Simulation of event-based control

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Abstract

The aim of this thesis is twofold. The first and major one is to develop an object-oriented Kaplan turbine model to support control design for hydropower systems. The model is developed in the Modelica language and implemented in the Dymola simulation environment. Using the equation-based modelling formalism of Modelica, the model exploits water flow and pressure balance equations to capture the relationship between the hydraulic and mechanical energy. A particular feature of the model is a detailed description of how the water flow and turbine torque are affected by the wicket gate and turbine runner blade angles. The second aim of the thesis is to demonstrate how the Kaplan turbine model can be used to design hydropower control systems. To this end, a relatively complete hydropower system model is developed using the model components and several dam control strategies are evaluated. This study clearly demonstrates the utility of the model in the design of hydropower control systems.
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Chapter 1

Introduction

1.1 Hydropower plant

Hydropower represented in 1999 19% of the world electricity production [1] and the development of hydroelectric power will be increased in the near future since there is an increased interest in renewable energy sources. The basic process of the hydropower plant is to convert the hydraulic energy to mechanical energy by using the water turbine and then transfer the mechanical energy to electrical energy by using a generator. Figure 1.1 shows a brief schematic structure of the hydropower plant. Hydropower plants rely on a dam that holds back water, creating a large reservoir. Gravity causes the water to fall through the penstock. At the end of the penstock there is a turbine propeller, which is turned by the moving water. The turns of the propeller will drive the generator to produce the alternating current, AC. The transformer inside the powerhouse takes the AC and converts it to the higher-voltage current. Power lines are connected to the generator that carry electricity to the customers. Outflow carries the used water through the pipe downstream.

Figure 1.1: Representation of a hydropower plant
1.2 Water turbine

In a hydropower plant water turbines are one important component. Water turbines were developed in the nineteenth century and now they are mostly used for electric power generators. Water turbines are turbines which utilize the water power. In a hydropower plant the potential or kinetic energy of the water is converted by means of the water turbine into mechanical energy, which turbine assistance of the flowing water shifted in turn. The turn of the turbine shaft can be used as mechanical achievement for the drive of a generator, which converts the rotational energy into electric current. Therefore the primary function of water turbines is to drive an electric generator. A Hydropower plant uses the energy of water falling through a Head that may vary between a few meters to several thousands meters. In order to manage this wide range of Head there are many different kinds of water turbines can be used and each one of which differs in its working components, according to the size of Head. In fluid dynamics, Head is the difference in elevation between two points in a column of fluid, and resulting pressure of the fluid at the lower point. Head is normally expressed by height, meter. The further description of Head will be presented in section 2.3.

There are two types of water turbines; impulse turbines and reaction turbines.

Impulse turbines
Impulse turbines change the velocity of a water jet. A nozzle transforms water under a high height into a powerful jet. The momentum of this jet is destroyed by striking the runner, which absorbs the resulting force. The driving energy of impulse turbines is supplied by the water only in kinetic form. Pelton turbines are this type of turbines. A Pelton turbine is used at very high heads, 400 meters or more.

Reaction turbines
Reaction turbines are acted on by water, when the water flows through the turbine’s runner blades which will transfer the hydraulic energy to mechanical energy. Turbines must be encased to contain the water pressure, or they must be fully submerged in the water flow. The driving energy of reaction turbines is supplied by the water partly in kinetic and partly in pressure form which is different than impulse turbines. Francis turbines and Kaplan turbines belong to this type of turbines. Francis turbines can cover a wide head range, from 20 meters to 700 meters. For Kaplan turbines they can cover much lower head compare to Francis turbines. Usually the head ranges for Kaplan turbines are between 6 meters to 60 meters.

Different turbine types are used to make the energy transformation as efficient as possible at different operating conditions, heads and water flow. For example the choice between reaction turbine type, Kaplan or Francis, is mainly depending on head but also partly on water flow. Kaplan turbines were an evolution of the Francis turbines. To compare with Francis turbines, Kaplan turbines can be used in low head applications that is not possible with Francis turbines.

Kaplan turbines play a major role in the hydroelectric production due to their extended range of application. The invention of the Kaplan turbine provides a possibility
of efficient power production up to 200 MW and the advantage of the Kaplan turbine is its usage at lower water fall height. Large Kaplan turbines are individually designed for each site to operate at the highest possible efficiency, typically over 90%. Figure 1.2 is the efficiency validation of different turbine types in Kroksströmmen power plant in Sweden which is provided by Kvaerner Turbin AB [17]. There are four different types of turbines used in this plant, Kaplan, Semikaplan, Francis and propeller and but here we will look at Francis and Kaplan only. In this plant there are two Francis turbines and one Kaplan turbine, which operate at the same head (58m). The flow of the two Francis turbines at full load is about half the maximum flow of the Kaplan turbine. The Francis-curve in the diagram represents both Francis units operating in parallel at the same flow. The diagram therefore gives a direct comparison between the efficiency for the Francis and the Kaplan turbine in general at the same head and flow.

Figure 1.2: Efficiency comparison of turbine types in Kroksströmmen power plant, Sweden

Table 1.1 shows some applications by using Kaplan turbines in different hydropower plants. One obvious observation of the data is that Kaplan turbine is possible to generate significant amounts of power with relatively low head, e.g. Lilla Edet, with a head of approximately 7 meters producing a full 39MW of power.

<table>
<thead>
<tr>
<th>Power plant</th>
<th>Country</th>
<th>Units</th>
<th>Height</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balbina</td>
<td>Brasil</td>
<td>5</td>
<td>24 m</td>
<td>58 MW</td>
</tr>
<tr>
<td>Kedung-Ombo</td>
<td>Indonesia</td>
<td>1</td>
<td>51 m</td>
<td>23.5MW</td>
</tr>
<tr>
<td>Lilla Edet</td>
<td>Sweden</td>
<td>4</td>
<td>6.5m</td>
<td>39MW</td>
</tr>
<tr>
<td>Otori</td>
<td>Japan</td>
<td>1</td>
<td>51 m</td>
<td>87,000kw</td>
</tr>
<tr>
<td>Palmar</td>
<td>Paraguay</td>
<td>3</td>
<td>32 m</td>
<td>113MW</td>
</tr>
<tr>
<td>Porto Primavera</td>
<td>Brasil</td>
<td>18</td>
<td>22 m</td>
<td>105MW</td>
</tr>
<tr>
<td>Zvornik</td>
<td>Serbia</td>
<td>4</td>
<td>19.3m</td>
<td>24MW</td>
</tr>
</tbody>
</table>

Table 1.1: Some applications of hydropower plants by using Kaplan turbines in different countries
1.3 Previous related studies

This section presents a review of previous studies related to this thesis. There exist many dissertations of Kaplan turbine models. One of them considers the effects of inlet boundary conditions of a Kaplan turbine. The task of the work focuses on the water flow through the penstock and wicket gate and intends to get initial boundary conditions for the simulation of the Kaplan turbine [10]. One thesis by studying how the water flow and the torque are effected by the deviations from the optimal combination curve of the angles of the wicket gate and runner blades in order to develop a new Kaplan turbine model [11]. Another one is modelling a hydropower plant for the purpose of the improvement of analysis of the hydropower plants [14].

To look at those previous related studies there is no such an object oriented model in a computational test bench for performing the virtual simulation of a hydropower plant. An object oriented model can be modified facilely and with its high extensibility additional computational components can easily be added in a test bench.

1.4 Thesis aim

The aim of this thesis is to create an object oriented dynamic process model [9] as a useful tool in order to control hydropower systems. With aid of this object oriented model an evaluation of different control strategies is facilitated. The functions and advantages of such an object-oriented model structure are supporting the understanding of the model, allowing easy navigation through the model, guaranteeing all model components are used consistently and supporting simple application of changes.

This thesis project consists of two objectives. The first and major one is modelling and simulation of an object-oriented Kaplan turbine model in a dynamic process of a hydropower plant in Dymola [16], by taking into account the nonlinear behaviors of the dynamic responses, leading to more realistic dynamic models of the hydraulic Kaplan turbine system. The other objective is a demonstration of an dam level controller design in the Dymola test bench which shows the utility of this object oriented model. Therefore this model can be easily represented to the automation industries for controller design. Using the test bench, a very versatile environment is achieved that can be modified for different needs and requirements. Novel in this thesis work is to create an object-oriented Kaplan turbine model structure for a hydropower plant.

