + u(t)^T R(t)u(t) \, dt + x(t_f)^T S x(t_f) \]  \hspace{1cm} (3.1)

subject to dynamic constraints as well as subject to an initial condition. That is

\[ J^* = J(u^*) = \min_{u} \int_{t_0}^{t_f} x(t)^T Q(t)x(t) + u(t)^T R(t)u(t) \, dt + x(t_f)^T S x(t_f) \]  \hspace{1cm} (3.2)

Subject to:

\[ \dot{x}(t) = A(t)x(t) + B(t)u(t), \]
with the initial condition defined as:
\[ x(t_0) = x_0. \]

The quadratic cost function introduces a way to minimize both the input, in this case the engine torque, and the error of the output, in this case the difference between the reference output and the actual output. This will be further clarified in Section 4.2.2. In order to ensure a physical meaning, the pair \((A, B)\) needs to be controllable and the matrices \(Q\) and \(R\) needs to be positive definite.

The proof of the solution to the general LQ problem is shown by completion of squares, i.e. a certain optimal cost is assumed to be given by:
\[ J^* = x(t)^TP(t)x(t) := V(t). \] (3.3)

In turn the derivate of (3.3) is:
\[
\dot{V}(t) = \dot{x}(t)^TP(t)x(t) + x(t)^T\dot{P}(t)x(t) + x(t)^TP(t)\dot{x}(t) = \\
= (Ax(t) + Bu(t))^TP(t)x(t) + x(t)^T\dot{P}(t)x(t) + x(t)^TP(t)(Ax(t) + Bu(t)) = \\
= x(t)^TA^TP(t)x(t) + u(t)^TB^TP(t)x(t) + \\
+ x(t)^T\dot{P}(t)x(t) + x(t)^TP(t)Ax(t) + x(t)^TP(t)Bu(t) = \\
= x(t)^T(A^TP(t)P(t) + P(t)A(t) + \dot{P}(t))x(t) + u(t)^TB^TP(t)x(t) + x(t)^TP(t)Bu(t)
\] (3.4)

The integration of (3.5) provides the expression:
\[
V(t_f) - V(t_0) = \int_{t_0}^{t_f} x(t)^T(A^TP(t)P(t) + P(t)A(t) + \dot{P}(t))x(t) + \\
+ u(t)^TB^TP(t)x(t) + x(t)^TP(t)Bu(t)\,dt
\]

If \(J(u) - V(t_0)\) is always greater than zero, then \(V(t_0)\) is in fact the minimal solution.

\[
J(u) - V(t_0) = J(u) + V(t_f) - V(t_0) - V(t_f) = \\
= \int_{t_0}^{t_f} x(t)^TQ(t)x(t) + u(t)^TR(t)u(t)\,dt + x(t_f)^TSx(t_f) + \\
+ \int_{t_0}^{t_f} x(t)^TA^TP(t)P(t) + P(t)A(t) + \dot{P}(t))x(t) + \\
+ u(t)^TB^TP(t)x(t) + x(t)^TP(t)Bu(t)\,dt
- x(t_f)^TP(t_f)x(t_f) = \\
= \int_{t_0}^{t_f} x(t)^T(Q + A^TP(t)P(t) + P(t)A(t) + \dot{P}(t))x(t)\,dt + \\
+ \int_{t_0}^{t_f} u(t)^TR(t)u(t) + u(t)^TB^TP(t)x(t) + x(t)^TP(t)Bu(t)\,dt + \\
+ x(t_f)^T(S - P(t_f))x(t_f)
\] (3.7)
3.2. OPTIMAL CONTROL

If $P(t)$ is defined by:

$$\dot{P}(t) = -A^TP(t) - P(t)A - Q + P(t)BR^{-1}B^TP(t)$$

(3.9) is the so called Riccati equation and has an unique solution that is positive semi-definite. With this definition (3.8) becomes:

$$J(u) - V(t_0) = \int_{t_0}^{t_f} (u(t) + R^{-1}B^TP(t)x(t))^T R(u + R^{-1}B^TP(t)x(t)) \, dt \geq 0$$

In turn, if $u^* = -R^{-1}B^TP(t)x(t)$ then $J(u^*) = V(t_0)$, hence $u^*$ is the optimal regulator and the solution to (3.2) provided $P(t)$ is the unique solution to (3.9).

That is:

$$u^* = -Kx(t)$$

$$K = -R^{-1}B^TP(t)$$

where $P(t)$ is the solution to the Riccati equation:

$$\dot{P}(t) = -A^TP(t) - P(t)A - Q + P(t)BR^{-1}B^TP(t)$$

The minimal cost is therefore:

$$J(u^*) = x_0^TP(t)x_0$$

A more simple set up than above is obtained when all the matrices $(A, B, Q, R)$ are constant and when studying the infinite horizon problem:

$$J^* = \min_u \int_0^{\infty} x(t)^TQx(t) + u(t)^TRu(t) \, dt$$

Subject to:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

Initial condition:

$$x(0) = x_0$$

This setup is the so called Linear Quadratic Regulator, (LQR), problem which implies that $P(t) = P = constant$ is the solution to the steady state Riccati Equation, also known as the Algebraic Riccati Equation, (ARE):

$$0 = -A^TP - PA - Q + PBR^{-1}B^TP$$

(3.10)
3.3 Robustness

Robust control is a branch of control theory that aims to achieve a robust performance and/or stability of the system.

A mathematical model of a plant provides a map from inputs to response. The quality of the model depends on how closely its response matches the response of the true plant. In practice, since no single model can respond exactly as the true plant one needs to find a model simple enough to facilitate the design, yet complex enough to give confidence that design based on the true model will work on the real plant. Methods have therefore been developed to handle these "unstructured uncertainties" as generic errors.

3.3.1 Identification of errors and disturbances

When it comes to a platoon of vehicles we have chosen to focus on:

- Model uncertainty - The most obvious uncertainties are that the model of the system is derived under assumptions that slope of the road is constantly zero, $\alpha_i = 0$ and the masses of the vehicles $m_i$ are exact and constant with respect to time. In the real world, the vehicles are affected by precipitation, wind, friction change of the road and leaning deviation, together with many other factors. Many of those disturbances can fairly be projected on the uncertainty in mass.

- Measurement disturbance - white noise in the measurement of the preceding vehicle's states. It has been showed in [8] that white noise do not affect stability of heavy vehicles traveling in platoons. This is due to the fact that heavy vehicles simply do not react fast enough to the frequent changes in measurement due to their inertia.

- Delays - there is always latency in communication and information processing together with other durations. For a longitudinal control system the delays can be split up in two main components: one is the time used for recognizing a hard brake in the regulator, in a typical system, the duration of this time delay is about 60 ms. The other is the time delay in the brake hardware, it ranges from 10 to 100 ms, therefore the total time delay can range from 70 to 160 ms, [5].

- Package loss - some of the measurements do not reach the processing stage.

- State deviation robustness - a vehicle measures the states of the preceding one. It lacks control of the vehicle in front and can only react to its actions. Therefore a robust analysis has been conducted to see how the deviation in the states of the preceding vehicle affects its own states.
Chapter 4

Methods

4.1 Chapter outline

In this chapter the derivation of the regulator and its subsequent results is presented. In accordance to the objective of this thesis the proposed algorithm, Algorithm 1, is based on the special type of communication topology where each vehicle only has the knowledge of the states of the preceding vehicle. In turn, this means in accordance with the platoon configuration that each vehicle only is able to measure the distance to and the velocity of the vehicle ahead. The following algorithm is therefore addressed to find a regulator that has the corresponding structural appearance.
4.2 LQR by a structural-decomposition algorithm

Consider the vehicle model with velocity and intermediate distance as state.

