On Optimal Energy-Efficient Multi-User MIMO*

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Abstract—Energy efficiency is becoming increasingly important for mobile devices because battery technology has not kept up with the growing demand of ubiquitous multimedia communications. Since multi-user multiple-input multiple-output (MU-MIMO) is a key technology in next-generation wireless communications, this paper addresses optimal energy-efficient design for MU-MIMO. The energy efficiency is measured by a classic metric, “throughput per Joule”, while both RF transmit power and device electronic circuit power are considered. We define the energy efficiency (EE) capacity for MU-MIMO and study the power allocation that achieves this capacity. We show that user antennas should be used only when the corresponding subchannels are sufficiently good and using them improves the overall network EE. Based on theoretical analysis, we further develop low-complexity yet globally optimal energy-efficient power allocation algorithms that converge to the optimum exponentially. Finally comprehensive simulation results are provided to demonstrate the significant gain in network energy efficiency.

Index Terms— energy efficiency, multi-user MIMO, power allocation, SDMA

I. INTRODUCTION

Energy efficiency is becoming increasingly important for mobile devices because battery technology has not kept up with the growing demand of ubiquitous multimedia communications [1]. In addition to energy saving, energy-efficient communications have the benefit of reducing interference to other co-channel users as well as lessening environmental impacts, e.g., heat dissipation and electronic pollution. Therefore, recent research has focused on energy-efficient wireless communication techniques [2]–[6]. When the transmission bandwidth approaches infinity, the minimum received signal energy per bit for reliable communication over additive white Gaussian noise (AWGN) channels approaches -1.59 dB [2]. Energy dissipation of both transmitter circuits and radio-frequency (RF) output is investigated in [7], where the modulation level is adapted to minimize the energy consumption based on simulation observations. In [3]–[6], optimal energy-efficient orthogonal frequency-division multiple access (OFDMA) is designed to balance the circuit power consumption as well as the transmit power consumption on all OFDM subchannels. Furthermore, it is shown in [8], [9] that energy-efficient power control in multi-cell networks improves not only energy efficiency but also spectral efficiency uniformly for all users because of the conservative nature of power optimization, which reduces other-cell interference to improve the overall network throughput. On the other hand, multiple-input multiple-output (MIMO) has been a key technology for wireless systems

because of its potential to achieve high capacity, increased diversity, and interference suppression. In a multi-user scenario, multi-user multiple-input multiple-output (MU-MIMO) systems can provide a substantial gain in networks by allowing multiple users to communicate in the same frequency and time slots [10], [11]. MU-MIMO takes the advantage of both high capacity achieved with MIMO processing and the benefits of space-division multiple access and has been accepted by major wireless standards like IEEE 802.16m and 3GPP Long Term Evolution (LTE). While there has been extensive research on improving the spectral efficiency of MU-MIMO, little effort in literature is focused on energy-efficient MU-MIMO systems.

In this paper, we address the energy-efficient design of MU-MIMO. We account for both circuit and transmit powers when designing power allocation schemes and emphasize energy efficiency over peak rates or throughput. The proposed scheme balances the energy consumption of circuit operations and RF transmissions of all users to achieve the maximum network energy efficiency, which is defined as the number of bits transmitted per Joule of energy across the whole network. We demonstrate the existence of a unique globally optimal power allocation that achieves the energy efficiency capacity. We also provide a one-dimensional algorithm to obtain this optimum.

The rest of the paper is organized as follows. In Section II, we formulate the problem and define energy-efficient MU-MIMO. In Section III and IV, we investigate the optimal condition and develop an algorithm to obtain the globally optimal solution. Simulation results are provided in Section V to demonstrate the performance improvement. Finally, we conclude the paper in Section VI.

Fig. 1: System Diagram of a Multi-User System

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II. ENERGY-EFFICIENT MU-MIMO

In this section, we introduce energy-efficient MU-MIMO. Throughout the paper, denote matrices by capital boldface letters, vectors by lowercase boldface, and scalars by either upper or lowercase letters without boldface.