This report is divided into a theoretical part where the theory behind the hydropower and Kaplan turbine is briefly explained, followed up by a section where the Dymola model of the Kaplan turbine is discussed and then a section which shows a demonstration of designing a feedback control based on this dynamic process model. Finally, the results are discussed in more detail.
Chapter 2

Kaplan turbines- general theory

2.1 History

The first known attempt to use an adjustable-blade turbine is evidenced by a United States Patent issued to O.W. Ludlow in 1867 [8]. There were no water-control gates on the turbine, and stationary guide vanes were used. Dr. Viktor Kaplan, of the Technische Hochschule in Vienna, was the first to apply the idea by using the advantage of adjusting the blades and the gates simultaneously. Patent applications were filed in Europe in 1913 and in the United States in 1914 [8]. The first large Kaplan turbine is installed at Lilla Edet in the south of Sweden and the efficiency is up to 92.3%. This caused the attention in the commercial market and now Kaplan turbines are widely used in the world.

Kaplan turbines are used for low head sites and compared to other turbines which have the range of heads between 10 to 1300 meters, Kaplan turbines have the lowest range of heads which are between 6 to 60 meters. In addition, Kaplan turbines usually have vertical shafts because this makes best use of the available head and makes installation of a generator more economical.

2.2 The components of Kaplan turbines

A Kaplan turbine is an inward flow reaction turbine which has adjustable wicket gate and runner blades. The inlet is a scroll-shaped tube that wraps around the turbine's wicket gate. Water is directed tangentially, through the wicket gate, and spirals on to a propeller shaped runner, causing it to spin. The outlet is a specially shaped draft tube that helps decelerate the water. This reduction of water speed will reduce the pressure on the outlet side, and in so increase the difference in water pressure between inlet and outlet side. This pressure difference will create an increased water flow through the turbine, thus increasing the force on the turbine blades and therefore the turbine speed, indirectly the generated power.

A schematic figure of vertical section through a Kaplan turbine is shown Figure 2.1.
A Kaplan turbine consists of some basic components. The water flows from the scroll casing inlet through the wicket gate, the runner blades and the draft tube into the tail water basin. Besides these components there are also some associated components together with the Kaplan turbine like dam, penstock and generator. The brief descriptions of the basic and associated components of Kaplan turbines are as the following.

### 2.2.1 Dam

The purposes of dams include water supply, and creating a reservoir of water to supply industrial uses or generating hydroelectric power. Most hydropower plants rely on a dam that holds back water, creating a large reservoir. For Kaplan turbine hydropower plant the hydroelectric power comes from the potential energy of the water in the dam which drives the turbine and generator. That is why the water height level in the dam will determine the hydraulic effect.

### 2.2.2 Penstock

A penstock is a large enclosed pipe which is the connecting components between a dam and a turbine. The installation of a penstock is normally equipped with a gate system. There are many different materials of penstocks such as wood, concrete and steel. Usually the length of a penstock is varying. It could be from several hundred meters to several kilometers. In a dynamic process of a hydropower plant the water inertia is proportional to the length of the penstock which can increase the dynamic height loss. This could influence the hydraulic effect since the hydraulic effect depends on the water height.
2.2.3 Wicket gate

Wicket gates are the special constructions of Kaplan turbines. A unit of a wicked gate consists of many small gates which are divided by numbers of guide vanes. The guide vanes are manufactured of steel plate material. The vanes design is purposely to obtain optimal hydraulic flow conditions, and they are given a smooth surface. A wicked gate is warped around by a scroll-shaped tube inlet. The function of a wicked gate is guiding the flowing water from the inlet to the runner blades while the adjustable angle of the guide vanes can regulate the volume flow rate of the flowing water. Stay vanes stand outside the guide vanes and are fixed.

2.2.4 Runner

The runner in a Kaplan turbine is a very challenging part to design. The propeller shaped runner is mounted vertically with several blades. The length and number of blades can determine the turbine’s rotational torque which can indirectly influence the hydraulic effect. Usually runners consisting of 4 blades can be used up to heads of 25-30 meters while 6 blades could be used for heads 60 meter Figure .

2.2.5 Draft tube

The outlet of a Kaplan turbine is a specially shaped draft tube. The function of draft tubes is to decelerate the water at the outlet of a turbine. Since the power extracted from a turbine is a directly function of the drop in pressure across it, the reduction of water velocity will reduce the pressure on the outlet side which can increase the difference in water pressure between inlet and outlet side. Usually the draft tubes are conical shaped which are similar to an inverted ice cream cone.

2.2.6 Generator

A generator is coupled to the turbine by a long shaft. The generator consists of a large, spinning rotor and a stator. The outer ring of the rotor is made up of a series of copper wound iron cells or poles [6] which act as an electromagnet. The stator is comprised of a series of vertically oriented copper coils nestled in the slots of an iron core. The shaft transmits the rotation to the rotor. As the rotor spins its magnetic field induces a current in the stator’s windings and generates electricity. The function of a generator is to convert the kinetic mechanical energy to the electrical energy.

Some components are shown illustrations. Right side of Figure 2.2 is the schematic figure of a Kaplan turbine. Wicket gate is warped around by a scroll-shaped tube inlet. Wicket gate’s guide vanes stand inside the stay vanes which are adjustable while the stay vanes are fixed. Left side of Figure 2.2 is the picture of turbines inlet. Figure 2.3 is the realistic turbines runner which is manufactured by Kvaerner Turbine AB in Sweden [8].
2.3 Basic definition

When the flowing water is directed on to the turbine's runner blades, causing the runner to spin, work will be generated by the force through the axis of the turbine. In this case energy is transferred from the hydraulic energy to the mechanical energy and the kinetic torque out of the turbine would drive the electric generator to produce electrical power. The runner blades angle decides the momentum of the turbine shaft and the angle of the wicket gate optimizes the water flow through the runner blades which means that the adjustable angles of runner blades and wicket gate could allow an efficient operation for a wide range of flow conditions.

2.3.1 Head

In fluid dynamics, Head is the difference in elevation between two points in a column of fluid, and resulting pressure of the fluid at the lower point[3]. Head is normally expressed by height, for example meters. The definition of Head is illustrated by Figure 2.4.
In Figure 2.4, according to the definition of Head, Head = $h_1 - h_2$.
In the bucket, $p_1$ is the atmospheric pressure and $p_2 = p_1 + \rho g (h_1 - h_2)$. By replacing $h_1 - h_2 = \text{Head}$, $p_2$ can be expressed as $p_2 = p_1 + \rho g \text{Head}$. It shows that the lower level pressure at the level _B can be described by Head.

There are some different types of head which are normally used in fluid dynamics [3].

- **Dynamic head**
  Dynamic head is due to the water kinetic energy by a motion of a fluid. According to Bernoulli’s equation [3]
  \[
  v = \sqrt{2gh}
  \]  
  (2.1)
  Here $v$ is the velocity, $h$ is the vertical distance, $g$ is the gravity acceleration and the equation 2.1 will leads to the equation of velocity head is
  \[
  H_{\text{dynamic}} = \frac{v^2}{2g}
  \]  
  (2.2)

- **Static head**
  The hydraulic static head is due to the water potential energy and the water pressure describes as
  \[
  p = \rho gh
  \]  
  (2.3)
In the equation 2.3 $\rho$ is the density of the water. Therefore the static head could be formulated as

$$H_{\text{static}} = \frac{p}{\rho g} \quad (2.4)$$

### 2.3.2 Efficiency

Efficiency is a dimensionless number, with a value between zero to one, which is defined as energy output devised by the input energy. [4]

**Hydraulic efficiency**

The hydraulic effect is composed by [4]

$$P_{\text{hydro}} = \rho g H_{\text{net}} Q \quad (2.5)$$

Here $\rho$ is the density of water, $g$ is the gravity acceleration, $H_{\text{net}}$ is defined at the inlet of the turbine referred to the level of the tail water of turbines and $Q$ is the volume flow rate of water.

**Turbine efficiency**

For turbine efficiency it is defined as the obtained mechanical energy in the turbine shaft divided by the supplied energy from the flowing water through the turbine. For the mechanical turbine effect is $P_{\text{tur}} = M \times w$. $M$ is torque and $w$ is the angular velocity. Since we know that the hydraulic effect is as equation 2.5, $P_{\text{hydro}} = \rho g H_{\text{net}} Q$.