\[
\begin{bmatrix}
\Delta \dot{v}_1 \\
\Delta \dot{d}_{2,1} \\
\Delta \dot{v}_2 \\
\Delta \dot{d}_{3,2} \\
\Delta \dot{v}_3 \\
\vdots \\
\Delta \dot{v}_{N-1} \\
\Delta \dot{d}_{N-1,N} \\
\Delta \dot{v}_N \\
\end{bmatrix}
= \begin{bmatrix}
\gamma_1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & \mu_2 & \gamma_2 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \mu_3 & \gamma_3 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & \gamma_{N-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \ldots & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & \mu_N & \gamma_N \\
\end{bmatrix}
\begin{bmatrix}
\Delta v_1 \\
\Delta d_{2,1} \\
\Delta v_2 \\
\Delta d_{3,2} \\
\Delta v_3 \\
\vdots \\
\Delta v_{N-1} \\
\Delta d_{N-1,N} \\
\Delta v_N \\
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
b_1 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & \ldots & 0 \\
0 & b_2 & 0 & \ldots & 0 \\
0 & 0 & 0 & \ldots & 0 \\
0 & 0 & b_3 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & \ldots & b_N \\
\end{bmatrix}
\begin{bmatrix}
\Delta T_1^e \\
\Delta T_2^e \\
\Delta T_3^e \\
\vdots \\
\Delta T_N^e \\
\end{bmatrix} = A\bar{x} + B\Delta \bar{T}_e
\]

Each and every vehicle can measure only the preceding vehicle’s velocity and relative distance to it, with an exception for vehicle number one. The problem can be decomposed into smaller subproblems connected through the velocity of the preceding vehicle.

\[
\begin{bmatrix}
\gamma_1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
1 & 0 & -1 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & \mu_2 & \gamma_2 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \mu_3 & \gamma_3 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & \gamma_{N-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \ldots & 1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & \ldots & 0 & \mu_N & \gamma_N \\
\end{bmatrix}
\begin{bmatrix}
\Delta v_1 \\
\Delta d_{2,1} \\
\Delta v_2 \\
\Delta d_{3,2} \\
\Delta v_3 \\
\vdots \\
\Delta v_{N-1} \\
\Delta d_{N-1,N} \\
\Delta v_N \\
\end{bmatrix}
\]

- Subsystem 1

\[
\Delta \dot{v}_1 = A_1 \Delta v_1 + B_1 \Delta T_1^e,
\]

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where $A_1 = \gamma_1$ and $B_1 = b_1$. The optimal state feedback $K_1 = (\ast) \in \mathbb{R}^{1 \times 1}$ can simply be found for this problem by solving (3.10), the linear quadratic cost function is defined by:

$$J^*_1 = \min_{\Delta T_e^1} \int_0^{\infty} \Delta v_1^T Q_1 \Delta v_1 + \Delta(T_e^1)^T R_1 \Delta T_e^1 dt$$

- Subsystem 2

The second vehicle has full knowledge of the dynamics of the preceding vehicle’s closed loop system. It is therefore fair to take this into concern when calculating the optimal feedback for Subsystem 2. Hence, the dynamics of the second subsystem become

$$\begin{bmatrix}
\dot{\Delta v}_1 \\
\dot{\Delta d}_{2,1} \\
\dot{\Delta v}_2
\end{bmatrix} = \hat{A}_2 \begin{bmatrix}
\Delta v_1 \\
\Delta d_{2,1} \\
\Delta v_2
\end{bmatrix} + B_2 \Delta T_e^2$$

where

$$\hat{A}_2 = A_2 - \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} K_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mu_2 \gamma_2 \end{bmatrix} - \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} K_1 & 0 & 0 \end{bmatrix},$$

and

$$B_2 = \begin{bmatrix} 0 \\ 0 \\ b_2 \end{bmatrix}.$$ 

The optimal state feedback $K_2 = (\ast \ast \ast) \in \mathbb{R}^{1 \times 3}$ is once again calculated from (3.10), where the linear quadratic cost function is defined as:

$$J^*_2 = \min_{\Delta T_e^2} \int_0^{\infty} x_2^T Q_2 x_2 + \Delta(T_e^2)^T R_2 \Delta T_e^2 dt$$

$$x_2 = \begin{bmatrix} \Delta v_1 & \Delta d_{2,1} & \Delta v_2 \end{bmatrix}^T$$

- Subsystem 3

In contrary to the second vehicle, the third vehicle has not full information of the dynamics of the preceding vehicle. This is due to the fact that it only can measure the states of the second vehicle and not of the first vehicle. In this case one need to estimate the closed loop dynamics of the coupled state,
\( \Delta v_2 \). This is done by the estimated regulator \( \tilde{K}_2 \) which implies the estimated dynamic of Subsystem 3 as

\[
\begin{bmatrix}
\Delta v_2 \\
\Delta d_{3,2} \\
\Delta \dot{v}_3
\end{bmatrix} = \hat{A}_3 \begin{bmatrix}
\Delta v_2 \\
\Delta d_{3,2} \\
\Delta v_3
\end{bmatrix} + B_3 \Delta T_e^3
\]

where

\[
\hat{A}_3 = A_3 - \begin{bmatrix}
b_2 \\
0 \\
0
\end{bmatrix}, \quad \tilde{K}_2 = \begin{bmatrix}
\gamma_2 & 0 & 0 \\
1 & 0 & -1 \\
0 & \mu_3 & \gamma_3
\end{bmatrix} - \begin{bmatrix}
b_2 \\
0 \\
0
\end{bmatrix} \tilde{K}_2,
\]

and

\[
B_3 = \begin{bmatrix}
0 \\
0 \\
b_3
\end{bmatrix}.
\]

The optimal state feedback \( K_3 = (\bullet \bullet \bullet) \in \mathbb{R}^{1 \times 3} \) is attained by solving (3.10), the linear quadratic cost function is defined as:

\[
J_3^* = \min_{\Delta T_e^3} \int_0^\infty x_3^T Q_3 x_3 + \Delta T_e^{3T} R_3 \Delta T_e^3 dt
\]

\[
x_3 = \begin{bmatrix}
\Delta v_2 \\
\Delta d_{3,2} \\
\Delta v_3
\end{bmatrix}^T
\]

... Subsystem N

In the same manner as subsystem 3, the dynamics of the \( N \) th subsystem become

\[
\begin{bmatrix}
\Delta \dot{v}_{N-1} \\
\Delta \dot{d}_{N,N-1} \\
\Delta \dot{v}_N
\end{bmatrix} = \hat{A}_N \begin{bmatrix}
\Delta v_{N-1} \\
\Delta d_{N,N-1} \\
\Delta v_N
\end{bmatrix} + B_N \Delta T_e^N
\]

where

\[
\hat{A}_N = A_N - \begin{bmatrix}
b_{N-1} \\
0 \\
0
\end{bmatrix}, \quad \tilde{K}_{N-1} = \begin{bmatrix}
\gamma_{N-1} & 0 & 0 \\
1 & 0 & -1 \\
0 & \mu_N & \gamma_N
\end{bmatrix} - \begin{bmatrix}
b_{N-1} \\
0 \\
0
\end{bmatrix} \tilde{K}_{N-1},
\]
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and

\[ B_N = \begin{bmatrix} 0 \\ 0 \\ b_N \end{bmatrix}. \]

The optimal state feedback \( K_N = (\ast \ast \ast) \in \mathbb{R}^{1 \times 3} \) is as previously attained by solving (3.10), where the linear quadratic cost function is defined as:

\[
J^*_N = \min_{\Delta T_N} \left( \int_0^\infty x_N^T Q_N x_N + \Delta(T_e^N)^T R_N \Delta T_e^N \, dt \right)
\]

\[ x_N = \begin{bmatrix} \Delta v_N \\ \Delta d_{N-1} \\ \Delta v_N \end{bmatrix}^T \]

Finally, the state feedback for the collected dynamics, with the desired sparse structure can be composed from the produced regulators, \( K_i = (\ast \ast \ast)|_{K_i} \):

\[
K = \begin{bmatrix}
\ast | K_1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\ast & \ast & \ast | K_2 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \ast & \ast & \ast | K_3 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & \ast & \ast & \ast | K_N
\end{bmatrix}
\]

This procedure can be summarized in the following algorithm:

