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Reliability-based ultimate limit state design of lime-cement columns

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Abstract

In this thesis, the stability of embankments founded on lime-cement (LC) columns is studied according to deterministic design and reliability-based design (RBD) perspectives. Two deterministic models and different RBD approaches for assessing stability and reliability of embankments are used. A number of sources of uncertainty associated with evaluation of the mean value of undrained shear strength of lime-cement columns ($\overline{c}_{u,col}$) is considered. It is shown that the uncertainty and the variability of $\overline{c}_{u,col}$ are the main parameters that affect the reliability and hence the stability of embankments. The study considers the sources of uncertainty that arise, mainly from the number of test samples (N, statistical uncertainty), testing procedure (measurement errors) and model transformation. The reliability of embankments is significantly improved by considering spatial variability with respect to $\overline{c}_{u,col}$. Since the uncertainty with respect to column strength property and performance can be highly uncertain, it is recommended that RBD design is conducted as a complement to the traditional deterministic design. A simple design procedure using partial factor design methods, as defined in Eurocode 7, is proposed for the design. A set of partial factors for $\overline{c}_{u,col}$ is suggested based on the variability and the degree of uncertainty. Partial factor design method is applied for the design of lime-cement columns at different conditions and it is considered to fulfil both deterministic and RBD requirements.

Currently, deterministic methods are used for the design of lime-cement columns due to their simplicity and the development that has taken place and the engineering experience built up over the years. Deterministic methods, however, lack the ability to handle the variability and uncertainty normally associated with different geotechnical soil properties in the design procedure. In this study, reliability-based ultimate limit state ULS design of LC columns is applied and the variability parameters of $c_{u,col}$ have been evaluated for two different sites. The following most important conclusions were drawn accordingly:

- Inherent variability of $c_{u,col}$ is very high and should be taken into consideration in the design of lime-cement columns.
- The reliability of embankments at ULS increases significantly when the effect of spatial variability is considered in the RBD design procedure.
- The reliability of embankments is influenced by the number of test samples. In order to perform a reliable design it is recommended that N should be at least 10.
- Transformation uncertainty has a significant influence on the reliability of embankments and hence in the evaluation of partial factors.
- Reliability of the design can be overestimated if the effect of positive correlation $\rho_{c_{u,col},c_{u,soil}}$ is ignored. However, the effect of $\rho_{c_{u,col},c_{u,soil}}$ on the reliability is practically insignificant for $\rho_{c_{u,col},c_{u,soil}}$ <+0.25.

Keywords: lime-cement columns; stability; variability; uncertainty; deterministic method; reliability-based design; partial factor design

Sammanfattning

I denna avhandling har stabiliteten för bankar grundlagda på kalkcementpelare studerats enligt deterministisk och tillförlitlighetsbaserad dimensionering (RBD). Två deterministiska modeller och olika RBD metoder för bedömning av stabilitet och säkerhetsnivå för bankar är använda. Ett antal källor till osäkerhet är beaktade i samband med utvärdering av medelvärdet av kalkcementpelares odränerade skjuvhållfasthet ($\overline{c}_{u,col}$). Det visas att osäkerheten och variationen relaterat till $\overline{c}_{u,col}$ har en signifikant inverkan på säkerhetsnivån och därmed stabiliteten hos bankar. I studien behandlas källor till osäkerhet som uppkommer, främst från antalet prover (N, statistisk osäkerhet), testprocedur (mätfel) och transformationsfel. Tillförlitligheten förbättras avsevärt genom att betrakta rumslig variabilitet med avseende på $\overline{c}_{u,col}$. Eftersom osäkerheten med avseende på osäkerheterna relaterat till pelarnas hållfasthet kan vara högt osäker, så rekommenderas att tillförlitlighetsbaserad dimensionering utförs som ett komplement till traditionell deterministisk dimensionering med totalsäkerhetsfaktorer. En enkel procedur för dimensionering föreslås med partialkoefficientmetoden enligt Eurokod 7. Uppsättningen av partialkoefficienter för $\overline{c}_{u,col}$ föreslås utifrån variationen och graden av osäkerhet. Partialkoefficientmetoden appliceras för dimensionering av kalkcementpelare för lika förhållanden som anses uppfylla krav enligt både deterministisk design och RBD.

För närvarande är det deterministiska metoder som används för dimensionering av kalkcementpelare på grund av deras enkelhet, den utveckling som har ägt rum och den tekniska erfarenhet som byggts upp under åren. Deterministiska metoder saknar dock förmåga att hantera variationer och osäkerheter som normalt förknippas med olika geotekniska egenskaper. I denna studie är tillförlitlighetsbaserad dimensionering i av kalkcementpelare i brottgränstillstånd tillämpad och variationen och osäkerheten avseende $c_{u,col}$ har utvärderats för två olika platser. Följande slutsatser drogs enligt följande:

- Inneboende naturliga variationer avseende $c_{u,col}$ är mycket hög och bör beaktas vid dimensionering av kalkcementpelare.
- Säkerhetsnivån för banker i brottgränstillstånd ökar signifikant då effekten av rumslig variation beaktas i dimensioneringsprocessen.
- Säkerhetsnivån för banker påverkas av antalet prover. För att göra en tillförlitlig konstruktion rekommenderas det att N bör vara minst 10.
- Säkerhetsnivån kan överskattas om effekten av positiv korrelation $\rho_{c_{u,col},c_{u,soil}}$ ignoreras. Emellertid är effekten av $\rho_{c_{u,col},c_{u,soil}}$ på tillförlitligheten praktiskt obetydlig för $\rho_{c_{u,col},c_{u,soil}} < 0,25$.

Preface

The work in this thesis is a complement to my research work that was started in October 2008 at the Division of Soil and Rock Mechanics, Department of Civil and Architectural Engineering, Royal Institute of Technology (KTH), Stockholm, Sweden. The work was supervised by Professor Stefan Larsson. My personal fund is partly granted by the Kurdistan regional government in Iraq. During the first two years, I conducted research and completed my licentiate period. From October 2010 until October 2012, the research work continued and this thesis is a product of my efforts.

I would like to take this opportunity to express my acknowledgement to the people who made the work of this thesis possible. I must express my appreciation to my supervisor Professor Stefan Larsson for his patient and valuable discussion and comments on my research work. Special thanks are due to Professor Håkan Stille for giving me an opportunity to do the present work at the Division of Soil and Rock Mechanics. I would also like to thank my colleagues from the Division of Soil and Rock Mechanics for their support and in particular my former colleague Lena Wennerlund for her kindness. Finally, I would like to express my special gratitude to my family, parents and parents-in-law for their continuous prayers over the period of my study. I dedicate the work in this thesis to the young geotechnical engineers and scientists in Iraq, especially those in Kurdistan, and also to my wife Heevy, my daughter Dalya, my sons Ahmed and Abdullah and to al-Naqshabandies all over the world.

Stockholm, November 2012

Mohammed Al-Naqshabandy

List of Publications

This thesis is written based on the research work presented in the following publications:

I Al-Naqshabandy, M.S., Bergman, N.S. and Larsson, S. (2012). "Strength variability in lime-cement columns based on cone penetration test data." Ground Improvement, Vol. 165, Issue GI1, 15-30.

II Al-Naqshabandy, M.S., Bergman, N.S. and Larsson, S. (2012). "Effect of spatial variability of the strength properties in lime-cement columns on embankment stability." Proc. 4th Int. Conf. on Grouting and Deep Mixing 2012, Marriott New Orleans. ASCE, Geotechnical Special Publication No. 228, Vol. 1, 231-242.

III Al-Naqshabandy, M.S., and Larsson, S. (2012). "Effect of uncertainties of improved soil shear strength on the reliability of embankments." Journal of Geotechnical and Geoenvironmental Engineering, ASCE. Posted ahead of print August 1, 2012. doi: 10.1061/(ASCE)GT.1943-5606.0000798.

IV Bergman, N.S., Al-Naqshabandy, M.S., and Larsson, S. "Variability of strength and deformation properties in lime-cement columns evaluated from CPT and KPS measurements." [Georisk 2012, article under review]

V Al-Naqshabandy, M.S. and Larsson, S. (2012). "Partial factor design for a highway embankment founded on lime-cement columns." Proc. of the Int. Sym. on Ground Improvement Works TC 211, IS-GI Brussels 2012 from 31 May to 1 June. Vol. III, 3-12.

VI Al-Naqshabandy, M.S., and Larsson, S. "Evaluation of partial factors for the undrained shear strength of lime-cement columns." [Submitted to Canadian Geotechnical Journal 2012].

The main research work, including data analysis and writing papers I, II, III, V and VI was done by Al-Naqshabandy. In paper IV, Al-Naqshabandy contributed to the paper by supervising and performing some of the field work as well as contributing to the discussion and commenting on the paper.

Content

Chapter 1: Introduction

1.1 Background

Ground improvement by deep mixing (DM) is a generic term used for a number of methods in which a binding agent, often lime and/or cement, is mechanically mixed with mainly soft soils to increase their engineering properties, i.e. permeability, deformation and strength (Porbaha, 1998; Larsson, 1999; Terashi, 2003; Broms, 2004). In Sweden, where exclusively dry binders are distributed in the soil with compressed air, the DM method is traditionally called lime-cement (LC) columns or dry deep mixing (Figure 1). The engineering properties of the improved soil depend mainly on the amount and characteristics of the binder, the characteristics of the soil, and the mixing and curing conditions. Lime-cement columns are mainly used to increase stability and reduce settlement of highway and railway embankments.