By the definition of the efficiency the turbine efficiency is

$$\eta = \frac{P_{\text{tur}}}{\rho g H_{\text{net}} Q} \quad (2.6)$$
Chapter 3

Object oriented Modelling of hydropower plant

An object oriented modelling can simplify to capture a large number of complex process configurations with a limited number of model blocks [9]. It provides a structure, computer-supported way of doing mathematical and equation-based modeling. Hydropower plant dynamic processes are well suited for object oriented modelling because of the extensive use of standard components, like dams, penstocks, turbines etc. Today’s modelling is normally based on simulation packages that use modern languages and established libraries of hydropower plant components. The models can help ensure in-depth knowledge of the hydropower plants behavior that is available directly from the computer. It can also be useful for the hydropower plants analysis, development and control implementation. This thesis uses Modelica as the object oriented modelling language to model the dynamic process in a hydropower plant. This dynamic process model is simulated in Dymola data environment [16]. Dymola is a simulation tool based on Modelica language. Modelica [16] is an object-oriented modelling language which provides a mathematical and equation-based modelling. The brief description of Modelica and Dymola are as below:

Modelica
Modelica is primarily a modelling language that allows specification of mathematical models of complex natural or man-made systems, e.g., for the purpose of computer simulation of dynamic systems where behavior evolves as a function of time. Modelica is also an object-oriented equation-based programming language, oriented toward computational applications with high complexity requiring high performance.

The main features of Modelica are [16]:

1. Modelica is primarily based on equations instead of assignment statements. In Modelica it is possible to write balance and other equations
in their natural forms as a system of differential-algebraic equations, DAE.

2. Modelica has multi-domain modelling capability, meaning that the model components corresponding to physical objects from several different domains such as, e.g., electrical, mechanical, thermodynamic, hydraulic, biological, and control applications can be described and connected.

3. Modelica is an object-oriented language with a general class concept [16] that unifies classes into a single language construct. This facilitates reuse of components and evolution of models.

**Dymola**

Dymola, Dynamic Modeling Laboratory, is a comprehensive industrial strength modelling and simulation environment for Modelica, with special emphasis on efficient real-time simulation. One of the main application areas is simulation in the automotive industry.

The structure of the Dymola tool contains the following main parts [16]:

1. A Modelica compiler including a symbolic optimizer to reduce the size of equation systems.
2. A simulation run-time system including a numeric solver for hybrid DAE equation systems.
3. A graphic user Interface including a connection editor.
4. A text editor for editing Modelica models.

In the following sections, it begins with a derivation dynamic process model, where an equation-based modelling is discussed, followed by a section, where an object oriented model is created in Dymola. In the end the simulation of this model is presented in Dymola.

### 3.1 The derivation model

In order to simplify a complex dynamic process of hydropower plants this object oriented modelling is divided into several components, dam, penstock, wicket gate and turbine. The method to approach these components is by computing the water flow and the pressure balance equations. The models of these components are described in the following sections, where it starts with the specified parameters and variables that are used for the equation-based modelling, followed by the formulation.

#### 3.1.1 Dam modelling

We look at a dam as an open tank see Figure 3.1. The parameters and variables that are utilized to this model are shown below.
Figure 3.1: A schematic figure of a dam

Parameters:

\( g \) : gravity acceleration [\( \frac{m}{s^2} \)]
\( \rho \) : the density of the fluid [\( \frac{kg}{m^3} \)]
\( \text{patm} \) : atmospheric pressure [\( Pa \)]

Variables:

\( m \) : the mass of the water \([kg]\)
\( h \) : the height of water in the dam \([m]\)
\( V \) : the volume of water in the dam \([m^3]\)
\( p1 \) : pressure \([Pa]\)
\( p_{out\_dam} \) : the pressure at the outlet of the dam \([Pa]\)
\( \text{min\_dam} \) : incoming mass flow rate \([\frac{kg}{s}]\)
\( \text{mout\_dam} \) : outgoing mass flow rate \([\frac{kg}{s}]\)
\( Q \) : volume flow rate \([\frac{m^3}{s}]\)

Equations:

\[ \text{patm} + p1 = p_{out\_dam} \]  \hspace{1cm} (3.1)

\[ \dot{m} = \text{min\_dam} - \text{mout\_dam} \]  \hspace{1cm} (3.2)

Equation 3.1 describes the pressure balance of a dam.
\( \text{patm} \) is the atmospheric pressure and \( p1 \) is the static pressure which depends on the height of the water in the dam. \( p1 \) is expressed as

\[ p1 = \rho gh \]  \hspace{1cm} (3.3)
\( p_{\text{out dam}} \) is the water pressure at the outlet of the dam and it is the sum of the atmospheric pressure and the water static pressure in the dam. Equation 3.2 represents the mass flow rate balance of the dam. \( \text{min}_{\text{dam}} \) is the incoming mass flow rate and \( \text{mout}_{\text{dam}} \) is the mass flow rate at the outlet of the dam.

### 3.1.2 Penstock modelling

A penstock is a large pipe which connects the dam’s outlet to the wicket gate’s inlet. The variables and parameters that are utilized to this equation-based modelling follows after an illustrated Figure 3.2.

![Figure 3.2: Penstock](image)

**Parameters:**

- \( A_{\text{pen}} \): the cross section area of penstock [m\(^2\)]
- \( l \): the length of the penstock [m]
- \( \rho \): water density [kg/m\(^3\)]
- \( g \): gravity acceleration [m/s\(^2\)]
- \( C_p \): the friction coefficient [Pa/m\(^2\)]
- \( m \): the mass of the water [kg]
- \( a \): acceleration [m/s\(^2\)]

**Variables:**

- \( p_{\text{in pen}} \): the pressure at the penstock’s inlet [Pa]
- \( p_{\text{out pen}} \): the pressure at the penstock’s outlet [Pa]
- \( p_{f \_pen} \): the pressure drop due to the friction [Pa]
- \( p_{i \_pen} \): the pressure drop due to the water inertia [Pa]
- \( \text{min}_{\text{pen}} \): incoming mass flow rate [kg/s]
- \( \text{mout}_{\text{pen}} \): outgoing mass flow rate [kg/s]
- \( Q \): volume flow rate [m\(^3\)/s]
Equations:

\[ p_{\text{in\_pen}} - \Delta p_{\text{pen}} - p_{\text{out\_pen}} = 0 \]  
\[ (3.4) \]

\[ \min_{\text{pen}} - m_{\text{out\_pen}} = 0 \]  
\[ (3.5) \]

Equation 3.5 describes the mass flow rate balance of the penstock. Equation 3.4 describes the pressure balance of penstock. \( p_{\text{in\_pen}} \) is the water pressure at the inlet of the penstock and \( p_{\text{out\_pen}} \) is the water pressure at the outlet of the penstock. \( \Delta p_{\text{pen}} \) is the pressure drop between \( p_{\text{in\_pen}} \) and \( p_{\text{out\_pen}} \) and in this case it is considered by two different types of the pressure drop.

\[ \Delta p_{\text{pen}} = p_{\text{f\_pen}} + p_{\text{i\_pen}} \]  
\[ (3.6) \]

\( p_{\text{f\_pen}} \) is the pressure drop due to the turbulent flow friction. \( p_{\text{i\_pen}} \) is the pressure drop due to the inertia of the water.

According to the Bernoulli equation [3] a turbulent flow friction in an enclosed tube could be modelled as

\[ p_{\text{f\_pen}} = C_{\text{f\_pen}} Q^2 \]  
\[ (3.7) \]

\( C_{\text{f\_pen}} \) is a friction coefficient.

The power of a fluid is expressed as [9]

\[ P(t) = p(t)Q(t) \]  
\[ (3.8) \]

Figure 3.2 shows that the water is moving from the inlet to the outlet of the penstock. The difference of the pressure, \( p_{\text{i\_pen}} = p_{\text{in\_pen}} - p_{\text{f\_pen}} - p_{\text{out\_pen}} \), would produce a force to accelerate the water. This force \( F \) can be described as \( F = A_{\text{pen}} p_{\text{i\_pen}} \) [9]. In this case, by using Newton’s law and Bernoulli’s equation [3], \( p_{\text{i\_pen}} \) is formulated as below

\[ F = A_{\text{pen}} p_{\text{i\_pen}} = ma \]
\[ A_{\text{pen}} p_{\text{i\_pen}} = l A_{\text{pen}} \rho \frac{d}{dt} v \]
\[ A_{\text{pen}} p_{\text{i\_pen}} = l A_{\text{pen}} \rho \frac{d}{dt} \frac{Q}{A_{\text{pen}}} \]

In the end the equation of \( p_{\text{i\_pen}} \) can be expressed as Equation 3.9.

\[ p_{\text{i\_pen}} = \frac{l \rho}{A_{\text{pen}}} \frac{dQ}{dt} \]  
\[ (3.9) \]
3.1.3 Wicket gate modelling

The wicket gate is located between the outlet of the penstock and the inlet of the Kaplan turbine. The variables and parameters of the equation-based modelling are followed by an Figure 3.3. Figure 3.3 is an illustration of the wicket gate’s angle, $\delta$. $\delta$ is the angle between the guide vane relative to the stay vane.