**Algorithm 1. LQR by a structural-decomposition**

1. **Calculate the CC for Subsystem 1, \( K_1 \).**

2. **For \( i=2 \)**
   - **Extract the corresponding subsystem dynamics from the collected dynamics, \( A \).**
     \[
     A_2 = \begin{bmatrix} \gamma_1 & 0 & 0 \\
     1 & 0 & -1 \\
     0 & \mu_2 & \gamma_2 \end{bmatrix}
     \]
   - **Take into concern the dynamics of the leading vehicle by subtraction of the CC regulator, \( K_1 \).**
     \[
     \hat{A}_2 = A_2 - \begin{bmatrix} b_1 \\
     0 \\
     0 \end{bmatrix} K_1
     \]
   - **Calculate the optimal AiCC regulator, \( K_2 \).**
3. For vehicle $i=3$ to $i=N$:
   - Extract the corresponding subsystem dynamics from the collected dynamics, $A_i$.
     
     $$A_i = \begin{bmatrix} \gamma_{i-1} & 0 & 0 \\ 1 & 0 & -1 \\ 0 & \mu_i & \gamma_i \end{bmatrix}$$

   - Estimate the dynamics of the preceding vehicle by subtraction of the estimated regulator, $\tilde{K}_{i-1}$
     
     $$\hat{A}_i = A_i - \begin{bmatrix} b_{i-1} \\ 0 \\ 0 \end{bmatrix} \tilde{K}_{i-1}$$

   - Calculate the optimal AiCC regulator, $K_i$

4.2.1 The choice of $\tilde{K}_i$

The estimation of the preceding vehicle’s closed loop dynamics,

$$\Delta \dot{v}_{i-1} = \begin{bmatrix} 0 & \mu_{i-1} & \gamma_{i-1} \end{bmatrix} \begin{bmatrix} \Delta v_{i-2} \\ \Delta d_{i-1,i-2} \\ \Delta v_{i-1} \end{bmatrix} - b_{i-1} \begin{bmatrix} K_{i-1}^1 \\ K_{i-1}^2 \\ K_{i-1}^3 \end{bmatrix}$$

$$\approx \begin{bmatrix} \gamma_{i-1} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta v_{i-1} \\ \Delta d_{i-1,i} \\ \Delta v_i \end{bmatrix} - b_{i-1} \tilde{K}_{i-1} \begin{bmatrix} \Delta v_{i-1} \\ \Delta d_{i,i-1} \\ \Delta v_i \end{bmatrix}$$

can be done in many ways for instance by:

- Approximating $\Delta v_{i-2}$ with $\Delta v_{i}$ and $\Delta d_{i-1,i-2}$ with $\Delta d_{i,i-1}$, this yields the regulator:

  $$\tilde{K}_{i-1} = \begin{bmatrix} K_{i-1}^3, & -\frac{\mu_{i-1}}{b_{i-1}} + K_{i-1}^2, & K_{i-1}^1 \end{bmatrix}$$ (4.1)

- Approximating $\Delta v_{i-2}$ with $\Delta v_{i-1}$ and $\Delta d_{i-1,i-2}$ with $\Delta d_{i,i-1}$ which implies the regulator:

  $$\tilde{K}_{i-1} = \begin{bmatrix} K_{i-1}^1 + K_{i-1}^3, & -\frac{\mu_{i-1}}{b_{i-1}} + K_{i-1}^2, & 0 \end{bmatrix}$$ (4.2)

- Truncating $\Delta v_{i-2}$ and $\Delta d_{i-1,i-2}$ which corresponds to the the regulator:

  $$\tilde{K}_{i-1} = \begin{bmatrix} K_{i-1}^3, & 0, & 0 \end{bmatrix}$$ (4.3)
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The simulations described in Chapter 5 favored the truncated estimation, (4.3). A feature of this estimated regulator is that it implies that the regulator of the $i$:th vehicle, $K_i$, is independent of the properties of all the vehicles in front of it.

**Theorem 1.** The regulator obtained through truncated estimation, (4.3), for vehicle $i$ is only dependent on the properties of vehicle $i - 1$.

**Proof:**

Assume that the positive definite solution to the Algebraic Riccati equation, (3.10) can be stated as:

$$P_{i-1} = \begin{bmatrix} p_{11}^{i-1} & p_{12}^{i-1} & p_{13}^{i-1} \\ p_{12}^{i-1} & p_{22}^{i-1} & p_{23}^{i-1} \\ p_{13}^{i-1} & p_{23}^{i-1} & p_{33}^{i-1} \end{bmatrix}$$

and the costs gain matrices:

$$Q_{i-1} = \begin{bmatrix} q_{11}^{i-1} & q_{12}^{i-1} & q_{13}^{i-1} \\ q_{12}^{i-1} & q_{22}^{i-1} & q_{23}^{i-1} \\ q_{13}^{i-1} & q_{23}^{i-1} & q_{33}^{i-1} \end{bmatrix}$$

$$R_{i-1} = r_{i-1}$$

Then equation (3.10) yields the following equations for the $i - 1$:th subsystem:

$$2(\gamma_i - 2p_{11}^{i-1} + p_{12}^{i-1}) + q_{11}^{i-1} - \frac{1}{r_{i-1}}(b_{i-1}p_{11}^{i-1})^2 = 0 \quad (4.4)$$

$$\gamma_i - 2p_{12}^{i-1} + p_{22}^{i-1} + b_{i-1}p_{13}^{i-1} + q_{12}^{i-1} - \frac{1}{r_{i-1}}p_{13}^{i-1}p_{23}^{i-1}b_{i-1}^2 = 0 \quad (4.5)$$

$$(\gamma_{i-2} + \gamma_i - 1)p_{13}^{i-1} - p_{12}^{i-1} + p_{23}^{i-1} + q_{13}^{i-1} - \frac{1}{r_{i-1}}p_{13}^{i-1}p_{23}^{i-1}b_{i-1}^2 = 0 \quad (4.6)$$

$$2b_{i-1}p_{23}^{i-1} + q_{22}^{i-1} + q_{23}^{i-1} - \frac{1}{r_{i-1}}(p_{23}^{i-1}b_{i-1})^2 = 0 \quad (4.7)$$

$$\mu_{i-1}p_{33}^{i-1} - p_{22}^{i-1} + \gamma_{i-1}p_{23}^{i-1} - \frac{1}{r_{i-1}}p_{23}^{i-1}p_{33}^{i-1}b_{i-1}^2 = 0 \quad (4.8)$$

$$2(\gamma_{i-1}p_{33}^{i-1} - p_{33}^{i-1}) + q_{33}^{i-1} - \frac{1}{r_{i-1}}(p_{33}^{i-1}b_{i-1})^2 = 0 \quad (4.9)$$

Since the optimal regulator is given by:

$$K_{i-1} = \frac{1}{r_{i-1}} \begin{bmatrix} b_{i-1}p_{13}^{i-1} & b_{i-1}p_{23}^{i-1} & b_{i-1}p_{33}^{i-1} \end{bmatrix}$$

and due to the fact that $\tilde{K}_{i-1} = \begin{bmatrix} K_{i-1}^2 & 0 & 0 \end{bmatrix}$ it is only necessary to show that the last element $p_{33}^{i-1}$ is independent of $\gamma_{i-2}$. 

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By equation (4.7) it is easily shown that:

\[ p_{23}^{i-1} = \frac{r_{i-1} \mu_{i-1}}{b_{i-1}^2} \pm \sqrt{\left( \frac{r_{i-1} \mu_{i-1}}{b_{i-1}^2} \right)^2 + \frac{r_{i-1} q_{22}^{i-1}}{b_{i-1}}} \]  \hspace{1cm} (4.10)

In the same way (4.9) yields:

\[ p_{33}^{i-1} = \frac{r_{i-1} \gamma_{i-1}}{b_{i-1}^2} \pm \sqrt{\left( \frac{r_{i-1} \gamma_{i-1}}{b_{i-1}^2} \right)^2 + \frac{r_{i-1}}{b_{i-1}^2} \left(2 \gamma_{i-1} p_{23}^{i-1} + q_{33}^{i-1}\right)} \]  \hspace{1cm} (4.11)

Insertion of equation 4.10 into equation 4.11:

\[ p_{33}^{i-1} = \frac{r_{i-1} \gamma_{i-1}}{b_{i-1}^2} \pm \sqrt{\left( \frac{r_{i-1} \gamma_{i-1}}{b_{i-1}^2} \right)^2 + \frac{r_{i-1}}{b_{i-1}^2} \left(2 \gamma_{i-1} \beta_{1}^{i-1} + q_{33}^{i-1}\right) + \frac{r_{i-1} q_{22}^{i-1}}{b_{i-1}}} \]  \hspace{1cm} (4.12)

\[ + \sqrt{\left( \frac{r_{i-1} \gamma_{i-1}}{b_{i-1}^2} \right)^2 + \frac{r_{i-1}}{b_{i-1}^2} \left(2 \gamma_{i-1} \beta_{1}^{i-1} + q_{33}^{i-1}\right) + \frac{r_{i-1} q_{22}^{i-1}}{b_{i-1}}} \]  \hspace{1cm} (4.13)

hence \( p_{33}^{i-1} \) is independent of \( \gamma_{i-2} \) and therefore constitutes that \( \tilde{K}_i \) only depends on the properties of the vehicle in front.

