Variable speed drive as an alternative solution for a micro-hydro power plant

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Master Thesis

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Abstract

This diploma work is mainly focused on developing the control strategy for a variable speed drive as an alternative solution to a micro-hydro power plant. The detailed mathematical model for a micro-hydro system including a Kaplan turbine, mechanical shaft and electrical machines is presented and validated through simulations. A control strategy for an autonomous operation of a doubly-fed induction machine-based drive is developed for a wide range of speed. The drive can operate at a unity power factor.

The possible applications of the analyzed system are also presented. As a positive side of the system, it is found that the direct interaction between the power electronic converters and the utility grid can be avoided by exploiting the proposed topology, which might lead to a better quality of the produced power in terms of harmonics. This could also lead to removal or reduction of the size of the harmonic filters that are being used in conventional doubly-fed induction generator installations.

As regards to the drawbacks of the system, a comparison of converter and generator ratings between the analyzed solution and the conventional solution was performed. While the converters rating remain the same, there is one more electrical machine and the doubly-fed generator rating is slightly increased. Losses are also slightly larger due to the presence of the second machine.
Sammanfattning


De möjliga tillämpningarna av det analyserade systemet presenteras också. Som en positiv sida av systemet, har det visat sig att den direkta interaktionen mellan kraft-elektroniska omvandlaren och distributionsnätet kan undvikas genom att utnyttja den föreslagna topologin, vilket kan leda till en bättre kvalité på den producerade effekten med hänsyn till övertoner. Det kan också leda till avlägsnande eller minskning av storleken på de övertonsfiltrer som används i de konventionella installationerna av dubbelmatade asynkrongeneratorer.

Avseende nackdelarna med systemet, gjordes en jämförelse av omvandlarens och generatorstypeffekter mellan den analyserade lösningen och den konventionella lösningen. Medan omvandlarens typeffekt förblir desamma, finns det ytterligare en elektriskmaskin och den dubbelmatade generatoreffekten är något större i den analyserade lösningen. Förluster är också något större på grund av närvaron av den andra maskinen.
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# TABLE OF CONTENTS

1 INTRODUCTION .......................................................................................................................... 1

1.1 BACKGROUND .......................................................................................................................... 2

1.2 PURPOSE OR OBJECTIVE ......................................................................................................... 3

1.3 STRUCTURE ............................................................................................................................... 4

1.4 DEFINITIONS ............................................................................................................................. 5

1.4.1 Symbols .................................................................................................................................. 5

1.4.2 Abbreviations ......................................................................................................................... 7

1.5 GLOSSARY ................................................................................................................................ 8

2 WORKING, ADVANTAGES AND APPLICATIONS ................................................................... 9

2.1 WORKING OF A DFIG ........................................................................................................... 9

2.1.1 Sub-synchronous mode ......................................................................................................... 10

2.1.2 Super-synchronous mode ..................................................................................................... 10

2.2 DFIG IN A TRADITIONAL DRIVE SYSTEM .......................................................................... 11

2.3 TOPOLOGY UNDER INVESTIGATION ............................................................................... 12

2.4 ADVANTAGES .......................................................................................................................... 14

2.5 APPLICATIONS ........................................................................................................................ 15

3 MATHEMATICAL MODELING OF THE SYSTEM .................................................................. 16

3.1 TURBINE MODEL ...................................................................................................................... 16

3.1.1 Impulse turbines .................................................................................................................... 16

3.1.2 Reaction turbines .................................................................................................................. 16

3.1.3 Kaplan turbine ...................................................................................................................... 16

3.1.4 Torque and speed calculations .............................................................................................. 17

3.1.5 HYGOV model ..................................................................................................................... 19

3.2 MECHANICAL SHAFT ............................................................................................................. 21

3.2.1 Drive train model ................................................................................................................. 21

3.2.2 Geared drive train ............................................................................................................... 21

3.2.3 Direct drive train ................................................................................................................ 22

3.2.4 The mechanical drive train model ....................................................................................... 22

3.2.5 Drive train resonance ......................................................................................................... 25

3.2.6 Harmonic modes ................................................................................................................. 25

3.2.7 Damping matrix D .............................................................................................................. 26

3.3 PERMANENT MAGNET SYNCHRONOUS MACHINE ....................................................... 27

3.3.1 Torque and power calculation for a PMSM machine ......................................................... 29

3.4 DFIG MODEL .......................................................................................................................... 30

3.4.1 Torque and power equations ............................................................................................... 34

3.4.2 Stator terminal power factor ............................................................................................... 35

3.5 DC-LINK MODEL ..................................................................................................................... 36

4 CONTROL DEVELOPMENT ................................................................................................. 38

4.1 TURBINE GOVERNING SYSTEM ......................................................................................... 38

4.2 TORQUE CONTROL FOR A PMSM .................................................................................. 40

4.3 DFIG CONTROL ...................................................................................................................... 41

4.3.1 Field oriented control for a DFIG ....................................................................................... 42

4.3.2 Field orientation .................................................................................................................. 42

4.3.3 Calculating transformation or stator flux angle ................................................................. 43

4.3.4 Reference signals .............................................................................................................. 45

4.3.5 Current controllers ............................................................................................................. 47

4.3.6 Cross-coupling terms ....................................................................................................... 47

4.3.7 Complete control system ................................................................................................. 47

4.4 BACK-TO-BACK PWM CONVERTERS AND SWITCHING PULSES ......................... 48

4.4.1 Sinusoidal PWM ............................................................................................................... 50

4.4.2 Space vector modulation SVM ......................................................................................... 52

4.5 DC-LINK VOLTAGE CONTROL ....................................................................................... 60

5 SIMULATION RESULTS ........................................................................................................ 62
5.1 CASE 1: SUB-SYNCHRONOUS MODE ............................................................. 62
5.2 CASE 2: SUPER-SYNCHRONOUS MODE .................................................... 70

6 VSD COMPONENT RATING COMPARISON ..................................................... 78
   6.1 BACK-TO-BACK PWM CONVERTER RATING ........................................... 78
   6.2 PMSM (EXCITER MACHINE) SIZE ......................................................... 80
   6.3 DFIG SIZE AND CONSIDERATION ON LOSSES ................................. 81
      6.3.1 Sub-synchronous mode ................................................................. 81
      6.3.2 Super-synchronous mode ............................................................... 81
   6.4 HARMONIC FILTERS ............................................................................. 81

7 CONCLUSIONS ............................................................................................... 86

8 FUTURE WORK .............................................................................................. 87

9 REFERENCES ................................................................................................... 88

10 APPENDICES .................................................................................................. 92
   10.1 APPENDIX A ......................................................................................... 92
   10.2 APPENDIX B ......................................................................................... 93
   10.3 APPENDIX C ......................................................................................... 96
1 INTRODUCTION

Hydroelectricity or hydro energy is one of the highly accepted form of energy source amongst the other renewable energy sources. The produced energy unit is cheaper compared to the other sources of energy such as coal or nuclear power plants [1]. It is also environment friendly as it does not emit carbon dioxide. It is the most widely used renewable source of energy as it gives 16% of the world electricity consumption [1] and contribute by 94% of the world's total renewable energy production [2].

Depending upon the amount of power produced the hydro power plants can be classified as large, small and micro-hydro power plants. There is no strict criterion for differentiating the size of the power plants and it also varies from country to country. Generally the power plants that have a capacity of 100’s of MW or 10’s of GW are classified as large hydro plants. The power plant that can produce 10’s of MW is considered as small and the range of a micro-hydro power plant is between 10’s of KW to 100’s of KW.

Usually for a large hydro power plant, a massive civil work is required for the construction of water reservoir that demands a large initial investment. Constructing a dam often causes the environmental and ecosystem changes like shifting of inhabitants. Micro-hydro power plants on the other hand are usually a “run-of-river” plants offering a cost effective solution as it requires less civil work and in most cases does not require any water reservoir (dam).

The European Union took the decision to increase the green electricity production from the renewable energy sources (wind, hydro, photovoltaic etc.) by 2020 in order to reduce the production of carbon dioxide. The European strategy named as 20-20-20 which requires that by 2020 each country must produce at least 20% of its energy through renewable energy sources [3]. Due to this strategy most of the countries (like Austria, Poland and Romania) are now utilizing their unused small or micro-hydro sites [3]. The major contribution from the hydro power to date is through large power stations which were built in last century. However nowadays, most of the large hydro sites in European and North American countries have been utilized and the ones which are available cannot be utilized because of the environmental concerns [2]. On the other hand, many micro-hydro sites are still available that need to be exploited and can be utilized in an efficient way for energy production.

In Europe the installed micro-hydro capacity ranges about 11500 MW sharing 1.7% in electricity production and 10% of total hydroelectricity [2]. European Small Hydropower Association (ESHA) on March 1, 2012 stated that there are many small hydro power sites available in all over the Europe and only less than half of these have been utilized [4]. The utilization of these micro-hydro sites can play an important role in the future as the fuel prices will increase further.

About 68% of the existing micro-hydro plants in Europe are based on ancient structures involving classical electromechanical sets [5]. The operation of the existing micro-hydro plants is based on fixed speed drives. Depending upon the load
and a governing system, a portion of the flow is allowed to pass through the turbine for regulating the frequency, thus only a portion of the total available power is being utilized [5].

1.1 Background

During the first half of the 20th century, fixed speed drives were commonly used in the generation system. The speed of the system was maintained constant using the governing system. In that way only the portion of the total available energy could be utilized resulting in the lower efficiency. With the progress in power electronic converters, the technology is shifting towards the variable speed drives with the aim of capturing the maximum available energy [6].

The variable speed drive can be obtained in several different ways. One of which is to use a direct drive system having no gear-box in the system. Usually the speed of a turbine is low (10’s of rad/sec) thus requires a bulky or heavy synchronous machine with large number of poles. This kind of drive requires a full rated power electronics converter at the stator of the machine thus increase the cost of the system. The variable speed drive can also be obtained using the squirrel cage induction machine as a generator. Such kind of a variable speed drive is shown in Figure 1.1.

![Variable speed drive using a synchronous generator.](image)

A variable speed drive can also be obtained using a gear-box called as geared drive train shown in Figure 1.2. This system consists of a wound rotor induction generator (DFIG) whose stator winding is directly connected to the transformer or the grid whereas the rotor winding is connected to the utility grid through the slip rings using a back-to-back PWM converter. This system has been widely accepted in the recent years because only the portion of the total power flows through the power electronic converter compared to the direct drive system. Usually 20-30% of the total power flows through the rotor. Hence the converters are rated for smaller power resulting in lower losses across the power electronic devices compared to the system having full rated power electronics. It also reduces the cost and the volume of the system. Therefore in this thesis work, variable speed drive for a micro-hydro generation system using a DFIG is studied.
Using a DFIG in a variable speed drives (VSD) reduces the power electronics rating but it has some complexities. It requires the excitation from external source which in most of the cases comes from the grid that makes the system to appear as a load on the grid. Therefore the operation of this kind of VSD also relates to the stability of the grid and will be affected if the fault occurs on the grid side. Because of that several different protections are also required in the drive system. For controlling the machine operation, a DFIG is supplied through the rotor slip rings and it requires the maintenance of these rotor slip rings at regular intervals of time. Mostly the applications of such kind of drives are in the rural or remote areas (offshore wind farm or micro-hydro application in far mountainous areas) making the maintenance work difficult and expensive.

Much of the research has been conducted in an area of VSD using a DFIG and resulted in several improvements. Issues like obtaining the excitation from the grid and slip ring maintenance has been a point of concern for the researchers over the last decade. Several efforts have been made in the past for developing the autonomous operation of a variable speed drive using a DFIG. Also there are several efforts for removing the rotor slip rings and avoiding the maintenance work. The research work presented in [2] and [6] shows promising results claiming the autonomous operation. In [6] an effort for removing the slip rings is also presented.

1.2 Purpose or objective

This thesis work is mainly focused on modeling and simulating a variable speed drive for a micro-hydro generation system. It is an effort for developing a variable speed drive as an alternative solution to the micro-hydro generation system offering the autonomous or independent operation with improved power quality. The main objectives of the entire work can be described as:

- To investigate about the advantages and application areas for the studied variable speed drive.
- To develop the mathematical model for observing the dynamics and the steady state operation of an autonomous micro-hydro generation system having variable speed drive. The objective is to model the hydro turbine, drive train, electrical machines and power electronic converters.
To develop the control strategy for an independent or autonomous variable speed drive (VSD) system producing power at unity power factor.

Performing the simulations and developing the strategy for efficient energy conversion over a wide range of speed.

To perform the comparison between the studied variable speed drive and the existing variable speed drives regarding the drive system components ratings and output power quality.

1.3 Structure
This section represents the structure or layout of the report.

Chapter 1: This chapter explains the background, purpose and scope of the report.

Chapter 2: This section explains the working of the autonomous or independent variable speed drive. As it is a newly developed scheme, so it is important to elaborate the basic operation of the drive. This section also describes the applications and advantages of the studied variable speed drive.

Chapter 3: In this chapter mathematical models for a micro-hydro generation system including a hydro turbine, drive train, electrical machines and converters are presented.

Chapter 4: Explains the control scheme or strategy for the system. It includes topics like control algorithms for wound rotor induction generator, permanent magnet machine, DC-link voltage control etc.

Chapter 5: Presents the simulation results and verified the developed control scheme.

Chapter 6: In this chapter a comparison is performed between the studied variable speed drive and the existing variable speed drives.

Chapter 7: Some conclusions drawn from the work are presented in this section.

Chapter 8: This section describes the future challenges and the work that need to be done in the future.
1.4 Definitions

1.4.1 Symbols

\( P_s \)  
DFIG stator active power

\( P_r \)  
DFIG rotor active power

\( s \)  
Slip

\( P_m \)  
Input mechanical power

\( \omega_s \)  
Electrical synchronous speed

\( Q_s \)  
DFIG stator reactive power

\( \omega_r \)  
Rotor electrical speed

\( T_m \)  
Input mechanical torque

\( P_{hydro} \)  
Available hydro power

\( Q \)  
Water flow

\( H \)  
Available water head

\( \rho \)  
Density of water

\( \delta \)  
Angle between guide vane and stay vane

\( P_{out} \)  
Output power of turbine

\( A \)  
Area of runner

\( v \)  
Velocity of water

\( D_t \)  
Diameter of runner blade at tip

\( D_h \)  
Diameter of runner blade at hub

\( v_f \)  
Flow velocity

\( x \)  
Flow ratio

\( k_u \)  
Velocity ratio

\( u \)  
Linear velocity

\( \omega \)  
Anglular velocity of shaft

\( F_{net} \)  
Net force on the fluid

\( L \)  
Length of penstock

\( G \)  
Gate position

\( q_{nl} \)  
Per-unit no load water flow

\( q \)  
Per unit water flow

\( A_t \)  
Turbine gain constant

\( D_{turb} \)  
Damping effect of a turbine

\( T_w \)  
Water starting time

\( T_e \)  
Electrical or load torque

\( T_{tg} \)  
Torque at shaft

\( \omega_g \)  
Angular speed of rotor

\( H_g \)  
Generator rotor inertia constant

\( H_t \)  
Turbine inertia constant

\( D_g \)  
Damping coefficient of generator

\( D_t \)  
Damping coefficient of turbine

\( D_{tg} \)  
Damping coefficient of shaft

\( K_{tg} \)  
Stiffness coefficient of shaft

\( \varphi_{tg} \)  
Shaft twist angle

\( N \)  
Gear ratio

\( f_{res} \)  
Resonance frequency

\( \phi_1, \phi_2 \)  
Eigen vectors
\( u_s \)  
Stator voltage vector

\( i_s \)  
Stator current vector

\( \psi_s \)  
Stator flux vector

\( u_{sd} \)  
d-axis stator voltage

\( u_{sq} \)  
q-axis stator voltage

\( R_s \)  
PMSM stator resistance

\( i_{sd} \)  
d-axis stator current

\( i_{sq} \)  
q-axis stator current

\( \psi_{sd} \)  
d-axis stator flux

\( \psi_{sq} \)  
q-axis stator flux

\( \psi_m \)  
Flux linkage between stator and rotor of a PMSM

\( L_{sd} \)  
d-axis stator inductance

\( L_{sq} \)  
q-axis stator inductance

\( p \)  
Number of pole pairs

\( p_s \)  
Spatial stator active power

\( q_s \)  
Spatial stator reactive power

\( r_s \)  
DFIG stator resistance

\( r_r \)  
DFIG rotor resistance

\( u_r \)  
Rotor voltage vector

\( u_{rd} \)  
d-axis rotor voltage

\( u_{rq} \)  
q-axis rotor voltage

\( i_r \)  
rotor current vector

\( i_{rd} \)  
d-axis rotor current vector

\( i_{rq} \)  
q-axis rotor current vector

\( i_a \)  
Phase a current

\( i_b \)  
Phase b current

\( i_c \)  
Phase c current

\( i_{ap} \)  
Phase a PMSM-side current

\( i_{bp} \)  
Phase b PMSM-side current

\( i_{cp} \)  
Phase c PMSM-side current

\( \psi_r \)  
Rotor flux vector

\( \psi_{rd} \)  
d-axis rotor flux

\( \psi_{rq} \)  
q-axis rotor flux

\( \omega_{s} \)  
Slip speed

\( L_s \)  
DFIG stator inductance

\( L_m \)  
DFIG magnetizing inductance

\( L_r \)  
DFIG rotor inductance

\( L_{lr} \)  
DFIG rotor leakage inductance

\( L_{ls} \)  
DFIG stator leakage inductance

\( S \)  
Apparent power

\( V_{DC} \)  
DC-link voltage

\( C_{DC} \)  
DC-link capacitance

\( i_1 \)  
DFIG-side current at DC-link

\( i_2 \)  
PMSM-side current at DC-link

\( i_{PM} \)  
Current between DC-link and PMSM

\( i_{DFIG} \)  
Current between DC-link and DFIG

\( i_C \)  
Capacitor current

\( W_{DC} \)  
Energy stored in capacitor
\[ P^* \] Reference power (Power requested at grid)
\[ P_{PM} \] PMSM active power
\[ P_{DFIG} \] DFIG rotor active power
\[ \omega^* \] Reference shaft speed
\[ i_{sd}^* \] Reference d-axis stator current
\[ i_{sq}^* \] Reference q-axis stator current
\[ i_{rd}^* \] Reference d-axis rotor current
\[ i_{rq}^* \] Reference q-axis rotor current
\[ T_{DFIG}^* \] Reference torque for a DFIG
\[ T^* \] Reference torque
\[ p_r \] Spatial rotor active power
\[ q_r \] Spatial rotor reactive power
\[ \theta_t \] Transformation angle
\[ T_s \] Switching period
\[ t_1 \] Switch on time for first non-zero space vector
\[ t_2 \] Switch on time for second non-zero space vector
\[ t_{7,8} \] Switch on time for zero vectors
\[ d_1 \] Duty cycle for first non-zero space vector
\[ d_2 \] Duty cycle for second non-zero space vector
\[ d_{7,8} \] Duty cycle for zero vectors

1.4.2 Abbreviations

CB Circuit breaker
GSC Grid side converter
RSC Rotor side converter
THD Total harmonic distortion
VSC Voltage source converter
VSD Variable speed drive
VSI Voltage source inverter
PWM Pulse width modulation
PM Permanent magnet machine
PMSM Permanent magnet synchronous machine
DFIG Doubly fed induction generator
SVM Space vector modulation
PMSC Permanent magnet machine side converter
DFIGSC Doubly fed induction generator side converter
HYGOV IEEE hydro turbine model
SMPM Surface mounted permanent magnet machine
PLL Phase locked loop
KCL Kirchhoff’s current law
1.5 Glossary

A small glossary is also given here for the assistance of an inexperienced reader that helps in understanding the different parts of a hydro turbine.