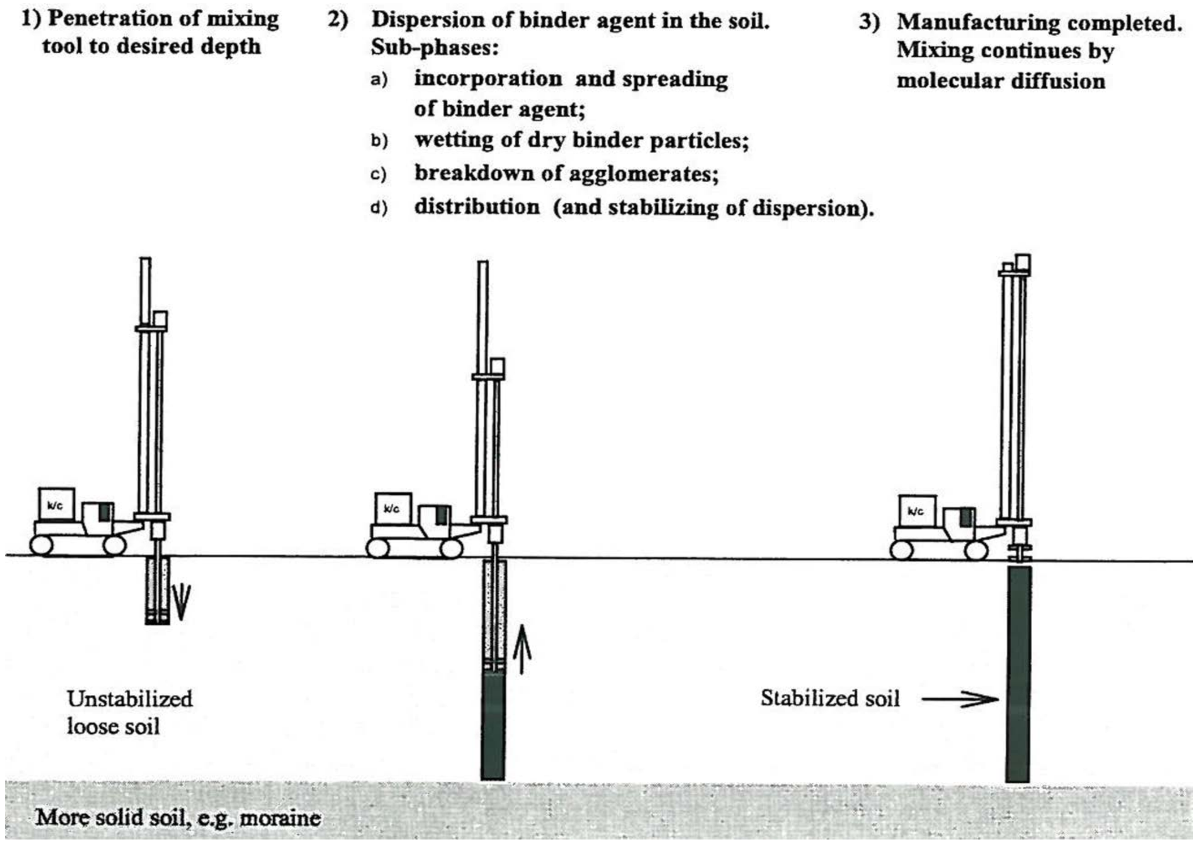

Figure 1. The dry deep mixing process (Larsson, 2003)

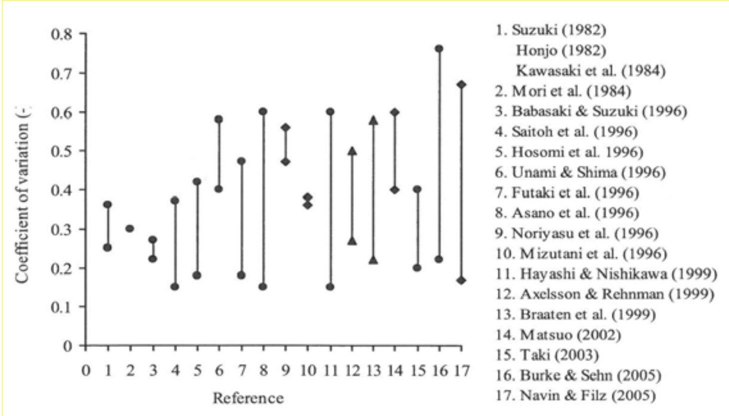

Figure 2. Inherent variability with respect to the coefficient of variation evaluated from compression tests in a number of DM projects (Larsson, 2005)

The inherent variability with respect to the engineering properties of the DM soils is high. Larsson (2005) summarised the results of variability evaluated from compression tests on samples taken from in-situ stabilised soil for a number of DM projects in terms of coefficient of variation (*COV*) as shown in Figure 2. The variability was found to be within the range of 14-75% with average range values of 25-50%. Extensive statistical analyses have been carried out by Filz and his research group to evaluate the variability parameters, i.e. the mean, *COV* and scale of fluctuation, of the unconfined compressive strength of 6,592 samples taken from variety of DM projects in the USA. Navin and Filz (2005) have shown interesting results regarding the shape of the probability distribution function (PDF), and they have also quantified the spatial variability of the unconfined compressive strength of DM wet columns. As shown in Figure 3, the evaluated *COV* ranged from 0.34 to 0.74 with an average value of 0.54. Furthermore, Filz and Navin (2010) have addressed the variability in the estimation of the design value of DM columns by proposing the so called *variability factor*.

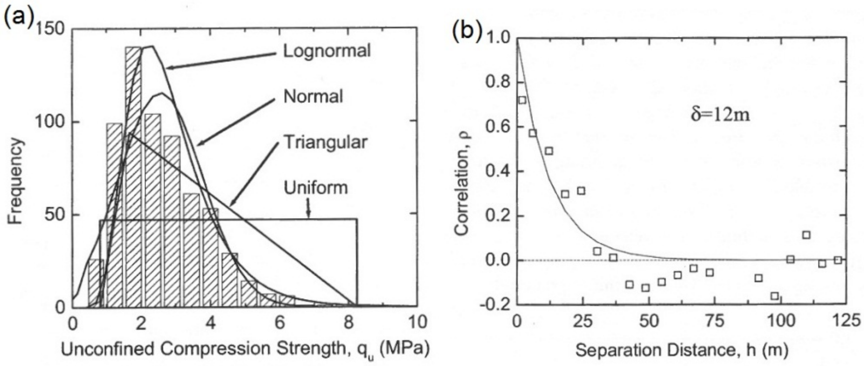

Figure 3. a) Probability distribution function of the unconfined compressive strength of DM columns, and b) scale of fluctuation of wet DM columns (Navin and Filz, 2005)

Although the significant improvements in the equipment and the method used in DM, the inherent variability of the strength property is still very high. This high variability is due to the high inherent variability of the natural soil and the complex mixing process. Several studies have been performed on natural soils in order to evaluate their inherent variability. The results of inherent variability of strength property for natural soils evaluated from different test methods are shown in Figure 4 (Phoon et al., 1995). As can be seen from Figure 4, the *COV* ranges from 2% to 80% with average range values between 10% and 50%. High inherent variability introduces uncertainties in the estimation of the design value and hence the performance of the geotechnical structure will be uncertain.

1.1.1 Deterministic design concept

According to the Swedish Transport Administration (2011), the safety of embankments is normally assessed in the ultimate limit state design (ULS) of LC columns using limit equilibrium methods of slices (LEM). Perfect interaction between LC columns and the surrounding soil is normally assumed, i.e. columns and soil fail simultaneously along the failure surface, and the failure surface is assumed to have a circular shape, as shown in Figure 5. The safety of embankments is normally evaluated by means of a total factor of safety (FS), which is defined as the ratio of a resisting force (R) and a driving force (L) evaluated along the most critical slip failure surface from the loads and soil parameters (x). In this design method, R and L parameters are considered deterministically, where only their specified characteristic values are normally considered in the design procedure, i.e. the samples' mean value ($\bar{x}$), and the design according to this method is called deterministic design.

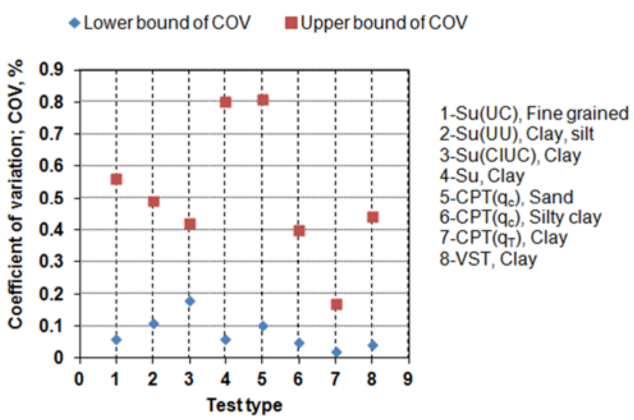

Figure 4. Inherent variability of different soil types evaluated from laboratory and in-situ tests in terms of COV. Data collected from Phoon et al., 1995