Consider a MU-MIMO system, as illustrated in Fig. 1, where one access point (AP) is serving K users. Both the AP and all users desire energy-efficient communications. The AP has N antennas. User i has k_i antennas and $\sum_{i=1}^K k_i \leq N$. The channel between the AP and users are predetermined earlier through either training pilots as in a time-division duplex system or a feedback channel as in a frequency-division duplex system. In a flat-fading propagation environment, the received signal at the AP is denoted by

$$\mathbf{y} = \mathbf{H} \cdot \mathbf{Q} \cdot \mathbf{P} \cdot \mathbf{x} + \mathbf{n} = \sum_{i=1}^K \mathbf{H}_i \cdot \mathbf{Q}_i \cdot \mathbf{P}_i \cdot \mathbf{x}_i + \mathbf{n}, \quad (1)$$

where $\mathbf{y} = [y_1, y_2, \dots, y_N]^T$. $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{ik_i}]^T$, is the transmitted symbols of User i and $E[|x_{ij}|^2] = 1$, where E is the expectation. Here $\mathbf{x}_i^T$ is the transpose of a vector. $\mathbf{P}_i = \text{diag}\{\sqrt{p_{i1}}, \sqrt{p_{i2}}, \dots, \sqrt{p_{ik_i}}\}$ is the power allocation matrix of User i . $\mathbf{Q}_i$ is the precoding matrix of User i . $\mathbf{H}_i$ is the $N \times k_i$ channel matrix of User i and is assumed to have rank k_i . $\mathbf{n}$ is the length- N noise vector, which is Gaussian distributed with a zero mean and a covariance matrix $\sigma^2 \mathbf{I}_N$, where $\mathbf{I}_N$ is the identity matrix of size N .

$$\begin{aligned} \mathbf{x} &= [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_K]^T, \\ \mathbf{P} &= \text{diag}\{\mathbf{P}_1, \mathbf{P}_2, \dots, \mathbf{P}_K\}, \\ \mathbf{Q} &= \text{diag}\{\mathbf{Q}_1, \mathbf{Q}_2, \dots, \mathbf{Q}_K\}, \end{aligned}$$

and

$$\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_K].$$

With a linear detector, the decision vector for the transmitted symbols is

$$\hat{\mathbf{x}} = \mathbf{w} \cdot \mathbf{y} = \mathbf{w} \cdot \mathbf{H} \cdot \mathbf{Q} \cdot \mathbf{P} \cdot \mathbf{x} + \mathbf{w} \cdot \mathbf{n}. \quad (2)$$

Using singular value decomposition (SVD),

$$\mathbf{H}_i = \mathbf{U}_i \begin{bmatrix} \mathbf{\Lambda}_i \\ \mathbf{0} \end{bmatrix} \mathbf{V}_i^H = [\dot{\mathbf{U}}_i \ddot{\mathbf{U}}_i] \begin{bmatrix} \mathbf{\Lambda}_i \\ \mathbf{0} \end{bmatrix} \mathbf{V}_i^H = \dot{\mathbf{U}}_i \mathbf{\Lambda}_i \mathbf{V}_i^H, \quad (3)$$

where $\mathbf{U}_i$ and $\mathbf{V}_i$ are $N \times N$ and $k_i \times k_i$ unitary matrices. $\dot{\mathbf{U}}_i$ consists of the first k_i columns of $\mathbf{U}_i$.

$$\mathbf{\Lambda}_i = \text{diag}\{\lambda_{i1}, \lambda_{i2}, \dots, \lambda_{ik_i}\}$$

where $\lambda_{ij} \geq 0$.