Penstock

A penstock is a large enclosed pipe which is used for connecting a water source (usually a dam) to the turbine. To provide with the governor control system it is also equipped with controlled gates.

Wicket gate

It is a special feature of a Kaplan turbine that makes it different from the simple propeller and helps the Kaplan turbine for controlling the input power. The wicket gate consists of many small vanes, some of these vanes are fixed and some can move. The function of a wicket gate is to guide the flowing water from the inlet to the runner blades.

Guide vanes

The moveable vanes of a wicket gate are called guide vanes as they are used for obtaining the optimal hydraulic flow. The flow of water can be adjusted by adjusting the angle of guide vanes in respect to the fixed or stay vanes.

Stay vanes

The fixed vanes of a wicket gate are called as stay vanes and along with the guide vanes it controls the water flow.

Runner

The propeller shaped runner is usually designed for a Kaplan turbine and always mounted vertically with several blades. Water from the wicket gate is made to flow vertically across the runner for producing the rotation.

Runner blades

The runner is provided with the numbers of blades for capturing the power from the water. Usually four or six numbers of blade are used in a Kaplan turbine. The turbine’s rotational torque largely depends on the length and the number of blades used in a runner.

Permanent droop

It is defined as the frequency deviation at a steady state in per unit caused by the per unit change in a gate position.
2 WORKING, ADVANTAGES AND APPLICATIONS

Before developing an understanding about the working, applications and advantages of the studied variable speed drive, it is necessary to understand the basic working of a doubly-fed induction generator (DFIG) in different operating conditions. The control strategy for the drive system will be based on the dynamics of the DFIG.

2.1 Working of a DFIG

A doubly-fed induction generator (DFIG) also known as wound rotor induction generator is popular now-a-days for generation applications with the limited variable speed range. In comparison to a squirrel cage induction machine, a DFIG has two windings, one on the stator side and other on the rotor side. The stator side winding consists of an insulated repeating three-phase winding. The repetition of stator three-phase winding depends on the desired number of poles.

Similar to the stator, the rotor is also equipped with an insulated repeating three-phase winding. The repetition of rotor three-phase winding depends on the desired number of poles. Depending on the mode of operation, the DFIG rotor winding either supply or recover power at a slip frequency. The rotor side windings are usually connected to the external stationary circuit (voltage supply) via slip rings and brushes. The rotor-side winding of a DFIG is fed through the controlled inverters, for controlling the speed, torque, stator-side frequency, active power and the reactive power. The control of these rotor currents will be explained in more details in chapter 4.

Because of the two-windings, a DFIG can produce or consume power through both the stator and the rotor of the machine depending upon the speed of the shaft. Considering the ideal machine and ignoring the copper and iron losses in the machine it can be written as [7]

\[ P_s + P_r = P_m \]  \hspace{1cm} (2.1)

\( P_s \) is the stator terminal active power and \( P_r \) is the rotor terminal active power of a DFIG. \( P_m \) is the input mechanical power. The relation between the rotor side power \( P_r \) and the mechanical power \( P_m \) is given as:

\[ P_r = -s \cdot P_m \]  \hspace{1cm} (2.2)

Slip \( s \) of a machine in (2.2) can be defined as a relation between the stator and the rotor angular velocities and it is given as [7]

\[ s = \frac{\omega_s - \omega_r}{\omega_s} \]  \hspace{1cm} (2.3)

\( \omega_s \) is the stator angular velocity, \( \omega_r \) is the rotor angular velocity and \( s \) is the slip. Depending upon the value of a slip \( s \), the operation of a DFIG can be differentiated into three different modes i.e.
Throughout this report, a motor convention is used i.e. consumed electrical power will have positive sign and produced electrical power will have negative sign. For explaining the power flow across the wound rotor induction machine, the machine is assumed to be operating as a generator.

### 2.1.1 Sub-synchronous mode

In a sub-synchronous mode the shaft speed is less than the synchronous speed which gives positive value of the slip. The detailed phasor diagram for a DFIG operating in a sub-synchronous mode is given in [7]. It is shown in [7] that operating as a generator in a sub-synchronous mode, the angle between the rotor voltage and the rotor current vector ranges between \( \frac{\pi}{2} \) to \( \pi \). This means that in a sub-synchronous mode, a DFIG consumes active power from the rotor-side supply. The amount of required rotor active power depends upon the slip of the machine.

![Figure 2.1: Power across a DFIG in a sub-synchronous mode.](image)

### 2.1.2 Super-synchronous mode

In a super-synchronous mode the shaft rotates at a speed higher than the synchronous speed which gives the negative value of a slip. The detailed phasor diagram for a DFIG operating in a super-synchronous mode is given in [7]. It is shown in [7] that operating as a generator in a super-synchronous mode, the angle between the rotor voltage and the current vector range between \( \frac{\pi}{2} \) to \( \pi \). This means that in a super-synchronous mode, a DFIG produce active power through both the rotor and the stator of the machine. The amount of produced rotor active power depends upon the slip of the machine.
Figure 2.1 and Figure 2.2 shows the direction of power across a wound rotor induction machine operating as a generator. The directions of all the powers get reversed when the machine is operated as a motor. Table 2.1 summarizes the direction of the stator and the rotor active power in different operating conditions.

<table>
<thead>
<tr>
<th>Operating condition</th>
<th>Operating mode</th>
<th>$P_m$</th>
<th>$P_s$</th>
<th>$P_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor ($T_m &lt; 0$)</td>
<td>Sub-synchronous</td>
<td>Produce</td>
<td>Consume</td>
<td>Produce</td>
</tr>
<tr>
<td>Generator ($T_m &gt; 0$)</td>
<td>Sub-synchronous</td>
<td>Consume</td>
<td>Produce</td>
<td>Consume</td>
</tr>
<tr>
<td>Motor ($T_m &lt; 0$)</td>
<td>Super-synchronous</td>
<td>Produce</td>
<td>Consume</td>
<td>Consume</td>
</tr>
<tr>
<td>Generator ($T_m &gt; 0$)</td>
<td>Super-synchronous</td>
<td>Consume</td>
<td>Produce</td>
<td>Produce</td>
</tr>
</tbody>
</table>

Table 2.1: Direction of power across a DFIG in different modes.

2.2 DFIG in a traditional drive system

As explained in section 2.1, for the operation of a DFIG, the slip power has to be supplied or recovered. For that it needs a supply from other system which can either be provided by the energy stored in the capacitors or directly from the grid.

Figure 2.3 shows a traditional generation system using a DFIG that is very common at the sites of variable input energy. The mechanical power from a turbine is captured and supplied to a DFIG that converts it to the electrical power. The stator of a DFIG is directly connected to the grid through a transformer whereas the rotor is connected to the transformer through the power electronic converters. The rotor side converter (RSC) provides controlled rotor currents to adjust the frequency, torque, active and reactive power at the stator terminals of the DFIG. The grid side converter (GSC) is used to control the DC-link voltage by supplying or consuming the rotor slip power. The power electronic components attached to the grid introduce harmonics.
in the current. A filter is attached between the GSC and the grid for improving the quality of the currents in terms of harmonic content.

The advantage of using this topology is the reduction in rating of power electronic components. The rating of the converters depends upon the slip of the shaft and usually it is rated between 25-30% of the total electrical power that is a big advantage compared to full generation systems in which full rated converters are required [8]. The reduction in converter rating reduces the overall cost of the system.

![Figure 2.3: Traditional generation system based on a DFIG [8].](image)

### 2.3 Topology under investigation

The system under study is proposed for a micro-hydro power application where the available water head is low with quite high flow rate. The complete topology is shown in Figure 2.4 which consists of a Kaplan turbine, gear-box, shaft, permanent-magnet synchronous machine, doubly-fed induction machine, back-to-back pulse width-modulated (PWM) converters and DC-link.

![Figure 2.4: Schematic of the topology (system) under study.](image)
Compared to the traditional system shown in Figure 2.3, it has an additional machine (PMSM) and the rotor of a DFIG is not connected to the grid. The DFIG feeds power to the grid only through the stator terminals having no power electronics directly attached to the grid. Both the machines are mechanically coupled on the same shaft and rotate with the same speed. The back-to-back PWM converter provides the electrical coupling between the DFIG rotor and the PMSM.

The working of this topology is quite different compared to the traditional topology as it has additional machine and the rotor of DFIG is not connected to the grid. The system behaves differently depending upon the shaft speed or depending upon the direction of rotor slip power.

In a sub-synchronous mode of operation, the DFIG rotor requires electrical power at a slip frequency for maintaining the constant frequency at the stator terminals. So the DFIG rotor will take the required electrical power from the DC-link through the PWM converter. For maintaining the DC-link voltage constant this amount of electrical power must be supplied to the DC-link by the PMSM. Therefore in a sub-synchronous mode, the PMSM operates as a generator and takes the required mechanical power from the shaft (turbine). Considering an ideal system, the required mechanical torque by the PMSM for providing the required slip power to the DFIG equals to \( s \cdot T_m \) and the available mechanical torque to the DFIG from the turbine can be given as \( (1 - s) \cdot T_m \). The power flow in the system is shown in Figure 2.5 a), red and blue arrows represent the mechanical and electrical power respectively. It can be seen that in a sub-synchronous mode, the DFIG and the PMSM takes mechanical power and produce electrical power. Assuming an ideal system, the power produced at the stator terminal of the DFIG must be equal to the total input mechanical power from the turbine, satisfying the energy conservation law.

In a super-synchronous mode of operation, the DFIG rotor produces electrical power at a slip frequency for maintaining the constant frequency at the stator terminals. So the rotor will supply the produced electrical power to the DC-link through PWM converters. For maintaining the DC-link voltage constant this amount of electrical power must be consumed by the PMSM. Therefore in a super-synchronous mode,
the PMSM operates as a motor and produces the mechanical power i.e. the PMSM adds mechanical power to the shaft along with the turbine. Considering the ideal system, the produced mechanical torque by the PMSM must be equal to the \( s \cdot T_m \) and the available mechanical torque to the DFIG can be given as \((1 + s) \cdot T_m\). The power flow in the system is shown in Figure 2.5 b), red and blue arrows represent the mechanical and electrical power respectively. It can be seen that in a super-synchronous mode, the DFIG has its mechanical power input from both the PMSM and the turbine. Assuming an ideal system, the power produced at the stator terminal of the DFIG in this case also must be equal to the total input mechanical power from the turbine, satisfying the energy conservation law.

In this new topology, the doubly-fed induction generator side converter (DFIGSC) provides controlled currents to DFIG rotor for controlling the operation of the DFIG. Using the (DFIGSC) the stator active and the reactive power can be controlled thus allows the control of the stator terminal power factor. The purpose of the permanent magnet machine side converter (PMSC) is to control the operation of PMSM, maintaining the DC-link voltage constant for smooth operation of machine.

2.4 Advantages

The proposed system can offer several numbers of advantages if used in the variable speed constant frequency generation system.

**Autonomous**

1. In the proposed topology, the PMSM provides excitation energy and supply/recover slip power, making the system autonomous in generation and can be connected to isolated loads [9].

2. In the earlier art, a DFIG consumes excitation current from the grid which can cause grid instability due to which some weak grid owners do not offer such a consumption of reactive power. In the proposed topology no reactive power is being consumed from the grid [9].

3. The size of the PMSM and used power electronics is expected to be 30% of the total plant capability which reduced the cost of the system compared to the system using full-power variable speed drive, in which size of the power electronics is same as that of total power capability of the plant [9].

4. Use of power electronics can provide better control of the system. Power factor at the stator terminals can be improved by supplying the excitation current through the rotor converter offering the possibility of operation at unity power factor. Also the active and the reactive power of a DFIG can be controlled independent of each other [9].

**Power quality**

5. In the proposed system, the grid receives energy only from the stator of the DFIG and there is no power electronics directly attached to the grid showing the possibility of improving the quality of power fed into the grid (free of harmonics) [9], [10].

6. The produced power has fewer harmonics in it, so the use of bulky filters can be decreased and it will also save the cost of the system [9], [10].
Grid Codes Fulfillment
7. It is easier to fulfill grid connection codes. Continuous operation of a DFIG is possible even if the grid is under fault [9], [10].
8. No extra elements are required for protection, as in case of grid fault which were required earlier like crow bars [9].

Electric Brakes
9. In case of wind turbines, the power produced by the PMSM can be used to provide electric brakes in case of an emergency stop [9].

Cost
10. The proposed system could also be cost effective considering the price of filters and protection equipment etc. [10].

Removal of slip rings
11. It is very easy to remove the slip rings for a DFIG using the studied VSD with little bit amendments [6].

2.5 Applications

The studied VSD can be utilized efficiently at the sites where input mechanical power varies. Some of the applications where it can be utilized are given below.

1. Wind energy system.
2. Hydropower applications.
3. Wave and tidal energy [9].
5. Geothermal energy [9].
6. Solar energy application [9].
3 MATHEMATICAL MODELING OF THE SYSTEM

In this chapter, the mathematical model for each component of the topology is presented. A proper derivation of mathematical model for the system is necessary to have a better analysis of the system dynamics in transients and steady-state operation. The coming sections cover the models for a Kaplan turbine, mechanical shaft, PMSM, DFIG and a DC-link.

3.1 Turbine model

The turbine is basically used to convert the hydro energy into mechanical energy that can later be converted into electrical energy. Depending upon the principle of operation, hydro turbines can be classified into two groups:

i. Impulse turbines.
ii. Reaction turbines.

3.1.1 Impulse turbines

This type of turbine is suitable for the sites where the available water flow rate is slow and water head is high. All the available potential energy in water head is completely converted to kinetic energy in the nozzles and the pressure inside the runner is constant at atmospheric level [12]. As the pressure is constant throughout the impulse turbine, therefore the velocity is the same at the inlet and outlet of the turbine. Pelton turbine is the practical example of impulse turbines that have been used in many high-head power applications.

3.1.2 Reaction turbines

Reaction turbines are different compared to the impulse turbines as potential energy is partly converted to kinetic energy at the start of guide vanes, while the remaining potential energy is gradually converted to kinetic energy in the runner. The pressure in the runner varies as flow rate varies in the runner tube [12]. As the pressure drops through the vane of the reaction turbine, therefore the velocity at the outlet is higher compared to the inlet. Depending on the direction of flow, reaction turbines are further classified as radial-flow and axial-flow turbines. Radial-flow turbines are suitable for medium-head and medium flow rate, whereas axial-flow turbines are suitable for low-head and fast flow rate. Francis turbines are a practical example of radial-flow turbines, while Kaplan turbines are an example of axial-flow reaction turbine.

The system under study is suggested for a micro-hydro power application where the flow rate is quite high with low water head, so the simplified model of a Kaplan turbine with some assumptions will be given.

3.1.3 Kaplan turbine

The Kaplan turbine is being widely used in low-head power applications. It is a highly efficient turbine and in some applications an efficiency of 90% or even higher is recorded [13]. A schematic diagram of a Kaplan turbine is presented in Figure 3.1, showing the wicket gate (guide vanes) and runner blades. The wicket gate is a
special feature of the Kaplan turbine making it different compared to a simple propeller. The wicket gate angle can be adjusted depending upon the desired flow.

The wicket gate structure for a Kaplan turbine is presented in Figure 3.2. The angle $\delta$ can be increased or decreased depending upon the value of flow rate. Angle $\delta$ can be varied between 0 and 80° whereas the most optimum angle is 25° as suggested in [14]. Because of the adjustment of runner blades and the guide vanes, Kaplan turbines offer constant efficiency over a wide range of flow. The efficiency of turbine starts decreasing above the specified 'constant efficiency flow rate'.

3.1.4 Torque and speed calculations

The speed and torque calculations for a Kaplan turbine are presented in [12] and [14]. The available hydro power depends on the available water head and the flow rate, i.e.

$$P_{\text{hydro}} = \rho \cdot g \cdot Q \cdot H \quad (3.1)$$

$$P_{\text{out}} = P_m = n \cdot P_{\text{hydro}} \quad (3.2)$$

Here $n$ is the efficiency, $Q$ is the flow rate, $H$ is the available water head and $\rho$ is the density of the water. Knowing the efficiency $P_{\text{out}}$ can be calculated using (3.2). For most of the Kaplan turbines, efficiency is usually 0.9 or higher [13], so an efficiency
of 0.9 is assumed here. The speed of the turbine propeller can be calculated using the following relations [13]. The flow in a tube is given as

\[ Q = A \cdot v \]  

(3.3)

Where \( A \) is the area of the runner and \( v \) is the velocity of the water in the runner.

\[ Q = \frac{\pi}{4} \cdot (D^2_t - D^2_h) \cdot V_f \]  

(3.4)

\( D_t = \) diameter at the tip, \( D_h = \) diameter at the hub and \( V_f = \) flow velocity.

\[ \frac{V_f}{\sqrt{2 \cdot g \cdot H}} = x \]  

(3.5)

\[ K_u = \frac{u}{\sqrt{2 \cdot g \cdot H}} \]  

(3.6)

\( x = \) flow ratio and \( K_u = \) velocity ratio. Using equations (3.3) to (3.6) \( u \) can be defined as:

\[ u = \frac{K_u \cdot 4 \cdot Q}{x \cdot \pi \cdot (D^2_t - D^2_h)} \]  

(3.7)

\( u \) is the linear velocity that can be converted to the angular velocity \( \omega \) in \((rpm)\) as

\[ \omega = \frac{u \cdot 60}{\pi \cdot D \cdot r} \]  

(3.8)

Here \( r \) is the average radius of the propeller. The mechanical torque produced by the turbine can be calculated using produced power i.e.

\[ P_{out} = T_m \cdot \omega \]  

(3.9)

Values of \( x, K_u, D_t \) and \( D_h \) are assumed using the reference [13] and given in Appendix B. \( P_{out} \) can be calculated using equation (3.2) for a constant efficiency range. After a certain flow rate and power, it is assumed here that guide vanes will change their position so that the output power of the turbine will remain constant and the efficiency of the turbine will decrease as a function of flow rate in order to maintain constant \( P_{out} \).

Figure 3.3 shows the characteristics of obtained Kaplan turbine for a range of water flow. It can be seen that below the flow rate of \( 81 \frac{m^3}{sec} \), the torque and the efficiency of the turbine is constant at 47755 Nm and 0.9 respectively. As the flow of the water increases, the output power of the turbine also increases and reaches a value of 5 MW at a flow rate of \( 81 \frac{m^3}{sec} \). It was desired to maintain the output power of the turbine at 5 MW, so for this purpose the angle \( \delta \) for guide vanes can be changed as shown in Figure 3.2. Thus by changing the position of guide vanes, the output power of the turbine can be controlled. To simplify the calculation and the model of the turbine it was assumed that as \( Q \) increases above \( 81 \frac{m^3}{sec} \), the angle \( \delta \) will be changed in such a way that the efficiency of the machine will be decreased to maintain the output power at 5 MW. So from Figure 3.3, it can be observed that as \( Q \) increases above 81
m³/sec, the efficiency and the torque of the turbine started decreasing whereas maintaining the constant output power.

Figure 3.3: Turbine characteristic curves for flow rate, torque, power and efficiency.

3.1.5 HYGOV model

HYGOV is the simplified standard IEEE model for hydro turbines and widely used for simulation purposes. There are both linear and non-linear models for hydro turbines. Non-linear models are used where the variation in the speed and power are large [15]. In this thesis work, only non-linear model is considered.