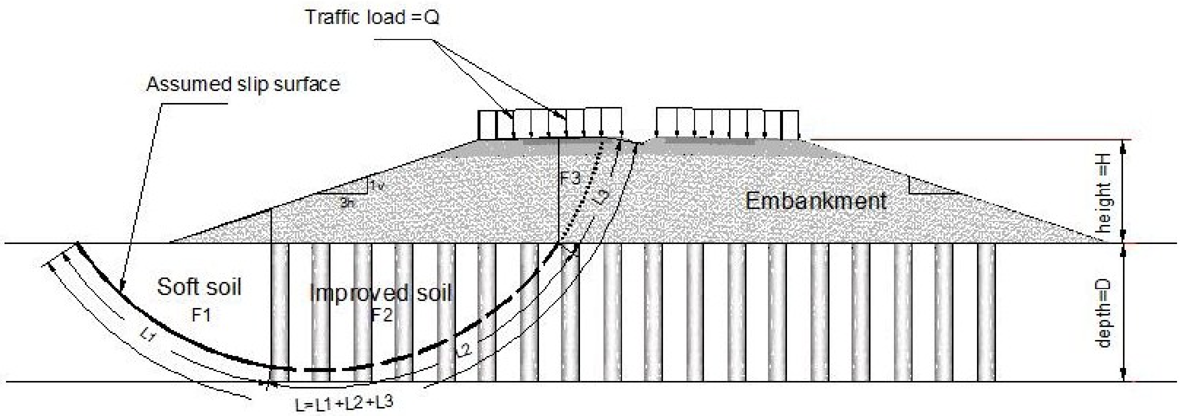

Figure 5. Cross-section of a typical embankment constructed on lime-cement columns

The principal concept of the deterministic design can be made more visual, see Figure 6. As shown in the figure, the design is truly dependent on the mean values (the thick lines) of the R and L parameters; no variability and uncertainty about the mean values of R and L can be seen. The main sources of uncertainty that are associated with the evaluation of $\overline{x}$ arise mainly from spatial variability, the number of test samples (statistical uncertainty), testing procedure (measurement errors) and model transformation uncertainties. Deterministic design does not account for the variability and the uncertainty in the design procedure. The safety of an embankment is normally assessed to ensure that the design reasonably fulfils the condition L<R by choosing a single value of FS that represents uncertainty in the whole system. The chosen value of FS is strongly correlated with engineering judgment and past experience, which both vary between different geotechnical engineers. The variability and the associated complex uncertainty with respect to the behaviour and strength of columns cannot be handled in the deterministic design in a rational way, since they are incorporated in one single value. The safety requirement for embankments improved with LC columns should normally fulfils FS>1.5 based on the critical state condition of undrained shear strength of the columns ($c_{u,col}$) (Broms, 2004). In order to provide a rational and reliable design for geotechnical systems, the variability and the concept of uncertainty should be incorporated directly in the design procedure (Ang and Tang, 2007).

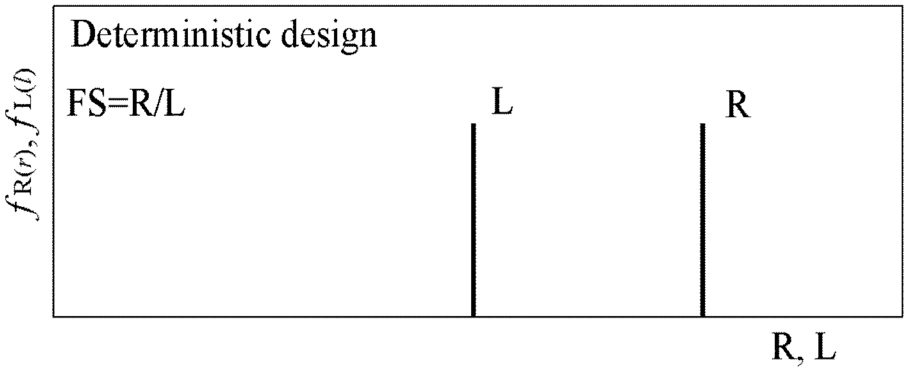

Figure 6. Principal concept of the deterministic design

1.1.2 Reliability-based design (RBD) concept

Reliability theory is based on the mathematical statistics for assessing system safety; R and L parameters can thereby be described as random variables. In order to perform RBD, knowledge is required about the type of PDF of the random variables and the correlation between them. This kind of information, however, is not available for some parameters in advance of the design of the LC columns, which increases the difficulty in applying RBD for DM. The design concept according to RBD is illustrated more visually in Figure 7. It can be seen from the figure that, with the presence of variability, there is always a risk that the unfavourable satiation L≥R will be reached.

Many years of research and great effort have been dedicated to evaluating the reliability of the design of different geotechnical systems. Knowledge about the variability and the uncertainty of geotechnical property parameters has been increased significantly. The limit state design, within the framework of RBD, has been used instead of the traditional working stress design (Baecher and Christian, 2003). Another important usage of RBD in geotechnical field, is that it can be used as a tool for risk assessment in many geotechnical subjects such as stability of earth slopes and embankments, bearing capacity of strip footings, laterally loaded piles and earth-retaining structures (Liang et al., 1999; Honjo and Suzoki, 2000; Griffiths et al., 2002; Loukidis et al., 2003; Dasaka et al., 2005; Foye et al., 2006; Cho, 2007; Haldar and Babu, 2008a, 2008b, 2009; Griffiths et al., 2009; Goh et al., 2009; Basha and Babu, 2010).

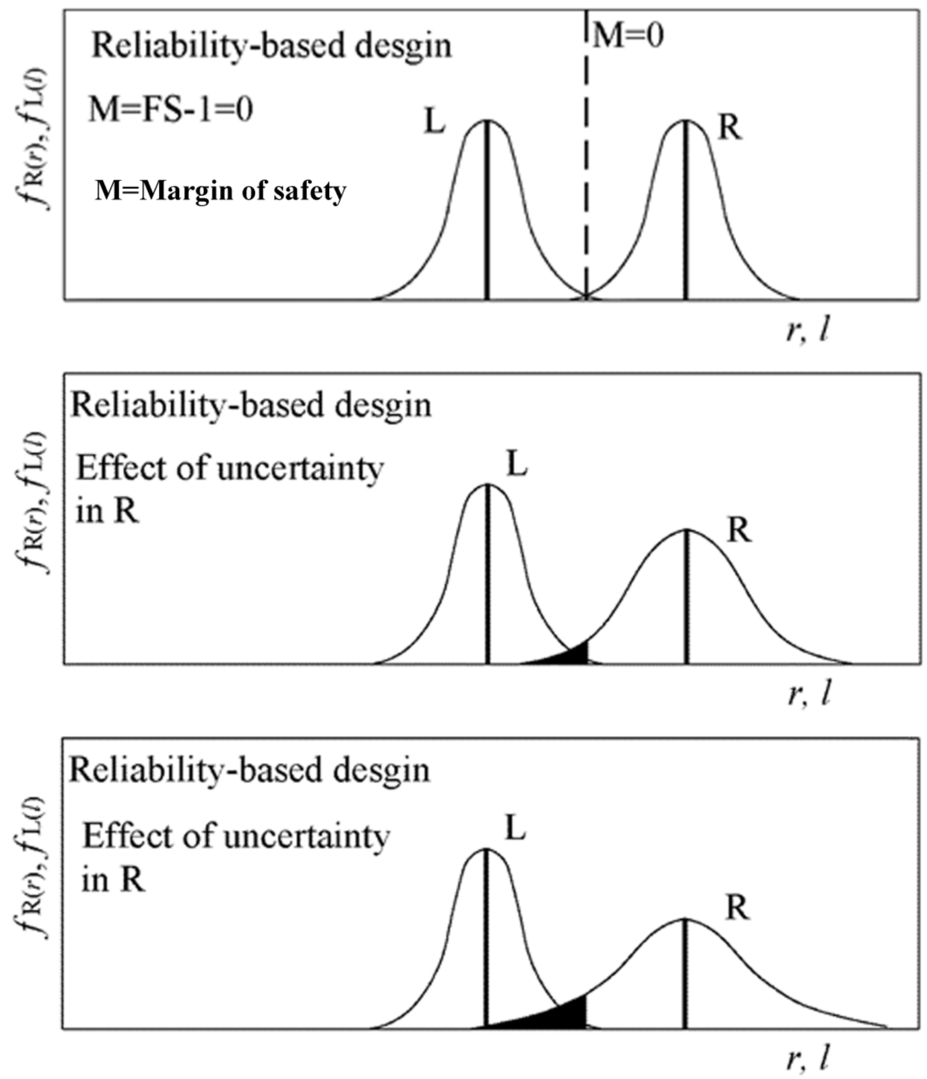

Figure 7. Illustration of reliability-based design concepts

1.1.2.1 Reliability-based design related to natural slopes and embankments

Extensive studies have been carried out on natural slopes and embankments in an attempt to incorporate the variability into the design procedure, and also to address its effect on the stability of slopes and embankments (Alonso, 1976; Vanmarcke, 1977; Grivas and Chowdhury, 1983; Li and Lumb, 1987; Christian et al., 1994; Alèn, 1998; El-Ramly et al., 2002, 2003; Phoon et al., 2003, among many others). Previous studies have come to almost the same conclusions, which are: first, stability of slopes and embankments are affected significantly by inherent variability and the degree of uncertainty associated with L and R parameters, and second, the use of probabilistic design approaches has been acknowledged to be rational design tools in terms of dealing with such variability and complex uncertainty.

1.1.2.2 Reliability-based design related to DM

The variability in DM has been extensively addressed previously; very little, however, has been published on the application of RBD related to DM design. An early study was made by Honjo (1982) where he utilized statistical methods to evaluate the shear strength and its variability of heterogeneous soil improved by DM methods. Larsson et al. (2005) studied the effect of the variability of the columns' strength property on the evaluated design values. It was emphasized that the variability and the concept of probability of failure can be considered in the design of LC columns by evaluating the design value using partial factor methods. In his PhD thesis, Navin (2005) assessed the stability of embankments constructed on improved soil with DM methods by using both a limit equilibrium method (LEM) and numerical analysis method (NM). Besides NM, Navin briefly discusses RBD application to DM and draws the conclusion that reliability analysis is necessary for DM design to address the impact of the variability of the improved soil properties and the variability of the other materials involved.