![Figure 3.3: An illustration of the wicket gate’s angle, $\delta$.](image)

Parameters:

- $\rho$ : the water density $[\text{kg/m}^3]$
- $\delta$ : the angle of the wicket gate $[\text{rad}]$
- $C_{f_{\text{gate}}}$ : friction coefficient $[\text{Pas}^2/\text{m}^6]$

Variables:

- $p_{\text{in}_{\text{gate}}}$ : the pressure at the wicket gate’s inlet $[Pa]$
- $p_{\text{out}_{\text{gate}}}$ : the pressure at the wicket gate’s outlet $[Pa]$
- $p_{f_{\text{gate}}}$ : the pressure drop due to turbulent friction $[Pa]$
- $m_{\text{in}_{\text{gate}}}$ : incoming mass flow rate $[\text{kg/s}]$
- $m_{\text{out}_{\text{gate}}}$ : outgoing mass flow rate $[\text{kg/s}]$

Equations:

$$p_{\text{in}_{\text{gate}}} - p_{f_{\text{gate}}} = p_{\text{out}_{\text{gate}}}$$ \hspace{1cm} (3.10)

$$m_{\text{in}_{\text{gate}}} - m_{\text{out}_{\text{gate}}} = 0$$ \hspace{1cm} (3.11)
Equation 3.11 describes the mass flow rate balance of the wicket gate, the incoming mass flow rate is equal to the outgoing mass flow rate in the wicket gate. Equation 3.10 describes the pressure balance of the wicket gate. $p_{\text{in}_{\text{gate}}}$ is the inflow pressure of the wicket gate and $p_{\text{out}_{\text{gate}}}$ is the outflow pressure of the wicket gate. $p_f_{\text{gate}}$ is the pressure drop when the water flows through the gate.

The angle $\delta$ determines the pressure drop of the wicket gate. $\delta$ is the angle between the guide vane relative to the stay vane and $\delta$ is limited between zero to eighty degrees. When $\delta$ is increased, the opening will decrease which results in a pressure drop between the outside of the vanes and the inside (where the turbine blades are located). On the other hand, if $\delta$ is close to zero, the opening is at its maximum and also here, there is a pressure drop through the gate. It is assumed in this report that the optimal $\delta$ angle is $25^\circ$ which results in the lowest pressure drop. This assumption is made with respect to reference [4].

The pressure drop is created by a turbulent fluid flow through the gate vanes. The turbulence occurs on the vane surface and increases with the fluid flow rate. This flow can be modelled by utilizing Bernoulli’s equation of turbulent flow friction in an enclosed tube. This will in this case be described as

$$p_{f_{\text{gate}}} = C_{f_{\text{gate}}} Q^2$$ (3.12)

Here $C_{f_{\text{gate}}}$ is a constant parameter. In this case equation 3.10 could be modelled as equation 3.13 which represents the pressure drop is determined by $\delta$. In equation 3.13 $\delta_0$ is the optimal angle, $25^\circ$. When $\delta$ is equal to $\delta_0$ then the pressure drop is at its minimum and results in the maximum $p_{\text{out}_{\text{gate}}}$.

$$p_{\text{in}_{\text{gate}}} - C_{f_{\text{gate}}}(2 - \cos(\delta - \delta_0))Q^2 = p_{\text{out}_{\text{gate}}}$$ (3.13)
3.1.4 Kaplan turbine modelling

The Kaplan turbine model has adjustable runner blades. The variables and parameters of the equation-based modelling are followed by an illustrated Figure 3.4.

![Figure 3.4: A schematic figure of the turbine](image)

Parameters:
- \( r \): the average radius of the turbine propeller [m]
- \( A_{\text{runner}} \): the cross sectional area of the turbine propeller \([m^2]\)
- \( \rho \): density \([\text{kg/m}^3]\)
- \( g \): gravity acceleration \([\text{m/s}^2]\)
- \( C_{f_{\text{tur}}} \): hydraulic friction coefficient \([\text{kg/m}^2]\)
- \( K_{m_{\text{tur}}} \): mechanical friction coefficient \([\text{kg/m}^2]\)

Variables:
- \( F \): force \([\text{kgm/s}^2]\)
- \( p_{f_{\text{tur}}} \): the friction pressure drop \([\text{Pa}]\)
- \( p_m \): pressure drop contributed to the turbine’s rotational momentum \([\text{Pa}]\)
- \( \theta \): the angle between the runner blades to the horizontal plan \([\text{rad}]\)
- \( w \): angular velocity \([\text{rad/s}]\)
- \( Q \): volume flow rate \([\text{m}^3/\text{s}]\)
- \( M \): rotational momentum \([\text{Nm}]\)
- \( M_m \): rotational momentum caused by hydraulic force \([\text{Nm}]\)
- \( M_j \): turbine inertia momentum \([\text{Nm}]\)
- \( M_f \): mechanical friction momentum \([\text{Nm}]\)
- \( M_{\text{gen}} \): kinetic rotational momentum \([\text{Nm}]\)
Equations:

\[ m_{in_{tur}} - m_{out_{tur}} = 0 \]  
(3.14)

\[ p_{in_{tur}} - \Delta p_{tur} = p_{out_{tur}} \]  
(3.15)

\[ M_{in} - M_f - M_{gen} = 0 \]  
(3.16)

In this model there are two different physical domains of concern. One is the hydraulics and the other one is mechanics.

- **Hydraulics domain**
  
  Equation 3.14 describes the mass flow rate balance of the turbine. Equation 3.15 describes the pressure balance of the turbine. \( p_{in_{tur}} \) is the inflow pressure of the turbine and \( p_{out_{tur}} \) is the outflow pressure of the turbine. \( \Delta p_{tur} \) is the pressure drop when the water flows through the turbine and in this model this pressure drop is considered by two types of the pressure drop as below.

\[ \Delta p_{tur} = p_{f_{tur}} + p_m \]  
(3.17)

\( p_{f_{tur}} \) is the turbulent flow friction and could be modelled as below as the same way as in Equation 3.12.

\[ p_{f_{tur}} = C_{f_{tur}} Q^2 \]  
(3.18)

\( p_m \) is the pressure drop due to the interaction of the flowing water with the runner blades transforming the hydraulic energy to the mechanical energy, i.e. the pressure difference between the pushing side and the vacuum side of the propeller. The angle \( \theta \) determines the size of the pressure drop. \( \theta \) is the angle between the runner blades to the horizontal plan. When \( \theta \) is closer to zero then the effective surface of the runner blades is maximum which gives the maximum pressure drop. On the other hand when \( \theta \) is increased then the effective surface of the runner blades is decreased which results in the lowest pressure drop.

- **Mechanics domain**
  
  Equation 3.16 describes the momentum balance of the turbine. \( M_{in} \) is the rotational momentum driven by the force of the flowing water and causing the generator shaft to rotate. \( M_{in} \) is the cross product of the force and the perpendicular distance to the force. According to Newton’s third law [3], which gives the relationship between mechanical variables, force \( (F) \) and velocity \( (v) \), and hydraulic variables, pressure \( (p_m) \) and volume flow rate\( (Q) \) are as below.

\[ F = p_m A_{runner} \]  
(3.19)
The relationship between the momentum and force is expressed as Equation 3.21

\[ M_{in} = r \times F \]  

Please note that since \( M_{in} \) is the cross product of \( r \) and \( M_{in} \), it is important to compute the component of the force which works perpendicularly to the surface of the runner blades. By combining equations 3.19 and 3.21 the formulation of \( M_{in} \) is as below

\[
M_{in} = r \times F \\
M_{in} = r F_\perp \\
M_{in} = r \sin(\theta)\cos(\theta)F \\
M_{in} = M_{in} = \frac{r \sin(\theta)\cos(\theta) p_m A_{runner}}{\sin(2\theta)} \\
F \\
F \\
\]

In the end relationship between the mechanical rotational momentum, \( M_{in} \), and the hydraulic pressure, \( p_m \), could be modelled as Equation 3.22.

\[ M_{in} = \frac{1}{2} r \sin(2\theta) A_{runner} p_m \]  

Equation 3.22 shows that the optimal angle \( \theta \) is 45° since the rotational momentum \( M_{in} \) is at its maximum when the runner blade angle is 45°. On the other hand, \( M_{in} \) is at its minimum when \( \theta \) is zero. The relationship between the hydraulic variable \( Q \) and the mechanical variable \( w \) could be expressed with the help of energy balance equation [9].

\[ p_m Q = M_{in} w \]  

By combining Equations 3.22 and 3.23, \( w \) can be formulated as below.

\[
w = p_m Q \frac{p_m Q}{M_{in}} \\
w = \frac{p_m Q}{0.5r \sin(2\theta) A_{runner} p_m} \\
M_{in} \\
M_{in} \\
\]

As a result, Equation 3.24 represents the relationship between the mechanical angular velocity, \( w \), and the hydraulic volume flow rate, \( Q \).