To get the mathematical model of the turbine, a fluid (water) in the turbine is considered as incompressible and the penstock of the turbine is assumed as a rigid conduit having length L and cross section area A. At steady state, the net force acting on the fluid in the penstock can be calculated using Newton’s second law of motion [16].

\[ F_{net} = \rho L \frac{dq}{dt} \]  

(3.10)

Also, \( F_{net} = ma \) gives

\[ F_{net} = \rho (H_s - H_1 - H)Ag \]  

(3.11)
Using (3.10) and (3.11)

\[
\frac{dq}{dt} = (H_s - H - H_1) \frac{gA}{L}
\]  

(3.12)

Where:

- \( q \) = Water flow rate, \( \frac{m^3}{sec} \)
- \( A \) = Penstock area, \( m^2 \)
- \( L \) = Penstock length, \( m \)
- \( g \) = Acceleration due to gravity, \( \frac{m}{sec^2} \)
- \( H_s \) = Static head of water column, \( m \)
- \( H \) = Head of turbine admission, \( m \)
- \( H_1 \) = Head loss due to friction in the conduit, \( m \)

Equation (3.12) can be converted to per-unit using base quantities \( h_{base} \) and \( q_{base} \). Mostly \( h_{base} \) is defined as the available static water head at the turbine gate. Ignoring head loss due to friction in the conduit, the equation (3.12) becomes:

\[
\frac{dq}{dt} = (1 - h) \frac{gA_{base}}{Lq_{base}} = \frac{(1-h)}{T_w}
\]  

(3.13)

The constant \( T_w = \frac{Lq_{base}}{gA_{base}} \) is the water starting time or water time constant. It gives the time that water takes at the head \( h_{base} \) to get the flow rate of \( q_{base} \). The base value for a gate position (G) can be defined as the maximum gate opening. The base water flow in a turbine can be given as a function of gate position and water head [15] i.e.

\[
q = f(G, h)
\]  

(3.14)

The per-unit flow rate in a turbine is given as:

\[
q = G \cdot \sqrt{h}
\]  

(3.15)

In case of an ideal turbine, the mechanical output power equals to the flow rate multiplied with the head. In reality, a turbine is not 100% efficient: this can be taken into account by subtracting the no-load water flow \( (q_{nl}) \) from the actual calculated water flow. Obtaining the difference as the effective value, which multiplied with the head \( h \) returns the mechanical power. There is also a speed deviation or damping effect because of the turbine damping \( D_{turb} \) which is a function of gate position. The per-unit mechanical output power \( P_m \) can be expressed as [15], [16]:

\[
P_m = A_t \cdot h \cdot (q - q_{nl}) - D_{turb} G \Delta \omega
\]  

(3.16)
The per-unit value of $D_{turb}$ ranges between 0-0.5 [17], where in this case 0 is assumed for the Kaplan turbine to make the model simple. $A_t$ is a proportional constant and can be obtained as a ration of the turbine rating in MW and the generator rating in MVA [16]. From the obtained mechanical power, the torque can be calculated using the expression (3.9). Figure 3.4 show the schematic of a HYGOV model obtained using the above presented equations.

![Figure 3.4: IEEE standard HYGOV model.](image)

### 3.2 Mechanical shaft

In this section, the mechanical interaction between a Kaplan turbine and the electrical machines is studied. The mathematical model for a mechanical shaft is given and the model is developed in MATLAB/SIMULINK. The un-damped natural frequencies for a mechanical drive train system are calculated in order to avoid the load problems and mechanical failures.

#### 3.2.1 Drive train model

The term mechanical drive train includes all the rotating parts from turbine propeller to the rotor of the generator including the mechanical shaft and gears. There are two basic types of drive train:

i. Geared drive train.

ii. Direct drive train.

#### 3.2.2 Geared drive train

This type of mechanical drive train is used where the rotation of the propeller of turbine is in tens of revolution per minute (rpm), whereas the rotation of electrical machines (generators) is in hundreds of rpm, so gears are used to transfer
mechanical energy from the low-speed shaft to the high-speed shaft, i.e. step-up gear-box is used. The use of a gear-box in the mechanical drive train has its own disadvantages e.g. regular maintenance, equipment cost, audible noise and losses. The losses in the gear box are comparable to the losses in the machines [19]. The advantage of using gear-boxes is that it transfers the mechanical energy from low speed to high speed, which allows the generator to be designed for higher speeds thus reduces the size of the generator i.e. reduces the weight and volume of the generator.

3.2.3 Direct drive train

Due to the possibility of designing the electrical machines with many numbers of poles, gear-less drive train can be used. In this case, the generator has to be designed with many numbers of poles for having the acceptable electrical speed or frequency at the output. The main disadvantage of using a gear-less drive train is that it results in bulky electrical machines and also full rated electronic converters are required. The advantages include the removal of gear-box therefore no maintenance work required for the gear-box and also it saves the losses across the gear-box.

3.2.4 The mechanical drive train model

The modeling of a mechanical drive train is not a straightforward process; different types of mathematical models such as lumped three-mass equivalent model, two-mass model and one-mass model are presented in [20]. The complexity of the drive train model varies depending upon the purpose of the study. The lumped two-mass model for a drive train is utilized here and shown in Figure 3.5. The mathematical model for a lumped two-mass model is given as [20].

\[
2. H_g \frac{d\omega_g}{dt} = -T_e + T_{tg} \tag{3.17}
\]

\[
2. H_t \frac{d\omega_t}{dt} = -T_{tg} + T_m \tag{3.18}
\]

\[
\frac{d\phi_{tg}}{dt} = \omega_t - \omega_g \tag{3.19}
\]

\[
T_{tg} = K_{tg} \phi_{tg} + D_{tg}(\omega_t - \omega_g) \tag{3.20}
\]

Here $H_t$, $D_t$ and $H_g$, $D_g$ are the inertia constants and damping coefficients of the turbine and the generator rotor respectively. $\omega_t$ and $\omega_g$ are the angular speeds of the turbine and the generator rotor respectively. $T_e$ is the electrical load torque and $T_m$ is the mechanical input torque. Throughout this thesis work $T_m$ is considered as positive in magnitude and $T_e$ is considered as negative in magnitude. $D_{tg}$, $K_{tg}$, $\phi_{tg}$ and $T_{tg}$ are the shaft damping coefficient, stiffness coefficient, twist angle and the torque at the shaft respectively.
The lumped one-mass model of a drive train can also be obtained neglecting the stiffness coefficient and the damping coefficient of the shaft. The one-mass drive train is shown in Figure 3.6 and the mathematical model for a lumped one-mass model is given as [20]:

\[ 2.H.\frac{d\omega_m}{dt} = -T_e + T_m + D_m \]  

\[ \frac{d\varphi_m}{dt} = \omega_m \]  

Here \( H \), \( D_m \), \( \omega_m \) and \( \varphi_m \) are respectively the inertia constant, damping coefficient, shaft angular speed and the shaft twist angle for the lumped one mass model whereas \( H = H_t + H_g \).

The topology under study uses a gear driven train in which a low speed mechanical energy is transferred to the high speed rotating shaft coupled with the PMSM and a DFIG (mounted on the same shaft). Therefore the lumped two-axis model needs little
modification in order to account for the step-up gear. The 3\textsuperscript{rd} order mathematical model for a two-mass lumped shaft model with gear ratio is given as [23].

\[ 2. H_T \frac{d\omega_T}{dt} = -T_e + \frac{1}{N} (T_{tg}) \quad (3.23) \]

\[ 2. H_T \frac{d\omega_T}{dt} = -T_{tg} + T_m \quad (3.24) \]

\[ T_{tg} = K_{tg} \varphi_{tg} + D_{tg} (\omega_t - \omega_y) \quad (3.25) \]

\[ \frac{d\varphi_{tg}}{dt} = \frac{d(\varphi_t - \varphi_y)}{dt} = \omega_t - \frac{1}{N} \cdot \omega_y \quad (3.26) \]

Here N is the gear ratio. The above set of equations can be expressed in matrix form for any order of model. The generalized form is given as [24]:

\[ M \ddot{\varphi} + D \dot{\varphi} + K \varphi = T \quad (3.27) \]

\[
\begin{bmatrix}
H_T & 0 \\
0 & H_g
\end{bmatrix},
K = \begin{bmatrix}
\frac{N^2}{K_{tg}} & \frac{-K_{tg}}{N} \\
\frac{-K_{tg}}{N} & \frac{K_{tg}}{N}
\end{bmatrix},
D = \begin{bmatrix}
\frac{D_{tg}}{N^2} & \frac{-D_{tg}}{N} \\
\frac{-D_{tg}}{N} & \frac{D_{tg}}{N}
\end{bmatrix},
\varphi = \begin{bmatrix}
\varphi_t \\
\varphi_y
\end{bmatrix}, T = \begin{bmatrix}
T_m \\
T_e
\end{bmatrix}
\]

Here \( M \) is the mass matrix containing the inertia of the system e.g. inertia of the generator and the turbine, \( K \) is the stiffness matrix representing the flexibility of the shaft, \( D \) is the damping matrix and \( T \) is the torque matrix. The size of these matrices depends on the order of used model. For two-order model, the matrices \( M, D \) and \( K \) are of 2 x 2 and \( T \) is of 2 x 1.

Using the above equations, the speed of the shaft can be calculated based upon the input mechanical torque and the load (electrical) torque. The acceleration of the generator rotor depends on the difference of the torques. If the difference is large, the rotor will accelerate at higher rate. If the difference is less, the speed rate will be less. If both the torque balances each other, the speed will remain constant. Using equations (3.23) to (3.26), the shaft model was developed in MATLAB/SIMULINK and shown in Figure 3.7.
3.2.5 Drive train resonance

The interaction between the mechanical and the electrical system can be implemented by using equation (3.27) if all the elements of the matrices are known. However, the analysis can be extended further to describe the drive train resonances. The method for determining the un-damped modes in the system is described in [24].

3.2.6 Harmonic modes

The un-damped harmonic modes for a mechanical system can be calculated from equation (3.27) by neglecting the damping and the torque terms, i.e.

$$M\ddot{\varphi} + K\varphi = 0$$  \hspace{1cm} (3.29)

Free vibration solutions of the structure harmonic motion are assumed so that:

$$\ddot{\varphi} = -\omega_{res}^2 \cdot \varphi$$  \hspace{1cm} (3.30)

That is valid if $\varphi = A \sin (\omega t)$. Substituting (3.30) in (3.29) yields...
This is a well-known form for calculations of eigen-values. The eigen-vectors and eigen-values for equation (3.31) can be found easily using MATLAB. From the eigen-values of $M^{-1}.K$ the un-damped modes can be calculated as:

$$\omega_{res,1} = \sqrt{N_1}$$

$$\vdots$$

$$\omega_{res,n} = \sqrt{N_n}$$

Here $n$ is the range of square matrix $M^{-1}.K$. Also resonant frequencies can be calculated from vector $\omega_{res}$ as:

$$f_{res} = \frac{\omega_{res}}{2.\pi}$$

From the resonance frequencies calculated from equation (3.33), it can be make sure that the designed drive train must not be operate at any of these frequencies, otherwise system may start vibrating and can also result in mechanical failure.

### 3.2.7 Damping matrix D

The matrix $D$ in equation (3.27) cannot be obtained as straight forward as matrices $M$ and $K$. Matrix $D$ can be calculate from modal damping matrix $D_m$ which is a diagonal matrix and whose elements are defined as [24].

$$D_i = 2.\varepsilon_i.\omega_{res,i}$$

$\varepsilon_i$ is called the damping factor and usually given in the percentage and mostly a value between 1% and 5% is used [24]. For a geared drive train $\varepsilon_i$ is considered as 5%. The damping matrix $D$ can be calculated from $D_i$ as:

$$D = (\varphi^T)^{-1}.D_i.\varphi^{-1}$$

$\varphi$ is the matrix consists of eigen-vectors obtained from the eigen-value analysis of equation (3.31). The mode shape vectors or eigen-vectors in equation (3.35) must also be normalized so that equation (3.36) holds.

$$\varphi^T.M.\varphi = I$$

Order of $\varphi$ depends on the type of the model that is considered for the modeling of drive train. If the lumped two-mass model is considered, the order of $\varphi$ will be $2 \times 2$ and if the lumped three-mass model is considered then $\varphi$ will be of $3 \times 3$. For a lumped two-mass model, matrix $\varphi$ is given as:
and $\Phi_2$ are the eigenvectors and $I$ is the identity matrix. The normalization factor for each eigen-vector can be obtained by solving equation (3.36) and using the normalized mode shape vectors or eigen-vectors, damping matrix $D$ can be calculated from equation (3.35).

For calculating the normalization factors for eigen-values, suppose that matrices $M$, $K$ and $D$ are of order $2 \times 2$. So there will be two eigen-values and two eigen-vectors. Two eigen-vectors are given as:

$$\Phi_1 = \begin{bmatrix} \Phi_{11} \\ \Phi_{12} \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} \Phi_{21} \\ \Phi_{22} \end{bmatrix}$$ (3.38)

Consider two normalization factors $n_1$ and $n_2$. Using these normalization factors in (3.38) and (3.37), equation (3.36) becomes

$$\begin{bmatrix} n_1 \cdot \Phi_{11} & n_1 \cdot \Phi_{12} \\ n_2 \cdot \Phi_{21} & n_2 \cdot \Phi_{22} \end{bmatrix} \cdot \begin{bmatrix} H_t & 0 \\ 0 & H_g \end{bmatrix} \cdot \begin{bmatrix} n_1 \cdot \Phi_{11} \\ n_1 \cdot \Phi_{12} \end{bmatrix} = I$$ (3.39)

Solving equation (3.39) gives normalization factors $n_1$ and $n_2$ as:

$$n_1 = \frac{1}{\sqrt{H_t \cdot \Phi_{11}^2 + H_g \cdot \Phi_{12}^2}}$$ (3.40)
$$n_2 = \frac{1}{\sqrt{H_t \cdot \Phi_{21}^2 + H_g \cdot \Phi_{22}^2}}$$ (3.41)

After normalizing the eigen-vectors using $n_1 \cdot \Phi_1$ and $n_2 \cdot \Phi_2$ and fulfilling the equation (3.36), the damping matrix $D$ can be found using equation (3.35).

### 3.3 Permanent magnet synchronous machine

In the studied variable speed drive, the PMSM is used to recover or supply the slip power for the DFIG through the PWM converters. For an electromagnetic machine, the stator voltage can be represented in space vector as [25],

$$u_s = \frac{2}{3} (u_a + a \cdot u_b + a^2 \cdot u_c)$$ (3.42)

Here, $a = e^{j \frac{2\pi}{3}}$ and $a^2 = e^{j \frac{2\pi}{3}} = e^{-j \frac{2\pi}{3}}$

Voltage equations for a three-phase machine can be written as:

$$u_a = R_s \cdot i_a + \frac{d \psi_a}{dt}$$ (3.43)
$$u_b = R_s \cdot i_b + \frac{d \psi_b}{dt}$$ (3.44)
\[ u_c = R_s \cdot i_c + \frac{d \psi_c}{dt} \] (3.45)

\( R_s \) is the per-phase stator winding resistance, \( u_a, u_b \) and \( u_c \) are the three-phase voltages, \( i_a, i_b \) and \( i_c \) are the phase currents and \( \psi_a, \psi_b \) and \( \psi_c \) are the three-phase fluxes. Using (3.43), (3.44) and (3.45), equation (3.42) can be expressed in terms of a current and flux special vectors. The resulting expression will be

\[ u_{s,abc} = R_s \cdot i_{s,abc} + \frac{d \psi_{s,abc}}{dt} \] (3.46)

Further the three-phase voltages can be transferred to the stator reference frame using the Clark’s transformation given in Appendix A. For better control of an electromagnetic machines, the voltage equations can be transferred to the rotating synchronous reference frame (named as \( dq \) reference frame) using the Cartesian transformation given in Appendix A. The stator voltage equations in synchronous reference frame are given as:

\[ u_{sdq} = R_s \cdot i_{sdq} + \frac{d \psi_{sdq}}{dt} + j \cdot \omega_s \cdot \psi_{sq} \] (3.47)

\( \omega_s \) is the electrical speed, can be obtained as \( p \cdot \omega_r \). \( \omega_r \) is the rotational speed of the rotor and \( p \) is the number of pole pairs. Equation (3.47) can further be sub-divided into the real and imaginary parts as:

\[ |u_s| = \sqrt{u_d^2 + u_q^2} \]

\[ u_d = R_s \cdot i_d + \frac{d \psi_d}{dt} - \omega_s \cdot \psi_q \] (3.48)

\[ u_q = R_s \cdot i_q + \frac{d \psi_q}{dt} + \omega_s \cdot \psi_d \] (3.49)

In matrix form it can be written as:

\[ \begin{bmatrix} u_d \\ u_q \end{bmatrix} = R_s \cdot \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} + \omega_s \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \psi_d \\ \psi_q \end{bmatrix} \] (3.50)

For a permanent magnet synchronous machine \( \psi_{sd} \) and \( \psi_{sq} \) can be calculated from \( i_{sd} \) and \( i_{sq} \) using the following equations [26].

\[ \begin{bmatrix} \psi_{sd} \\ \psi_{sq} \end{bmatrix} = \begin{bmatrix} L_{sd} & 0 \\ 0 & L_{sq} \end{bmatrix} \cdot \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} \psi_m \end{bmatrix} \] (3.51)

Here \( L_{sd} \) and \( L_{sq} \) are the \( d- \) and \( q- \) axis inductances, \( \psi_m \) is the rotor magnetic flux linking the stator. For observing the dynamics of the machine, equation (3.48) and (3.49) can be solved for \( i_{sd} \) and \( i_{sq} \) using the Laplace transform. The resulting equations for \( d- \) and \( q- \) axis currents can be used for the modeling of a permanent magnet machine.

\[ i_{sd} = \frac{u_{sd} + \omega_s \cdot L_{sq} \cdot i_{sq}}{R_s + s \cdot L_{sd}} \] (3.52)

\[ i_{sq} = \frac{u_{sq} - \omega_s \cdot L_{sd} \cdot i_{sd} - \omega_s \cdot \psi_m}{R_s + s \cdot L_{sq}} \] (3.53)
3.3.1 Torque and power calculation for a PMSM machine

For a machine with salient rotor, the electromagnetic torque can be calculated using equation (3.54) [27]. There are two components of the torque,

\[ T = \frac{3p}{2} \cdot (\psi_m i_{sq} + (L_{sd} - L_{sq}) \cdot i_{sd} \cdot i_{sq}) \]  (3.54)

The first part of the torque is because of the interaction between the magnet flux and the \( q \)-axis stator current and the second part of the torque (often called reluctance torque) is due to the saliency of the machine i.e. because of the difference of inductances in \( d \)- and \( q \)-axis of the rotor. In Figure 3.8, \( \beta \) is the electrical angle between the stator current vector and the magnet flux or \( d \)-axis. The rectangular components of the current in \( d \)- and \( q \)-axis are given as:

\[ i_{sd} = i_s \cdot \cos (\beta) \]  (3.55)

\[ i_{sq} = i_s \cdot \sin (\beta) \]  (3.56)

where \( i_s \) is the stator current vector. The permeability of a magnetic material is almost the same as that of the air, therefore in case of the surface mounted permanent magnet (SMPM) machines, rotor \( d \)-axis and \( q \)-axis inductances are almost equal leading to the fact that the second term in the torque equation disappears. The torque equation for a surface mounted PM machine becomes:

\[ T = \frac{3p}{2} \cdot (\psi_m i_{sq}) \]  (3.57)

In this thesis work, the SMPM is considered for recovering the slip power of the DFIG. For any value of the current and in order to maximize the torque, the stator current angle can be changed in such a way that it coincides with the \( q \)-axis of the rotor, making the \( d \)-axis current zero and it also reduces the copper losses of the machine [27].
Power in a three-phase system can be calculated using the relation [25].

\[ p_s = u_a \cdot i_a + u_b \cdot i_b + u_c \cdot i_c \]  

(3.58)

Converting the above equation to voltage and current vectors, it becomes:

\[ p_s = \frac{3}{2} \text{Re} \left( u_{sdq}^* i_{sdq} \right) \]  

(3.59)

Equation (3.59) can be represented in any frame of reference i.e. stator, rotor or synchronous reference frame provided that both the voltage and the current vectors must be in the same reference frame.