With local channel knowledge $\mathbf{H}_i$, User i sets the precoding matrix $\mathbf{Q}_i = \mathbf{V}_i$. Denote

$$\mathbf{U} = [\dot{\mathbf{U}}_1, \dot{\mathbf{U}}_2, \dots, \dot{\mathbf{U}}_K]$$

and

$$\mathbf{\Lambda} = \text{diag}\{\mathbf{\Lambda}_1, \mathbf{\Lambda}_2, \dots, \mathbf{\Lambda}_K\}.$$

It is easy to see the decision vector at the AP is

$$\hat{\mathbf{x}} = \mathbf{w} \cdot \mathbf{U} \cdot \mathbf{\Lambda} \cdot \mathbf{P} \cdot \mathbf{x} + \mathbf{w} \cdot \mathbf{n}. \quad (4)$$

While there are many ways of designing the linear receiver $\mathbf{w}$, we focus on the zero-forcing receiver [12], i.e.,

$$\mathbf{w} = (\mathbf{U}^H \mathbf{U})^{-1} \mathbf{U}^H, \quad (5)$$

for its simplicity. Then the decision vector is

$$\hat{\mathbf{x}} = \mathbf{\Lambda} \cdot \mathbf{P} \cdot \mathbf{x} + \hat{\mathbf{n}}, \quad (6)$$

where $\hat{\mathbf{n}} = (\mathbf{U}^H \mathbf{U})^{-1} \mathbf{U}^H \cdot \mathbf{n}$, which is also Gaussian distributed with a zero mean and a covariance matrix

$$E[\hat{\mathbf{n}} \hat{\mathbf{n}}^H] = \sigma^2 [(\mathbf{U}^H \mathbf{U})^{-1}]^H, \quad (7)$$

with all elements in the diagonal to be σ^2 .

From (6), the transmissions of different users are uncoupled. The AP can detect each symbol independently and the achieved *signal-to-noise ratio* (SNR) of all the symbols for User i is

$$\eta_i = \left[\frac{p_{i1} \lambda_{i1}^2}{\sigma^2}, \frac{p_{i2} \lambda_{i2}^2}{\sigma^2}, \dots, \frac{p_{ik_i} \lambda_{ik_i}^2}{\sigma^2} \right]^T. \quad (8)$$

Given the transceiver structure and the channel state, each user determines the optimal data rate and power on each antenna. Denote the data rate vector of User i , $\mathbf{R}_i = [r_{i1}, r_{i2}, \dots, r_{ik_i}]$. Correspondingly, the overall data rate is

$$R_i = \sum_{k=1}^{k_i} r_{ik}. \quad (9)$$

Denote B as the system bandwidth. The achievable data rate r_{ik} is determined by [13]

$$r_{ik} = B \log(1 + \frac{\eta_{ik}}{\Gamma}), \quad (10)$$

where $\eta_{ik} = \frac{p_{ik} \lambda_{ik}^2}{\sigma^2}$ and Γ is the SNR gap that defines the gap between the channel capacity and a practical coding and modulation scheme. The SNR gap depends on the modulation, coding, and the target probability of error. For a coded *quadrature amplitude modulation* (QAM) system, the gap is given by [13]

$$\Gamma = 9.8 + \gamma_m - \gamma_c \text{ (dB)}, \quad (11)$$

where γ_m is the system design margin and γ_c is the coding gain. For Shannon capacity [14], $\Gamma = 0$ dB. Denote the overall transmit power of User i to be P_{Ti} and

$$P_{Ti} = \frac{\sum_{k=1}^{k_i} p_{ik}}{\zeta}, \quad (12)$$

where $\zeta \in [0, 1]$ is the power amplifier efficiency and depends on transmitter design and implementation.

In addition to transmit power, mobile devices also incur additional circuit power consumption owing to inevitable electronic operations which is relatively independent of the radio frequency (RF) transmission [7], [15]. While the transmit power models all the power used for reliable data transmission, the circuit power represents the average energy consumption of operating device electronics, such as mixers, filters, and digital-to-analog converters, and this portion of energy consumption excludes that of the power amplifier and is

independent of the transmission state. Denote the circuit power of User i as P_{Ci} , the overall power consumption of User i will be

$$P_i = P_{Ci} + P_{Ti}. \quad (13)$$

In addition to the energy consumption at the user side, the AP also consumes electronic circuit energy to receive and decode signals. Denote the receiving power to be P_r . Similar to the circuit power, P_r models the average energy consumption of AP device electronics, such as mixers, filters and analog-to-digital converters.