4.2.2 The cost function

The cost function of an optimal control predicts how and why a system perform as it does. More precise a cost function specifies how the input should be chosen in order to minimize the total cost:

\[ J(u) = \int_0^\infty x(t)^T Q x(t) + u(t)^T R u(t) \, dt \]

This implies that the matrices, \( Q \) and \( R \) should be chosen such that the desired performance is achieved.

In coordination with the previous section each subsystem has an unique cost function, \( J(\Delta T_i)_i \). The first subsystem, \( i = 1 \) has only one state, namely \( \Delta v_1 \), since the corresponding vehicle is the leading vehicle which utilizing a Cruise Control. In order to achieve a good performance for this control an actual velocity close to the reference velocity, \( \Delta v_1 \) will be rewarded as well as a small input torque, \( \Delta T_1 \). This implies the following cost function

\[ J(\Delta T_1)_1 = \int_0^\infty \beta_1^{\Delta v_1} \Delta v_1^2 + \beta_1^{\Delta T_1} \Delta T_1^2 \, dt, \]  \hspace{1cm} (4.14)
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where

\[ Q_1 = \beta_1^{\Delta v}, \]
\[ R_1 = \beta_1^{\Delta T}. \]

In the contrary to the first subsystem the remaining subsystems, \( i > 0 \) utilize an AiCC. This provides the opportunity to use a more sophisticated cost function than (4.14). In this case it is desirable to find a good balance between a small inter vehicle distance and a small input torque while at the same time the safety has to be addressed. In other words the cost function should be rewarded when the inter vehicle distance is close to the desired time gap distance. That is when equation (2.2) is small:

\[
\Delta d_{i,i-1} = d_{i,i-1} - \tau v_{i-1} = \\
= \Delta d_{i,i-1} + \tau v_{ref} - \tau (\Delta v_{i-1} + v_{ref}) = \\
= \Delta d_{i,i-1} - \tau \Delta v_{i-1}
\]

For safety reasons the cost function should be more penalized when (2.2) is greater than zero than when (2.2) is less or equal to zero. A way of achieving this is to also reward a small deviation from the reference inter vehicle distance:

\[
\Delta d_{i,i-1} = d_{i,i-1} - \tau v_{ref}
\]

In the case where the vehicle ahead hits the brake the difference will be larger if the \( i \):th vehicle crosses (2.2) than if it is not hence the input rather pulls the \( i \):th vehicle backwards than towards the vehicle ahead.

Furthermore it might be a good idea to encourage a small difference of the \( i \):th and the \( i-1 \):th vehicle’s velocities:

\[
v_{i-1} - v_i = \Delta v_{i-1} + v_{ref} - (\Delta v_i + v_{ref}) = \Delta v_{i-1} - \Delta v_i
\]

It might also be a good idea to reward a small deviation from the reference velocity:

\[
\Delta v_i = v_i - v_{ref}
\]

However this might push the vehicle closer to the vehicle in front and counter act the reward of a small deviation from the reference distance.

Of course a large input torque, \( \Delta T_i \) needs to be punished as well.

Summing up the above reasoning, the cost function of each subsystem will have the following appearance:

\[
J(\Delta T_i)_i = \int_0^\infty \beta_i^{\Delta T} (\Delta d_{i,i-1}^2 + \Delta v_{i-1}^2 + \Delta T_i^2) dt
\]
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In matrix form this becomes:

\[
J(\Delta T_i) = \int_0^\infty \begin{bmatrix} \Delta v_{i-1} & \Delta d_{i,i-1} & \Delta v_i \end{bmatrix} Q \begin{bmatrix} \Delta v_{i-1} \\ \Delta d_{i,i-1} \\ \Delta v_i \end{bmatrix} dt
\]

where:

\[
Q = \begin{bmatrix}
\beta_i^v + \tau^2 \beta_i^T & -\tau \beta_i^T & -\beta_i^v \\
-\tau \beta_i^T & \beta_i^T + \beta_i^{\Delta d} & 0 \\
-\beta_i^v & 0 & \beta_i^v + \beta_i^{\Delta v}
\end{bmatrix}
\]

(4.15)

\[
R = \beta_i^{\Delta T}
\]

(4.16)

4.2.3 State robustness

We have chosen to study how a deviation from the reference velocity of the vehicle ahead, \(\Delta v_{i-1}\), most affects the deviation from the reference inter vehicle distance, \(\Delta d_{i,i-1}\), as well as the deviation from the reference velocity, \(\Delta v_i\). This can be interpreted as studying the robustness of the states, namely the \(H_\infty\) norm of the corresponding transfer functions in the frequency domain. The \(H_\infty\) norm is defined as:

\[
||G||_\infty = \max_\omega |G(j\omega)| = \alpha
\]

This implies that if the input is a sinus function with frequency \(\omega_{max}\) and amplitude 1 the maximum amplitude of the output will be \(\alpha\). In this case the maximum peak response, \(\alpha\), will be used to investigate how the state \(\Delta v_{i-1}\) at most affects the states \(\Delta d_{i,i-1}\) and \(\Delta v_i\). That is:

\[
|\Delta d_{i,i-1}| \leq \alpha_i^d |\Delta v_{i-1}|
\]

(4.17)

\[
|\Delta v_i| \leq \alpha_i^v |\Delta v_{i-1}|
\]

(4.18)

The dynamic for each vehicle in the time domain is:

\[
\Delta v_i = \mu_i \Delta d_{i,i-1} + \gamma_i \Delta v_i - b_i(k_i^1 \Delta v_{i-1} + k_i^2 \Delta d_{i,i-1} + k_i^3 \Delta v_i)
\]

and the corresponding inter vehicle distance is:

\[
\Delta d_{i,i-1} = \Delta v_{i-1} - \Delta v_i
\]

Hence the dynamics of each vehicle in the Laplace domain is:

\[
\Delta V_i = \frac{1}{s} [\mu_i \Delta D_{i,i-1} + \gamma_i \Delta V_i - b_i(k_i^1 \Delta V_{i-1} + k_i^2 \Delta D_{i,i-1} + k_i^3 \Delta V_i)] =
\]

\[
= \frac{1}{s} [-b_i k_i^1 \Delta V_{i-1} + (\mu_i - b_i k_i^2) \Delta D_{i,i-1} + (\gamma_i - b_i k_i^3) \Delta V_i]
\]

(4.19)
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\[
\Delta D_{i,i-1} = \frac{1}{s}(\Delta V_{i-1} - \Delta V_i) \quad (4.20)
\]

Extracting \(\Delta V_i\) from equation (4.19) gives:

\[
\Delta V_i = \frac{1}{s - (\gamma_i - b_i k_i^3)} \left[ -b_i k_i^1 \Delta V_{i-1} + (\mu_i - b_i k_i^2) \Delta D_{i,i-1} \right] \quad (4.21)
\]

By inserting (4.21) in (4.20) and extracting \(\Delta D_{i,i-1}\) one obtain the transfer function from \(\Delta V_{i-1}\) to \(\Delta D_{i-1,i}\), \(G_d^i(s)\):

\[
\Delta D_{i,i-1} = \frac{s - (\gamma_i - b_i k_i^3)}{s^2 - (\gamma_i - b_i k_i^3)s + \mu_i - b_i k_i^2} \left( 1 + \frac{b_i k_i^1}{s - (\gamma_i - b_i k_i^3)} \right) \Delta V_{i-1} = G_d^i(s) \Delta V_{i-1}
\]

where

\[
G_d^i(s) = \frac{s - (\gamma_i - b_i k_i^3)}{s^2 - (\gamma_i - b_i k_i^3)s + \mu_i - b_i k_i^2} \left( 1 + \frac{b_i k_i^1}{s - (\gamma_i - b_i k_i^3)} \right)
\]