Dynamical model of a permanent magnet synchronous machine was developed in MATLAB/SIMULINK using equation (3.52) and (3.53) and shown in Figure 3.9.

![Figure 3.9: Permanent magnet synchronous machine model.](image)

3.4 DFIG model

The dynamical model of a DFIG can be derived in a similar way to the conventional induction machine. The only difference is that in a DFIG, rotor voltages are not zero but can be controlled through a converter. Thus along with the stator voltage equations, the rotor voltages also need to be considered. The dynamics of a DFIG can be obtained using 5th order dynamic model. It is easier to control the machine in a synchronous reference frame; therefore mathematical model for a DFIG is also given in \( dq \) reference frame. The mathematical equations for the stator and the rotor voltages are given as [8]
In reference frame it can be written as:

\[ u_{s,dq} = r_s \cdot i_{s,dq} + \frac{d\psi_{s,dq}}{dt} + j \cdot \omega_s \cdot \psi_{s,dq} \]  \hspace{1cm} (3.60) \\
\[ u_{r,dq} = r_r \cdot i_{r,dq} + \frac{d\psi_{r,dq}}{dt} + j \cdot (\omega_s - \omega_r) \cdot \psi_{r,dq} \] \hspace{1cm} (3.61) 

In \( dq \) reference frame it can be written as:

\[ u_{sd} = r_s \cdot i_{sd} + \frac{d\psi_{sd}}{dt} - \omega_s \cdot \psi_{sq} \] \hspace{1cm} (3.62) \\
\[ u_{sq} = r_s \cdot i_{sq} + \frac{d\psi_{sq}}{dt} + \omega_s \cdot \psi_{sd} \] \hspace{1cm} (3.63) \\
\[ u_{rd} = r_r \cdot i_{rd} + \frac{d\psi_{rd}}{dt} - (\omega_s - \omega_r) \cdot \psi_{rq} \] \hspace{1cm} (3.64) \\
\[ u_{rq} = r_r \cdot i_{rq} + \frac{d\psi_{rq}}{dt} + (\omega_s - \omega_r) \cdot \psi_{rd} \] \hspace{1cm} (3.65) 

where \( u_s, u_r, u_{sd}, u_{sq}, u_{rd} \) and \( u_{rq} \) are respectively the stator voltage vector, rotor voltage vector, \( d \)-axis stator voltage component, \( q \)-axis stator voltage component, \( d \)-axis rotor voltage component and \( q \)-axis rotor voltage component. \( i_s, i_r, i_{sd}, i_{sq}, i_{rd} \) and \( i_{rq} \) are respectively the stator current vector, rotor current vector, \( d \)-axis stator current component, \( q \)-axis stator current component, \( d \)-axis rotor current component and \( q \)-axis rotor current component. \( \psi_s, \psi_r, \psi_{sd}, \psi_{sq}, \psi_{rd} \) and \( \psi_{rq} \) are respectively the stator flux linkage vector, rotor flux linkage vector, \( d \)-axis stator flux linkage vector, \( q \)-axis stator flux linkage component, \( d \)-axis rotor flux linkage vector and \( q \)-axis rotor flux linkage component.

\( \omega_s = \) angular velocity of the stator voltages and currents \( \frac{rad}{sec} \) \\
\( \omega_r = \) angular velocity of the rotor \( \frac{rad}{sec} \).

\( \omega_r = p \cdot \Omega_m, \) \( p \) is the number of pole pairs and \( \Omega_m \) is the mechanical angular velocity of the rotor. The detailed operation and the modeling of a DFIG is presented in [7], induced voltage on the rotor-side depends on the relation between the stator flux angular velocity and the rotor flux angular velocity. The angular velocity of the induced rotor voltages and the currents can be obtained as [7]:

\[ \omega_{slip} = \omega_s - \omega_r \] \hspace{1cm} (3.66) 

In this thesis work, T-model for a DFIG is considered. The name T-model is based upon the number of inductances that are considered in the steady state circuit of a DFIG. The stator leakage inductance, rotor leakage inductance and the magnetization inductance is considered as shown in Figure 3.10 [8]. The rotor-side resistance and voltages are divided by the slip in order to refer them to the stator-side. The slip \( s \) is defined in the coming section.
Figure 3.10: Single phase steady state equivalent T–model for a DFIG referred to the stator [8].

The $dq$-axis flux linkages in equation (3.62), (3.63), (3.64) and (3.65) can be calculated as [7]:

\begin{align}
\psi_{sd} &= L_s \cdot i_{sd} + L_m \cdot i_{rd} \\
\psi_{sq} &= L_s \cdot i_{sq} + L_m \cdot i_{rq} \\
\psi_{rd} &= L_r \cdot i_{rd} + L_m \cdot i_{sd} \\
\psi_{rq} &= L_r \cdot i_{rq} + L_m \cdot i_{sq}
\end{align}

(3.67)  
(3.68)  
(3.69)  
(3.70)

Here $L_s$, $L_r$ and $L_m$ are respectively the stator, rotor and the mutual inductances. The stator inductance is defined as $L_s = L_{ls} + L_{l_m}$ and the rotor inductance $L_r$ is defined as $L_r = L_{lr} + L_m$. $L_{ls}$ is the stator leakage inductance and $L_{lr}$ is the rotor leakage inductance.

From equation (3.61), (3.64) and (3.65) it is clear that for a normal and steady-state operation of a DFIG at fixed stator frequency, the rotor side voltages and currents must be applied at $\omega_{slip}$. It is also useful to define a relation between the stator and the rotor angular velocities and it is given as [7]:

\[ s = \frac{\omega_s - \omega_r}{\omega_s} \]  

(3.71)

where $s$ is defined as the slip. Now, from (3.66) and (3.71), a direct relation between rotor currents frequency and stator frequency can be obtained as:

\[ \omega_{slip} = s \cdot \omega_s \]  

(3.72)

Solving equation (3.67) and (3.69) $i_{sd}$ can be obtained in terms of flux components and similarly solving equation (3.68) and (3.70), $i_{sq}$ can be obtained in terms of flux components. $i_{sd}$ and $i_{sq}$ are given below where $\sigma = \frac{L_s \cdot L_r - L_m^2}{L_s \cdot L_r}$.

\[ i_{sd} = \frac{\psi_{sd}}{\sigma \cdot L_s} \cdot \frac{L_m \cdot \psi_{rd}}{L_{ts} \cdot L_r} = \frac{\psi_{sd}}{L_{ts} \cdot L_s} \cdot \frac{L_m \cdot \psi_{rd}}{L_{ts} \cdot L_r} \]  

(3.73)
In equation (3.73) and (3.74) $L_{ts} = \sigma \cdot L_s$, similarly from equations (3.67) to (3.70) $i_{rd}$ and $i_{rq}$ can also be obtained in terms of flux components and given as:

\[
\begin{align*}
    i_{rd} &= \frac{\psi_{rd}}{\sigma \cdot L_r} - \frac{L_m \cdot \psi_{rd}}{\sigma \cdot L_s \cdot L_r} = \frac{\psi_{rd}}{L_{tr}} - \frac{L_m \cdot \psi_{rd}}{L_{tr} \cdot L_s} \\
    i_{rq} &= \frac{\psi_{rq}}{\sigma \cdot L_r} - \frac{L_m \cdot \psi_{rq}}{\sigma \cdot L_s \cdot L_r} = \frac{\psi_{rq}}{L_{tr}} - \frac{L_m \cdot \psi_{rq}}{L_{tr} \cdot L_s} \tag{3.75}
\end{align*}
\]

where $L_{tr} = \sigma \cdot L_r$. In a DFIG model the current or the flux of a machine is used as a state variable. The model for a DFIG is derived in the synchronously rotating reference frame and flux is considered as the state variable. By substituting the current equations from (3.73) to (3.76) into the voltage equations (3.62) to (3.65), the dynamics of a DFIG can be obtained both in transient state and steady state. The resulting differential equations are given as:

\[
\begin{align*}
    \frac{d\psi_{sd}}{dt} &= u_{sd} - r_s \cdot \frac{\psi_{sd}}{\sigma \cdot L_s} + r_s \cdot \frac{L_m \cdot \psi_{rd}}{\sigma \cdot L_s \cdot L_r} + \omega_s \cdot \frac{\psi_{sq}}{L_{tr}} \\
    \frac{d\psi_{sq}}{dt} &= u_{sq} - r_s \cdot \frac{\psi_{sq}}{\sigma \cdot L_s} + r_s \cdot \frac{L_m \cdot \psi_{rq}}{\sigma \cdot L_s \cdot L_r} - \omega_s \cdot \frac{\psi_{sd}}{L_{tr}} \\
    \frac{d\psi_{rd}}{dt} &= u_{rd} - r_r \cdot \frac{\psi_{rd}}{\sigma \cdot L_r} + r_r \cdot \frac{L_m \cdot \psi_{sd}}{\sigma \cdot L_s \cdot L_r} + (\omega_s - \omega_r) \cdot \frac{\psi_{rq}}{L_{tr}} \\
    \frac{d\psi_{rq}}{dt} &= u_{rq} - r_r \cdot \frac{\psi_{rq}}{\sigma \cdot L_r} + r_r \cdot \frac{L_m \cdot \psi_{sq}}{\sigma \cdot L_s \cdot L_r} - (\omega_s - \omega_r) \cdot \frac{\psi_{rd}}{L_{tr}} \tag{3.79}
\end{align*}
\]

Equations (3.77) to (3.80) can be summarized in matrix form as:

\[
\begin{bmatrix}
    \frac{d\psi_{sd}}{dt} \\
    \frac{d\psi_{sq}}{dt} \\
    \frac{d\psi_{rd}}{dt} \\
    \frac{d\psi_{rq}}{dt}
\end{bmatrix} =
\begin{bmatrix}
    -r_s \cdot \frac{1}{\sigma \cdot L_s} & \omega_s & -r_s \cdot \frac{L_m}{\sigma \cdot L_s \cdot L_r} & 0 \\
    -\omega_s & -r_s \cdot \frac{1}{\sigma \cdot L_s} & 0 & -r_s \cdot \frac{L_m}{\sigma \cdot L_s \cdot L_r} \\
    r_r \cdot \frac{L_m}{\sigma \cdot L_s \cdot L_r} & 0 & -r_r \cdot \frac{1}{\sigma \cdot L_r} & \omega_s - \omega_r \\
    0 & r_r \cdot \frac{L_m}{\sigma \cdot L_s \cdot L_r} & -(\omega_s - \omega_r) & -r_r \cdot \frac{1}{\sigma \cdot L_r}
\end{bmatrix} \begin{bmatrix}
    \psi_{sd} \\
    \psi_{sq} \\
    \psi_{rd} \\
    \psi_{rq}
\end{bmatrix} +
\begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
    u_{sd} \\
    u_{sq} \\
    u_{rd} \\
    u_{rq}
\end{bmatrix} \tag{3.81}
\]

The stator and the rotor flux dynamics can be obtained using equation (3.81) and from the flux dynamics, the current dynamics can be calculated using equations (3.73) to (3.76). The developed DFIG model in MATLAB/SIMULINK using the above equations is shown in Figure 3.11.
3.4.1 Torque and power equations

The well-known expression for calculating the electromagnetic torque of an induction machine is given in the form of stator and rotor current vectors [28].

\[ T_e = \frac{3}{2} \cdot p \cdot L_m \cdot I_m (i_{s,dq} \cdot i_{r,dq}) \]  \hspace{1cm} (3.82)

The machine torque can be expressed in several other forms using the current and flux equations (3.67) to (3.70). Depending upon the available information, torque can be expressed in only stator variables or in only rotor variables. In stator variables it is given as

\[ T_e = \frac{3}{2} \cdot p \cdot I_m (i_{s,dq} \cdot \psi_{s,dq}) \]  \hspace{1cm} (3.83)

In rotor variables it is given as:

\[ T_e = \frac{3}{2} \cdot p \cdot I_m (i_{r,dq}^* \cdot \psi_{r,dq}) \]  \hspace{1cm} (3.84)
In a DFIG, the electrical power flows through both the stator and the rotor terminals. The direction of the power through a DFIG rotor depends on the speed of the shaft. The stator active power in a three-phase system can be calculated using the relation [25].

\[ p_s = u_{as}i_{as} + u_{bs}i_{bs} + u_{cs}i_{cs} \]  

(3.85)

The stator active power \( p_s \) and reactive powers \( q_s \) can be represented in terms of space vectors as:

\[ p_s = \frac{3}{2} Re(u_{s,dq}^*i_{s,dq}) \]  

(3.86)

\[ q_s = \frac{3}{2} Im(u_{s,dq}^*i_{s,dq}) \]  

(3.87)

In \( dq \) reference frame, the stator active and reactive power can be written as:

\[ p_s = \frac{3}{2} (u_{sd}i_{sd} + u_{sq}i_{sq}) \]  

(3.88)

\[ q_s = \frac{3}{2} (u_{sd}i_{sq} - u_{sq}i_{sd}) \]  

(3.89)

Similarly for the rotor side power equations are given as:

\[ p_r = \frac{3}{2} (u_{rd}i_{rd} + u_{rq}i_{rq}) \]  

(3.90)

\[ q_r = \frac{3}{2} (u_{rd}i_{rq} - u_{rq}i_{rd}) \]  

(3.91)

### 3.4.2 Stator terminal power factor

The power factor at the stator terminal of a DFIG can be calculated using the power triangle shown in Figure 3.12. The apparent power \( S \) at the stator terminals can be calculated from the stator active \( p_s \) and reactive power \( q_s \) as:

\[ S = \sqrt{p_s^2 + q_s^2} \]  

(3.92)

\[ \cos \theta = \frac{p_s}{S} \]  

(3.93)

**Figure 3.12: Power triangle.**
3.5 DC-link model

The capacitor of DC part of a back-to-back PWM converter is often called the DC-link. Due to the energy storing capability of the capacitor it is possible to maintain a constant voltage $V_{dc}$ at its terminal. It provides the electrical connection between the permanent magnet synchronous machine and the doubly-fed induction machine through two PWM converters named as PMSC and DFIGSC. PMSC is the permanent magnet machine side converter that takes or delivers the power to the PMSM whereas DFIGSC is the doubly-fed induction machine side converter that delivers or takes the slip power from the rotor of a DFIG depending upon the speed of the shaft. Figure 3.13 shows a back-to-back PWM converter with a DC-link in between two converters.

![Figure 3.13: Back-to-back PWM converter with DC-link.](image1)

The mathematical model for a DC-link can be derived using the current equation of the capacitor that is given as:

$$i_c = C_{dc} \frac{dV_{dc}}{dt} \tag{3.94}$$

$$V_{dc} = \frac{1}{C_{dc}} \int i_c \, dt \tag{3.95}$$

The capacitor current $i_c$ can be found by applying the KCL at the capacitor node. Figure 3.14 shows the current flowing across the node. Assuming the instant when current from PMSM enters into the node whereas current leaves to the capacitor and DFIG. Using KCL at the node gives:

$$i_c = i_{PM} - i_{DFIG} \tag{3.96}$$
Here $i_c$ is the current flowing through the capacitor, $i_{PM}$ is the PMSM-side current and $i_{DFIG}$ is the DFIG-side current. DC current $i_{PM}$ and $i_{DFIG}$ can be found using the switching states of the PWM converters [29].

$$i_{DFIG} = S_{a,DFIG} \cdot i_a + S_{b,DFIG} \cdot i_b + S_{c,DFIG} \cdot i_c$$

$$i_{PM} = S_{a,PM} \cdot i_{ap} + S_{b,PM} \cdot i_{bp} + S_{c,PM} \cdot i_{cp}$$

$S_{a,PM}$, $S_{b,PM}$, $S_{c,PM}$ and $S_{a,DFIG}$, $S_{b,DFIG}$, $S_{c,DFIG}$ are the switching states of the converter switches that can be calculated depending upon the type of modulation strategy used. The methods for obtaining the switching states are explained in chapter 4. Based upon the above given equations (3.95) to (3.98), a DC-link model is shown in Figure 3.15.

Figure 3.15: DC-link model base on currents.

The model for a DC-link presented above is based on the currents, whereas the model for a DC-link can also be obtained from the power flow across the capacitor [30]. The energy stored in the DC-link capacitor $C_{DC}$ can be calculated using (3.99), where $W_{DC}$ is the capacitor energy.

$$W_{DC} = \frac{1}{2} \cdot C_{DC} \cdot V_{DC}^2$$

Considering an ideal converter and switches, the energy in a DC-link depends on the flow of energy to/from the DFIG rotor from/to the PMSM and given as [31]:

$$\frac{dW_{DC}}{dt} = \frac{1}{2} \cdot C_{DC} \cdot \frac{dV_{DC}^2}{dt} = P_{PM} - P_{rDFIG}$$

This gives the dynamics of a DC-link as:

$$C_{DC} \cdot V_{DC} \cdot \frac{dV_{DC}}{dt} = P_{PM} - P_{rDFIG}$$
4 CONTROL DEVELOPMENT

It is very important to have a stable control algorithm for each component of a micro-hydro generation system. In this chapter the speed control for a turbine governor system, torque or power control for a PMSM and DFIG is given. Also the calculation of reference signals for operating the DFIG at a unity power factor is presented. The method for obtaining the switching signals for a PWM converter using the SVM is also presented at the end of this chapter.

4.1 Turbine governing system

In section 3.1, two mathematical models are presented for a hydro turbine. The IEEE standard HYGOV model is utilized for simulating the whole model and only the control related to the HYGOV model is presented.

The frequency of a generation system is a key factor and should remain constant, typically ± 0.1% deviation of the nominal frequency is allowed at the maximum. The frequency control system ensures that the speed of the electrical machine remains constant. The frequency deviation relates closely to the balance between produced and consumed (load) power. This means that if a load changes in the network at a certain time, then this change in a production and demand have to be balanced instantly through the frequency control. So the governor control system makes sure that at every instant, the produced power balanced by the load (demand power). Figure 4.1 shows a basic scheme for a governor control system.

![Figure 4.1: Speed governor system [24].](image)

Whenever the demand changes in the system, a mismatch occurs between the active power (electrical power) and the mechanical power causing the variation in the speed. The governor system compares the actual rotating speed with the reference value and based on the error gives a signal to the gate valves. The gate valves open or close in order to increase or decrease the (flow) mechanical power so that it balances with the load.

In this thesis work, the governor system presented in [17] and [32] is used and shown in Figure 4.2. The governor system has two inputs, the reference speed \( \omega_{ref} \), and the speed deviation (actual speed) \( \omega \). The speed error is then fed to the governor block that calculates the desired gate position based on the speed deviation. The output of the governor block is then used as an input to the servo motor that operates the gate valves. Considering the real time operation, there are two limits in the governor operation. The first limit is the gate opening limit, for which the maximum gate opening position and the minimum gate opening position (closed position) has to be defined. The maximum gate opening can be defined as 1 and the minimum gate opening can be defined as 0. Secondly, the velocity is limited by the servo motor speed. These two limits must be considered in the model.
In Figure 4.2, the hydraulic turbine model is also shown, which takes the gate position as an input from the governor system. Using (3.13), (3.15) and (3.16) the per-unit hydraulic flow, per-unit water head and per-unit mechanical power can be calculated respectively. Depending upon the required electrical power and to maintain the speed constant, the governor controls the gate position. This gate position serves as an input to the hydraulic turbine and thus regulates the output mechanical power of the turbine. In Figure 4.2, the used constants and their typical values are given in Table 4.1 [17].