\[ w = \frac{Q}{0.5r \sin(2\theta) A_{runner}} \]  

The mechanical friction \( M_f \) can be expressed [9] as

\[ M_f = K_m \tau_r w \]
$M_j$ is the inertia momentum of the turbine and since the generator’s rotor is attached to the turbine the inertia of rotor is considered. The formula of $M_j$ is [9]

$$M_j = J_{(turbine+rotor)} \frac{dw}{dt} \tag{3.26}$$

$M_{gen}$, the rotational kinetic momentum, will determine the size of the energy to the generator. A larger kinetic momentum will result in more electrical power. The formulation of $M_{gen}$ can be describes as [9]

$$M_{gen} = \frac{1}{2} J_{(turbine+rotor)} w^2 \tag{3.27}$$

## 3.2 Creating an object oriented model in Dymola

This object oriented model in Dymola is based on the derivation model in section 3.1. There exist Modelica standard Libraries which are available for several technical applications, like electrical, mechanical and thermo-hydraulic systems as well as control of such systems. Under Modelica standard libraries there are some user packages for the different purposes. Blocks with continuous and discrete input/output blocks, filters, and sources. Constants provides constants from mathematics, machine dependent constants and constants from nature. Electrical provides electric and electronic components such as resistor, diode and transistor. Icons provides common graphical layouts. Math gives access to mathematical functions such as sin, cos and log. Mechanics includes one-dimensional translational and rotational components such as gearbox, bearing friction and clutch. SIunits with about 450 type definitions with units, such as Angle, Voltage, and Inertia. Thermal provides models for heat-transfer. Although there exists already Modelica libraries it is also good to insert developed components into a library in order to reach the requirement of the model.

### 3.2.1 Connectors

In Dymola the function of the connectors is to combine components in order to create a dynamic process model. There are two types of connectors, hydraulic connectors `flowport_a`, `flowport_b` and mechanical connectors `flange_a`, `flange_b`. The description of the connectors are as below:

#### Mechanical connectors

There are two mechanical connectors, `flange_a` and `flange_b`. The variables of these connectors are torque [Nm] and angle [rad].
*flange* _a_ is a connector for rotational mechanical system and models a mechanical flange. There is a second connector for flanges: *Flange* _b_. The connectors *Flange* _a_ and *Flange* _b_ are completely identical. There is only one difference in these icons that is the sign conventions due to these variables are basically elements of vectors, i.e., have a direction.

**Hydraulic connectors**

There are two hydraulic connectors, *flowport* _a_ and *flowport* _b_. The variables of these connectors are pressure [Pa] and mass flow rate [kg/s]. *flowport* _a_ and *flowport* _b_ are also completely identical but with the different direction of flow.

### 3.2.2 Components in Dymola

There are five components, dam, penstock, wicket gate, turbine and turbine inertia that are created under the library. Each component consists of connectors in order to be combined with the other components. These components are based on the derivation model in section 3.1.

**Dam**

Figure 3.5 is a dam component with connectors *flowport* _a_, *flowport* _b_.

![Figure 3.5: Dam component in Dymola](image)

In Dymola the dam model is created by the following equations:

\[ flowport\_a\_pressure = p_{atm} \]
\[ flowport\_a\_massflowrate = \text{min}_{dam} \]
\[ flowport\_b\_pressure = p_{out\_dam} \]
\[ flowport\_b\_massflowrate = m_{out\_dam} \]
\[ \frac{dm}{dt} = flowport\_a\_massflowrate + flowport\_b\_massflowrate \]
Penstock

Figure 3.6 describes a penstock component with connectors `flowport_a` and `flowport_b`. In Dymola this component is created mainly by the following equations.

![Figure 3.6: Penstock component in Dymola](image)

\[
\text{flowport}_a\_\text{pressure} = p_{in\_pen} \\
\text{flowport}_a\_\text{massflowrate} = \text{min}_{pen} \\
\text{flowport}_b\_\text{pressure} = p_{out\_pen} \\
\text{flowport}_b\_\text{massflowrate} = m_{out\_pen} \\
\text{min}_{pen} + m_{out\_pen} = 0 \\
d(m_{out\_pen}) = \frac{A_{pen}}{l} p_{in\_pen} - p_{out\_pen} - C_f\_pen(\frac{\text{min}_{pen}}{\rho})^2
\]
**Wicket gate**

Figure 3.7 is the wicket gate component with connectors `flowport_a`, `flowport_b` and a real input.

![Wicket gate component in Dymola](image)

In Dymola this component is created mainly by the following equations:

- $flowport\_a\_pressure = p_{in\_gate}$
- $flowport\_a\_massflowrate = \text{min}_{gate}$
- $flowport\_b\_pressure = p_{out\_gate}$
- $flowport\_b\_massflowrate = \text{mout}_{gate}$
- $\text{min}_{gate} + \text{mout}_{gate} = 0$
- $p_{out\_gate} = p_{in\_gate} - C_{g\_gate}(2 - \cos(\delta - \delta_0))(\frac{\text{min}_{gate}}{\rho})^2$
Turbine

Figure 3.8 is the turbine component with connector flowport_a, flowport_b, flange_b and a real input. In Dymola this turbine component is based on the following equations:

\[
\begin{align*}
\text{flowport}_a\_\text{pressure} &= p_{\text{in\_tur}} \\
\text{flowport}_a\_\text{massflowrate} &= m_{\text{tur}} \\
\text{flowport}_b\_\text{pressure} &= p_{\text{out\_tur}} \\
\text{flowport}_b\_\text{massflowrate} &= m_{\text{out\_tur}} \\
\text{min\_gate} + m_{\text{out\_gate}} &= 0 \\
p_{\text{out\_gate}} &= p_{\text{in\_gate}} - C_{g\_gate}(2 - \cos(\delta - \delta_0))(\frac{m_{\text{min\_gate}}}{\rho})^2 \\
\text{flange}_b\_\text{torque} &= M_{\text{in}} \\
\text{flange}_b\_\text{angle} &= \angle \\
\frac{d}{dt}\angle &= w \\
0.5w A_{\text{tur}} r \sin(2\theta) &= \frac{m_{\text{tur}}}{\rho}
\end{align*}
\]

Figure 3.8: The turbine1 component in Dymola
Turbine inertia

Figure 3.9 is the turbine inertia component with connectors `flange_a`, `flange_b`. This component is based on the following equations in Dymola:

- $flange\_a\_torque = M_{in}$
- $flowport\_a\_angle = \text{angle}$
- $flowport\_b\_torque = M_{gen}$
- $flowport\_b\_angle = \text{angle}$
- $\frac{d}{dt}\text{angle} = w$
- $d(w) = a$
- $M_f = K_mw$
- $J_{totala} = M_{in} + M_{gen} - M_f$

In the end the diagram of this created object oriented dynamic process model in Dymola is shown Figure 3.10. In the diagram the input of this system is the water source and the output is the torque and pressure out of the turbine.
Figure 3.10: The object oriented dynamic process model in Dymola
3.2.3 Parameters computation

In this model some parameters can be specified, like the water density, but there are some unspecified parameters which need to be computed, like the frictional coefficients. In order to compute those unspecified parameters there are some assumption parameters need to be decided in the beginning. These assumption are partly based on the reference [14] and [8].

Constant parameters

Table 3.1 constant parameters used in the Dymola test bench.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>gravity acceleration</td>
<td>9.8 [m/s²]</td>
</tr>
<tr>
<td>ρ</td>
<td>water density</td>
<td>1000 [kg/m³]</td>
</tr>
<tr>
<td>p atm</td>
<td>atmosphere pressure</td>
<td>100000 [Pa]</td>
</tr>
</tbody>
</table>

Table 3.1: Constant parameters of the object oriented model

Assumption parameters

Table 3.2 assumption variables and parameters of the object oriented model, used in the Dymola test bench [8] [14].