In turn, by inserting (4.20) in (4.21) and extracting \(\Delta V_i\) one obtain the transfer function, \(G_v^i(s)\) from \(\Delta V_{i-1}\) to \(\Delta V_i\):

\[
\Delta V_i = \frac{-b_i k_i^1 s + \mu_i - b_i k_i^2}{s^2 - (\gamma_i - b_i k_i^3)s + \mu_i - b_i k_i^2} \Delta V_{i-1} = G_v^i(s) \Delta V_{i-1},
\]

where

\[
G_v^i(s) = \frac{-b_i k_i^1 s + \mu_i - b_i k_i^2}{s^2 - (\gamma_i - b_i k_i^3)s + \mu_i - b_i k_i^2}
\]

The maximum peak response for each transfer function is given by:

\[
\alpha_d^i = \|G_d^i\|_\infty \quad (4.22)
\]

\[
\alpha_v^i = \|G_v^i\|_\infty \quad (4.23)
\]
4.2.4 Time gap constraint

In order to guarantee that the $i$th vehicle is not closer to the vehicle in front than what is allowed by the specified time gap (2.1), a constraint on the time gap is developed by adopting the maximal $\Delta d_{i,i-1}$ of (4.17)

If the vehicle ahead experience a deviation from the reference velocity, $\Delta v_{i-1} \neq 0$ the required minimum distance between the $i-1$th and the $i$th vehicle will change according to:

$$\tau |v_{\text{ref}} - \tau (v_{\text{ref}} - \Delta v_{i-1}) = \tau \Delta v_{i-1}$$

If the deviation $\Delta v_{i-1}$ implies a maximum deviation, $|\Delta d_{i,i-1}^{\text{max}}| = \alpha_i d |\Delta v_{i-1}|$, which is greater than the change of the minimum distance it is obvious that it exists a possibility that the $i$th vehicle crosses this minimum distance.

![Figure 4.1. Time gap, $\tau \Delta v_{i-1}$, versus the deviation from the distance where the linearization was performed around, $\Delta d_{i,i-1}$.](image)

Hence, the following condition must be fulfilled in order to guarantee the time gap:

$$\tau |\Delta v_{i-1}| \geq \alpha_i d |\Delta v_{i-1}| \Rightarrow$$

$$\tau \geq \alpha_i d$$
Chapter 5

Simulations

5.1 Chapter outline

This chapter presents the simulation results when vehicle platoons utilize the proposed regulator. In addition to these results, this chapter further presents the corresponding analysis which is done in accordance to the theory of robustness, presented and developed throughout Chapters 3 and 4. The Tuning procedure, Section 5.2, describes how the tuning of the regulator was conducted and why the following results is presented. The Scenario overview in Section 5.3, presents how the simulation was conducted. The remaining sections, Extreme mass scenarios in Section 5.4, and Delay and packet loss disturbances in Section 5.5, together with State robustness in Section 5.6, and Model uncertainty robustness in Section 5.7, presents the simulation results where the later two are focusing on the robustness.

5.2 Tuning procedure

The tuning of the $Q$ and $R$ matrices was performed by trail and error due to the complexity of choosing their values by physical reasoning. In this study it is assumed that the realistic maximum number of vehicles in a platoon is 7, which yields a total platoon length of about a quarter of a kilometer. The results of the simulations is analyzed by comparing the plots of the inter vehicle distances and the velocities to the provided maximum and minimum torque.

The penalty matrices, $Q$, $R$, have been chosen with respect to the following objectives:

- Decrease the velocity overshoot:
  \[
  \max\{v_i - v_{ref}\}
  \]

- Decrease the time gap overshoot:
  \[
  \min\{\tau(v_{i-1} - d_{i,i-1})\} \tag{5.1}
  \]

27
• Decrease the input of the system, the torque, $T_i^e$.

Besides these objectives there are also some physical constraints which needed to be addressed:

• The maximum accelerating torque is assumed to be 2300 Nm.
• The maximum produced decelerating torque in a HDV can be up to $\sim 60000$ Nm per axle.
• The maximum acceleration is $0.05 \text{ m/s}^2$ of a HDV on a flat road, described in [7].
• A very rough brake is $6.00 \text{ m/s}^2$, described in [3].

5.3 Scenario overview

It is of great importance to consider real life scenarios. Therefore packet loss and system delays are introduced as model disturbances.

• Packet loss - The packet loss is defined by how probable it is for a measured signal to be lost at some time instance, $k$ and for a particular vehicle, $i$. The probability that at a certain time instance a packet will be lost is:

$$P_k(k \in t_{pl}) = \sqrt{p}$$

and at a given time instance, the probability for a vehicle to lose a packet is

$$P_i(i \in Vehicles_{pl}(k)) = \sqrt{p},$$

Where $t_{pl}$ consists of $100\sqrt{p} \%$ randomly chosen instances of the set $t \in [0 : timestep : T]$ and $Vehicles_{pl}(k)$ consists of $100\sqrt{p} \%$ randomly chosen vehicles at a particular time instance, $k$. This provides the probability, $p$, that a particular vehicle at a particular time instance loosing the measurement of the vehicle in front:

$$P_{k,i}(k \in t_{pl} \cap i \in Vehicles_{pl}) = P_k(k \in t_{pl})P_i(i \in Vehicles_{pl}(k)) = \sqrt{p}\sqrt{p} = p$$

If a packet is lost, the vehicle will assume that the distance to, as well as the velocity of the vehicle in front is the same as it was the last time a packet was received.

• Delay - The delay is defined by the time latency from the time instance a signal is measured and the new torque is calculated to the time instance when the vehicle actually reacts. Unlike the packet loss, a delay will take place for every vehicle at every time instance.
5.4. EXTREME MASS SCENARIOS

The simulations together with the robustness analysis were conducted in two stages. Since a heavy duty vehicle has a mass range of 40–60 tons, the first stage will present how good the regulator can be tuned for three different extreme mass scenarios within this interval. System uncertainty analysis is addressed in the second stage where mass disturbance and other model uncertainties are discussed. A tough maneuver, which is closer described in Section 5.3.1, has been used throughout the entire chapter.

5.3.1 Maneuver overview: Hard brake, hold, regain

A platoon of 7 trucks traveling in 80 km/h when the leading vehicle hits the brake, e.g. to avoid a collision with a slow vehicle that suddenly emerges ahead of it, which results in a very rough deceleration of 6 m/s$^2$. The leading vehicle is forced to keep a new fixed velocity, where after the speed obstacle disappears and the leading vehicle is able to regain to its reference velocity of 80 km/h as fast as possible. The simulations will be conducted for three different mass scenarios, with all heavy weight trucks, with all light weight trucks and with mixed masses. At each simulation the time gap is set to, $\tau = 1$ s and the linearization point is chosen to be $v_{ref} = 80$ km/h.

5.4 Extreme mass scenarios

5.4.1 Slow dynamics

Masses of all the 7 vehicles are set as 60 ton which results in slow dynamics. Table 5.1 presents the result of the simulation. As can be seen the maximum torques required are far below the maximum limit while the minimum torques required are far above the minimum limit for all the vehicles. None of the vehicles cross the initial reference velocity of 80 km/h, however all the vehicles trespass the time gap distance but since these overshoots are only a few centimeters they can be neglected in the reality.

Figure 5.1 depicts how the vehicles behave during this scenario in terms of velocity. The first vehicle performs a deceleration of 6 m/s$^2$ with an artificial trajectory for a period of 1.85 seconds. This rapid brake manages to take the vehicle down to a new reference velocity of 41.2 km/h. This new reference velocity is to be held for a time of 50 seconds. One should keep in mind that the deceleration of the first vehicle is not done with regards to its dynamics, it is only set in a ramp motion. On the other hand the rest of the vehicles in the platoon are keeping their dynamics under the entire simulation and provides the valuable information over their behavior. Later on, the first vehicle is able to regain to its initial reference velocity of 80 km/h with a true trajectory, i.e. with the true dynamics.