![Block diagram of a Governor and a hydro turbine (HYGOV).](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Representation</th>
<th>Min</th>
<th>Max</th>
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</thead>
<tbody>
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<td>$R$</td>
<td>Permanent droop</td>
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<td>0.1</td>
</tr>
<tr>
<td>$r$</td>
<td>Temporary droop</td>
<td>0.02</td>
<td>0.4</td>
</tr>
<tr>
<td>$T_r$</td>
<td>Governor time constant</td>
<td>0.02</td>
<td>10</td>
</tr>
<tr>
<td>$T_g$</td>
<td>Servo time constant</td>
<td>0.01</td>
<td>1</td>
</tr>
<tr>
<td>$T_w$</td>
<td>Water time constant</td>
<td>-1</td>
<td>100</td>
</tr>
<tr>
<td>$G$</td>
<td>Gate position</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$A_t$</td>
<td>Turbine gain</td>
<td>0.5</td>
<td>3</td>
</tr>
<tr>
<td>---------</td>
<td>--------------</td>
<td>-----</td>
<td>-----------</td>
</tr>
<tr>
<td>$D_{turb}$</td>
<td>Turbine damping</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>$q_{nl}$</td>
<td>No load flow</td>
<td>0.01</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 4.1: HYGOV turbine constants and used values [17].

### 4.2 Torque control for a PMSM

In the studied topology the operation of a permanent magnet synchronous machine depends on the slip power of a DFIG. Therefore it is necessary to develop a torque control loop for the PMSM in order to maintain the DC-link voltage constant and to accommodate the DFIG slip power.

Field oriented vector control algorithm is used for controlling and maximizing the torque of a permanent magnet machine. The torque equation for a surface mounted permanent magnet machine is given in chapter 3 and repeated here as:

$$T = \frac{3}{2} \cdot p \cdot \psi_m \cdot i_{sq}$$  \hspace{1cm} (4.1)

The only variable quantity in equation (4.1) for controlling the torque is the stator current, rest of the quantities are constant. The stator current has $d$- and $q$-axis components in the synchronous reference frame.

By proper orientation of the reference frame, the permanent magnet synchronous machine can be controlled in a same way as that of DC machines i.e. independently controlling the flux with $d$-axis stator current and the torque with $q$-axis stator current. The flux interaction $\psi_m$ between the rotor and the stator can be controlled by $d$-axis stator current. Injecting a positive $i_{sd}$, the flux gets strengthened and it can be weaken by injecting the negative $i_{sd}$. This is a very useful concept especially when a PMSM is operated in a flux weakening range (to weaken the flux produce by the magnet). For surface mounted permanent magnet machines $i_{sd}$ is usually set at 0, so that all the stator current aligns with $q$-axis i.e. maximizing the torque.

To transform the three-phase rotating quantities as DC quantities in a synchronous rotating frame, a proper transformation angle is required. In case of a SMPM the rotor quantities and the stator quantities rotate with the same speed having no slip. It means that the transformation angle can be found by integrating the rotor speed $\omega_r$.

Because of the rotating reference frame, there exist the cross coupling terms in the stator voltage equations which causes the variation in one axis component due to the change in another component. In order to fully de-couple the $d$- and $q$-axis terms, the coupling terms should be subtracted from the calculated $d$- and $q$-axis voltages.

The block diagram showing the control strategy for a SMPM is shown in Figure 4.3. $i_{sd}^*$ = 0 is given as a reference signal, which is compared with the actual or real machine $d$-axis current and error is fed to a PI controller which calculated the required voltage that should be fed to the machine. Similarly $T^*$ is given as a reference signal to the $q$-axis and based on the reference torque, $q$-axis current $i_{sq}^*$ can be calculated using (4.2). The reference $q$-axis current $i_{sq}^*$ is also compared to
the actual $q$-axis machine current and based on the error $PI$ regulator calculates the required $q$-axis voltage. The transformation blocks from $dq$-axis to $\alpha\beta$-axis and then from $\alpha\beta$ to $abc$ and vice versa are obtained using the matrices given in Appendix A.

$$i_{sq}^* = \frac{r^*}{1.5\psi_{m,p}} \quad (4.2)$$

4.3 DFIG control

The DFIG control regulates the magnitude of the torque, stator frequency, stator active power, reactive power and the power factor. The control strategy along with the pulse width-modulator generates the switching pulses depending upon the reference signal.

The parameters that are desired here to be controlled for a DFIG are the stator terminal active power $P_s$, reactive power $Q_s$ and the stator terminal power factor $\cos(\theta)$. It is desired to operate the machine at a unity power factor i.e. all the reactive power must be handled at the rotor side. Two widely used control strategies for a DFIG are the vector control also called field oriented control (FOC) and the direct torque control technique (DTC). In this study work, field oriented control (FOC) method is studied and used.

Figure 4.3: Control strategy for a PMSM.
4.3.1 Field oriented control for a DFIG

The basic principle of field oriented control (FOC) for a DFIG is similar to the FOC for a squirrel cage induction machine. In case of a squirrel cage induction machine, the rotating $dq$ reference frame is aligned with the rotor flux vector as shown in Figure 4.4. The machine flux and the electromagnetic torque for an induction machine can be controlled separately by aligning the reference frame to the rotor flux vector [33]. In this way a squirrel cage induction machine can be controlled similar to a DC machine.

A grid connected DFIG can be controlled in a similar way, the only difference is that in a DFIG the rotating $dq$ reference frame is aligned with the stator flux vector instead of the rotor flux vector. By using this orientation, the stator active and reactive power of a DFIG can be controlled independently by regulating the quadrature ($q$) and the direct ($d$) axis currents respectively. A comparison between the FOC for a squirrel cage induction machine and a DFIG is shown in Figure 4.4.

![Vector Control Diagram](image)

**Figure 4.4: Comparison between the field oriented control for a squirrel cage induction machine and a DFIG [33].**

4.3.2 Field orientation

As mentioned earlier, in order to control $P_s$ and $Q_s$ for a DFIG a proper orientation of rotating $dq$ reference with the stator flux is required. This can be done by calculating the angle for the stator flux vector $\mathbf{\psi}_s$. There are two ways for obtaining the transformation angle called the stator flux orientation and the grid flux orientation (stator voltage orientation). If the stator resistance is neglected then the stator flux orientation also gives the orientation along the stator voltage vector without any significant error [30]. Mathematically it can be shown as:

$$ u_s = R_s i_s + \frac{d\psi_s}{dt} + j \omega_s \psi_s \quad (4.3) $$

In a steady state and ignoring the stator resistance i.e. $R_s = 0$, it gives:

$$ u_s = j \omega_s \psi_s \quad (4.4) $$

4.3.2.1 Stator flux orientation

The transformation angle $\theta_s$ can be found by measuring the stator flux vector $\mathbf{\psi}_s$ i.e.
Using $\theta_r$, the stator flux $\psi_s$ in a stator reference frame can be transferred to the flux oriented reference frame in such a way that it aligns with the $d$-axis having no imaginary component. A stator flux vector can be measured by measuring the stator voltages and the currents [30].

### 4.3.2.2 Grid flux orientation

In this method, a virtual grid flux $\psi_s^g$ is assumed and the angle of grid flux is obtained from the grid voltage vector $u_s$, this angle can be used for obtaining the transformation angle as:

$$\angle u_s = \theta_g$$

(4.6)

$$\theta_r = \angle \frac{u_s}{j} = \theta_g - \frac{\pi}{2}$$

(4.7)

Using the obtained transformation angle $\theta_r$, the stator terminal voltage $u_s$ can be transferred to the rotating reference frame. For a perfect grid flux orientation $u_s$ will have only the imaginary component i.e. $u_s$ aligned along $q$-axis [30]. The transformation angle for a grid flux orientation can be found directly from the stator voltage using phase locked loop technique that is explained later.

Comparing the stator flux orientation and the grid flux orientation, it can be observed that for a perfect transformation in case of a stator flux orientation, $\psi_s$ has only real component (aligned to the $d$-axis) and in case of a perfect grid flux orientation, $u_s$ has only imaginary component. But there exist a small angular difference for a stator terminal voltage and stator flux space vector in a stator flux orientation and a grid flux orientation respectively. The comparison of a stator flux vector orientation and a grid flux vector orientation is shown in Figure 4.5. This small difference in angular position is basically because of the stator resistance [29], [30].

![Comparison between a stator flux orientation (a) and a grid flux orientation (b) [33].](image)

**Figure 4.5: Comparison between a stator flux orientation (a) and a grid flux orientation (b) [33].**

### 4.3.3 Calculating transformation or stator flux angle

There are three different methods for obtaining the transformation angle $\theta_r$ that are studied here and named as:

1. Phase locked loop (PLL).
2. Stator voltage and current method.
3. EMF method.
4.3.3.1 Phase locked loop (PLL)

PLL is a tool for calculating the transformation angle. There are several different algorithms for PLL but here a simple and an efficient one is used and explained as [29]:

In the used strategy, the PLL synchronizes with the three-phase signal (grid voltage) using the $dq$ voltage axis. Based on the closed loop logic, the PLL synchronizes with the grid voltage in such a way that it aligns to the $d$-axis having no imaginary component. So the PLL modifies $\theta_g$ until the grid voltage vector completely aligns with the $d$-axis. The transformation angle $\theta_t$ can be found by subtracting $\frac{\pi}{2}$ from $\theta_g$. The block diagram for PLL is shown in Figure 4.6.

![Figure 4.6: PLL block diagram.](image)

This method offers a simple calculation for the transformation angle $\theta_t$ but it uses integration in the loop, so if there exists a small error somewhere in the system then that error will be integrated on its way. Also the above explained method of using the stator terminal voltages does not consider the voltage drop across the stator resistance which can cause problems in a real time application.

In the used strategy, the dynamics of a PLL depends largely on the tuning of PI regulator [29]. The tuning of PI regulator depends largely on the applied voltage magnitude [29] and can vary largely with the applied voltage. This problem can be solved if the input quantities are expressed in per-unit system.

4.3.3.2 Stator voltage and current method

The transformation angle can be found by measuring the stator terminal voltages and currents. Usually two line-to-line voltages are measured from which the three-phase voltages can be obtained and similarly the currents can be obtained in the same way. The stator terminal voltage in a stator reference frame is given as:

$$u_{s,\alpha\beta} = R_s i_{s,\alpha\beta} + \frac{d\psi_{s,\alpha\beta}}{dt}$$

(4.8)

The stator flux vector (magnitude and angle) can be calculated as:

$$\psi_{s,\alpha\beta} = \int (u_{s,\alpha\beta} - R_s i_{s,\alpha\beta}) dt$$

(4.9)

$$\angle \psi_s = Tan^{-1} \left( \frac{\psi_{s,\beta}}{\psi_{s,\alpha}} \right)$$

(4.10)
In this method, the stator flux orientation is used as it gives the angular position for $\psi_s$ directly. Comparing this to the last method, it considers the voltage drop across $R_s$ but still it uses integration in the loop.

4.3.3.3 EMF method

The integration problem in the stator voltage and current method can be avoided if the EMF angular position is measured i.e.

$$e_{\alpha\beta} = u_{\alpha\beta} - R_s i_{\alpha\beta} \quad (4.11)$$

$$\angle e_{\alpha\beta} = \tan^{-1} \left( \frac{e_{\beta}}{e_{\alpha}} \right) \quad (4.12)$$

Subtracting $\frac{\pi}{2}$ from $\angle e_{\alpha\beta}$ gives the stator flux angle. Using this method, integration can be avoided and also it considers voltage drop across the stator resistance. Therefore this method will be used for obtaining the transformation angle throughout this study work. The block diagram presenting this method is shown in Figure 4.7.

![Block diagram](image)

**Figure 4.7: Calculation of transformation angle $\theta_t$ through EMF.**

4.3.4 Reference signals

The active and the reactive power of a DFIG at the stator terminals can be controlled independently if the rotating $dq$ reference frame is properly aligned with the stator flux. Under this orientation the relation between the flux and the currents can be obtained from the DFIG flux equations given in chapter 3 as:

$$\psi_{sd} = L_s i_{sd} + L_m i_{rd} \quad (4.13)$$

$$\psi_{sq} = L_s i_{sq} + L_m i_{rq} = 0 \quad (4.14)$$

The $d$- and $q$-axis stator current can be derived from (4.13) and (4.14) as:

$$i_{sd} = \frac{\psi_{sd} - L_m i_{rd}}{L_s} \quad (4.15)$$

$$i_{sq} = -\frac{L_m i_{rq}}{L_s} \quad (4.16)$$
The reference values of $i_{rd}^*$ and $i_{rq}^*$ for controlling the stator active power $p_s$ and the stator reactive power $q_s$ can be derived from the power equations that are given in chapter 3 as:

$$p_s = \frac{3}{2} (u_{sd} \cdot i_{sd} + u_{sq} \cdot i_{sq})$$  \hspace{1cm} (4.17)

As explained earlier, with the reference frame aligned with the stator flux, the stator voltage gets align with the imaginary axis having only imaginary component i.e.

$$p_s = \frac{3}{2} i_{sq} \cdot i_{sq}$$  \hspace{1cm} (4.18)

Using (4.16) and (4.18), the reference $q$-axis rotor current $i_{rq}^*$ can be derived as:

$$i_{rq}^* = \frac{p_s^*}{\frac{3}{2} u_{sq} \cdot \frac{L_m}{L_s}}$$  \hspace{1cm} (4.19)

Also the reference $q$-axis rotor current $i_{rq}^*$ can be obtained in terms of reference torque i.e.

$$p_s = \omega_r \cdot T_{elec} = \frac{\omega_s}{p} T_{elec}$$  \hspace{1cm} (4.20)

Equation (4.19) can be modified using (4.20) i.e.

$$i_{rq}^* = \frac{\omega_s T_{elec}^*}{\frac{3}{2} p u_{sq} \cdot \frac{L_m}{L_s}}$$  \hspace{1cm} (4.21)

Similarly, the reference $d$-axis rotor current $i_{rd}^*$ can be obtained from the required stator reactive power $q_s$. $q_s$ in the rotating reference frame terms can be given as:

$$q_s = \frac{3}{2} (u_{sd} \cdot i_{sd} - u_{sq} \cdot i_{sq})$$  \hspace{1cm} (4.22)

With the alignment of the rotating reference frame to the stator flux vector it gives $q_s$ as:

$$q_s = -\frac{3}{2} u_{sq} \cdot i_{sd}$$  \hspace{1cm} (4.23)

Using (4.15) and (4.23) it gives,

$$q_s = -\frac{3}{2} u_{sq} \cdot \left(\psi_{sd} - \frac{L_m \cdot i_{rd}^*}{L_s}\right)$$  \hspace{1cm} (4.24)

In steady state operation with the stator flux orientation and ignoring the stator resistance $\psi_s \approx j \omega_s \cdot \psi_s$ [30], it gives:

$$\psi_s = \frac{u_{sq}}{\omega_s}$$  \hspace{1cm} (4.25)

Using (4.24) and (4.25), the reference $d$-axis rotor current $i_{rd}^*$ can be found for any required value of the stator reactive power $q_s$. In this study work, the objective is to operate the DFIG at a unity power factor i.e. at $q_s = 0$, for $\cos(\theta) = 1$. So for a unity power factor, $i_{rd}^*$ can be obtained as:

$$0 = -\frac{3}{2} u_{sq} \cdot \left(\frac{u_{sq}}{\omega_s} \cdot \frac{L_m \cdot i_{rd}^*}{L_s}\right)$$  \hspace{1cm} (4.26)
\[ i_{rd}^* = \frac{u_{sq}}{L_m \omega_s} \]  
\[ i_{rd}^* = \frac{u_{sq}}{L_m \omega_s} \]  

4.3.5 Current controllers

After calculating the reference currents and the transformation angle for the rotating reference frame, the current controllers are used to make sure that the DFIG currents properly follow the reference currents. It is possible to implement the controller in any frame of reference but practically it is better to use same reference frame in which the reference currents are calculated [33]. The proportional integral (PI) controllers are used because of their relative simplicity. The anti-wind up phenomenon is also adopted here to limit the output of the regulators to the actual possible values. In the rotating dq reference frame the three-phase quantities appear as DC quantities which make the control process much simple.

4.3.6 Cross-coupling terms

The voltage at the rotor terminal in dq reference frame is given by equations (3.64) and (3.65), also the rotor flux in dq reference frame is given by equations (3.69) and (3.70). Further using \( i_{sd}, i_{sq} \) from equation (4.15) and (4.16) and placing in (3.69) and (3.70) respectively gives the fluxes as:

\[ \psi_{rd} = (L_r - \frac{L_m^2}{L_s}) \cdot i_{rd} + \frac{L_m}{L_s} \cdot \psi_{sd} \]  
\[ \psi_{rq} = (L_r - \frac{L_m^2}{L_s}) \cdot i_{rq} \]  

Defining \( \sigma \cdot L_r = L_r - \frac{L_m^2}{L_s} \) and using (4.28) and (4.29) in (3.64) and (3.65) respectively gives:

\[ u_{rd} = r_r \cdot i_{rd} - \omega_r \cdot \sigma \cdot L_r \cdot i_{rd} + \sigma \cdot L_r \frac{d}{dt} i_{rd} + \frac{L_m}{L_s} \frac{d}{dt} \psi_{sd} \]  
\[ u_{rq} = r_r \cdot i_{rq} + \omega_r \cdot \sigma \cdot L_r \cdot i_{rd} + \sigma \cdot L_r \frac{d}{dt} i_{rd} + \omega_r \frac{L_m}{L_s} \psi_{sd} \]  

Under the normal operating conditions, the grid voltage remains constant in amplitude giving \( \frac{d}{dt} \psi_{sd} = 0 \). So the last terms of (4.30) and (4.31) disappear (but this term will not disappear if there is variation in the grid voltage). From the control point of view the term \( \omega_r \frac{L_m}{L_s} \psi_{sd} \) is constant and depends on the stator flux (grid voltage), independent of the control loop, so it can also be ignored [33]. The terms \( \omega_r \cdot \sigma \cdot L_r \cdot i_{rd} \) and \( \omega_r \cdot \sigma \cdot L_r \cdot i_{rq} \) are the cross-coupling terms and exist because of the rotating reference frame. These terms give constant values under the constant references and do not affect the normal control operation but these terms cause effect on each other during transients. So for compensating there effects during transients, these terms are subtracted from the reference calculated voltages. By doing so, two-axes become independent of each other.

4.3.7 Complete control system

The method for calculating the transformation angle \( \theta_t \), the reference currents, current control and the removal of cross coupling terms is given above. Having all these values, the conventional vector control for a DFIG can be implemented. The
block diagram showing the DFIG control is given in Figure 4.8. The rotor currents measured from the machine and transferred to the stator flux oriented reference frame using $\theta_t$. In a stator flux oriented frame the measured currents are compared with the reference values and the PI controllers based on the error produces the required voltages which further transformed to the three-phase quantities before applying to the inverter and the machine. The transformation angle can be calculated using any of the method explained. In this work, $\theta_t$ is found using EMF method. The cross-coupling terms are also subtracted to have the independent control of $p_s$ and $q_s$. The reference current values are calculated using equation (4.19) and (4.27) as shown in Figure 4.8. The proper tuning of PI controllers and the calculation of initial values for the DFIG fluxes are very important for getting the smooth curves.