A number of conference papers have been published recently on the use of RBD related to DM. Kasama et al. (2009) used probabilistic methods to assess the reliability of the bearing capacity of cement-treated ground considering spatial variability. The main conclusion from this study was that the bearing capacity of the cement-treated ground decreases with increasing variability. Tokunaga et al. (2009) performed parametric studies on RBD applied in DM, and the conclusion was that the RBD can be established for any failure mechanisms, i.e. sliding, overturning, bearing capacity, and internal and extrusion failures. Filz and Navin (2010) proposed statistical analyses for evaluating the design value of $c_{u,col}$ that support embankments. The proposed method takes account of the inherent variability associated with $c_{u,col}$ and $c_{u,soil}$ by introducing a factor called *variability factor*, which is the ratio of the design strength of the DM ground to the specified strength of the DM ground.

According to this review some important notes can be highlighted as follows:

1- Previous studies were useful in terms of identifying the adverse consequences that arise from the high variability in DM strength properties.
2- The variability parameters of the improved soil properties have been evaluated in a number of DM projects. Knowledge about scales of fluctuation is available and can be used at other similar DM conditions.

3- In order to take better account of the variability and uncertainty in the strength properties of DM columns in design, it has been realized that the existing design tool for DM design needs to be developed.

Despite the fact that previous researchers have addressed the effect of the inherent variability on the mechanical system, the effect of the other sources of uncertainty has not been addressed. Furthermore, evaluation of the spatial variability along the failure surface has not been treated.

1.2 Aim and objectives

In practice, dealing with the variability and the uncertainty in the design is a difficult task. The design method currently used for the design of lime-cement columns is based on deterministic design, where uncertainties are not treated rationally. Rational estimation of the improved soil properties, and incorporating their variability and uncertainty into the design of LC columns, are important to find the reliability of the design. The main aim of this study is to develop a design tool that takes account of the complex uncertainty and satisfies both reliability and safety requirements of LC columns and which can be used practically by geotechnical engineers. The objectives that were planned in the present study to achieve this aim are as follows:

1. Increase the empirical knowledge concerning the inherent variability and identify the sources of uncertainty that are associated with the estimation of LC columns' strength property. (Paper I)
2. Evaluation of the spatial variability parameters of LC columns' strength property at different test sites. (Paper IV)
3. Investigate the effect of the spatial variability and other sources of uncertainty on the reliability of embankments. (Paper II)
4. Investigate the impact of different test methods used for the evaluation of the design value of LC columns strength property. (Paper IV)
5. Assess the reliability of highway embankments improved with lime-cement columns using RBD approaches. (Paper III)
6. Develop RBD as a design tool for the design of LC columns at the ULS that meets both deterministic and reliability requirements. (Paper V and VI)

1.3 Scope and Limitations

The study in this thesis focuses mainly on demonstrating the effect of the inherent variability and the sources of uncertainty associated with the evaluation of $c_{u,col}$ on the reliability of embankments at the ULS. Studying the effect of the variability and the uncertainty on the serviceability limit state (SLS) is beyond the scope of this study. The analyses were performed on low-strength columns in accordance with the current Swedish design practice, where the improved and unimproved soils are assumed to interact perfectly at failure according to Mohr-Coulomb failure criteria. Only the internal stability of LC columns with an assumption of shear failure mode is assessed. Other failure modes that have been addressed by Kivelö (1998), Broms (1999a), Kitazume et al. (2000), Kitazume and Maruyama (2006; 2007), Filz et al. (2012) and Larsson et al. (2012) and progressive failure as discussed by Broms (1999b) are not considered.

The stability of embankments was analysed under undrained conditions with respect to $c_{u,col}$ and $c_{u,soil}$, where $c_{u,col}$ is evaluated based on cone penetration test (CPT) measurements executed shortly after LC columns' installation. Strength properties of LC columns normally increase with time and thus affect the reliability. This particular issue, however, is beyond the scope of this study.

1.4 Research contribution

The variability and uncertainty associated with LC columns' strength property have always been an important issue that affects evaluation of the design value. However, very little has been published with regard to this particular subject. The effect of the inherent variability and the degree of uncertainty on the stability of highway embankments is unknown.

From scientific and practical points of view, the main contributions of this study are as follows:

- Increase empirical knowledge about the variability parameters of $c_{u,col}$ and of the other soil parameters involved in the ULS design of embankments.
- Present extensive statistical analyses and methods for evaluating the variability parameters of $c_{u,col}$.

- Provide information about the spatial variability of $c_{u,col}$ within the volume of LC columns.
- The study connects current traditional deterministic FS design with the more sophisticated RBD in a simple manner.
- Knowledge about the type of PDF of the undrained shear strength of LC columns is provided to be used in RBD.
- In practice, the spatial variability can only be evaluated in horizontal and in vertical directions. Since a potential failure surface is rarely perfectly straight in the horizontal or vertical direction, knowledge about evaluation of the spatial variability over the failure surface is provided in this study.
- The sensitivity factors are estimated for $c_{u,col}$ as well as for the other L and R parameters involved in the limit state function. The most important parameters affecting the ULS function are identified.
- The significant influence of different sources of uncertainty on the reliability of embankments is shown.
- Partial factors have been suggested for $c_{u,col}$ and for the other parameters involved in the limit state function. The evaluated partial factors consider the spatial variability and other sources of uncertainty associated with L and R parameters.

1.5 Outline of the thesis

This thesis consists of a summary of the research work in addition to six appended papers. The summary section contains a number of chapters that describe the methodologies that were used, as well as an individual summary of each appended paper and the main conclusions drawn from them, and finally further work that needs to be considered in future research is highlighted.

Chapter 2: Data and Methods

2.1 Data

The variability parameters, of L and R parameters, were evaluated in this study based on two types of data; the first dataset was related to the improved and unimproved soil properties (LC columns and the surrounding soil) based on in-situ measurements. The second dataset used for the other soil and load parameters involved in the mechanical system in Figure 5 consists of the collected in previous studies.

2.1.1 Data obtained from LC columns

The data used to evaluate the variability parameters of LC columns were obtained from two in-situ tests. Cone penetration tests (CPT) were used to test 60 columns; 30 at Lidatorp south of Stockholm and 30 in Kista, 10 km north of Stockholm. Column penetration tests (KPS) were used to test 30 columns at the test site in Kista. Both CPT and KPS tests are similar in concept, since both provide measurements of their cone tip resistances ($q_{c,CPT}$ and $q_{c,KPS}$) rather than the actual soil properties. They were used in this study for comparison purposes. However, empirical relations are normally used to infer $c_{u,col}$ from measurements of $q_{c,CPT}$ and $q_{c,KPS}$ respectively as follows:

$$c_{u,col} = \frac{q_{c,CPT} - \sigma_{v_0}}{N_{k,CPT}} \quad 1$$

$$c_{u,col} = \frac{q_{c,KPS}}{N_{k,KPS}} \quad 2$$

where, σ_{v_0} is the initial total vertical in-situ stress, and N_k is the cone factor. The values of N_k factors are normally assessed by calibrating $q_{c,CPT}$ and $q_{c,KPS}$ with unconfined compression tests conducted on samples taken from columns. The data obtained from CPT tests at the Lidatorp and Kista test sites are presented in Figure 8 and those from the KPS tests at the Kista test site in Figure 9.

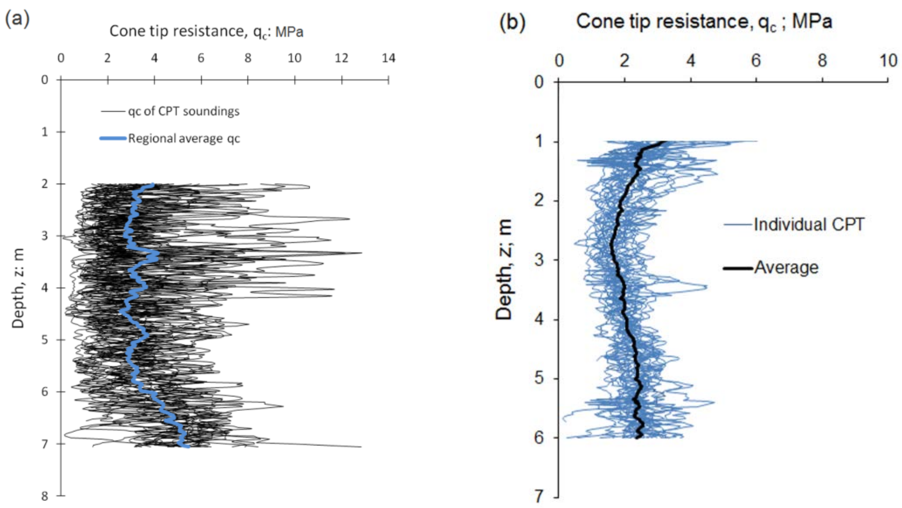

Figure 8. Measurements of CPT tests in 60 LC columns a) 30 tests conducted at the Lidatorp test site and b) 30 tests at the Kista test site

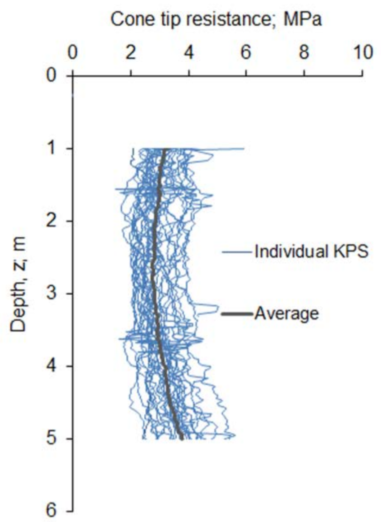

Figure 9. Measurements of KPS tests in 30 LC columns at the Kista test site

2.1.2 Data obtained from soft soil

The strength properties of the soft soil surrounding columns are normally obtained by conducting soundings and routine tests on samples. In order to evaluate the variability in the soft soil, one CPT was performed at the Kista test site. The results obtained from the fall cone test which was conducted to evaluate $c_{u,soil}$ at the Lidatorp and Kista test sites are presented in Figure 10 (a) and (b). The results of q_c measurements obtained from CPT tests are presented in Figure 10 (c).