By looking closer at the stabilization of the vehicles velocities around the new refer-
ence velocity it becomes clear that they have a strict inter-vehicle reaction hierarchy. The second vehicle reacts fastest when the first vehicle changes its velocity, meanwhile the last vehicle is the slowest to react throughout the entire simulation. On the other hand this is not that peculiar since the informational flow of the first vehicle’s change of state has to propagate throughout the entire platoon.

Another effect that can be revealed in Figure 5.1, is that a vehicle in the formation reaches a lower speed than its neighbor in front during the stabilization phase. At the same time one can observe from the inter-vehicle trajectories in Figure 5.2, that they stabilizes around the same distance meanwhile it is observed from Table 5.1 that the time gap overshoot, (5.1), decreases with the index of the vehicle. The effect that the vehicles with larger indexes are decreasing their velocities more rapidly is due to the dependence of the time gap in the cost matrix, $Q$, described in Section 4.2.2, hence a tendency to sustain the fixed time gap. One can also observe that due to the heavy masses of all the vehicles it takes about 200 seconds for the platoon to regain to its initial reference velocity of 80 km/h. That is done with no velocity overshoot and with an average acceleration of $\frac{\Delta v}{\Delta t} \approx 0.05$ m/s$^2$.

In turn, Figure 5.2 shows how the intermediate distances are affected within a platoon of heavy vehicles. Figure 5.2 reveals a good performance where the distances decreases when the speed of the vehicle ahead decreases while in contrast the distances increases when the velocity of the vehicle ahead increases. During the new reference velocity of 41.2 km/h the intermediate distances are slightly above the corresponding time gap: $\frac{41.2}{\Delta t} = 11.44$ m.

Another interesting result of the simulation is that there is no disturbance attenuation throughout the platoon, this will be further discussed in Section 5.4.4.
5.4. EXTREME MASS SCENARIOS

<table>
<thead>
<tr>
<th></th>
<th>Vehicle 1</th>
<th>Vehicle 2</th>
<th>Vehicle 3</th>
<th>Vehicle 4</th>
<th>unit</th>
</tr>
</thead>
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<td>60000</td>
<td>60000</td>
<td>60000</td>
<td>kg</td>
</tr>
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<td>879.6032</td>
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<td>Nm</td>
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<td>-38550</td>
<td>-32935</td>
<td>Nm</td>
</tr>
<tr>
<td>Velocity overshoot</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>km/h</td>
</tr>
<tr>
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<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>Time gap overshoot:</td>
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<td>-0.1160</td>
<td>-0.0824</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>$\alpha_1^q$ :</td>
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<td>1.0084</td>
<td>1.0084</td>
<td></td>
<td></td>
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<tr>
<td></td>
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<td>Vehicle 7</td>
<td></td>
<td>unit</td>
</tr>
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<td>60000</td>
<td>60000</td>
<td></td>
<td>kg</td>
</tr>
<tr>
<td>Max. torque required:</td>
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<td>872.4792</td>
<td>869.3770</td>
<td></td>
<td>Nm</td>
</tr>
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<td>Min. torque required:</td>
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<td>-27220</td>
<td>-25444</td>
<td></td>
<td>Nm</td>
</tr>
<tr>
<td>Velocity overshoot</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td></td>
<td>km/h</td>
</tr>
<tr>
<td>$\alpha_1^i$ :</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time gap overshoot:</td>
<td>-0.0622</td>
<td>-0.0482</td>
<td>-0.0373</td>
<td></td>
<td>m</td>
</tr>
<tr>
<td>$\alpha_1^q$ :</td>
<td>1.0084</td>
<td>1.0084</td>
<td>1.0084</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1. Simulation result, slow dynamics. *Note: first vehicle is an artificial trajectory.

Figure 5.1. Velocities of the members of a platoon with slow dynamics.
5.4.2 Fast dynamics

In this simulation all the vehicles in the platoon have a mass of 40 tons which implies that the dynamics of the platoon reacts fast. Table 5.2 shows the collected results of this simulation. Once again the maximum torques required are far below the maximum limit while the minimum torques required are far above the minimum limit for all the vehicles. Neither this time does none of the vehicles cross the reference velocity of 80 km/h and the time gap overshoots are still only a few centimeters. It is obvious that a truck with lower weight has faster dynamics than one with a higher weight provided that the mass is the only parameter that differs. Figure 5.3 supports this statement and shows that the velocities of the vehicles in the platoon regain to the reference velocity quicker than in the case of the previous simulation. Figure 5.4 shows the intermediate distances for this mass setup. After the stabilizing phase, at around 12 seconds, the new achieved reference distances are on the safe side of the corresponding time gap distance in the same manner as in the previous simulation. Figure 5.4 as well as Figure 5.2 shows how the vehicles closely follow each other on the same distances, this is expected when the vehicles are equally heavy and utilizing the same regulator.
5.4. EXTREME MASS SCENARIOS

<table>
<thead>
<tr>
<th>Vehicle 1</th>
<th>Vehicle 2</th>
<th>Vehicle 3</th>
<th>Vehicle 4</th>
<th>unit</th>
</tr>
</thead>
<tbody>
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<td>Mass:</td>
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<td>40000</td>
<td>40000</td>
<td>kg</td>
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<tr>
<td>Max. torque required:</td>
<td>854.8634</td>
<td>818.5287</td>
<td>904.1688</td>
<td>899.6078</td>
</tr>
<tr>
<td>Min. torque required:</td>
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<td>-25972</td>
<td>-22208</td>
<td>Nm</td>
</tr>
<tr>
<td>Velocity overshoot</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>km/h</td>
</tr>
<tr>
<td>$\alpha_v^i$:</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>Time gap overshoot:</td>
<td>-0.1826</td>
<td>-0.0927</td>
<td>-0.0593</td>
<td>m</td>
</tr>
<tr>
<td>$\alpha_{d}^i$:</td>
<td>1.0051</td>
<td>1.0060</td>
<td>1.0060</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Vehicle 5</th>
<th>Vehicle 6</th>
<th>Vehicle 7</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
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<td>40000</td>
<td>kg</td>
</tr>
<tr>
<td>Max. torque required:</td>
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<td>889.8014</td>
<td>884.9697</td>
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<td>Min. torque required:</td>
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<tr>
<td>Velocity overshoot</td>
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<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_v^i$:</td>
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<td>1.0000</td>
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<tr>
<td>Time gap overshoot:</td>
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<td>-0.0201</td>
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<tr>
<td>$\alpha_{d}^i$:</td>
<td>1.0060</td>
<td>1.0060</td>
<td>1.0060</td>
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</table>

Table 5.2. Simulation result, fast dynamics. *Note: first vehicle is an artificial trajectory.

![Figure 5.3](image_url)  
**Figure 5.3.** Velocities of the members of a platoon with fast dynamics.
CHAPTER 5. SIMULATIONS

5.4.3 Mixed dynamics

During this simulation, a platoon where the first six vehicles weigh 40 ton and a heavy vehicle of 60 ton is placed at the end is investigated. Table 5.3 shows the results. In accordance with the previous simulations all the vehicles manage to keep their torque within the limits. However this time the last vehicle is forced to increase its input in order to follow up the light weight vehicles in front, hence a larger maximum torque is required. Since the vehicles only measure what is front of it, Section 2.2, the first six columns shows the same results as in Table 5.2. Figure 5.5 shows the velocities within the platoon with mixed masses. Due to the higher input torque the seventh vehicle is able to follow up the preceding vehicles and performs as good as the seventh vehicle in the platoon with fast dynamics.

Figure 5.6 shows that even though the last vehicle is much heavier than the vehicle in front it manage to keep up with the rest and keep its desired distance, this is of course correlated with the vehicle’s good ability of keeping its desired velocity. Another phenomena that occurs in this and the previous cases is that the time gap overshoot decreases with the index of the vehicle. That is, the time gap overshoot of vehicle \( i \) is less than the time gap overshoot of vehicle \( i - 1 \).
5.4. EXTREME MASS SCENARIOS

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Mass (kg)</th>
<th>Max. torque required (Nm)</th>
<th>Min. torque required (Nm)</th>
<th>Velocity overshoot (km/h)</th>
<th>α^v:</th>
<th>Time gap overshoot (m)</th>
<th>α^d:</th>
</tr>
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<tr>
<td>1</td>
<td>40000</td>
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<td>-</td>
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</tr>
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<td>4</td>
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<td>1.0000</td>
<td>-0.0034</td>
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</tr>
</tbody>
</table>

Table 5.3. Simulation result, mixed dynamics. *Note: first vehicle is an artificial trajectory.