Figure 4.8: Complete FOC control logic for DFIG.

4.4 Back-to-back PWM converters and switching pulses

Back-to-back pulse width-modulated converter (PWM) consists of two three-phase PWM converters connected to each other through a DC-link shown in Figure 4.9. The back-to-back PWM converter provides the electrical coupling between two electrical machines. The back-to-back PWM converter is shown in Figure 4.9. It consists of semiconductor power switches and the freewheeling diodes. Most of the modern converters use IGBTs as power switches as it offers a high switching speed
and less conduction losses. The freewheeling diode is used for bi-directional flow of the power through the converter [8].

![Back-to-Back PWM Converter](image)

Figure 4.9: Back-to-Back PWM Converter.

Basically one of the two back-to-back PWM converters, depending upon the mode of the operation will receive electrical power from one of the electrical machine and convert it to DC power and then the other converter will supply the controlled voltage and frequency (power) to the other electrical machine. So the purpose of the converter is to control the flow of AC power between the machines keeping the DC-link voltage constant.

The output of a converter can be adjusted by controlling the switching of the IGBTs in a proper way. The output voltage of the converter is a pulse train that switches between $+\frac{V_{dc}}{2}$ and $-\frac{V_{dc}}{2}$. Therefore along with the desired fundamental component, the output contains many unwanted components called harmonic components that affect the performance of both the converter and the machines. There are several control switching algorithms or techniques such as six-step inverter, hysteresis current control, sinusoidal pulse width-modulation and the space vector pulse width-modulation (SVPWM). Each algorithm has its own advantages and disadvantages [34]. Mainly there are three different performance parameters for a converter output signal. The harmonic contents or the measure of the total harmonic distortion (THD) in the output, the low value of a THD is always desired. The second parameter is the switching frequency; high switching frequency is always aimed as harmonic contents shift towards higher frequencies which are easy to filter out. The switching frequency of a converter is limited by the higher switching losses across converter and also it is limited by the switching capability and the dead time of the devices. Third constraint is the best utilization of the available $V_{dc}$.

The six-step inverter and the hysteresis current controller offer simple control algorithm but contains lower order harmonics compared to the PWM, resulting in a higher value of THD [34]. Therefore in this work, only Sinusoidal PWM and SVPWM algorithms are utilized.

As the aim of the thesis is to evaluate the new topology for a variable speed drive using a DFIG, therefore a simple two-level voltage source converter (VSC) is
considered in this case. Also throughout this thesis, an ideal converter model (ideal switches) is considered for the modeling. The dead time, switching delay, conduction losses and switching losses are neglected.

4.4.1 Sinusoidal PWM

This is the most popular and well-known strategy that is used for controlling the switches. In this method, a triangular signal (carrier) at high frequency called the switching frequency is compared with the low frequency reference (required) signal. It controls the phase leg switches such that if the value of the reference signal is higher than the carrier signal then upper switch will be on and if the reference signal is lower than the carrier signal then lower switch of phase leg will get on (bipolar switching). An important thing that must be considered here is that both the switches of a phase leg must not be switched on at an instant to avoid the short circuit. Consider the phase leg A for the PMSM side converter shown in Figure 4.9 and suppose at some instant the carrier signal is lower than the reference signal. The control scheme will generate logic 1 and at that instant switch a1 shown in Figure 4.9 will be on and the switch a2 will be off, where on the contrary if carrier signal is higher than the reference signal then the control scheme will generate -1 logic and at that instant switch a1 will be off and switch a2 will be on. The sinusoidal PWM converter modeled in SIMULINK is shown in Figure 4.10 and the basic principal and logic curves for the control of switches are shown in Figure 4.11.

The control scheme for a sinusoidal PWM is easy to implement using analog integrators and comparators [34]. Because of the variation of the reference signal, the relation between the reference signal and the carrier signal varies and results in varying the switching instants causing the unwanted harmonics [34]. Also V_{DC} is not being fully utilized in simple sinusoidal PWM, for this an addition of 1/6\textsuperscript{th} of third harmonic is required to go beyond the modulation index of 1 while staying in the linear range of modulation index [29].
Figure 4.10: MATLAB/SIMULINK converter model, triangular and reference signal for sinusoidal PWM converter.
4.4.2 Space vector modulation SVM

Space vector modulation is a widely accepted control strategy for power electronic converters and a lot of research had been conducted in this area. It reduces most of the drawbacks that existence in a sinusoidal PWM such as generate fewer harmonics, and make an efficient use of the available $V_{DC}$.

In SVM, a voltage space vector is processed as a whole whereas in sinusoidal PWM a separate modulator for each phase is required [34]. In this section, the basic working of SVM and the pulse generation for a space vector modulated converter is presented.

4.4.2.1 Voltage space vectors

The basic working principle of a three-phase space vector modulated VSI is based on the representation of three-phase quantities to two dimensional quantities $(\alpha, \beta)$. Three-phase voltages can be converted to $(\alpha, \beta)$ plane using Clark’s transformation matrix also given here as:

$$
\begin{bmatrix}
V_{s\alpha} \\
V_{s\beta}
\end{bmatrix} =
\begin{bmatrix}
\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\
0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}}
\end{bmatrix}
\begin{bmatrix}
V_a \\
V_b \\
V_c
\end{bmatrix}
\quad (4.32)
$$

Above given matrix of transformation is valid only in a case when $\alpha$ axis is aligned with $a$ axis of a three-phase supply as shown in Figure 4.12.
4.4.2.2 Switching algorithm

In order to understand that how voltage space vectors are related to the switching of a voltage source inverters, consider the single voltage source inverter as shown in Figure 4.13. For a particular instant, the upper switch of phase leg A and the lower switches of phase leg B and C are closed. With this switching configuration the line voltages $V_{ab}$, $V_{bc}$ and $V_{ca}$ are given as:

\[ V_{ab} = V_{DC} \]  \hspace{1cm} (4.33)
\[ V_{bc} = 0 \]  \hspace{1cm} (4.34)
\[ V_{ca} = -V_{DC} \]  \hspace{1cm} (4.35)

With the given values of $V_{ab}$, $V_{bc}$ and $V_{ca}$ the effective voltage space vector $V_1$ is also shown in Figure 4.13. The magnitude and the direction of vector $V_1$ can be adjusted by controlling the switching of the converter switches. A two-level VSC with six switches having two switches in each phase leg can produce eight possible switching combinations making sure that two switches of any phase leg must not be switched on or off together at an instant. All the possible eight combinations for a two-level VSC are shown in Figure 4.14.
Each switching topology or combination shown in Figure 4.14 corresponds to a unique voltage space vector. The voltage space vectors correspond to above combinations are shown in Figure 4.15. The states 1 or 0 shown above each voltage vector indicate the switching combination. Switching states only for the upper switches of the legs are given. For example 110 for $V_2$ in Figure 4.15 represents that the upper switches of phase leg A and B are on whereas for C it is off. Eight space vectors are termed as $V_1, V_2, V_3, V_4, V_5, V_6, V_7$ and $V_8$. Two out of these eight vectors are zero vectors as all the upper switches in $V_7$ and $V_8$ are closed and open respectively producing the zero voltage. Figure 4.15 b) shows different segments within the hexagon numbered as 1 – 6 (area enclosed between two adjacent vectors is defined as segment).
Figure 4.15: a) Voltage space vectors for SVM, b) Sectors between voltage vectors.

Now the required (reference) three-phase AC signal can also be represented as a rotating space vector $V^*$. The reference space vector can be obtained by controlled switching of the nearest three voltage space vectors [29]. Depending on the segment of the reference voltage vector, two different voltage vectors are injected whereas the third injected voltage vector is always the zero vector either $V_7$ or $V_8$. For example, if the reference voltage vector $V^*$ is located in sector 1 as shown in Figure 4.15, the two non-zero vectors that will be used in this case are $V_1$ and $V_2$ combined with the zero vector as a third voltage vector. For properly controlling the magnitude of the actual generated vector, the two active vectors must be switched for calculated time of interval during each switching period. The required voltage vector $V^*$ as a whole in terms of these three vectors can be presented as [20]:

$$V^* = V_1 \cdot d_1 + V_2 \cdot d_2 + V_0 \cdot (1 - d_1 - d_2) \quad (4.36)$$

$d_1$ and $d_2$ are the duty cycles for two active vectors. The relations for calculating the duty cycles are given in [20] as:

$$d_1 = m_n \cdot (\cos \theta_n - \frac{\sin \theta_n}{\sqrt{3}}) \quad (4.37)$$

$$d_2 = m_n \cdot 2 \cdot (\frac{\sin \theta_n}{\sqrt{3}}) \quad (4.38)$$

Remaining of the time in a switching period is dedicated to the zero voltage vectors that can be calculated as:

$$d_{7,8} = 1 - d_1 - d_2 \quad (4.39)$$

$\theta_n$ is the equivalent angle of the reference voltage vector in first segment and its relationship is given in Table 4.2. $m_n$ is used for normalization of the reference voltage vector with the available DC-link voltage and given as [20]:

$$m_n = \frac{|V^*|}{\frac{2}{3}V_{dc}} \quad (4.40)$$
Table 4.2: Equivalent angle $\theta_n$ in the first segment.

<table>
<thead>
<tr>
<th>Segments</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual angle $\theta$</td>
<td>$(0, \frac{\pi}{3})$</td>
<td>$(\frac{\pi}{3}, \frac{2\pi}{3})$</td>
<td>$(2\frac{\pi}{3}, \pi)$</td>
<td>$(\pi, 4\frac{\pi}{3})$</td>
<td>$(4\frac{\pi}{3}, 5\frac{\pi}{3})$</td>
<td>$(5\frac{\pi}{3}, 2\pi)$</td>
</tr>
<tr>
<td>Equivalent angle $\theta_n$</td>
<td>$\theta$</td>
<td>$\theta + \frac{2\pi}{3}$</td>
<td>$\theta - \frac{2\pi}{3}$</td>
<td>$\theta + 4\frac{\pi}{3}$</td>
<td>$\theta - 4\frac{\pi}{3}$</td>
<td>$-\theta + 6\frac{\pi}{3}$</td>
</tr>
</tbody>
</table>

Once the duty cycles are calculated, the time for all three vectors can be calculated using the switching time period i.e.

\[
t_1 = d_1 \cdot T_s \\
t_2 = d_2 \cdot T_s \\
t_{7,8} = d_{7,8} \cdot T_s
\] (4.41) (4.42) (4.43)

Depending upon the requirement, there are three different ways of switching for a SVM voltage source inverter. These schemes are differentiated depending upon the placement of zero voltage vectors and named as:

1. Right-aligned sequence SVM
2. Symmetric sequence SVM
3. Alternating zero vector sequence SVM

Three patterns are shown in Figure 4.16 and the comparison between them is given in [35], [36]. The number of switching is the least in right-aligned sequence SVM and maximum in symmetric sequence SVM. In terms of harmonics or THD measures, the symmetric sequence SVM shows the best performance [34], [35], [36]. Therefore, to have better harmonic performance symmetric sequence SVM is studied and implemented here.
Consequently when the switching times are obtained using (4.41), (4.42) and (4.43) the next step is to obtain the gate pulses for controlling the output of the inverter. The switching pulses for the gates can be obtained by comparing the calculated switching times to a carrier (triangular) signal. The frequency of the carrier (triangular) signal is defined by the switching frequency of the inverter. Comparison and the pulse generation for one switching period is shown in Figure 4.17.

It should be noticed here that the time calculated for zero vectors is equally divided between \( V_7 \) and \( V_8 \). Half of \( t_{7,8} \) is given to \( V_8 \) and half is given to \( V_7 \). \( V_9 \) is injected symmetrically at the beginning and at the end of each switching period where as \( V_7 \) is just injected in the middle of each switching period. The other two non-zero vectors can be injected symmetrically according to the calculation of \( t_1 \) and \( t_2 \) as shown in Figure 4.17.

Compared to the sinusoidal PWM, the gate pulses in SVM are generated by comparing one triangular waveform to three constant values obtained by switching times using (4.41), (4.42) and (4.43). The switching signals \( S_a, S_b \) and \( S_c \) for upper switches of three phase legs are shown in Figure 4.17 whereas the switching states for lower switches will always be the opposite to the upper ones.

---

**Figure 4.16: Three general switching patterns for SVM.**
Figure 4.17: Pulse generation for a controlled switching of the inverter.

The value of $t_1$, $t_2$ or $d_1$, $d_2$ are altered or adjusted in every switching cycle according to the required rotating reference space vector. After calculating the pulses in equivalent first segment, the actual or real gate pulses depending upon the real segment must be generated. The summary of conversion from equivalent first segment to the real segments is given in Table 4.3.

<table>
<thead>
<tr>
<th>Segment</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase leg A upper switch</td>
<td>$S_{ar} = S_a$</td>
<td>$S_{ar} = S_b$</td>
<td>$S_{ar} = S_c$</td>
<td>$S_{ar} = S_b$</td>
<td>$S_{ar} = S_a$</td>
<td>$S_{ar} = S_b$</td>
</tr>
<tr>
<td>Phase leg B upper switch</td>
<td>$S_{br} = S_b$</td>
<td>$S_{br} = S_a$</td>
<td>$S_{br} = S_b$</td>
<td>$S_{br} = S_c$</td>
<td>$S_{br} = S_c$</td>
<td>$S_{br} = S_b$</td>
</tr>
<tr>
<td>Phase leg C upper switch</td>
<td>$S_{cr} = S_c$</td>
<td>$S_{cr} = S_c$</td>
<td>$S_{cr} = S_b$</td>
<td>$S_{cr} = S_a$</td>
<td>$S_{cr} = S_a$</td>
<td>$S_{cr} = S_b$</td>
</tr>
</tbody>
</table>

The above generated switching states $S_{ar}$, $S_{br}$ and $S_{cr}$ can now be applied to the gates of the switches. The maximum achievable fundamental voltage in the linear range of modulation index with SVM is equivalent to the sinusoidal PWM with 1/6th of the third harmonic injection in the carrier signal [29]. The simplified summary of the SVM strategy is shown in Figure 4.18.
Figure 4.18: Block diagram of the implemented SVM.
4.5 DC-link voltage control

It is very important to have a stable DC-bus voltage for a smooth operation of machines. The capacitor current can be given as:

\[ i_c = C_{DC} \frac{dV_{DC}}{dt} \quad (4.44) \]

\[ V_{DC} = \frac{1}{C_{DC}} \int i_c \, dt \quad (4.45) \]

If the current flows out of the capacitor (discharging) then capacitor terminal voltage decreases whereas if the current flows to the capacitor (charging) then terminal voltage increases. To have the constant \( V_{DC} \), it must be ensured that the energy flowing into the capacitor must be equal to the energy flowing out of the capacitor or in terms of current, current entering the capacitor must be balanced by the current leaving it.

In case of the studied variable speed drive (VSD), for a constant DC-link voltage the amount of the power that is produced or consumed by the DFIG rotor must be consumed or supplied by the PMSM respectively. For example, if the DFIG is operating in sub-synchronous mode then rotor will consume active power \( P_r \). The DFIG will take this power through the DC-link by a PWM converter. For maintaining the DC-link voltage constant, the PMSM should supply this amount of power to the DC-link. The power flow across the DC-link and back-to-back PWM converter operating in a sub-synchronous mode is shown in Error! Reference source not found.. In the figure for maintaining the DC-link voltage constant, the power flowing into the box of back-to-back PWM converter must be equal to the power flowing out of the box. The currents at the DC-link node are shown in Figure 4.19.

![Figure 4.19: Current at DC-link node.](image)

To maintain a constant \( V_{DC} \), the reference signal for the control of a PMSM can be generated from the DFIG rotor-side active power in such a way that if the rotor is consuming power then PMSM must generate that power and if a DFIG rotor is producing power then the PMSM must consume that electrical power to produce the mechanical power. Assuming the ideal converters (lossless), the DFIG rotor power and the PMSM power at a DC-link terminal can be interpreted in terms of DC voltage \( V_{DC} \) as:

\[ P_{rDFIG} = V_{DC} \cdot i_1 \quad (4.46) \]

\[ P_{PM} = V_{DC} \cdot i_2 \quad (4.47) \]
Note that the direction of power for $P_{rDFIG}$ and $P_{PM}$ is always opposite of each other. $P_{rDFIG}$ can be measured by using power equation (3.90) given in chapter 3. For controlling the power of a PMSM, the reference value $i_{sq}^*$ should be known and it can easily be calculated. As mentioned in the section 4.2, for maximizing the torque in a surface mounted permanent magnet machine the stator current vector must be aligned to the $q$-axes having $i_{sd}^* = 0$ i.e. the PMSM active power can be calculated using $i_{sq}$ as:

$$P_{PM} = T_{elec} \cdot \omega_r$$  \hspace{1cm} (4.48)  \\
$$P_{PM} = \frac{3}{2} \cdot p \cdot \psi_m \cdot i_{sq} \cdot \omega_r$$  \hspace{1cm} (4.49)  

The current form the DFIG to the DC-link can be calculated using (4.46) and (3.92) whereas the current between the DC-link and the PMSM can be calculated using (4.47) and (4.49). According to Kirchhoff’s circuit law it can be written as:

$$i_1 + i_2 = i_c$$  \hspace{1cm} (4.50)  

The complete control strategy for a DC-link voltage control is shown in Figure 4.20. The current $i_{sq}$ measured from the machine and the PMSM power is calculated using (4.49). From the calculated power, the current flowing from PMSM to DC-link $i_2$ is calculated using (4.47). The capacitor current $i_c$ is calculated using KCL and it is assumed that $i_1$ and $i_2$ is always opposite. The dynamics of a DC-link voltage are calculated using (4.45) and compared with the reference (required) $V_{DC^*}$. The difference between the values fed to a $PI$ regulator that generates the required current $i_2^*$. $i_{sq}^*$ can be obtained from calculated $i_2^*$ as:

$$i_{sq}^* = \frac{2}{3} \cdot \frac{V_{DC} \cdot i_2}{p \cdot \psi_m \cdot \omega_r}$$  \hspace{1cm} (4.51)  

![Figure 4.20: Control strategy for DC-link voltage.](image)

The initial value of $V_{DC}$ is set to zero so that there is no stored energy in the capacitor, therefore the PMSM machine should charge the capacitor to the required voltage before the operation of a DFIG at the load. This can be done before the shaft reaches the minimum specified speed for the energy production.
5 SIMULATION RESULTS

The modeling and the control strategies for different components of the studied VSD have been explained in chapter 3 and 4. It is very important here to have stable control of the whole system as it has both the mechanical and the electrical coupling. In this chapter, the developed models and the control strategies are verified under different operating speeds (sub- and super-synchronous modes) using the MATLAB/SIMULINK.

The scheme for simulating the topology as a whole is shown in Figure 5.1. The hydro turbine model and the governing system is presented with HYGOV block, mechanical shaft block represents the low-speed and high-speed shaft. \( \omega_s \) is the turbine-side shaft speed before the gear-box and \( \omega_r \) is the rotor-side shaft speed. The shaft speed is taken as a feedback to the turbine governor for controlling the rotation of the turbine. Two machines are coupled mechanically on the same shaft having the same speed \( \omega_r \). \( T_{mech} \) is the input mechanical torque produced by the micro-hydro turbine and \( T_{elec} \) is the load (electrical) torque on the electrical machines that is obtained as a sum of the electrical torque both from the PMSM and the DFIG.