It is desirable to maximize the amount of data sent with a given amount of energy. The amount of energy Δe consumed in a small duration, Δt , is

$$\Delta e = \Delta t \left(\alpha \sum_i P_i + \beta P_r \right),$$

where the weights $\alpha \in [0, 1]$ and $\beta \in [0, 1]$ characterize the priorities of transmitter and receiver power consumptions. For example $\alpha = 1$ and $\beta = 0$ indicate that the receiver power consumption is not considered.

The MU-MIMO system wants to send a maximum amount of data by choosing the optimal power allocation to maximize

$$\frac{\sum_i R_i \Delta t}{\Delta e}, \quad (14)$$

which is equivalent to maximizing

$$U(\mathbf{P}) = \frac{\sum_i R_i}{\alpha \sum_i P_i + \beta P_r}. \quad (15)$$

U is called the energy efficiency of MU-MIMO. The unit of the energy efficiency is bits per Joule, which has been frequently used in literature for energy-efficient communications [2], [16]–[19]. With metric (15), the energy used for sending each information bit is minimized. The energy efficiency capacity of MU-MIMO is defined as

$$U^* = \max_{\mathbf{P}} \frac{\sum_i R_i}{\alpha \sum_i (P_{Ti} + P_{Ci}) + \beta P_r}, \quad (16)$$

and the optimal energy-efficient power allocation achieving the energy efficiency capacity is

$$\mathbf{P}^* = \arg \max_{\mathbf{P}} U = \arg \max_{\mathbf{P}} \frac{\sum_i R_i}{\alpha \sum_i (P_{Ti} + P_{Ci}) + \beta P_r}. \quad (17)$$

Note that when $K = 1$, (16) and (17) give the energy efficiency capacity and the optimal power allocation for a point-to-point MIMO system. Therefore the results in this paper are also applicable to MIMO systems.

III. PRINCIPLES OF ENERGY-EFFICIENT MU-MIMO POWER ALLOCATION

In the following, we demonstrate that a unique globally optimal power allocation always exists and give the necessary and sufficient conditions for a power allocation scheme to achieve the energy efficiency capacity.

The concept of quasi-concavity is defined as [20].

Definition 1. A function f , which maps from a convex set of real n -dimensional vectors, $\mathcal{D}$, to a real number, is called

strictly quasi-concave if for any $\mathbf{x}_1, \mathbf{x}_2 \in \mathcal{D}$ and $\mathbf{x}_1 \neq \mathbf{x}_2$,

$$f(\lambda \mathbf{x}_1 + (1 - \lambda) \mathbf{x}_2) > \min\{f(\mathbf{x}_1), f(\mathbf{x}_2)\}, \quad (18)$$

for any $0 < \lambda < 1$.

Any strictly concave function is strictly quasi-concave but the reverse may not be true. An example is given in Fig. 2.

Fig. 2: An example of strictly quasi-concave function.

It is proved in Appendix I that U has the following properties.

Lemma 1. U is strictly quasi-concave in $\mathbf{P}$.

Fig. 2 is indeed an example of U when $\mathbf{P}$ is a 2×2 diagonal matrix.

For a strictly quasi-concave function, if a local maximum exists, it is also globally optimal [20]. Hence, a unique globally optimal power allocation always exists and is summarized in Theorem 1 according to the proof in Appendix I.

Theorem 1. There exists a unique globally optimal energy-efficient power allocation $\mathbf{P}^*$ that achieves the energy efficiency capacity, where p_{ik}^* is given by

$$p_{ik}^* = \begin{cases} \frac{B\zeta}{\alpha U^*} - \frac{\Gamma \sigma^2}{\lambda_{ik}^2} & \text{if } \frac{B\zeta \lambda_{ik}^2}{\alpha \Gamma \sigma^2} > U^*, \\ 0 & \text{otherwise,} \end{cases} \quad (19)$$

Correspondingly, the energy efficiency capacity is

$$U^* = U(\mathbf{P}^*). \quad (20)$$

Theorem 1 says that the k th antenna of User i should be used only when the corresponding subchannel, characterized by λ_{ik}^2 , is sufficiently good such that using it improves the overall network energy efficiency. The power allocation is indeed a water-filling solution and the water level is determined by the energy efficiency capacity. The relative difference of power allocation of different users on different antennas depends on the channel gains of those subchannels.