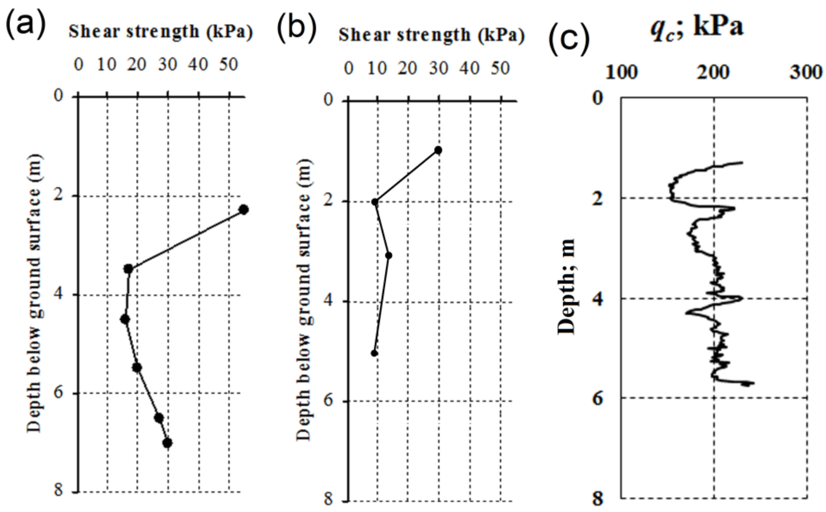

Figure 10. Evaluation of the undrained shear strength for the soft soil, $c_{u,soil}$; a) results of fall cone test from the Lidatorp test site, b) results of fall cone test from the Kista test site and c) results of q_c from one CPT test at the Kista test site

2.1.3 Data used for the other L and R parameters

The L and R parameters that are included in the mechanical system in Figure 5 are those which are related to the mechanical system's components, i.e. embankment, soil, LC columns and traffic load. The load parameters are; embankment's unit weight (γ_{emb}), column's unit weight (γ_{col}), soil's unit weight (γ_{soil}) and traffic loadings (q). The resistance parameters are; column's strength ($c_{u,col}$), soil's strength ($c_{u,soil}$) and embankment's friction angle (ϕ_{emb}).

The data for the main R parameters ($c_{u,col}$ and $c_{u,soil}$) were presented in sections 2.1.1 and 2.1.2, respectively. In this section, only the data for the other parameters that were used in the analyses are presented. The data were collected in accordance with either design codes or recorded reports and previous studies. The input data used in the analyses are presented in Table 1.

2.2 Methods

Stability of the embankments was evaluated in this study by using the traditional deterministic design, represented by total FS and the RBD design methods. This section is devoted to describing both methods. Special emphasis is placed on describing some important definitions of the parameters involved in the design procedure, in particular RBD.

2.2.1 Deterministic design methods

Generally, stability of embankments can be evaluated deterministically in terms of the total FS by using either numerical methods (NM), e.g. Finite element methods or limit equilibrium (LEM) design methods. In this study, LEM refers to the method of slices (Duncan, 1996). In the design of LC columns, the most applied method for assessing the stability is LEM methods since they are considered to be the most accustomed design methods used by geotechnical designers. They are normally simpler in their application than NM, very handy for stability calculations, and are based on simple assumptions. In addition, extensive experience has been built up over the years and these methods are widely used around the world (Figure 6 and Eq.3). In spite of the differences between the two methods, Han et al. (2005) and Filz and Navin (2006) have evaluated the safety of embankments using both methods. Their studies reveal good agreement between the methods in the case of low-strength columns, although for high-strength columns LEMs tend to overestimate the safety of DM columns.

The most adopted deterministic design method for the design of LC columns is Bishop's simplified method of slices (Broms, 1999b; Terashi and Kitazume, 2011). This method was therefore used as the main method in this study. According to Bishop (1955), the FS can be evaluated as follows:

$$FS = \left(\sum_{i=1}^{i=j} \frac{l_i \cos\theta_i c_i' + \left(w_i - u_i l_i \cos\theta_i\right)\tan\phi_i'}{m_{(\alpha)}} \right) \Big/ \sum_{i=1}^{i=j} w_i \sin\theta_i \qquad 3$$

Table 1: The in-data used in the analyses

Components of the mechanical system	Property parameter	Symbol	Unit	Statistical parameters * Mean (μ)	** COV	References
LCC columns	Unit weight	γ_{col}	kN/m^3	16	9%	*Åhnberg (2006), **Phoon and Kulhawy (1999a)
Soft soil	Unit weight	γ_{soil}	kN/m^3	16	9%	* Åhnberg (2006), ** Phoon and Kulhawy (1999a)
Embankment	Unit weight	γ_{emb}	kN/m^3	18	10%	* Swedish Transport Administration (2011), ** Phoon and Kulhawy (1999a)
Embankment	Friction angle	ϕ	(°)	35	-	* Swedish Transport Administration (2011)
Traffic	Static load	q	kN/m^2	13	5%	* Swedish Transport Administration (2011), ** Sundquist (2010)

where i is the slice number (*i=1,2,3……j*); j is the total number of slices; l_i, θ_i and w_i are the length, the inclination angle and the weight of the i^{th} slices respectively; c' and ϕ' are the soil's effective cohesion and friction angle respectively; and u is the pore water pressure on the base of a slice.

For the analysis of the mechanical system in Figure 5 at the undrained condition ($c_{u,col}$ and $c_{u,soil}$), and due to the inhomogeneity in the system, Bishop's method can be rewritten to fit the mechanical system in this study as follows:

$$FS = \frac{R}{L} = \frac{\sum_{i=1}^{i=j_{soil}} l_{soil} c_{u,soil} + \sum_{i=1}^{i=j_{comp}} l_{comp} c_{u,comp} + \sum_{i=1}^{i=j_{emb}} \left(w_{emb} \tan \phi_{emb} \right) m_{(\alpha)_{emb}} + \sum_{i=1}^{i=j_{emb}} \left(q b_{emb} \tan \phi_{emb} \right) m_{(\alpha)_{emb}}}{\sum_{i=1}^{i=j_{soil}} w_{soil} \sin \theta_{soil} + \sum_{i=1}^{i=j_{comp}} w_{comp} \sin \theta_{comp} + \sum_{i=1}^{i=j_{emb}} w_{emb} \sin \theta_{emb} + \sum_{i=1}^{i=j_q} q b_q \sin \theta_q} \quad \text{4-a}$$

$$c_{u,comp} = A c_{u,col} + (1-A) c_{u,soil} \quad \text{4-b}$$

$$w_{comp} = A w_{col} + (1-A) w_{soil} \quad \text{4-c}$$

$$m_{(\alpha)} = \left(\cos \theta_i + \left(\sin \theta_i \tan \phi_i / FS \right) \right)^{-1} \quad \text{4-d}$$

where c_u and ϕ are the undrained shear strength and friction angle at the base of a slice; q is the traffic load, which is uniformly distributed; w is the weight of slices calculated from $w = \gamma bh$, where γ, b and h are the unit weight, the width and the central height of a slice, respectively. The subscripts *soil*, *emb* and *comp* denote the material properties and the failure geometry in the soil, embankment and composite material, i.e. LC columns and surrounding soft soil, respectively. Since the term FS is present on both sides of Eq.4, the iteration process is required in order to solve the equation with respect to FS. A Microsoft Excel spreadsheet was developed in this study for this purpose.

2.2.2 Reliability-based design (RBD)

Like many other structures, geotechnical structures such as highway embankments need to be safe. However, due to the variability and uncertainty in L and R parameters, there will be a probability that the structures will not perform as intended, as shown in Figure 7. Although the risk of failure of geotechnical structures is generally very low, there is a need to control the probability of reaching unintended performance in a rational way (dealing with associated uncertainty). It has been acknowledged that RBD can be used as a powerful tool to evaluate the performance of the geotechnical systems, i.e. probability of failure, p_f. Reliability theory is the method based on mathematical statistics whereby L and R parameters, the variability and the uncertainty can be described by a random process. The parameters that are involved in the system are called *random variables*. A random variable is a variable that can take on multiple values. The domain of a random variable is the outcome set and its range is the set of possible values. Although RBD can be particularly useful for identifying the effect of different sources of uncertainty on the performance function, in some application the uncertainty is highly uncertain this makes RBD give a subjective probability of failure.

2.2.2.1 Levels of reliability

Depending on the importance of the structure, i.e. the adverse consequences of the failure, different levels of reliability analysis can be used in RBD methods. According to Thoft-Christensen and Baker (1982), the methods used for structural reliability can be divided into three levels of safety checking. Each level is characterized by the extent of information about the problem that is used and provided. The levels of reliability methods can be summarized as follows:

Level 1 method: The probabilistic aspect of a random variable, x, is taken into account by introducing suitable characteristic values, x_k. The design value, x_d, is evaluated from the predefined x_k values of L and R parameters after being factorized by a set of partial factors, γ_{x_i}. These factors should be evaluated from probabilistic methods to ensure appropriate level of reliability of the design. Partial factor design (PFD) or load resistance factor design (LRFD) methods are covered by this category.

Level 2 method: Reliability methods in which two parameters of a random variable are involved in the iteration calculation in order to obtain an approximate solution of the system's failure function. Methods like the first order reliability method (FORM), the first order second moment method (FOSM) and the point estimation methods fall into this category.