![Figure 5.5](image.png)

Figure 5.5. Velocities of the members of a platoon with mixed dynamics.
Figure 5.6. Inter vehicle distances between the members of a platoon with mixed dynamics.

5.4.4 Summary

In these three cases of different mass distribution one can see that the regulator manages to decelerate with $6 \text{ m/s}^2$. Hold a new reference velocity and regain to the first one with an acceleration that is greater than $0.05 \text{ m/s}^2$ without overpassing the maximum limit of the input torque.

In the case of all heavy trucks, the largest engine torque is observed for vehicle number 3, 879.6 Nm, which is 38.6\% of its maximum value. The largest brake torque is in turn 52496 Nm and was obtained for the second vehicle, which is 43.7\% of the maximum braking torque (vehicle with two axis).

For the light vehicle case, the largest engine torque is almost the same compared to the heavy case. It reaches only 39.7\% (vehicle 3) of its maximum capacity. On the
5.4. EXTREME MASS SCENARIOS

Other hand, the braking torque reaches at most 29.4% (vehicle 2), which is 14.3% less than for the heavy scenario. It is expected that heavy vehicles should stress the inputs harder but one can observe that as long as the change in acceleration is small the regulator is not sensitive to different masses, provided the mass is homogeneously distributed.

The case of mixed masses is supposed to represent the worst scenario of mass distribution. The six first vehicles are light and therefore move with fast dynamics, the last one is heavier and therefore harder to maneuver. To be able to keep track with the other vehicles in front, the last truck has to put in 64.9% of the maximum torque in the acceleration. The second truck has to brake hardest even in this case, with 29.4% of the maximal braking torque.

Table 5.1 to Table 5.3 shows that the braking torque decreases with the index of the vehicle, presumed equal weights. This is a desired result with regard to the fuel consumption. A related result is that in all these three cases, the error, in the sense of the time gap overshoot, is decreasing throughout the platoon. Figures 5.7, 5.8 and 5.9 shows a simulation with 50 vehicles and where all the masses equal 50 ton each, together with the corresponding time gap overshoots. These figures emphasize the decreasing time gap overshoot and suggests that the proposed regulator may be string stable when the number of vehicles goes to infinity.

![Figure 5.7. Velocities when the number of vehicles= 50 and $m_i = 50$ ton.](image_url)
Figure 5.8. Inter vehicle distances when the number of vehicles= 50 and $m_i = 50$ ton.

Figure 5.9. Time gap overshoot when the number of vehicles= 50 and $m_i = 50$ ton.
5.5 Delay and packet loss disturbances

To illustrate how much disturbances caused of delays and packet loss, Section 5.3, affect the robustness, the trajectories of the last vehicle in the mixed masses scenario has been further analyzed. This is the worst case trajectory due to the demanding mass distribution in addition to the fact that disturbance caused of delays and packet loss tends to propagate throughout the platoon. The simulations are carried out with a delay of 100 ms, a packet loss disturbance with a 10% probability that a packet is lost as well as carried out without disturbances. We assumed that a proper communication setup should not have a greater packet loss then one in ten packets while using a sample time of 100 ms.

Figure 5.10 shows how disturbances are affecting the velocity of the seventh vehicle while Figure 5.11 shows how the disturbances affecting its distance to the vehicle in front. By comparing the three curves it can be seen that delays causing the most oscillating trajectory. It deviates greatest from the trajectories with no disturbance with at most 2 km/h in velocity and 1 m in distance. By studying the impact of packet loss, one can notice that these trajectories are almost identical to the case without disturbances.

![Graph showing velocity of vehicles over time with three trajectories: 10% packet loss, 100ms delay, and no disturbances.](image)

**Figure 5.10.** Disturbances caused by delays for the 7th vehicle in the mixed mass scenario, zoomed in on the part of the plot where the effect are most obvious.
5.6 State robustness

By studying Table 5.1 to Table 5.3 the correlation between the state robustness, \( \alpha_i^d \), (4.22), and the time gap overshoot is clear: When \( \alpha_i^d \) is greater than the time gap, \( \tau \), a time gap overshoot is more likely to occur. Since \( \alpha_i^v \), (4.23), is constant one there is a perfect correlation between the velocity of the \( i \):th and the \( i - 1 \):th vehicle. The state robustness is a good measurement of the possible time gap overshoot, hence a good tuning parameter, but it is also fruitful to further investigate a larger spectra of frequencies. This can be done by studying the Bode plots, Figures 5.12, 5.13 and 5.14, of the above mass scenarios. By doing this it is possible to draw the following conclusions:

- The period when the amplitude of \( G(j\omega) \) exceeds \(-3\) dB is around \( T\) dB ≈ 4 s.
- The cut off period of \( G(j\omega) \) is: \( T_c \approx 20 \) s.
- The corresponding phase margin of \( G(j\omega) \) is: \( \alpha_m \approx 170 \) degrees.

These results further emphasize the good performance shown in Figure 5.1 to Figure 5.11.
5.6. STATE ROBUSTNESS

Figure 5.12. $m_{i+1} = 60\text{ton}, m_i = 60\text{ton}, G(j\omega)$.

Figure 5.13. $m_{i-1} = 40\text{ton}, m_i = 40\text{ton}, G(j\omega)$. 
5.7 Model uncertainty robustness

In contrary to the previous test this test is created to observe how sensitive the system is to fluctuation in mass or external forces. Applied external forces, $F$, can be simulated through fluctuation in the mass:

$$F \pm \epsilon_F = ma \pm m \epsilon_a = (m \pm m_e)a,$$

(5.2)
since one always can express any bounded force $\epsilon_F$ as the product of the given acceleration $a$ and some mass $m_e$.

5.7.1 Nominal regulator versus tuned regulator

In order to demonstrate how sensitive a regulator is against model uncertainties, described in Section 3.3.1, a nominal regulator, $K^{\text{nom}}$, has been tuned for a convoy established of vehicles that weigh 50 tons and have been applied to neighboring vehicles for which mass is assumed to fluctuate. Earlier in Section 4.2.1, it has been shown that the proposed regulator is only dependent on the masses of the $i$ : th and the $i-1$ : th vehicle:

$$K_i^{\text{tuned}} = K_i^{\text{tuned}}(m_{i-1}, m_i, Q_i, R(m_{i-1}, m_i))$$

While tuning the regulator for a mass span between $30-70$ tons (81 different mass combinations i.e. $m_{i-1} = \{30, 35, ..., 70\}$ and $m_i = \{30, 35, ..., 70\}$), it was observed...
5.7. MODEL UNCERTAINTY ROBUSTNESS

that $Q_i = Q = \text{constant}$, (4.15), and $R_i(m_{i-1}, m_i) \approx R(m_i)$, (4.16). This implies:

$$K_{i}^{\text{tuned}} \approx K_i(m_i, m_{i-1}, Q, R(m_i))$$

in this case

$$K_i^{\text{nom}} = K_i(50, 50, Q, R(50))$$

Four different measures have been produced to define a framework for model uncertainty sensitivity:

- Largest velocity deviations:
  $$\Delta v_{\text{max}}(m_{i-1}, m_i) = \max_t (v_i(t)|_{K_i^{\text{nom}}} - v_i(t)|_{K_i^{\text{tuned}}})$$
  $$\Delta v_{\text{min}} = \min_t (v_i(t)|_{K_i^{\text{nom}}} - v_i(t)|_{K_i^{\text{tuned}}})$$

- Largest intermediate distance deviations:
  $$\Delta d_{\text{max}}(m_{i-1}, m_i) = \max_t (d(t)|_{K_i^{\text{nom}}} - d(t)|_{K_i^{\text{tuned}}})$$
  $$\Delta d_{\text{min}} = \min_t (d(t)|_{K_i^{\text{nom}}} - d(t)|_{K_i^{\text{tuned}}})$$

Robustness of velocity

The deviation of the velocity when applying the nominal regulator with regard to fluctuation in mass deviation is presented in Figure 5.15, 5.16 and 5.17. Figure 5.15 reveals that the maximal undershoot, $\min(\Delta v_{\text{min}}(m_{i-1}, m_i))$, occurs when the mass of the $i$:th vehicle and the mass of the $i-1$:th vehicle equals 30 ton. In turn, the maximal overshoot, $\max(\Delta v_{\text{max}}(m_{i-1}, m_i))$, occurs when the mass of the $i$:th vehicle and the mass of the $i-1$:th vehicle equals 70 ton.