Based on Theorem 1, we have the following basic properties of power allocation.

Proposition 1. The energy efficiency capacity decreases strictly, while the optimal allocated power on each subchannel,

if nonzero, increases strictly with the circuit power of any user.

Proof: Denote $\mathbf{P}^*$ to be the optimal power allocation given a set of circuit power conditions $\{P_{C_i}\}$. The achieved energy efficiency is U^* . Suppose any P_{C_i} decreases a certain amount to be $P_{C_i} - \Delta P_C$. With the same power allocation $\mathbf{P}^*$, the energy efficiency is higher than U^* . Hence the energy efficiency capacity increases. Therefore energy efficiency capacity decreases strictly with circuit power. Furthermore, according to (19), the optimal power on each subchannel, if nonzero, decreases strictly with energy efficiency capacity. The proposition follows immediately. ■

The main intuition behind Proposition 1 is that as circuit power increases, higher power should be allocated to achieve higher data rate such that each information bit can be transmitted faster and less circuit energy is consumed. Similarly we have Proposition 2.

Proposition 2. *When receiving power is considered ($\beta > 0$), the energy efficiency capacity decreases strictly, while the optimal allocated power on each subchannel, if nonzero, increases strictly with the receiving power.*

IV. A ONE-DIMENSIONAL LOW COMPLEXITY ALGORITHM

Theorem 1 provides the necessary and sufficient condition for a power allocation to be the unique and globally optimum one. However, it is difficult to directly solve the joint nonlinear equations in Theorem 1. Therefore, we develop an iterative method to search the optimal $\mathbf{P}^*$ based on the analysis of the optimal power allocation in Theorem 1.

Denote $p_{ik}(\mu) = \left[\mu - \frac{\Gamma\sigma^2}{\lambda_{ik}^2} \right]^+$, where $[x]^+ = \max(x, 0)$, and the corresponding power allocation matrix to be $\mathbf{P}(\mu)$. Clearly when $\mu = \frac{B\zeta}{\alpha U^*}$, $\mathbf{P}(\mu) = \mathbf{P}^*$. Denote

$$f(\mu) = U(\mathbf{P}(\mu)) \quad (21)$$

and it is easy to see that the optimal μ^* that maximizes $f(\mu)$ equals $\frac{B\zeta}{\alpha U^*}$. Therefore we only need to find μ^* such that

$$\mu^* = \arg \max_{\mu} f(\mu). \quad (22)$$

As shown in Appendix II, when $f(\mu) > 0$, $f(\mu)$ is strictly quasi-concave in μ . Hence a unique globally optimal μ^* exists such that for any $\mu < \mu^*$, $f'(\mu) > 0$, and for any $\mu > \mu^*$, $f'(\mu) < 0$. Assume $\mu_1 \leq \mu^* \leq \mu_2$. To determine μ^* , let $\hat{\mu} = \frac{\mu_1 + \mu_2}{2}$. If $f'(\mu)|_{\hat{\mu}} = 0$, μ^* is found. If $f'(\mu)|_{\hat{\mu}} < 0$, then $\mu_1 < \mu^* < \hat{\mu}$ and replace μ_2 with $\hat{\mu}$; otherwise, replace μ_1 with $\hat{\mu}$. This iteration continues until in one iteration, $\mu_2 - \mu_1$ is sufficiently small to meet the convergence requirement. This energy-efficient MU-MIMO power allocation (EMMPA) algorithm is summarized in Table I.

The global convergence to the optimal power allocation of EMMPA is guaranteed by the strict quasi-concavity of $f(\mu)$ [21] and it can be easily proved that the convergence rate is characterized by Proposition 3.