Level 3 method: Full information on the random variables, i.e. the mean, the variance and the type of PDF, is involved in this design level. A complete analysis is made of the multidimensional joint PDF of the random variables to determine p_f by means of performing direct integration or Monte-Carlo simulation (MC).

However, the three levels of safety checking are actually connected to each other, where level 2 methods are an approximation to level 3 methods, and level 1 methods are a discretization of level 2 methods. In order to use reliability analyses in practice, it is necessary to use a method that is computationally fast and effective, so that the expected performance level can be estimated with the desired degree of accuracy. Level 2 methods are widely used in structural engineering and satisfy the requirements (Baecher and Christian, 2003; Thoft-Christensen and Baker, 1982). However, in geotechnical engineering level 1 methods are designated to be used in practice by the geotechnical design codes (Eurocode7; Backer, 2006; Fenton and Griffiths, 2008). In this study, level 2 methods are used by means of FORM and FOSM methods to analyse the safety of the embankments and also to find the effect of the spatial variability and the uncertainty on the reliability of the embankments. The rigorousness of FORM analyses, in terms of problems related to linearization approximation of the limit state function, is assessed by means of a level 3 method, i.e. MC. Finally, a level 1 method (the PFD method) is proposed to be adopted in the design of lime-cement columns to satisfy both safety and reliability requirements. Furthermore, the partial factors for the most important variables affecting the ULS function are suggested based on the degree of variability and uncertainty. In the following section, the aforementioned RBD methods will be described.

2.2.2.2 General RBD method of analysis

The outcome of the level 3 and level 2 methods described above is p_f and a reliability index (β). For a basic reliability problem with only two independent normal random variables, L and R, the joint PDF of the R and L parameters is shown in Figure 11. The performance of the geotechnical system is described herein by means of a linear margin of safety (M), which is the difference between R and L (i.e. M = R-L). The boundary between the safe and the unsafe regions is identified when M=0. The limit state is therefore defined at M=0. The system will, theoretically, fail if M ≤ 0. Since the mechanical system consists of multiple random variables (x_i), which can be represented by a victor X, it is reasonable to describe M as a function of stochastic input parameters of the random variables, $g(X)$:

$$M = g(X) = g(x_1, x_2, x_3, \ldots\ldots, x_n) \quad 5$$

where, n is the number of random variables in the system.

The ULS function for the mechanical system in Figure 5 can be described as follow:

$$M = R - L = \left(\sum_{i=1}^{i=j_{soil}} l_{soil} c_{u,soil} + \sum_{i=1}^{i=j_{comp}} l_{comp} c_{u,comp} + \sum_{i=1}^{i=j_{emb}} \left(w_{emb} \tan\phi_{emb} \right) m_{(\alpha)_{emb}} + \sum_{i=1}^{i=j_{emb}} \left(q b_{emb} \tan\phi_{emb} \right) m_{(\alpha)_{emb}} \right)$$
$$- \left(\sum_{i=1}^{i=j_{soil}} w_{soil} \sin\theta_{soil} + \sum_{i=1}^{i=j_{comp}} w_{comp} \sin\theta_{comp} + \sum_{i=1}^{i=j_{emb}} w_{emb} \sin\theta_{emb} + \sum_{i=1}^{i=j_q} q b_q \sin\theta_q \right) = 0 \qquad 6$$

Reliability analyses were performed on the most critical slip surface, defined as the surface with the lowest FS evaluated from Eq.4. The geometry of the failure surface (i.e. slices' inclination angle, length, width and height) will be known during RBD analyses. The geometry of the failure surface can therefore be considered a deterministic parameter, and Eq.6 can be simplified to the following equation:

$$M = \left(c_{u,soil} R_1 + c_{u,comp} R_2 + \gamma_{emb} R_3 + q R_4 \right) - \left(\gamma_{soil} L_1 + \gamma_{comp} L_2 + \gamma_{emb} L_3 + q L_4 \right) = 0 \qquad 7$$

where, (R_1, R_2, R_3, R_4) and (L_1, L_2, L_3, L_4) are the parameters representing the geometry constants of the resistance and the load parameters in the performance function. The other parameters shown in Eq.7 are considered as random variable during RBD analyses.

The probability of failure, i.e. the shaded area in Figure 11-a, of the entire system can be defined as a probability of the system's performance function being less than or equal to zero. This can be calculated for a simple RBD problem with two normal random variables by direct integration of the joint PDF of R and L variables as shown in Figure 11-c or by using statistical methods as follows:

$$p_f = \Phi \left(\frac{0 - \left(\mu_R - \mu_L \right)}{\sqrt{\sigma_R^2 + \sigma_L^2 - 2 \rho_{RL} \sigma_R \sigma_L}} \right) \qquad 8$$

where Φ is the standard normal joint probability distribution function of R and L, ρ_{RL} is the correlation coefficient of R and L, and μ, σ and σ^2 are the mean, the standard deviation and the variance of R and L. The numerator and the denominator in Eq.8 represent μ and σ of the margin of safety (μ_M) and (σ_M), respectively.

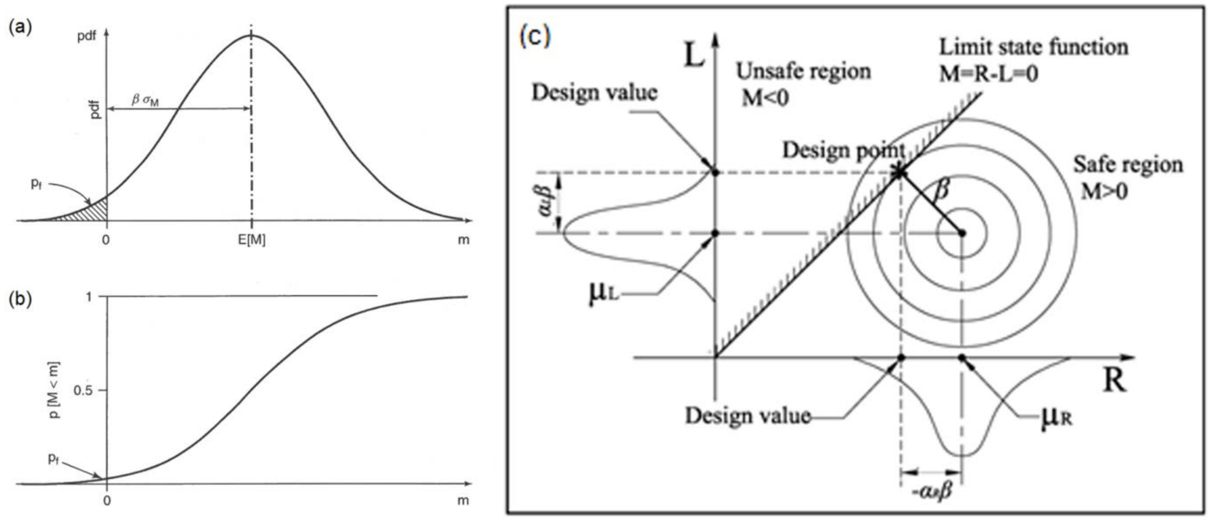

Figure 11. Schematic diagram showing in a) and b) the definition of β and p_f , and c) the concept of the design value and the design point in two-dimensional in the joint PDF of R and L parameters and for linear limit state function (Baecher and Christian, 2003)

The reliability index, β, is defined as the number of standard deviations between the mean margin of safety and the limit state function (Figure 11). Generally, evaluation of β can be performed reasonably for the linear limit state function with the uncorrelated variables as follows:

$$\beta = \frac{\mu_M}{\sigma_M} = \frac{\mu_R - \mu_L}{\sqrt{\sigma_R^2 + \sigma_L^2}} \qquad 9$$

In geotechnical engineering, it is more acceptable to talk about β than p_f. In this study, the index β will be most often used as a relative measure of p_f. However, both of these indices are related to each other as follows:

$$\beta = \Phi^{-1}(1 - pf) \qquad 10$$

$$pf = \Phi(-\beta) \qquad 11$$

A low value of p_f will correspond to a large value of β, i.e. a big margin of μ_M or a small uncertainty, σ_M).

As mentioned earlier, the main task of RBD is to evaluate the safety of a system or a component in the system, which is achieved by calculating either β or p_f. The complexity in RBD calculations, however, depends on many factors, such as the importance of the system, type of the limit state function (safety model), number of random variables, type of their PDF, and the correlation between them.

Although the direct evaluation of p_f and β according to Eqs.8 and 9 is simple in many cases. However, the main problem with this evaluation is that M is not invariant. Different representations of M will provide different results of β or p_f, e.g. the joint PDF of the linear M with normal R and L parameters will also be normal, but when M represents the ratio of R and L, the normality of their joint PDF is questionable. Several RBD methods have therefore been developed over the years in order to capture these issues in RBD evaluations.

2.2.2.3 Monte-Carlo simulation (MC)

Monte-Carlo simulation (MC) is considered to be the most robust simulation method, as it considers all the information of the PDF of each random variable and not just the first two moments (Ang and Tang, 2007). The mathematical expression of the limit state function does not have any influence on the simulation results. This method is based on generating the number of sets (N) of random values for the variables, based on the variables' two moments and type of PDF. The performance function is evaluated for each of these sets of random values. The number of failure events (η) is calculated when $g(X) \leq 0$. The mean and the coefficient of variation of p_f can then be estimated by the following equations:

$$\overline{p}_f = \frac{1}{N}\sum_{i=1}^{i=N}\eta \qquad 12$$

$$COV_{p_f} = \sqrt{\frac{1-p_f}{p_f N}} \qquad 13$$

The reliability index (β_{MC}) for the linear limit state function and evaluated from MC can be determined as follow:

$$\beta_{MC} \approx \Phi^{-1}(1-\overline{p}_f) \qquad 14$$

The sets of N random numbers generations were chosen in such a way that $COV_{p_f} \leq 10\%$ and $p_f = 8\times10^{-5}$ correspond to the target reliability index $\beta_t = 3.8$.