Figure 5.15 doesn’t show where these overshoots take place, therefore Figure 5.16 and Figure 5.17 presents the time line for the relevant part of these extreme cases. Figure 5.16 reveals that the greatest difference occurs during the braking part where the nominal regulator acts quicker than the tuned one. Utilizing the nominal regulator when the vehicle’s actual mass is lower than what it is tuned for implies that the regulator applies more force than necessary when braking the vehicle, which results in this effect. On the contrary, when the vehicle utilizing the nominal regulator, is heavier than what it is tuned for, this effect will be opposite, shown in Figure 5.17.

Another effect visualized by Figure 5.15 is that there will be just a small deviation in velocity as long as:

$$m_{i-1} + m_i = 2m_{\text{nom}} = 2 \times 50000 \text{ kg} = 100 \text{ ton}$$
In other words, as long as the masses are placed on the horizontal diagonal of Figure 5.15 stretching from $m_i = 30$ ton, $m_{i-1} = 70$ ton to $m_i = 70$ ton, $m_{i-1} = 30$ ton. The sloping appearance of the surfaces in Figure 5.15 is due to the fact that when the regulator is tuned for a weight lower than the vehicle actually has, the regulator applies less torque than what is required during the braking phase, Figure 5.17. This means that the vehicle reacts slower than what it should have done utilizing a tuned regulator. In turn, this implies that the most positive velocity deviation is larger the heavier it is while the most negative velocity deviation is smaller, the lighter the vehicle is. Meanwhile when the vehicle in front weighs less than what its regulator is tuned for, it will react faster than necessary which means that the vehicle behind is forced to brake quicker. This implies that the effect is almost canceled out when $m_i = 30$ ton and $m_{i-1} = 70$ ton or $m_i = 70$ ton and $m_{i-1} = 30$ ton. Even though there are obvious differences in performance between the tuned and the nominal regulator, Figure 5.15 shows that this difference is at most 3.5 km/h during this particular scenario. One can imagine that this effect, for instance occurs when both the $i$th and the $(i-1)$th vehicle rapidly brakes while traveling on a steep slope.

**Figure 5.15.** 3D plot showing the deviation of the velocity, with respect to $m_i$ and $m_{i-1}$. The upper red upper surface visualizes $\Delta v_{\text{max}}(m)$ while the blue lower surface visualizes $\Delta v_{\text{min}}(m)$. 

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Figure 5.16. Trajectories of the velocity when $m_i = m_{i-1} = 30$ ton.

Figure 5.17. Trajectories of the velocity when $m_i = m_{i-1} = 70$ ton.
Robustness of intermediate distance

The deviation of the intermediate distance by applying the nominal regulator with regard to fluctuation in mass is represented in Figure 5.18, 5.19 and 5.20. In Figure 5.18 one can clearly observe that the surfaces is fairly symmetric, which implies that uncertainty in distance increases when $|m_{i-1} - m_{nom}|$ or $|m_i - m_{nom}|$ increases. The largest deviation of $\Delta d_{\text{min}}$ is under 1.5 m and is obtained when $m_i$ increases towards 70 ton, while the largest deviation of $\Delta d_{\text{max}}$ is slightly above 1 m and is obtained when $m_i$ and $m_{i-1}$ equals 70 ton. Since these deviations are quite small it is only necessary to study, $\Delta d_{\text{min}}$ which in fact affects the safety of the driver. Figure 5.19 further visualize that the maximum undershoot occurs when the mass of the $i-1$th vehicle equals 70 ton as well. In turn, Figure 5.20 shows that this undershoot occurs in the stabilizing phase, just after the braking of the first vehicle is completed. Even though Figure 5.20 reveals an oscillating behavior, this might in practice be neglected due to inertia of the system that is not accounted for in this model.

![Model uncertainty robustness](image)

**Figure 5.18.** 3D plot showing the deviation of the intermediate distance, with respect to $m_i$ and $m_{i-1}$. The red upper surface visualizes $\Delta d_{\text{max}}(m)$ while the blue lower surface visualizes $\Delta d_{\text{min}}(m)$.
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Figure 5.19. 2D plots showing the deviation of the intermediate distance, with respect to $m_{i-1}$. The red upper surface visualizes $\Delta d_{\text{max}}(m)$ while the blue lower surface visualizes $\Delta d_{\text{min}}(m)$.

Figure 5.20. Trajectories of the inter vehicle distance when $m_i = m_{i-1} = 70$ ton.
CHAPTER 5. SIMULATIONS

Time gap overshoot robustness

Figure 5.21 represents the minimum time gap overshoot when applying the nominal regulator contra the tuned regulator. In accordance to Figure 5.18, Figure 5.21 shows that the time gap overshoot is the largest when \( m_i = 70 \text{ ton} \) and \( m_{i-1} = 70 \text{ ton} \) and when the vehicles utilizing the nominal regulator. When a platoon utilizing nominal regulators, the maximal time gap overshoot is about 0.045 s. This is the most a vehicle within the platoon will trespass the time gap distance, (2.1), even though it is exposed to heavy forces.

When the vehicle is tuned for a weight lower than the actual weight, the braking force applied by the regulator is too low which implies that the vehicle brakes slower than what it should have. This implies that the vehicle is more likely to cross the time gap distance the heavier it is, hence the appearance of Figure 5.21.

![Model uncertainty robustness](image)

**Figure 5.21.** 3D plot of the time gap overshoot robustness, with respect to \( m_i \) and \( m_{i-1} \). The blue surface represents the tuned regulator while the green surface represents the nominal regulator.
Chapter 6

Discussion

This thesis discusses the desired behavior of a vehicle platoon and enlightens the reader on how this behavior can be transferred to a cost function minimization problem. The LQR-method that is proposed is simulated for a platoon of seven vehicles and exposed to demanding traffic scenarios where it demonstrates very fast responses together with a smooth stabilization of all the seven vehicles with almost no overshoots. No disturbance attenuation throughout the platoon is observed during the simulations. Model uncertainty robustness together with state robustness is particularly discussed with a solid outcome. These results indicate that a platoon of vehicles can prosperously be controlled by the proposed LQR-method.

Observations on how one should tune the regulator are made and commented throughout the thesis. One should take notice that the regulator works best if the weight matrices are specially chosen with respect to the time gap. However, this fact does not result in any practical problems, since one regulator can be computed for each time gap.

The objective of this thesis is to find a regulator that minimizes the energy consumption meanwhile it takes the slipstream effect into concern, is robust and is able to withstand realistic strenuous scenarios. The proposed regulator takes the air drag reduction into account while minimizing the energy consumption in the sense that a LQR approach for the estimated subsystems are employed. It is also shown that the regulator is robust against model uncertainties, in terms of the mass of the vehicle, during realistic strenuous scenarios. With regard to this, the objective of this thesis is fulfilled.

The regulator seems to have smoother, more robust trajectories then the PID regulator used in [8], this may indicate that it is less active and therefore uses less energy, which in turn means better fuel reduction capabilities. However, no effort has been made to show how optimal the proposed regulator is with regard to these particular structural constraints. Only by implementing the proposed regulator in
a field test, like [4], one can certainly draw the conclusion about its fuel reducing properties.

The vehicle model that has been studied has only two states, acceleration and intermediate distance. This makes it hard to introduce inertia and other latencies to the acceleration of the vehicles. The possibility of expanding the method to have acceleration as a third state can be worth investigating in the future. Introducing the centripetal force which is clearly present every time a vehicle is facing a curve may also be a good feature.
Bibliography