The reference torque or the requested torque \( T^* \) for the model is obtained by dividing the required electrical power \( P^* \) by the rotating speed of the shaft. The reference torque \( T^*_{DFIG} \) signal for a DFIG is obtained as the difference of the total input torque from the turbine and the PMSM torque. The reference for the PMSM machine is obtained based on the control strategy. The only external reference that is required for the operation of the topology is the required electrical power that can be defined by the grid owner. The reference for the control of a PMSM is obtained automatically based on the principle of DC-link voltage control explained in chapter 4. The q-axis reference current \( i^*_q \) for controlling the PMSM is obtained using the DFIG rotor active power and DC-link voltage control as shown in Figure 5.1. For obtaining the smooth reference signal for a DFIG, a low-pass filter is used on the way of reference calculation for getting rid of the DFIG dynamics, otherwise there may be a chance of sustaining oscillations in the system. The transfer function of used low-pass filter is given as:

[5.1]

\[
t . f = \frac{s}{s + 30}
\]

Electrical connection between the machines is provided by using the back-to-back PWM inverters, included in the respective machine blocks and the DC voltage \( V_{dc} \) from the DC voltage block is taken as an input for the inverters.

The system can be operated in a wide range of speed depending upon the variation of the flow. Therefore the system was simulated under two different cases, named as case 1 and case 2.

5.1 Case 1: Sub-synchronous mode

In this case, the complete model was simulated at a slip of +0.2 i.e. \( \omega_r = \frac{rad}{\sec} \). The parameters of the used PMSM and the DFIG are given in the Appendix B and also the other parameters like DC-link capacitance, parameters for HYGOV and mechanical shaft are provided in Appendix B. The detailed control
blocks and all the schematics from modeling in SIMULINK are given in the Appendix C. The simulation settings for case 1 are listed in Table 5.1. Further it was assumed that until the shaft reached the speed of \( \frac{60 \text{ rad}}{\text{sec}} \), the rotor terminals were open-circuited i.e. no input and output from the DFIG.

**Figure 5.1: Complete simulation model for the whole system.**
Table 5.1: Simulating conditions at sub-synchronous mode.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchronous speed</td>
<td>$157.08 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Grid frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Operation speed</td>
<td>$125.664 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Slip</td>
<td>+0.2</td>
</tr>
<tr>
<td>Load torque</td>
<td>250 Nm</td>
</tr>
<tr>
<td>Requested power</td>
<td>31.416 kW</td>
</tr>
</tbody>
</table>

It was supposed that the turbine and the shaft were at rest and at $t = 0$ the turbine started its operation. At the start it was assumed that there was no electrical load at the stator terminal of the DFIG, so this allowed the shaft speed to increase from 0 to the specified or reference speed. The shaft or the rotor speed is shown in Figure 5.2. The shaft reached the desired speed at about $t = 10.5 \text{ sec}$. Still there was no electrical load applied at the stator of the DFIG and the system continued its operation at no load. At the start, the slope of the speed curve is less because the control here was designed in such a way that at the start PMSM had to deliver the power to charge the capacitor to the specified DC-link voltage. In this case the system was subjected to an electrical load at $t = 16 \text{ sec}$.

Figure 5.2: Rotor speed simulated at sub-synchronous mode with $s=0.2$.

The torque curves from the turbine, DFIG and the PMSM are shown in Figure 5.3. The mechanical torque curve is showing the total input torque from the turbine to the
system. At the start, the turbine is giving its maximum torque to reach the reference speed. After reaching the specified speed $\omega_r$, the turbine torque decreases to the no load torque. Between $t = 10.5$ to $16$ sec, the turbine looks noisy because of the pulsations in the torque introduced by the PWM converters.

Figure 5.3: Input mechanical, total load, PMSM load and DFIG load torques. At $t = 16$ sec, the system was subjected to an electrical load or a power of 31.416 kW was requested at the grid. The requested power was divided by the operating speed of 125.664 rad/sec in order to get the reference torque for the model. At $t = 16$ sec, the turbine torque increased to balance the load torque in order to maintain...
the rotational speed constant. The load curve is showing the total electrical load (torque) on the system that was obtained as a sum of the load on the PMSM and the DFIG. At $t = 0$ sec there was a load torque of $-600$ Nm which was taken by the PMSM for charging the DC-link capacitor. At $t = 16$ sec, an electrical load of $-250$ Nm was applied that was equally supplied by the turbine. From the curves, it can be observed that the electrical dynamics (response) is much faster compared to the mechanical dynamics. The PMSM and the DFIG load torque curves are also shown in Figure 5.3, as the system was simulated in sub-synchronous mode therefore the DFIG rotor required an electrical power at a slip frequency and that power must be supplied by the PMSM. So for providing the slip power to DFIG rotor, the PMSM took slip torque from the turbine and the available torque for the DFIG was the difference of the total input turbine torque and the PMSM load torque. At $+0.2$ slip, PMSM load torque was $250 \cdot 0.2 = 50$ Nm and the input for a DFIG was $200$ Nm. There was a small difference in the calculated and the simulated torques and that was because of the losses across the rotor and the stator resistances of a DFIG.

![DFIG Stator Active Power](image)

**Figure 5.4: DFIG stator active and reactive power.**

Figure 5.4 is showing the simulated stator active and the reactive power of a DFIG. The total electrical output of the system appeared only at the stator terminal of the DFIG and it was equal to the total input mechanical power. The simulated active power $p_a$ at the stator terminal was $3.023 \cdot 10^4$ watts, which was very close to the total system input power $3.1416 \cdot 10^4$ watts. The difference in the input and the output is because of the resistive losses in the electrical machines. The objective of the control strategy was to operate the DFIG at a unity power factor $\cos(\theta) = 1$. The stator reactive power was just $-77.69$ watts compared to the stator active power that gives a power factor of $-0.999976$. The negative sign appearing in the power factor is because of the used definition for the power. The power factor for a DFIG is shown in Figure 5.5.
The DC-link voltage is also shown in Figure 5.5. At the start of the simulation capacitor was charged to 800 V, and then maintained at a constant value for the rest of operation.

![Power Factor @ Sub - Synchronous Mode](image1)

![DC - Link Voltage @ Sub - Synchronous mode](image2)

Figure 5.6 shows the PMSM and the DFIG rotor active power. As explained in chapter 4 for maintaining the DC-link voltage constant, the PMSM must recover or deliver the slip power. In a sub-synchronous mode, the DFIG rotor takes the power from the DC-link and the PMSM must have to provide that power for maintaining the DC-link voltage constant. From Figure 5.6 it can be seen that the PMSM was controlled in such a way that it always produced opposite power to that of a DFIG rotor. For instance at \( t = 20.02 \text{ sec} \) the DFIG rotor took 7372 watts and this amount of power was supplied by the PMSM (−7263 watts).

The model was simulated at \( s = +0.2 \) i.e. \( \omega_r = 125.6 \frac{\text{rad}}{\text{sec}} \). The total input mechanical power to the model was \( P_m = \omega_r \cdot T_m = 31416 \) watts. Therefore ideally, in a sub-synchronous mode the DFIG rotor should have taken \( s \cdot P_{\text{mech}} = 6283.2 \) watts. But in simulations, the rotor took more power than the ideally calculated because of the resistive losses across the stator and the rotor resistance of a DFIG. For having the stator flux frequency at a constant grid frequency, the DFIG rotor asked for a certain EMF at a slip frequency. This can be explained as; to develop this required rotor EMF, the power supplied at the rotor terminals should be greater than the slip power in order to compensate the rotor resistive losses. Because of rotor resistive losses, the rotor took 7372 watts instead of 6283.2 watts. This difference was mainly because of the rotor resistance and it can be shown by calculating the losses across \( r_r \). The rotor resistance \( r_r \) of used DFIG was 49.45 m-Ohm and the RMS rotor current was 81.33 amps. The calculated rotor resistive losses were 981.271 watts. The difference in the simulated and the ideally calculated slip power is 1088 watts which is very close to the rotor resistive losses.
Figure 5.6: PMSM and DFIG rotor active power.

Figure 5.7: DFIG rotor active and reactive power.

Figure 5.8 shows the d-axis rotor current. As it is explained in chapter 3 and 4, for maintaining the DFIG stator frequency at a constant grid frequency (50 Hz), the rotor must be supplied with the currents at a slip frequency ($s \cdot f_s$). In this case, the shaft
was rotating at \( s = +0.2 \), which gave \( s \cdot f_s = 10 \) Hz. From the small window in Figure 5.8, it can be observed that the rotor currents were injected at a frequency of 10 Hz.

![Figure 5.8: Injected rotor d-axis currents at a slip frequency.](image1)

The DFIG stator terminal phase a current is shown in Figure 5.9. The frequency of the current is exactly maintained at 50 Hz. The current waveform is not perfectly sinusoidal because of the PWM converters and presence of current ripples can be observed. The ripple in the current appeared because of the harmonics in the rotor flux caused by the rotor currents injected through the PWM converter.

![Figure 5.9: DFIG stator terminal phase a current.](image2)
5.2 Case 2: Super-synchronous mode

In this case the model was simulated at a slip of $-0.2$ i.e. $\omega_r = 188.496 \text{ rad/sec}$, the shaft speed was higher than the synchronous speed. The simulation settings for case 2 are listed in Table 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchronous speed</td>
<td>$157.08 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Grid frequency</td>
<td>$50 \text{ Hz}$</td>
</tr>
<tr>
<td>Operation speed</td>
<td>$188.496 \text{ rad/sec}$</td>
</tr>
<tr>
<td>Slip</td>
<td>$-0.2$</td>
</tr>
<tr>
<td>Load torque</td>
<td>$250 \text{ Nm}$</td>
</tr>
<tr>
<td>Requested power</td>
<td>$47.124 \text{ kW}$</td>
</tr>
</tbody>
</table>

Table 5.2: Simulating condition at super-synchronous mode.

Similarly, it was supposed here that turbine and the shaft were at rest. The turbine started its operation at $t = 0$. At the start it was assumed that there was no electrical load at the stator terminal of the DFIG, so this allowed the shaft speed to increase from 0 to the specified or the reference speed. The shaft or the rotor speed is shown in Figure 5.10. The shaft reached the desired speed at about $t = 16 \text{ sec}$. Still the load was not applied at the stator of the DFIG and the system continued its operation at no load. At the start, the slope of the speed curve was less because the control was designed in such a way that at the start of the operation the PMSM had to deliver the power to charge the capacitor to the specified DC-link voltage. In this case the system was subjected to electrical load at $t = 18 \text{ sec}$. 
The torque curves from the turbine, DFIG and the PMSM are shown in Figure 5.11. The turbine torque curve is showing the total input torque from the turbine to the system. At the start, the turbine was giving its maximum torque to reach the reference speed. After reaching the specified speed $\omega_r$, the turbine torque decreased to the no load torque. At $t = 18$ sec, the system was subjected to an electrical load or a power of 47.124 kW was requested at the grid. The requested power was divided by the operating speed of 188.496 rad/sec in order to get the reference torque for the model. At $t = 18$ sec, the turbine torque increased to balance the load torque in order to maintain the rotational speed constant. The load curve is showing the total electrical load (torque) on the system that was obtained as a sum of the PMSM and the DFIG torques. At $t = 0$ sec, there was a load torque of $-600$ Nm which was taken by the PMSM for charging the DC-link capacitor. At $t = 18$ sec, an electrical load of $-250$ Nm was applied that was equally supplied by the turbine. The PMSM and the DFIG load torque curves are also shown in Figure 5.11. As the system was simulated in a super-synchronous mode, therefore the DFIG rotor produced electrical power at a slip frequency and that power was consumed by the PMSM. So from the slip power that was produced at a DFIG rotor terminal, the PMSM produced mechanical torque equal to the slip torque. The mechanical torque produced by the PMSM machine also acted on the shaft and therefore the available torque for a DFIG was the sum of the total input turbine torque and the PMSM produced torque. Operating at $-0.2$ slip, the PMSM should produce torque equals to $250 \times 0.2 = 50$ Nm, and the total input torque to the DFIG for this case becomes 300 Nm. The difference in the calculated and the simulated torques were because of the losses that appeared across the rotor and the stator resistances of the machines.

Figure 5.10: Rotor speed at super-synchronous mode at $s = -0.2$. 

The torque curves from the turbine, DFIG and the PMSM are shown in Figure 5.11. The turbine torque curve is showing the total input torque from the turbine to the system. At the start, the turbine was giving its maximum torque to reach the reference speed. After reaching the specified speed $\omega_r$, the turbine torque decreased to the no load torque. At $t = 18$ sec, the system was subjected to an electrical load or a power of 47.124 kW was requested at the grid. The requested power was divided by the operating speed of 188.496 rad/sec in order to get the reference torque for the model. At $t = 18$ sec, the turbine torque increased to balance the load torque in order to maintain the rotational speed constant. The load curve is showing the total electrical load (torque) on the system that was obtained as a sum of the PMSM and the DFIG torques. At $t = 0$ sec, there was a load torque of $-600$ Nm which was taken by the PMSM for charging the DC-link capacitor. At $t = 18$ sec, an electrical load of $-250$ Nm was applied that was equally supplied by the turbine. The PMSM and the DFIG load torque curves are also shown in Figure 5.11. As the system was simulated in a super-synchronous mode, therefore the DFIG rotor produced electrical power at a slip frequency and that power was consumed by the PMSM. So from the slip power that was produced at a DFIG rotor terminal, the PMSM produced mechanical torque equal to the slip torque. The mechanical torque produced by the PMSM machine also acted on the shaft and therefore the available torque for a DFIG was the sum of the total input turbine torque and the PMSM produced torque. Operating at $-0.2$ slip, the PMSM should produce torque equals to $250 \times 0.2 = 50$ Nm, and the total input torque to the DFIG for this case becomes 300 Nm. The difference in the calculated and the simulated torques were because of the losses that appeared across the rotor and the stator resistances of the machines.
Figure 5.11: Input mechanical, total load, PMSM produced and DFIG load torques in super-synchronous mode.

Figure 5.12 is showing the simulated active and the reactive power at the DFIG stator terminals. The total electrical output of the system appeared at the stator terminals of the DFIG and ideally it should be equal to the total input mechanical power. The simulated active power $p_s$ at the stator terminal was $4.662 \cdot 10^4$ watts, which was very close to the total system input mechanical power that was $4.7124 \cdot 10^4$ watts. The difference in the input and the output was because of the resistive
losses in the electrical machines. The objective of the control strategy was to operate the DFIG at a unity power factor \((\cos(\theta) = 1)\). The stator reactive power in this case was just 634.2 watts compared to the stator active power. It gives a power factor of -0.9999757. The power factor for DFIG is shown in Figure 5.14. The negative sign in the power factor is just the matter of defining the conventions for the power flow.

\[
\cos(\theta) = 1
\]

The model was simulated at \(s = -0.2\) i.e. \(\omega_r = 188.5 \frac{rad}{sec}\). The total input mechanical power to the model was \(P_m = \omega_r \cdot T_m = 47124\) watts. Therefore ideally, in a super-synchronous mode the DFIG rotor should deliver \(s \cdot P_m = 9424.8\) watts. But in the simulation results, the rotor delivered less power than the ideally calculated because of the resistive losses across the rotor resistance. This can be explained as; for having the stator flux frequency at a constant grid frequency, the DFIG rotor has to deliver power at a certain rotor EMF. In a super-synchronous mode the rotor side also acts as a generator therefore the power that appears at the rotor terminals should be less than the slip power compensating the rotor resistive losses. Because of resistive losses, the rotor produced 8063 watts of power instead of 9424.8 watts. This difference was mainly because of the rotor resistance and it can be shown by calculating the losses across \(r_r\). The RMS rotor current in this case was 90.5 amps. The calculated rotor resistive losses were 1215.34 watts. The difference in the

![Figure 5.12: DFIG stator active and reactive power.](image-url)

Figure 5.12 shows the PMSM and the DFIG rotor active power. For maintaining the DC-link voltage constant, the PMSM must recover or deliver the slip power. In a super-synchronous mode, along with the stator terminals the DFIG rotor also produces the active power and the PMSM must have to consume that power for maintaining the DC-link voltage constant. From Figure 5.13 it can be seen that the PMSM was controlled in such a way that it always produced opposite power to that of a DFIG rotor. For instance at \(t = 22.42\) sec the DFIG rotor produced 8049 watts and this amount of power was consumed by the PMSM (8063 watts).

![Figure 5.13: PMSM and DFIG rotor active power.](image-url)
simulated and the ideally calculated slip power is 1361.8 watts which is very close to the rotor resistive losses.

DC-link voltage is also shown in Figure 5.14. At the start of the simulations the capacitor was charged to 800 V, and then maintained at a constant value of 800 V for a smooth operation of the machines.
**Figure 5.14:** DFIG stator terminal power factor and DC-link voltage

**Figure 5.15:** DFIG rotor active and reactive power
Figure 5.16 shows the d-axis rotor current. For maintaining the DFIG stator frequency at a constant grid frequency ($f_g$, $50 \text{ Hz}$), the DFIG rotor must be supplied with currents at a slip frequency ($s \cdot f_s$). In this case, the shaft was rotating at $s = -0.2$, which gave $s \cdot f_s = 10 \text{ Hz}$. From the small window in Figure 5.16 it can be observed that the rotor currents were injected at a frequency of 10 Hz.

One important thing that must be noted here is that between $t = 12$ to $13 \text{ sec}$, the rotor shaft was rotating at a speed that was very close to the synchronous and also during that time, the rotor shaft crossed the synchronous speed. At a synchronous speed the slip becomes 0, so the rotor currents must be injected at 0 frequencies (DC quantity). It can be verified from Figure 5.10 and Figure 5.16.

The stator terminal phase a current is shown in Figure 5.17. The frequency of the current was exactly maintained at 50 Hz. The current waveform is not perfectly sinusoidal because of the PWM converters. The ripple in the stator currents appeared because of the harmonics in the rotor flux caused by the rotor currents injected through the PWM converter.
Figure 5.17: DFIG stator phase A current.
6 VSD COMPONENT RATING COMPARISON

The simulation results presented in chapter 5 verified the developed control strategy and showed that the topology can work in a wide range of speed. Also there are many advantages like autonomous operation and improved power quality that are claimed in [9], [10]. In this chapter some thoughts regarding the ratings of system components like converters, exciter machine (PMSM), DFIG and harmonic filters are presented and comparison with the existing variable speed drives system is presented.

6.1 Back-to-back PWM converter rating

In this section comparison on the size of back-to-back PWM converter for different VSD’s is presented. As explained in the earlier sections, a DFIG supplies/consumes fraction of total power through the rotor terminals depending upon the mode of operation. The supplied/consumed rotor power equals to the operating slip multiplied with the total input power. The power flow across the studied variable speed drive (VSD), considering the ideal case is shown in Figure 6.1 and Figure 6.2. Figure 6.1 is for sub-synchronous mode whereas Figure 6.2 is in super-synchronous mode. In figures PWM converter block is representing the back-to-back PWM converter that provides the electrical connection between the machines. In the figures $P_m$ is the input mechanical power from the turbine, $P_s$ is the stator electrical power and $s$ is the slip. In an ideal case, the power flowing across the converter in both the modes equals to $s \cdot P_m$, so the converters should be able to handle this amount of power for a smooth operation of the drive.

If the resistive losses across the machines are considered, the size of the converter can vary a bit. As shown in the simulation results presented in chapter 5, during a sub-synchronous mode the DFIG rotor asks for some extra power because of the resistive losses across the rotor resistance. So in a sub-synchronous mode the converter must be able to provide the slip power plus the resistive losses across the rotor resistance.