Proposition 3. *EMMPA converges to the globally optimal μ^* . Any μ , which satisfies $|\mu - \mu^*| \leq \epsilon$, can be found within at most $\lceil \log_2 \left(\frac{(\alpha-1)\mu^*}{\epsilon} - 1 \right) \rceil$ iterations.*

Algorithm Energy-Efficient MU-MIMO Power Allocation

1. $\mu_1 \leftarrow \min_{i,j} \frac{\Gamma\sigma^2}{\lambda_{ij}^2}$, $\alpha \leftarrow 10$
2. $\mu_2 \leftarrow \mu_1 * \alpha$
3. **while** $f'(\mu_2) > 0$
4. **do** $\mu_1 \leftarrow \mu_2$, $\mu_2 \leftarrow \mu_2 * \alpha$
5. **while** no convergence
 (* search the optimum iteratively *)
6. **do** $\mu \leftarrow \frac{\mu_2 + \mu_1}{2}$;
7. **if** $f'(\mu) > 0$
8. **then** $\mu_1 \leftarrow \mu$;
9. **else** $\mu_2 \leftarrow \mu$
10. **return** μ and $p_{ik} \leftarrow \left[\mu - \frac{\Gamma\sigma^2}{\lambda_{ik}^2} \right]^+$.

TABLE I: Energy-Efficient MU-MIMO Power Allocation

V. SIMULATION RESULTS FOR ENERGY-EFFICIENT MU-MIMO

The proposed energy-efficient MU-MIMO can be applied in different types of wireless networks to improve the network energy efficiency. In this section, we provide simulation results for a single-cell cellular network to demonstrate the performance of energy-efficient MU-MIMO. System parameters are listed in Table II. In each trial, users are dropped uniformly within 250 meters from the AP. The performance below is the average over all trials. Each user consumes a fixed amount, 100 mW, of circuit power.

Fig. 3 gives the average energy efficiency capacity when there are two users in the network and each user has 1, 2, 3, or 4 antennas. The average energy efficiency capacity is an average of multiple user droppings and channel realizations. The number of AP antennas is varied to observe its impact on energy efficiency capacity. On the other hand, Fig. 4 compares the average energy efficiency capacity when the AP has 64 antennas. We can see that without circuit management, more users and more antennas always help improve the energy efficiency capacity of MU-MIMO. Fig. 5 compares the energy efficiency of EMMPA and that of the fixed power allocation (FPA). With the fixed power allocation, each user employs a fixed amount of transmit power, given by the value in the x axis, and allocates it equally on all subchannels. Two scenarios are considered. In one scenario, there are four users in the network, each with two antennas, and the AP has eight antennas. In the second scenario, there is only one user with four antennas in the network and the AP has eight antennas. As shown in Fig. 5, significant gain in energy efficiency can be observed by using EMMPA. The gain is even larger when there are multiple users because EMMPA effectively exploits multi-user diversity in the network to improve energy efficiency.

VI. CONCLUSION

In this paper, we investigated the optimal energy-efficient MU-MIMO. Both electronic circuit and RF transmit power consumptions are considered. We first analyzed an MU-MIMO system based on distributed SVD decomposition of the channels of all users and derived the achieved SNR conditions for all users. Then we proposed the concept of energy-efficient MU-MIMO and defined the energy efficiency capacity for

Fig. 3: Relationship between energy efficiency capacity, transmit antennas, and receive antennas.

Fig. 4: Relationship between energy efficiency capacity, users, and transmit antennas.

MU-MIMO. We demonstrated the existence of a uniquely globally optimal power allocation that could achieve this energy efficiency capacity. A one-dimensional low-complexity algorithm was developed to obtain the globally optimal power allocation and this algorithm converges to the global optimum at an exponential speed. Comprehensive simulation results were provided to demonstrate the algorithm performance and the significant gain in energy efficiency for a cellular network.