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>water flow rate</td>
<td>200 [m³/s]</td>
</tr>
<tr>
<td>P</td>
<td>power</td>
<td>24 [MW]</td>
</tr>
<tr>
<td>w</td>
<td>turbines angular velocity</td>
<td>15.7 [rad/s]</td>
</tr>
<tr>
<td>h</td>
<td>water height in the dam</td>
<td>18 [m]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_m_tur</td>
<td>the mechanical friction</td>
<td>1 [N.ms/rad]</td>
</tr>
<tr>
<td>l</td>
<td>the length of the penstock</td>
<td>1000 [m]</td>
</tr>
<tr>
<td>A_pen</td>
<td>the area of the penstock</td>
<td>28 [m²]</td>
</tr>
<tr>
<td>A_dam</td>
<td>the area of the dam</td>
<td>190000 [m²]</td>
</tr>
<tr>
<td>r</td>
<td>the average radius of the propeller</td>
<td>2 [m]</td>
</tr>
<tr>
<td>A_runner</td>
<td>the cross section area of the propellers</td>
<td>12.6 [m²]</td>
</tr>
</tbody>
</table>

Table 3.2: Assumption parameters of the object oriented model

The parameters of the dam, penstock and turbine are assumed according to reference [8]. The variables of water flow rate, power, turbine angular velocity and the height are assumed according to one simulation models of hydropower plants[14]. We also assume the mechanical friction momentum is small and put the value 1 to the mechanical friction coefficient, \(K_m_{tur}\).
Computational parameters

With the constant and assumption variables and parameters in Table 3.1 and 3.2, the frictional coefficient and the moment of inertia of the turbine can be computed as below.

\[ C_{f\_pen} \] the friction coefficient of the penstock model

The friction head loss in a tube can be formulated as Equation 3.28 according to the reference [19].

\[ h_f = \frac{Lv^2}{M^2R^4} \tag{3.28} \]

\( v \) = the water’s velocity [\( \frac{m}{s} \)]

\( L \) = the length of the tube [m]

\( h_f \) = the friction height loss [m]

\( M \) = the material coefficient of the tube [no unit] [19]

\( R \) = hydraulic average depth [m] = penstock’s section area / penstock’s section area circumference [19]

In this case, \( L = 1000 \text{m} \), \( M = 90 \) (we choose the material coefficient of the rough surface of concrete tube [19]), \( v = \frac{Q_{\text{pen}}}{A_{\text{pen}}} = 5.5357 \), \( R = \frac{A_{\text{pen}}}{2\pi r_{\text{pen}}} = 1.4927 \), put them in the Equation 3.28, the friction head loss in the penstock is \( h_f = 3.6923 \text{m} \).

When we have the friction head loss the friction pressure loss/drop of the penstock can be calculated as

\[ p_{f\_pen} = \rho gh_f = 36209 \text{[Pa]} \]

Finally by using Equation 3.7, \( C_{f\_pen} \) can be expressed as

\[ C_{f\_pen} = p_{f\_pen}/C_{f\_pen}Q^2 \]

Result: \( C_{f\_pen} = 1.0 \text{[\( \frac{\text{Pa}\_s^2}{\text{m}^2\_s^2} \)]} \)

\[ C_{f\_gate} \] the friction coefficient of the wicket gate model

The friction head loss in a gate can be formulated as Equation 3.29 according to reference [19].

\[ \Delta H_f = \beta \sin\alpha \left(\frac{d}{a}\right)^{1.25} \frac{v^2}{2g} \tag{3.29} \]

\( \Delta H_f \) = head loss [m]

\( v \) = water’s velocity [m/s]

\( \alpha \) = the angle of the wicket gate [degree]

\( d \) = the thickness of the gate vanes [mm] [19]

\( \beta \) = coefficient depends on the shape of the gate vane [19]

\( a \) = the free distance between the gate vanes [mm] [19]

In this case, \( \beta = 2.42 \) (we choose rectangular shape gate vanes [19]), \( \alpha = 25^\circ \) (we choose the optimal wicket gate angle, \( \alpha = 25^\circ \)), \( d = 15 \text{mm} \) (assumption), \( a = \)
20 (assumption), \( v = \frac{Q}{A_{pen}} = 5.5357 \), put them in the Equation 3.29, the friction head loss in the wicket gate is \( H_f = 1.8567 \) m.

The pressure loss/drop in the wicket gate can be composed as

\[
p_{f_{\text{gate}}} = \rho g \Delta H_f = 18208 \text{ [Pa]}. \]

In the end, \( C_{f_{\text{gate}}} \) can be computed by Equation 3.12,

\[
C_{f_{\text{gate}}} = \frac{p_{f_{\text{gate}}}}{Q^2}. \]

**Result:** \( C_{f_{\text{gate}}} = 0.5 \) \([\frac{\text{Pa}^2}{\text{m}^6}]\)

\([C_{f_{\text{tur}}}]\) the turbulent friction coefficient of the turbine

This coefficient is computed by the calculation of the turbine’s pressure drop. According to Equation 3.15, the turbine’s pressure drop, \( \Delta p_{\text{tur}} = p_{\text{in}_{\text{tur}}} - p_{\text{out}_{\text{tur}}} \). Equation 3.17 gives that \( \Delta p_{\text{tur}} \) consists of \( p_{f_{\text{tur}}} \), the turbulent friction pressure drop, and \( p_m \), the pressure drop contributes to the turbine’s torque. \( C_f \) is the coefficient of this pressure drop, \( p_{f_{\text{tur}}} \). In order to compute \( C_{f_{\text{tur}}} \) we have to calculate \( p_{\text{in}_{\text{tur}}} \), \( p_{\text{out}_{\text{tur}}} \) and \( p_m \) first. The formulation is as below:

**\( p_{\text{in}_{\text{tur}}} \) the pressure at the inlet of the turbine**

The pressure at the inlet of the turbine is the pressure at the dam’s outlet deducts the pressure drops of the penstock and the wicket gate, see illustration Figure 3.11. The pressure at the dam’s outlet, \( p_{\text{out}_{\text{dam}}} = p_{\text{atm}} + \rho gh \), with the assumption data \( h = 18 \) [m], the pressure at the dam’s outlet is, \( p_{\text{out}_{\text{dam}}} = 276520 \) [Pa]. By using the computed data of the pressure drops of the penstock and the wicket gate, \( p_{\text{in}_{\text{tur}}} \), the pressure at the turbine’s inlet, can be computed as

\[
p_{\text{in}_{\text{tur}}} = p_{\text{out}_{\text{dam}}} - p_{f_{\text{pen}}} - p_{f_{\text{gate}}} = 222103 \text{ [Pa]}. \]

**\( p_{\text{out}_{\text{tur}}} \) the pressure at the outlet of the turbine**

We assume that the pressure at the outlet of the turbine is the atmosphere pressure, \( p_{\text{out}_{\text{tur}}} = 100000 \) [Pa].

**\( p_m \) the pressure drop which contributes to the turbine's torque**

Recall the section 3.1.4, \( p_m \) can be computed by Equation 3.23, \( p_m Q = M_{in} w \).

Since \( Q \) and \( w \) are given as the assumption parameters there is only \( M_{in} \) needed to be solved in order to compute \( p_m \). Equation 3.21 gives that \( M_{in} = M_f + M_j + M_{gen} \). When \( w \) is constant then \( M_j \), turbines inertia momentum, is zero. It results in \( M_{in} = M_f + M_{gen} \). The mechanical power, \( P = M_{in} w \) [3], is the power generated by the turbines torque. In this case \( M_{gen} = P / w \). With the data, \( P = 24 \) [MW], \( w = 15.7 \) [rad/s] and \( K_f = 1 \frac{\text{Nm}}{\text{rad}} \), we get \( M_{gen} = 1527900 \) [Nm], \( M_f = K_f w = 15.7 \) [Nm], which results in \( M_{in} = M_{gen} + M_f = 1529715 \) [Nm]. Therefore the pressure drop which contributes to the turbine’s torque is , \( p_m = M_{in} w / Q = 120143 \) [Pa].

Finally, the friction pressure drop of the turbine, \( p_{f_{\text{tur}}} = \Delta p_{\text{tur}} - p_m = p_{\text{in}_{\text{tur}}} - p_{\text{out}_{\text{tur}}} - p_m = 1960 \) [Pa] and \( C_f \) can be computed as \( C_{f_{\text{tur}}} = p_{f_{\text{tur}}} / Q^2 \), Equation 3.18.
Result: \( C_{f_{\text{tur}}} = 0.05 \left[ \frac{P_{\text{fr}}s^2}{m^2} \right] \).

Figure 3.11: Illustration of the pressure drop of the turbine

\( [J] \): the turbine’s momentum inertia

A kinetic rotational momentum (\( T \)) can be expressed as \( T = \frac{1}{2}Jw^2 \) [9]. In this case the kinetic rotational momentum of the turbine, \( M_{\text{gen}} = 1527900 \) [N.m]. With the assumption parameter of \( w \), \( J \) is computed as \( 2M_{\text{gen}}/w^2 \).

Result: \( J = 12385 \) [kgm\(^2\)]
3.3 Simulation

There is no measurement data available for this model. The simulation of this object oriented dynamic process model is carried out by different experiments in the Dymola test bench. The simulation of the open loop dynamic process model is plotted from Dymola Figure 3.12.

Figure 3.12: The simulation of the object oriented dynamic process model in Dymola with input water flow $Q=200 \, m^3/s$

Figure 3.12 is simulated by using the data in section 3.2.3 with the constant angels of the wicket gate and the runner blade. The simulation shows that the water level of the dam is decreasing which results in the source of the water static pressure reduces. With the decreasing pressure source the rotational momentum of the turbine is reduced. In order to regulate this system a control system is needed.