In a super-synchronous mode, the DFIG also delivers power through the rotor along with the stator terminals. In a real case, the power available at the rotor terminals in super-synchronous mode will be slightly smaller than the ideal case because of the rotor resistive losses. So in a super-synchronous mode the converter can be dimensioned for a power rating slightly smaller than the slip power.

The power flow across traditional VSD using a DFIG is also given in Figure 6.3. It can be observed that power flow across the converter in both the drives is similar. The only difference is that in traditional case a slip power is being taken from or supplied to the grid. So comparing to the traditional VSD the studied drive system can operate autonomously using the converters of same ratings and it also avoids the direct connection between power electronics and grid.
As the studied drive system utilizes the converters of the same rating as that of a traditional VSD therefore it also offers the similar benefits in comparison to the VSD using a synchronous machine or a permanent magnet machine. In case of the
studied VSD, the converters are rated to handle only the slip power whereas the
collectors in a VSD using a synchronous machine must be rated to handle the total
produced power thereby reduces the overall cost of the system. In addition to that,
the studied VSD also offers better quality of produced power compared to the VSD
using a synchronous machine. Figure 1.1 in chapter 1 shows the VSD with a
synchronous machine and a full rated converter connected to the transformer.

6.2 PMSM (exciter machine) size

As shown in the simulation results and also from the mathematical modeling, the
PMSM is used for supplying or consuming the rotor active power. For maintaining
the DC-link voltage constant, the exciter machine (PMSM) must be capable of
handling the rotor slip power in both the modes. This is an additional machine in the
studied VSD compared to the traditional VSD, so it has to be dimensioned carefully
as it adds to the cost of the drive. The size of a PMSM mainly depends on the
operating slip i.e. larger the slip value, larger will be the slip power and it will cause
the higher rating for a PMSM. Close to the synchronous speed there is very less slip
causin the less slip power and it will ultimately reduce the size of the required
PMSM. Usually the variable speed drives are designed to operate between the slip
range of +0.25 to −0.25 which means that PMSM should be rated for 25% of the
total input power for this value of slip range.

6.3 DFIG size and consideration on losses

It is important here to investigate the required size of a DFIG in the studied VSD and
to compare it with the traditional VSD. For that purpose power flow across the
studied VSD in a sub-synchronous and super-synchronous mode is shown in Figure
6.1 and Figure 6.2 respectively whereas the power flow across the traditional VSD is
shown in Figure 6.3. Assuming the same input mechanical power in both the modes
of operation, the required size of a DFIG remains same for the studied VSD whereas
in a traditional VSD it varies depending upon the mode of operation. The size of a
required DFIG is discussed separately in two modes operation.

6.3.1 Sub-synchronous mode

In the studied VSD, there is no connection between the grid and the rotor side of a
DFIG therefore all the power flows to the grid through the stator of a DFIG as shown
in Figure 6.1. In Figure 6.1, \( P_m \) is the input mechanical power, part of the input power
\( s \cdot P_m \) is taken by the PMSM and supplied to the rotor of the DFIG whereas remaining
part of the power \((1-s) \cdot P_m\) flows as the air-gap power. Comparing it to the traditional
VSD in a sub-synchronous mode shown in Figure 6.3 a), the required slip power is
being taken from the grid through the converters and the total power that appears at
the stator of a DFIG becomes the slip power plus the input mechanical power,
\((1+s) \cdot P_m\). This power flow leads to the fact that in case of a traditional VSD operating
at a sub-synchronous mode, a DFIG having a stator rating of \((1+s) \cdot P_m\) is required
whereas in case of the studied VSD a DFIG having a stator rating of \( P_m \) is required.

Now considering the electrical losses that will appear across the VSD, it can be seen
that the losses across the converters will be equal in both the drives. In case of
electrical machines, the studied VSD will have electrical losses corresponding to
power $P_m$ at the stator side and losses corresponding to $s \cdot P_m$ at the rotor side of a DFIG. Along with that the studied VSD has a PMSM machine that will also produce the losses corresponding to the slip power i.e. $s \cdot P_m$. So on a total, across the electrical machines there will be the losses corresponding to the power $(1+2\cdot s)\cdot P_m$. In case of a traditional VSD, the losses appearing at the stator of a DFIG will be corresponding to $(1+s)\cdot P_m$ and on the rotor side it will be corresponding to $s \cdot P_m$. On a total, there will be the losses corresponding to the $(1+2\cdot s)\cdot P_m$. So there are equal electrical losses appearing across both the drives in a sub-synchronous mode.

6.3.2 Super-synchronous mode

The power flow across the studied VSD in a super-synchronous mode is shown in Figure 6.2. In this mode, the DFIG rotor also delivers power along with the stator terminal. The PMSM takes this electrical power and convert it to the mechanical power, but the total output power at the stator terminals of a DFIG remain equals to the input mechanical power from the turbine. So in this case, a DFIG having a stator rating of $P_m$ is required. Now comparing it to the traditional VSD operating in a super-synchronous mode as shown in Figure 6.3 b) a DFIG having a stator rating of $(1-s)\cdot P_m$ is required. This means that the studied VSD requires a DFIG of higher ratings compared to the traditional one.

From the losses point of view, it can be seen that the losses across the converters will be equal in both the drives. In case of electrical machines, the studied VSD will have electrical losses corresponding to power $P_m$ at the stator side and losses corresponding to $s \cdot P_m$ at the rotor side of a DFIG. Along with that it will also have losses across PMSM corresponding to the slip power i.e. $s \cdot P_m$. So on a total, there will be the losses corresponding to the power $(1+2\cdot s)\cdot P_m$ across the electrical machines. In case of a traditional VSD operating in a super-synchronous mode, the losses appearing at the stator of a DFIG will be corresponding to the power $(1-s)\cdot P_m$ and on the rotor side it will be corresponding to power $s \cdot P_m$. So in a traditional VSD operating in a super-synchronous mode there will be the losses corresponding to the power $P_m$ only.

It shows that operating in a super-synchronous mode the studied VSD requires a DFIG of higher rating and it will also produce extra electrical losses across the drive compared to the traditional one.

6.4 Harmonic filters

The use of power electronics in a generation system produces unwanted harmonic content in the currents along with the required fundamental component. These unwanted harmonics need to be filtered out before feeding the currents into the grid.

In a traditional variable speed drive shown in Figure 2.3, the power is fed to the grid from both the stator and the rotor of a DFIG. The power electronics is used on the rotor-side for controlling the operation of the machine. It makes a direct interaction between the utility grid and the power electronics. Therefore bulky inductive filters are required between the grid and the power electronics for filtering the unwanted harmonics.

Now in case of a studied variable speed drive shown in Figure 2.4, the produced power is fed to the grid only through the stator of a DFIG. The rotor-side converter is
also used here for controlling the machine operation but instead of grid it is connected to a PMSM. There is no power electronics attached directly to the grid. This lead to a fact of removal of rotor-side filter that is required in case of a traditional VSD.

In most of the reviewed literature like [8], [29], [30] etc harmonic filters are shown only on the rotor side of a traditional VSD. Considering the schematic of a traditional VSD presented in Figure 2.3, it can be concluded that using the studied variable speed drive can eliminate the requirement of harmonic filters.

Just for the curiosity, it is tried to study the effect of rotor flux harmonics on the stator terminal currents. This effect can be analyzed using the transfer function of a DFIG between the rotor voltages and the stator currents.

The transfer function between the rotor voltages and stator currents can be obtained by representing the DFIG model in terms of state space matrices. The state variables in a DFIG are the stator and rotor fluxes. The output matrix for a DFIG contains the stator and rotor currents, whereas the input matrix consists of stator and rotor voltages. In terms of state space representation it can be written as:

\[
\dot{x}(t) = A(t).x(t) + B(t).u(t) \quad (6.1)
\]

\[
Y(t) = C(t).x(t) + D(t).u(t) \quad (6.2)
\]

\(x\) is called the 'state vector'.

\(Y\) is called the 'output vector'.

\(u\) is called the 'input vector'.

\(A\) is called the 'state matrix'.

\(B\) is called the 'input matrix'.

\(C\) is called the 'output matrix'.

\(D\) is the 'feed through or feed forward matrix'.

The transfer function in terms of state space matrices can be given as [37]:

\[
G(s) = C(sI - A)^{-1}.B + D \quad (6.3)
\]

The matrices \(A\), \(B\) and \(C\) for a DFIG can be obtained using the DFIG model presented in chapter 3. For a DFIG, \(D\) is a 0 matrix as there is no feed forward and \(I\) is identity matrix of 4 x 4. The matrices \(A\), \(B\) and \(C\) are obtained from the DFIG equations and given as:
Using the above given matrices and equation (6.3) the transfer function between rotor voltage and the stator current can be found. Using the parameters of a 40 kW DFIG given in Appendix B, the transfer function was found and bode plots are plotted between the currents and the rotor voltages. Figure 6.4 shows bode plot between the rotor voltages and the stator currents whereas Figure 6.5 shows the bode plot between rotor voltages and rotor currents. By carefully examining the plots, it can be found that the attenuation for stator currents is bit higher compared to the attenuation for the rotor currents. The difference in attenuation is very small but can be significant for the harmonics. This shows that the magnitude of current harmonics at the stator side can be smaller than the rotor side harmonics. The attenuation for stator current at 20 kHz is $-93.4$ whereas for rotor currents it is $-93.13$. This decrease of 0.17 dB for stator currents gives a reduction of 1% for stator-side current harmonics compared to rotor-side current harmonics. The attenuation values are compared at 20 kHz as the current ripple in symmetrical SVM appears at double the switching frequency. In chapter 5, a switching frequency of 10 kHz was used for simulating the model.

Bode diagrams shown in Figure 6.4 and Figure 6.5 are obtained without considering the effect of grid inductance. The presence of grid inductance on the stator side can also help in smoothening of the currents. The final statement about the use of filters at the stator-side of a DFIG can be given after the detailed Fourier analysis of the stator currents and also it depends on the grid codes and the maximum short circuit current of the grid. The Fourier analysis and the grid code study is considered beyond the scope of this thesis work.
Figure 6.4: Bode plot between rotor voltages and stator currents.
Figure 6.5: Bode plot between rotor voltages and rotor currents.
7 CONCLUSIONS

During this research work, control logic for a variable speed drive for a limited speed range is presented as an alternative solution for a micro-hydro generation system. It is found that the studied variable speed drive can be used in an autonomous generation system or can supply isolated loads.

Comparing the studied variable speed drive to a traditional variable speed drive, it was found that it can work with the power electronics of similar ratings.

The slip power is not fed to the utility grid, in fact it is passed through another machine and all the power is fed to the grid only through the stator of a DFIG. In this way a direct interaction between power electronic converters and the grid can be avoided resulting in a better quality of produced power.

The use of rotor-side harmonics filters can be omitted using the studied variable speed drive. This feature can also make it easy to fulfill the required grid codes in terms of harmonics.

While developing the control strategy for the studied variable speed drive, it was found that the generation of reference signal for the exciter machine needs a low pass filter in order to filter out the dynamics from a DFIG. Otherwise there may be a chance of sustaining the dynamics in the system.

For accommodating the slip power for a DFIG, the studied variable speed drive utilizes an extra electrical machine having the ratings of slip power. This adds to the weight and volume of drive trains, especially in case of nacelle for the wind power applications.

From the simulations it was found that all the generated power flows to the grid through the stator terminals of a DFIG. Operating in a super-synchronous mode it results in increase of the required size of a DFIG compared to a traditional VSD.

Having a kind of power loop in the drive and operating in a super-synchronous mode, the studied variable speed drive produces extra electrical losses across the electrical machines compared to a traditional VSD.
8 FUTURE WORK

The research in the field of developing an autonomous variable speed drive using a DFIG is at its initial stages and there are lots of aspects that can be studied in the future. An experimental validation of the developed control strategy on a test bench can be an interesting work in the future for addressing the real time challenges in using the studied variable speed drive.

As shown in the previous chapters that there is a kind of power loop between the two machines and these two machines are coupled through power electronics. So it can be interesting to perform the harmonic analysis and to study the effect of harmonics from one machine to the operation of the other machine.

As the machines are mechanically coupled on the same shaft, so it can cause shear stresses and can produce fatigue in a shaft which can ultimately reduce the life of the shaft. This effect on the life of shaft needs to be investigated further.

It can also be interesting to work on the effect of cogging torque produced from the PMSM on the mechanical resonance of the drive train.

In this work the field weakening for the PMSM is not considered but it would be interesting to develop a control for a PMSM based on the field weakening principle so that the required power can be produced with the smaller available DC-link voltage even at higher speeds.

The maintenance of the slip rings for a DFIG rotor is a quite challenging task especially at remote areas. Using the studied VSD, there is a possibility of removal of DFIG rotor slip rings. The control development in this case will be the similar as presented above but it needs to investigate the other aspects like how to get the power for the operation of the micro-controller. Also it needs to investigate about the required cohesive force for the drives as drives will be rotating along the shaft.

An additional machine is used in the drive compared to the traditional system. It will add to the cost of the system. On the other hand, removal of harmonic filters can save some cost. Therefore it can also be interesting to perform the detailed cost analysis of the studied drive system.
9 REFERENCES


[17] Siemens PTI-software solutions, “BOSL controllers-Standards 1”. 


89


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10 APPENDICES

10.1 Appendix A
When dealing with the control of electrical quantities like currents or voltages, it is always desired to transfer sinusoidal varying signals to constant value signals. For this three phase quantities can be transferred to two phase complex plane (stator reference frame). These stator frame quantities (vectors) in space rotate with the applied frequency therefore it is further required to transfer the stator frame quantities to a rotating reference frame in such a way that they appear as constant or DC quantities in that frame. Definition for each frame is given as

Stator (stationary) reference frame
This frame is always at rest and called as stator reference frame. Three phase quantities can be transferred to stator reference frame as rotating vectors. Matrix used for transformation from three phase to two phase is given below.

Rotating (dq) reference frame
As mentioned earlier quantities in stator frame are rotating vectors but it is desired to have constant or DC values for better control. This problem can be solved by assuming a frame that is rotating with the same speed as that of rotating vector. In this way rotating quantities become DC quantities. This frame of reference is called rotating reference frame. All that is required for the transformation of quantities is to find the proper transformation angle to get align to that rotating vector. Matrix used for transformation from stator frame to rotating frame is given below.

Transformations:
Transformation matrices from one frame to other, that are used through this thesis work are given here as

Three phase system to two phase system (abc to αβ):
Three phase sinusoidally varying quantities can be transferred to the stator reference frame using the Clark's transformation matrix given as

\[
\begin{bmatrix}
u_a \\ v_b \\ v_c
\end{bmatrix} = 2 \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\
-\frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{2} & \frac{\sqrt{2}}{2}
\end{bmatrix} \begin{bmatrix}
u_a \\ v_b \\ v_c
\end{bmatrix}
\]

In above equation it is assumed that α axis is aligned with a axis of three phase system. Also symmetrical machines are considered here, so the third term \(v_y = 0\) in this case. Equation can be presented as.
Inverse transformation from two phase system to three phase system is given in (3).

\[
\begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
\frac{-1}{2} & \frac{\sqrt{3}}{2} & \frac{-1}{2} \\
\frac{1}{2} & \frac{-\sqrt{3}}{2} & \frac{1}{2}
\end{bmatrix} \begin{bmatrix}
u_a \\
u_b \\
u_c
\end{bmatrix}
\]

Above equations are valid for all the three phase quantities voltages, currents and fluxes.

**Stationary frame to rotating reference frame (αβ to dq):**
Transformation matrix from αβ to dq frame is given as

\[
\begin{bmatrix}
u_d \\
u_q
\end{bmatrix} = \begin{bmatrix}
cos\theta_t & sin\theta_t \\
-sin\theta_t & cos\theta_t
\end{bmatrix} \begin{bmatrix}
u_a \\
u_b
\end{bmatrix}
\]

Inverse transformation from rotating reference frame dq to stator reference frame αβ is given as

\[
\begin{bmatrix}
u_a \\
u_b
\end{bmatrix} = \begin{bmatrix}
cos\theta_t & -sin\theta_t \\
sin\theta_t & cos\theta_t
\end{bmatrix} \begin{bmatrix}
u_d \\
u_q
\end{bmatrix}
\]

θ_t is the transformation angle (angle between d axes and space vector quantity), required to align the dq reference frame with the rotating vector. There are several different ways of finding θ_t depending upon the type of machine. For DFIG method for obtaining θ_t is explained in chapter 4.

### 10.2 Appendix B

**Table A-1: Parameters for 5 MW Kaplan turbine**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water density</td>
<td>1000 Kg/m³</td>
</tr>
<tr>
<td>Available water head</td>
<td>7 m</td>
</tr>
<tr>
<td>Gravitational force (g)</td>
<td>9.8 m/s²</td>
</tr>
<tr>
<td>Flow ratio</td>
<td>0.65</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>----------------------------</td>
<td>---------</td>
</tr>
<tr>
<td>Velocity ratio</td>
<td>1.6</td>
</tr>
<tr>
<td>$D_t$</td>
<td>1.5 m</td>
</tr>
<tr>
<td>$D_h$</td>
<td>0.75 m</td>
</tr>
<tr>
<td>Water flow at 5 MW</td>
<td>81 m$^3$/s</td>
</tr>
</tbody>
</table>

**Table A-2: Two-mass shaft model parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear ratio</td>
<td>8.849</td>
</tr>
<tr>
<td>Turbine inertia</td>
<td>400</td>
</tr>
<tr>
<td>Generator rotor inertia</td>
<td>50</td>
</tr>
</tbody>
</table>

**Table A-3: Parameters of the simulated DFIG**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator resistance</td>
<td>28 m$\Omega$</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>49.450 m$\Omega$</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>0.1591 mH</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>0.1726 mH</td>
</tr>
<tr>
<td>Magnetizing inductance</td>
<td>12.71 mH</td>
</tr>
<tr>
<td>Stator voltage, RMS (Star)</td>
<td>400 V</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>2</td>
</tr>
</tbody>
</table>

**Table A-4: Parameters of the simulated PMSM**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>---------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>28 mΩ</td>
</tr>
<tr>
<td>Rotor magnetic flux</td>
<td>1 T</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>4 mH</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>2</td>
</tr>
</tbody>
</table>

Table A- 5: Values for different constants used in HYGOV model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent droop $R$</td>
<td>0.02</td>
</tr>
<tr>
<td>Temporary droop $r$</td>
<td>0.05</td>
</tr>
<tr>
<td>Governor time constant $T_r$</td>
<td>0.02</td>
</tr>
<tr>
<td>Servo time constant $T_g$</td>
<td>0.01</td>
</tr>
<tr>
<td>Water time constant $T_w$</td>
<td>0.05</td>
</tr>
<tr>
<td>Gate position $G_{min}$</td>
<td>1</td>
</tr>
<tr>
<td>Gate position $G_{max}$</td>
<td>0</td>
</tr>
<tr>
<td>Turbine gain $A_t$</td>
<td>2.5</td>
</tr>
<tr>
<td>Turbine damping $D_{turb}$</td>
<td>0</td>
</tr>
<tr>
<td>No load flow $q_{nl}$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

DC-link capacitance = 1 mF.
Switching frequency = 10 kHz.
DC-link voltage = 800 V.
10.3 Appendix C
Developed models from MATLAB/SIMULINK

Figure A-1: Complete control for the system from MATLAB/SIMULINK
Figure A- 2: HYGOV model

Figure A- 3: Two-axis mechanical shaft model
Figure A-6: Closed loop control of a PMSM

Figure A-7: DFIG model from MATLAB/SIMULINK
Figure A-8: Closed loop control and transformation angle estimation for a DFIG