APPENDIX I PROOF OF LEMMA 1

Proof: According to Section II,

$$U(\mathbf{P}) = \frac{\sum_i R_i}{\alpha \sum_i (P_{Ti} + P_{Ci}) + \beta P_r} = \frac{\sum_{i=1}^K \sum_{k=1}^{k_i} B \log \left(1 + \frac{p_{ik} \lambda_{ik}^2}{\Gamma \sigma^2} \right)}{\alpha \sum_{i=1}^K P_{Ci} + \beta P_r + \frac{\alpha}{\zeta} \sum_{i=1}^K \sum_{k=1}^{k_i} p_{ik}}. \quad (I.23)$$

Fig. 5: Comparison between EMMPA and Fixed Power Allocation (Scenario 1: $N = 8$, $K = 4$, $k_i = 2$; Scenario 2: $N = 8$, $K = 1$, $k_i = 4$).

TABLE II: Simulation Parameters

Carrier frequency	1.5 GHz
System bandwidth	10 kHz
Thermal noise power, N_o	-141 dBW/MHz
User antenna height	1.6 m
BS antenna height	40 m
Environment	Macro cell in urban area
Receiver power, P_r	1000 mW
Propagation Model	Okumura-Hata model
Shadowing	10 dB lognormal
Fading	Rayleigh flat fading
Power amplifier efficiency, ζ	0.5
SNR gap, Γ	0 dB
α	1
β	1

Denote the upper contour sets of $U(\mathbf{P})$ as

$$S_\mu = \{\mathbf{P} \succeq \mathbf{0} | U(\mathbf{P}) \geq \mu\}, \quad (I.24)$$

where symbol $\succeq$ denotes matrix inequality and $\mathbf{P} \succeq \mathbf{0}$ means each element of $\mathbf{P}$ is nonnegative. According to Proposition C.9 of [20], $U(\mathbf{P})$ is strictly quasi-concave if and only if S_μ is strictly convex for any real number μ . When $\mu < 0$, no points exist on the contour $U(\mathbf{P}) = \mu$. When $\mu = 0$, only $\mathbf{0}$ is on the contour $U(\mathbf{0}) = \mu$. Hence, S_μ is strictly convex when $\mu \leq 0$. Now we investigate the case when $\mu > 0$. S_μ is equivalent to

$$S_\mu = \left\{ \mathbf{P} \succeq \mathbf{0} | \mu \alpha \sum_{i=1}^K P_{Ci} + \mu \beta P_r + \frac{\mu \alpha}{\zeta} \sum_{i=1}^K \sum_{k=1}^{k_i} p_{ik} - \sum_{i=1}^K \sum_{k=1}^{k_i} B \log \left(1 + \frac{p_{ik} \lambda_{ik}^2}{\Gamma \sigma^2} \right) \leq 0 \right\}. \quad (I.25)$$

It is easy to see that S_μ is strictly convex. Hence, we have the strict quasi-concavity of $U(\mathbf{P})$.

It is easy to see that at the local maximum of $U(\mathbf{P})$, which is also the global maximum because of the strict quasi-concavity, all $p_{ik} < \infty$ as otherwise $U(\mathbf{P}) = 0$. Hence at the local maximum, p_{ik} is positive or $p_{ik} = 0$. If p_{ik} is positive, it can be obtained by setting the partial derivative of $U(\mathbf{P})$ with

respect to p_{ik} to be zero, i.e.,

$$\left. \frac{\partial U(\mathbf{P})}{\partial p_{ik}} \right|_{\mathbf{P}=\mathbf{P}^*} = 0, \quad (\text{I.26})$$

and we have

$$p_{ik}^* = \frac{B\zeta}{\alpha U^*} - \frac{\Gamma\sigma^2}{\lambda_{ik}^2}, \quad (\text{I.27})$$

Hence, the unique optimal energy-efficient power allocation is given by

$$p_{ik}^* = \begin{cases} \frac{B\zeta}{\alpha U^*} - \frac{\Gamma\sigma^2}{\lambda_{ik}^2} & \text{if } \frac{B\zeta\lambda_{ik}^2}{\alpha\Gamma\sigma^2} > U^*, \\ 0 & \text{otherwise.} \end{cases} \quad (\text{I.28})$$