2.2.2.4 First order second moment method (FOSM)

When the margin of safety, M, is linear and only consists of two random variables (L and R), β can be reasonably evaluated from Eq.9. However, when M consists of multiple random variables described with a nonlinear function as

$M = FS = g(X) = g(x_1, x_2, x_3, ..., x_n)$, the first two moments of this function, i.e. the expected value $\mathrm{E}[FS]$ and the variance σ_{FS}^2, can be determined using Taylor series expansion of FS about the mean value. The first two moments of FS can be evaluated as follows:

$$\mathrm{E}[FS] = \mu_{FS} \approx FS(x_1, x_2, x_3 ....., x_n) \quad 15$$

$$\sigma_{FS}^2 \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial FS}{\partial x_i} \frac{\partial FS}{\partial x_j} \rho_{x_i x_j} \sigma_{x_i} \sigma_{x_j} \quad 16$$

where n is the number of random variables, σ_{x_i} is the standard deviation of the random variable *i*, $\rho_{x_i x_j}$ is the correlation coefficient between random variables *i* and *j*, and ∂ stands for the partial derivative notation. Note that the type of PDF is not required in order to evaluate the values of $\mathrm{E}[FS]$ and σ_{FS}^2. The type of the joint PDF of FS according to the FOSM method will therefore be unknown. However, in order to determine β, assumptions need to be made regarding the type of PDF of FS. For the normal and log-normal assumptions, β can be calculated respectively as follows:

$$\beta = \frac{\mathrm{E}[FS] - 1}{\sigma_{FS}} \quad 17$$

$$\beta = \frac{\mu_{\ln_{FS}}}{\sigma_{\ln_{FS}}} = \frac{\ln \mu_{FS} - 0.5 \ln\left(1 + COV_{FS}^2\right)}{\sqrt{\ln\left(1 + COV_{FS}^2\right)}} \quad 18$$

2.2.2.5 First order reliability method (FORM)

This reliability method is also called the Hasofer-Lind reliability method. The evaluated β according to this method will be denoted by (β_{HL}), which can be defined geometrically as a minimum distance in the number of standard deviations between the mean margin of safety and the limit state function, Figure 11. The significance of β_{HL} is that it identifies the coordinates of the failure point for each random variable when its sensitivity factor (α_i) is known. α_i is the factor that describes the significance of the random variables to the limit state function. For the L and R parameters, α_i takes positive and negative signs respectively. Evaluation of β_{HL} by the FORM method is independent of the mathematical formulation of the limit state function. According to FORM, all random variables should be transformed to their standard form (i.e. zero mean and unit standard deviation). For the limit state function which consists of *n* normal random variables (x_i), each variable x_i is defined as $x_i \in Normal(\mu_{x_i}, \sigma_{x_i})$. A primed variable x_i' can then be defined as a standard form of the original random variable x_i as follows:

$$x_i' = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \tag{19}$$

$$x_i = \mu_{x_i} + x_i' \, \sigma_{x_i} \tag{20}$$

At the limit state $g(X) = 0$, the primed and the original variables will therefore be defined on the failure line. The superscript (*) is used to identify that x_i and x_i' are evaluated at the failure line; the design value (x_i^*) and its coordinate ($x_i'^* = \alpha_i \beta_{HL}$) will then be identified (see Figure 11). For the log-normal PDF of the random variables, the design value can be determined as follows:

$$x_i^* = Exp(\mu_{\ln x_i} + x_i'^* \sigma_{\ln x_i}) \tag{21}$$

$$\mu_{\ln x_i} = \ln \mu_{x_i} - 0.5\sigma_{\ln x_i}^2 \tag{22}$$

$$\sigma_{\ln x_i}^2 = \ln\left[1 + \left(COV_{x_i}\right)^2\right] \tag{23}$$

where $\mu_{\ln x_i}$ and $\sigma_{\ln x_i}^2$ are the log-normal distribution's parameters and COV_{x_i} is the coefficient of variation of the original variable *xi*, defined as $COV_{x_i} = \sigma_{x_i} / \mu_{x_i}$. Accordingly, the performance function of the mechanical system in Figure 5 according to Eq.7 can be reformulated and β can be evaluated for the uncorrelated variables respectively as follows:

$$M = g\left(X_i^*\right) = \left[Exp\left(\mu_{c_{u,soil}} + x_{c_{u,soil}}'^* \sigma_{c_{u,soil}}\right)R_1 + Exp\left(\mu_{c_{u,comp}} + x_{c_{u,comp}}'^* \sigma_{c_{u,comp}}\right)R_2 + Exp\left(\mu_{\gamma_{emb}} + x_{\gamma_{emb}}'^* \sigma_{\gamma_{emb}}\right)R_3 + R_4\right] -$$

$$\left[Exp\left(\mu_{\gamma_{soil}} + x_{\gamma_{soil}}'^* \sigma_{\gamma_{soil}}\right)L_1 + Exp\left(\mu_{\gamma_{comp}} + x_{\gamma_{comp}}'^* \sigma_{\gamma_{comp}}\right)L_2 + Exp\left(\mu_{\gamma_{emb}} + x_{\gamma_{emb}}'^* \sigma_{\gamma_{emb}}\right)L_3 + L_4\right] = 0 \tag{24}$$

$$\beta_{HL} = \min_{x_i'^* \in *}\left(\mathrm{X}'^{*\mathrm{T}}\mathrm{X}'^*\right)^{1/2} \quad \text{Subjected to } g\left(X_i^*\right) = 0 \tag{25}$$

where X'^* and T are the matrix of the design point for uncorrelated variables and its transpose. α_i is evaluated according to Baecher and Christian (2003) as follows:

$$\alpha_i = \frac{\left(\dfrac{\partial g}{\partial x_i'^*}\right)}{\sqrt{\sum_{i=1}^{i=n}\left(\dfrac{\partial g}{\partial x_i'^*}\right)^2}} \qquad 26$$

where $\left(\dfrac{\partial g}{\partial x_i'^*}\right)$ is the partial derivative of the limit state function (Eq. 24) with respect to the failure coordinates of the random variables. It is obvious from Eqs. (24 and 25) that β_{HL} can be defined by the optimisation technique (iterations) up to limit state function converges, i.e. $M = g(X_i) = 0$. This requires evaluation of α_i in every iteration. Accordingly, the optimisation algorithm proposed by Rackwitz and Fiessler (1978) was used to evaluate β_{HL} in Eq. (25) and α_i in Eq. (26). A Microsoft Excel spreadsheet was developed in this study for the iterative calculations. For more information about the FORM method and methods of β_{HL} calculations, the reader is referred to (among others, Thoft-Christensen and Baker, 1982; Baecher and Christian, 2003).

The aforementioned methodology for evaluating β_{HL} was for the uncorrelated variables. However, in reality some sort of correlation (ρ) may exist between some random variables in the system. The correlated variables will thus be defined by a symbol y_i in the system instead of x_i. The design value, the limit state function and β_{HL} of the correlated variables can be evaluated as follows:

$$y_i^* = Exp(\mu_{\ln x_i} + y_i'^* \sigma_{\ln x_i}) \qquad 27$$

$$M = g\left(Y_i^*\right) = \left[Exp\left(\mu_{c_{u,soil}} + y'^*_{c_{u,soil}}\sigma_{c_{u,soil}}\right)R_1 + Exp\left(\mu_{c_{u,comp}} + y'^*_{c_{u,comp}}\sigma_{c_{u,comp}}\right)R_2 + Exp\left(\mu_{\gamma_{emb}} + y'^*_{\gamma_{emb}}\sigma_{\gamma_{emb}}\right)R_3 + R_4\right] -$$

$$\left[Exp\left(\mu_{\gamma_{soil}} + y'^*_{\gamma_{soil}}\sigma_{\gamma_{soil}}\right)L_1 + Exp\left(\mu_{\gamma_{comp}} + y'^*_{\gamma_{comp}}\sigma_{\gamma_{comp}}\right)L_2 + Exp\left(\mu_{\gamma_{emb}} + y'^*_{\gamma_{emb}}\sigma_{\gamma_{emb}}\right)L_3 + L_4\right] = 0 \qquad 28$$

$$\beta_{HL} = \min_{y_i'^* \in *}\left(Y'^{*T} Y'^*\right)^{1/2} \qquad \text{Subject to } g\left(Y_i^*\right) = 0 \qquad 29$$

where y_i^* and $y_i'^*$ are the design value and the design point of the correlated variables respectively, and Y their matrix. However, in order to calculate $y_i'^*$, the correlation matrix (K) should be identified first. Baecher and Christian (2003) and Phoon (2008) used the technique called Cholesky decomposition to evaluate $y_i'^*$ of the correlated variables from the standard space of $x_i'^*$ as follows:

$$Y'^* = SX'^* \qquad 30$$

where S is the matrix of Cholesky decomposition of K. In this study Microsoft Excel spreadsheet is used to solve β_{HL} for the correlated variables and it has also been used to calculate S and Y'^*. Cholesky decomposition matrix can be calculated numerically according to Baecher and Christian (2003) as follows:

$$S_{ii} = \left(K_{ii} - \sum_{k=1}^{i-1} S_{ik}^2 \right)^{1/2} \qquad i = 1, \ldots\ldots, n \qquad 31\text{-a}$$