To study the characteristics of this model there are some experiments have been done in the Dymola test bench. These experiments are executed by changing only one parameter at a time. The results are discussed below.
3.3.1 Different water flow inputs

In this experiment all data are fixed as same as in Section 3.2.3, while the input of the water flow is changed. The comparison of two different flow inputs, $300m^3/s$ and $500m^3/s$ is plotted below.

![Figure 3.13: The comparison of the water level of the dam with the different water flow input, 300$m^3/s$ upper and 500 $m^3/s$ below](image1)

![Figure 3.14: The comparison of the turbine rotational momentum with the different water flow input, 300$m^3/s$ upper and 500 $m^3/s$ below](image2)

Figure 3.13 shows that the lowest water level of the dam is higher when the inflow of the dam is larger. It also shows that the slope of the water level of the inflow 500 $m^3/s$ is a positive sign that’s because the inflow volume flow rate is higher than the outflow volume flow rate of the dam. Figure 3.14 shows that the turbine’s rotational momentum is a direct function of the input of the water flow. The increasing water flow enhances the turbine rotational momentum.
3.3.2 Different cross sectional areas of the dam

In this experiment we test two different cross section areas of the dams, 100000 $m^2$ and 500000 $m^2$ while the other parameters and variables are fixed as in Section 3.2.3. The water flow input is 200 $m^3/s$. The result of the simulation is plotted below.

Figure 3.15: The comparison of the water level of the dam with the different cross sectional areas of the dams, 100000$m^2$ upper and 50000 $m^2$ below. The water flow input is 200 $m^3/s$.

Figure 3.16: The comparison of the turbine rotational momentum with the different cross sectional area of the dam. 100000$m^2$ upper and 500000 $m^2$ below. The water flow input is 200 $m^3/s$.

Figure 3.15 and Figure 3.16 show that the size of the cross section area of the dam affects the rapidity of the stability of the water flow of the dam. Larger size of the dam results in the water level reduces slowly to a stable condition. On the other hand the
turbine rotational momentum is not influenced by the size of the cross section areas of
the dam.

3.3.3 Different lengths of the penstock

In this experiment all data are fixed as same as in Section 3.2.3, while the lengths of
the penstocks are 10000 meters and 100000 meters. The comparison plots are shown
below.

Figure 3.17: The comparison of the water level of the dam with the different length
of the penstocks. 10000 m upper and 100000 m below. The water flow input is 200
m$^3$/s.

Figure 3.18: The comparison of the turbine rotational momentum with the different
length of the penstocks. 10000 m upper and 100000 m below. The water flow input is
200 m$^3$/s.

Figure 3.17 shows that the increasing length of the penstock results in the slower
rapidity of the stability of the water in the dam and it affects the turbine rotational
momentum. The longer penstock occurs the delay of the stability of the turbine rota-
tional momentum and the aggressive acceleration of the turbine rotational momentum Figure 3.18.

3.3.4 Different lengths of the average radius of the turbine propeller

We experiment two different average radius of the turbine propeller, 2m and 5m, while the other data are fixed as same as in Section 3.2.3.

![Figure 3.19: The comparison of the water level of the dam with the different average radius of the turbine propeller, 2m upper and 5 m below. The water flow input is 200 m$^3$/s.](image)

![Figure 3.20: The comparison of the turbine rotational momentum with the different average radius of the turbine propeller, 2 m upper and 5 m below. The water flow input is 200 m$^3$/s.](image)

The result shows that the difference of the lowest water level of the dam is very small. It is hard to see from the Figure 3.19 but the lowest water level of the 5 m
propeller radius is 0.5 m lower than the 2 m propeller radius. The result of the turbine rotational momentum is quite different. Figure 3.20 shows that the turbine rotational momentum decreases over 90000 Nm by lengthening the radius of the turbine propeller from 2 m to 5 m. Equation 3.22 gives that the turbine rotational momentum depends on the average radius of the turbine propeller and the pressure. As the result in Figure 3.19, the water level of the dam decreases when the radius of the turbine propeller lengthens, which results in the source of the hydraulic static pressure reduces and the pressure that transmits to the turbine rotational momentum decreases. In consequence of this effect the turbine rotational momentum is reducing while the source of the hydraulic static pressure is decreasing. Even though the turbine rotational momentum increases proportionally by the radius of the turbine propeller, the bigger radius does not increase the turbine rotational momentum any way since the term of the decreasing pressure of the turbine is larger than the average radius of the turbine propeller.

As the results of the experiments above, it shows that the pressure and the water flow play the major roles in this system and they interact the water level in the dam and the turbine rotational momentum. The hydraulic static pressure and the water flow rate determine the turbine rotational momentum and indirect to the electrical power, therefore it is important to control the water level of the dam and water flow rate which can be done by a control system design.
Chapter 4

Demonstration - a dam level controller design in the Dymola test bench

This chapter presents the utility of this object oriented model by a demonstration of an dam level controller design in the Dymola test bench. Before the demonstration a general information of hydropower plant controllers is described.

4.1 General hydropower plant controllers

The main function of a hydropower plant controller is to regulate the turbine speed, voltage frequency and the active power. This function requires information of the turbine rotor speed and of the electric power in order to determine the appropriate opening of the wicket gate. The problem of a hydropower plant controller design is the non-linear behaviour of the dynamic process and it may vary strongly with the operation point. Generally turbines controllers are usually PID [12] based and their implementation may include from pure mechanical controllers or electric-mechanical controllers to full electronic controllers. Many electro hydraulic controllers are based on PID, Proportional Integral Derivative, controllers. The proportional function is correlated to the controller speed. The integral function can be used to eliminate the error of output. The derivative function is to enhance the stability [12]. The normal used hydropower plant controllers are Mechanical Hydraulic Controllers, Electric Hydraulic Controller, Digital Electro hydraulic controllers. For older hydroelectric units they use mechanical hydraulic control, MHC. The basic elements of the control system are a speed controller, speed relays and hydraulic servomotors and their functions are carried out through mechanical components. [13]. Electric hydraulic control, EHC, was introduced in the 1960s and the system uses electronic circuits in place of mechanical components associated with the MHC in the low-power portions. EHC systems offer more flexibility and improve the performance since they take into account the time lags [13]. Modern digital electro hydraulic (DEH) controllers, these controllers have several
operating modes and functions and provide facile operating and flexibility, which is not possible with old mechanical controllers [13].

4.2 A dam level controller design

A demonstration of a dam level controller design in the Dymola test bench is represented in this section. The main problem for the controller design is that the behavior of the dynamic model is a nonlinear system. Compared to linear systems, this makes it much more difficult to create a reliable and working model. A common way to solve this is to simplify the system by linearizing it around a defined operating point. Within the limits of operation, the system thus becomes a linear one, and around the operating point, linear models may be used.

4.2.1 Background

In order to design a control system, a method called loop shaping is used. The idea of this method is to form the loop gain for the open loop with the help of a controller in order to achieve the wanted properties for the closed loop.

In Figure 4.1, u(t) is the control signal sent to the system, y(t) is the measured output, r(t) is the desired output and e(t)=r(t)-y(t) is the tracking error. A PI controller will be used as a feedback control design. "PI" means Proportional, Integrating. With a PI controller, the relationship between the control error e(t) and the control signal u(t) is as Equation 4.1.

\[ u(t) = K_P e(t) + K_I \int_0^t e(s) \, ds \] (4.1)

A desired closed-loop dynamics is obtained by adjusting the parameters \( K_P \) and \( K_I \). There are different effects related with each one of them. \( K_P \) is correlated to the controller speed, proportional gain, but sometimes too high gain will also reduce the stability. \( K_I \) is the gain factor for the integrator used to eliminate the error of output by integrating the difference between r(t) and y(t).
4.2.2 Loop shape method

The physical implementation of the PI controller could be written as:

\[ F = K (1 + \frac{1}{sT_i}) \]  \hspace{1cm} (4.2)

As in Figure 4.1 a loop gain is given by \( L = GF \). The specifications of the loop shape method is that at the desired phase margin \( \varphi \) and cross-over frequency \( w_c \) at unity gain, the following constrains should be fulfilled:

\[ |G(iw_c)F(iw_c)| = 1 \]  \hspace{1cm} (4.3)

\[ \text{arg}G(iw_c) + \text{arg}F(iw_c) - \varphi = - \pi \]  \hspace{1cm} (4.4)

In this case Equation 4.4 is equivalent to Equation 4.5

\[ \text{arg}G(iw_c) + \arctan(w_cT_i) - \frac{\pi}{2} - \varphi = - \pi \]  \hspace{1cm} (4.5)

If system(G) is known then \( \text{arg}G(iw_c) \) could be read from Bode-diagram of system(G) and \( T_i \) could then be solved out by Equation 4.5. K could be solved out by Equation 4.6

\[ K = \frac{1}{|L(iw_c)|} \]  \hspace{1cm} (4.6)

where \( |L(iw_c)| \) could be read from \( L(iw_c) \) Bode-diagram where \( w_c \) is the cross-over frequency at unity gain.