Lemma 1 and Theorem 1 are readily obtained. ■

APPENDIX II

PROOF OF QUASI-CONCAVITY OF $f(\mu)$

Proof: Similar to the proof in Appendix I, denote the upper contour sets of $f(\mu)$ as

$$S_\nu = \{\mu > 0 | f(\mu) \geq \nu\}. \quad (\text{II.29})$$

We need to show S_ν is strictly convex for any real number ν , which is obvious when $\nu \leq 0$. When $\nu > 0$, S_ν is equivalent to

$$\begin{aligned} S_\nu = & \left\{ \mu > 0 \left| \nu\alpha \sum_{i=1}^K P_{Ci} + \nu\beta P_r + \frac{\nu\alpha}{\zeta} \sum_{i=1}^K \sum_{k=1}^{k_i} \left[\mu - \frac{\Gamma\sigma^2}{\lambda_{ik}^2} \right]^+ \right. \right. \\ & \left. \left. - \sum_{i=1}^K \sum_{k=1}^{k_i} B \log \left(1 + \frac{\left[\mu - \frac{\Gamma\sigma^2}{\lambda_{ik}^2} \right]^+ \lambda_{ik}^2}{\Gamma\sigma^2} \right) \leq 0 \right\} \\ & \triangleq \{\nu > 0 | M(\mu) \leq 0\}. \end{aligned} \quad (\text{II.30})$$

The second order derivative of $M(\mu)$ is

$$\begin{aligned} M''(\mu) = & -d^2 \sum_{i=1}^K \sum_{k=1}^{k_i} B \log \left(1 + \frac{\left[\mu - \frac{\Gamma\sigma^2}{\lambda_{ik}^2} \right]^+ \lambda_{ik}^2}{\Gamma\sigma^2} \right) / d\mu^2 \\ = & \sum_{i,k: \mu > \frac{\Gamma\sigma^2}{\lambda_{ik}^2}} \frac{B\lambda_{ik}^4}{\left[\Gamma\sigma^2 + \lambda_{ik}^2 \left(\mu - \frac{\Gamma\sigma^2}{\lambda_{ik}^2} \right) \right]}. \end{aligned} \quad (\text{II.31})$$

At least one antenna of one user should have positive power allocation as otherwise $f(\mu) = 0$, which is clearly not optimal. Therefore there exists at least one i and k such that $\mu > \frac{\Gamma\sigma^2}{\lambda_{ik}^2}$. Hence $M''(\mu) > 0$ for all feasible μ of interest and S_ν is strictly convex and we have the strict quasi-concavity of $f(\mu)$ when $f(\mu) > 0$. ■

APPENDIX III

PROOF OF QUASI-CONCAVITY OF $f(\mu)$

Proof: $\frac{\partial f(p_{ij})}{\partial p_{ij}} = -\frac{\lambda_{ij}^2}{\Gamma\sigma^2} \frac{\alpha}{\zeta} \left(R_{ij}^o + B \log(1 + \frac{p_{ij}\lambda_{ij}^2}{\Gamma\sigma^2}) \right) < 0$. Therefore $f(p_{ij})$ is strictly decreasing. It is easy to see that as $p_{ij} \rightarrow +\infty$, $f(p_{ij}) \rightarrow -\infty$. If when $p_{ij} \rightarrow 0$, $f(p_{ij}) > 0$,

then there exists a unique $p_{ij} > 0$ such that $f(p_{ij}) = 0$. Otherwise, Antenna (i, j) should be turned off.

$$\lim_{p_{ij} \rightarrow 0} f(p_{ij}) = B \frac{\lambda_{ij}^2}{\Gamma\sigma^2} (P_{ij}^o + P_{Cu}(k_i^o)) - \frac{\alpha}{\zeta} R_{ij}^o > 0.$$

Therefore, Antenna (i, j) should be turned on when $\frac{\lambda_{ij}^2}{\sigma^2} > \frac{R_{ij}^o \Gamma \alpha}{(P_{ij}^o + P_{Cu}(k_i^o)) B \zeta}$. ■

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