4.2.3 PI feedback controller design

The function of this PI feedback controller design is to govern the water level of the dam. By controlling the \( \delta \) angle of the wicket gate to stabilize the the water level of the dam around a defined operating point. In this model the defined operating is chosen as 6 meter. The input of this system is \( \delta \) angle, and the output is the water level of the dam. The diagram of the feedback control system in the Dymola test bench is shown as Figure 4.2
Figure 4.2: A diagram of the feedback control system in the Dymola test bench
Linearize the dynamic system model

The system of this object oriented dynamic model we call \( (G_0) \) in the Dymola test bench. When linearizing the system model \( (G_0) \) around an operating point, the first order of system model \( (G_0) \) can be described with the following transfer function.

\[
\Delta y(\text{output}) = \frac{k}{1 + \tau s} \Delta u(\text{input})
\]

\( k \) and \( \tau \) can be determined from the step response of the open loop system.

\( k = \frac{\Delta y}{\Delta u} \)

\( \tau \) is the time duration for the step response to reach 63% of its full value.

The input and output step responses of the open loop system from Dymola is plotted in the following graphs. Figure 4.3, 4.4.

![Figure 4.3: Step response of the system input with step 0.1.](image)

![Figure 4.4: Step response of the system output, the water level of the dam](image)

\[
\Delta u = 1.347 - 1.247 = 0.1
\]

\[
\Delta y = 6.25 - 6.10 = 0.15
\]
\[ k = \frac{\Delta y}{\Delta u} = 1.5 \]
\[ \tau = 63400 - 50000 = 13400(s) \]
Which give the first order system \( G \) is \( \frac{1.5}{1+13400s} \).

**PI controller**

- Estimate parameters \( K \) and \( T_i \)
  
  According to Equation 4.2, the physical equation of PI controller is \( F = K(1 + \frac{1}{sT_i}) \). Parameters \( K \) and \( T_i \) are obtained by using the loop shape method. Here \( G = \frac{1.5}{1+13400s} \). At the cross-over frequency \( w_c = 0.001 \) and \( \varphi = 60 \) degrees with the help of MATLAB program it gives the results as \( K = 7.4 \) and \( T_i = 1467.7 \).

- Sensitivity and robustness
  
  A good feedback controller design would be with respect to sensitivity and robustness. For a closed-loop system a sensitivity function \((S)\) is the transfer function between the error and the output which describes as

  \[
  S(s) = \frac{1}{1 + G(s)F(s)} \quad (4.7)
  \]

  For a good controller design a sensitivity function ought to have a small value for low frequencies and concerning the system robustness, a complementary sensitivity function \((T)\) is considered and it ought to have a small value for high frequencies. A complementary sensitivity function comes from robustness criterion. It represents the stability of the model and could be described as

  \[
  T(s) = \frac{F(s)G(s)}{1 + G(s)F(s)} \quad (4.8)
  \]

  With the help of MATLAB program the performance of the sensitivity and robustness of the closed-loop system by using \( F = 7.4(1 + \frac{1}{3400s}) \) are shown in the figures below.

![Singular value of sensitivity function S](image1)

![Singular value of complementary sensitivity function T](image2)

Figure 4.5: Singular value of \( S \) and \( T \) of the feedback controller system.
From the MATLAB plots Figure 4.5, it can be seen that the performance of the sensitivity is quite good but not for the robustness. Since the singular value of the sensitivity function is quite small at low frequencies which is good but the singular value of the complementary sensitivity function is a little bit big at high frequencies.

- Simulation PI feedback controller in the Dymola test bench

The simulation of the water level of the dam and the control input to the wicket gate with the PI feedback controller in the Dymola test bench are as below Figure 4.6, 4.7.

Figure 4.6: The simulation of the water level of the dam with PI feedback controller.

Figure 4.7: The simulation of the control input to the wicket gate.

Figure 4.6 is the simulation of the water level of the dam by using this PI feedback controller. The result shows that there is an obvious peak in the plot. It is because of the constrain of the wicket gate. According to the characteristic of the wicket gate Section 3.1.3, it describes that the pressure difference between input and output of the wicket gate has got a minimum at the optimum $\delta$ angle, and the slope of the pressure curve has got a sign change when passing through this mini ma. Also, discussed earlier, we want to linearize the control system. Thus, only the part of the pressure curve with positive sign will be used and for linearization purpose treated as a straight line. In Figure 4.7, it can be seen that the maximum controller output is limited to a value of 1.4. This corresponds to maximum open
guide vane. Since the $\delta$ of the guide vane is limited to 80 degrees, or 1.4 rad, the maximum value of PI controller output is set to 1.4. The minimum value for the controller output is related to the optimum opening angle for the guide vane of 25 degrees and set to 24 degrees, or 0.43 rad, in the model for linearization purpose. From Figure 4.7 it can be seen that the gain of the PI controller is too big and this is the reason the output hits the value 1.4 in the beginning. The peak of the Figure 4.6 is affected consequently. In order to solve this problem an improved PI controller is implemented by tuning the parameters $K$ and $T_i$. By doing this in the Dymola test bench, optimized values for $K$ and $T_i$ were found to be $K=0.00003$ and $T_i = 2$. The simulation of the water level of the dam and the control input to the wicket gate with the improved PI feedback controller is shown as Figure 4.8, 4.9.

![Figure 4.8: The simulation of the water level of the dam with the improved PI feedback controller.](image)

![Figure 4.9: The simulation of the control input to the wicket gate.](image)

Figure 4.9 shows that the result is quite good since the problem we had in the earlier PI controller is solved and The output of the improved PT controller has been pressed under the value of 1.4 and it can be seen in Figure 4.8 that the previous peak appeared before is gone. However, a trade-off is made to reach system stability and this is the time to equilibrium, which has increased.
Chapter 5

Conclusion and Discussion

As a result of the simulation of this object oriented dynamic process model in the Dymola test bench, it can be seen that the performance of the Kaplan turbine is strongly influenced by the water flow and the pressure that feed the turbine. Since the driving energy of a generator in a hydropower plant is the rotational momentum of the turbine, an in-depth knowledge of the characteristics of the turbine is useful for the analysis and development of a hydropower plant. An object oriented model is one utilized tool that can meet these requirements. As we mentioned before, an evaluation of different control strategies is facilitated by the aid of this object oriented model. This has been proved by our demonstration. With no difficulty a dam level PI feedback controller is improved with the help of this object oriented model Figure 4.8. At the same time, by the advantage of using an object oriented model structure, the problem of the PI feedback controller caused by the constraint of the wicket gate is discovered directly from the Dymola test bench Figure 4.6.

As a conclusion, the objectives of this thesis work are fairly well achieved. Perhaps there are inaccurate formulations of the components or incorrect assumption parameters of this model. This is however easily modified in the components of this object oriented model, which is the primary advantage of an object oriented modelling. In addition to this, an object oriented model can be easily represented to the automation industries for controller design.

There is one interesting issue discovered by designing a dam level controller in the Dymola test bench. According to the demonstration it addresses a problem of the feedback control system design, namely the wicket gate angle. As we mentioned in section 3.1.3 the function of the wicket gate system is nonlinear, approximately U-shaped. This results in a non-linear control system which is very complex to design. The non-linear characteristic of the wicket gate is one of the main issues for the controller design for the hydropower plant. A key finding in this study is that the wicket gate function may be linearized and linear control theory may be
applied for the system. This facilitates the usage of well known controllers, as in this study where a PI controller has been utilized. It should be noted that, due to the constraints of the possible angles of the wicket gate, the control loop may go into saturation if the design is not carefully made. This can be avoided, but with penalties on the step response time, at least when, as in this report, a PI regulator is used. One of the key conditions for the system is that there exists an optimal angle of the wicket gate. There are many CFD, Computational Fluid Dynamics, studies about the wicket gate and the model in this report is using the results from these studies in order to find this optimum. It should also be noted that the technical complexity behind the wicket gate modelling is greater than the description in this report.

Further work of this thesis is recommended to extend this object oriented dynamic process model with more components, like the generator, and to correlate this model to a real world situation. In the meanwhile an optimal control system design in order to develop a high efficient Kaplan turbine model is the final target.
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