$$S_{ji} = \left(K_{ji} - \sum_{k=1}^{i-1} S_{ik} S_{jk} \right) \Big/ S_{ii} \qquad j = i+1, \ldots\ldots, n \qquad 31\text{-b}$$

$$S_{ij} = 0 \qquad j = i+1, \ldots\ldots, n \qquad 31\text{-c}$$

2.2.2.6 Partial factor design method (PFD)

The basic principle of the PFD is that it replaces the single value of total FS with a set of partial safety factors (γ_{x_i}) for a variable x_i. This can be achieved on the bases of the degree of uncertainty associated with the evaluation of μ_{x_i} for individual parameters of R and L based on the evaluation of their sample mean values ($\overline{x}_i$). The uncertainty associated with $\overline{x}_i$ is described extensively in the next section. In this section, the emphasis will be on describing PFD and the related parameters. The partial factors for R and L parameters, (γ_{R_i}) and (γ_{L_i}), can be calculated according to Thoft-Christensen and Baker (1982) as follows

$$\gamma_{R_i} = \frac{x_{Rk_i}}{x_{Rd_i}^*} \qquad 32$$

$$\gamma_{L_i} = \frac{x_{Ld_i}^*}{x_{Lk_i}} \qquad 33$$

where x_{Rk_i}, x_{Lk_i} and $x^*_{Rd_i}$, $x^*_{Ld_i}$ are the characteristic and the design values of *i*th number of random variable (*i=1, 2, 3 ....n*) of R and L, respectively. The concept of the design value and the design point in PFD can be more visualized in Figure 11. In the PFD design procedure, the ULS for the mechanical system (Figure 5) considered in this study is calculated as follow

$$FS(x_i^*, A) = FS(x_{k_i}\gamma_{x_i}, A) \geq 1 \qquad 34$$

where, $x_{k_i}\gamma_{x_i}$ are the factored parameters, i.e. design values, and A is the area ratio defined as the ratio of the total area of LC columns to the total area of the improved ground. The parameter A is also called *design variable* in this study, because its value is subject to change during the PFD procedure.

According to the Swedish design code (Swedish Transportation Administration, 2011), lime-cement columns should be designed according to the limiting values of $\bar{c}_{u,col}$ ≤150 kPa, due to the assumption that the columns behave as an elastic perfectly plastic material. In practice, $\bar{c}_{u,col}$ is estimated from a finite number of test samples, normally evaluated from KPS but occasionally from CPT tests. Since the mechanical properties of the columns cannot be determined before installation and testing on-site, it is difficult in design to choose a proper characteristic value (c_{u,col_K}). In this study, c_{u,col_K} was considered to be equal to $\bar{c}_{u,col}$ suggested by the Swedish design code for the design. In order to investigate the effect of $\bar{c}_{u,col}$ on the evaluation of $\gamma_{c_{u,col}}$ beyond the Swedish design limits, the range of $\bar{c}_{u,col}$ were extended to $\bar{c}_{u,col}$ =200 kPa. The characteristic values of the other random variables (x_{Rk_i}, x_{Lk_i}) were also considered to be equal to their sample mean values ($\bar{x}_{R_i}$, $\bar{x}_{L_i}$). The uncertainty and the variability associated with evaluation of $\bar{c}_{u,col}$ was considered in the evaluation of the design value (c^*_{u,col_d}). This procedure is also valid for the other random variables. The design values of the random variables were evaluated by FORM in such a way that the design fulfils certain performance level. In this study, the intended level of performance is set to β_t =3.8 as suggested by Eurocode for the ULS design (Eurocode, 2002).

The calculation procedure for the evaluation of γ_{x_i} is shown in Figure 12. The design procedure by PFD is then presented as steps in the flowchart, as shown in Figure 13. The advantage of the PFD for the design of lime-cement columns can be clearly seen in the figure, since both the traditional deterministic design and the reliability-based design are combined into one single design procedure. The rationality in the evaluation of $\gamma_{c_{u,col}}$ is truly dependent on the rational evaluation of their design value. In the analyses, the variables were assumed to be uncorrelated and to follow a log-normal probability density function (PDF). The assumption of the log-normal PDFs is reasonable since it prevents random variables from becoming negative in addition to the related uncertainty. The assumption of the non-correlation between L and R variables is considered to be reasonable and has been adopted by many authors for similar applications (Babu and Singh, 2011; Ching and Phoon, 2011; Murakami et al., 2011). However, unimproved soil properties may have an influence on the improved soil properties and a correlation may therefore exist to some extent between $c_{u,col}$ and $c_{u,soil}$. However, the complex mixing process has a significant influence on the variability and may reduce the correlation. The degree of correlation is unknown and therefore the effect of correlation on the evaluation of $\gamma_{c_{u,col}}$ is shown and discussed in this study.

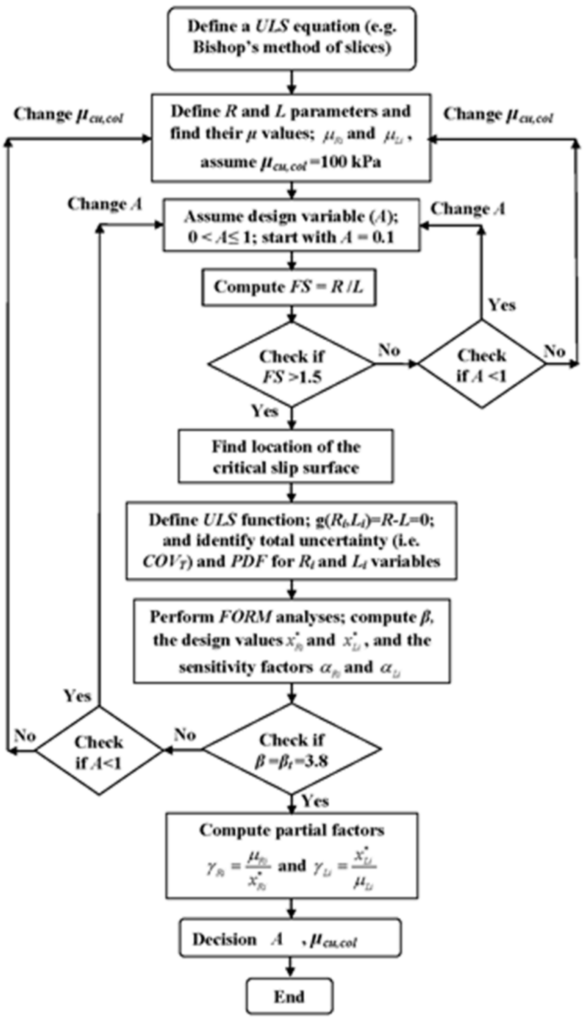

Figure 12. Flow chart illustrating the calculation procedure for determination of partial factors for undrained shear strength of lime-cement columns ($\gamma_{c_{u,col}}$)

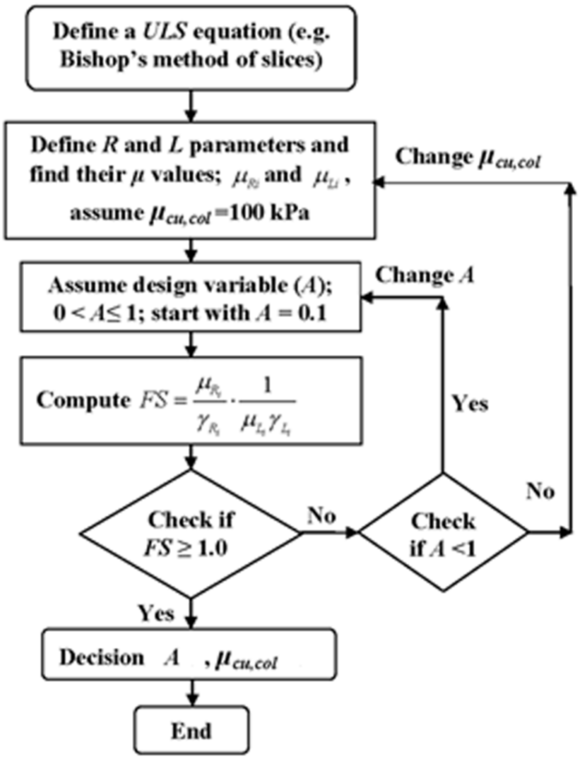

Figure 13. Flow chart illustrating the PFD procedure applied for the design of lime-cement columns for determination of design variables A and $\mu_{c_{u,col}}$

2.2.3 Variability and uncertainty

Soil properties are normally evaluated by either laboratory, penetration or in-situ tests. Evaluating "real" soil parameters' values from test samples will always be associated with uncertainties that arise mainly from two sources, *data scatter* and *systematic error* (Christian et al., 1994), as shown in Figure 14.

The first term (data scatter) is also called *aleatory* uncertainty and is the combination of the uncertainty in the *inherent* randomness of natural process manifesting as variability over space at different locations, i.e. spatial variability, and the uncertainty from measurement errors, i.e. equipment, procedure and random testing errors. In the design of any geotechnical system, inherent variability is usually considered to be the major and unavoidable source of uncertainty, as a consequence of the complex geological process involved with the deposition and formation of soils and rocks (Orchant et al., 1988; Christian et al., 1994; Phoon and Kulhawy, 1999b). Inherent variability for a set of data can be described by defining the first two moments, i.e. μ and σ^2. In order to better understand the spatial variability, i.e. the variation of a property at different locations, there is a need to evaluate a third parameter suggested by Vanmarcke (1977) called scale of fluctuation (δ), which is the distance within which a soil property shows strong correlation. Beyond this distance, a soil property is considered to be uncorrelated. The significant of δ is that it helps in site investigation when it comes to sampling, e.g. evaluation of μ based on test samples taken within the space covered by δ will be misleading, since it will only provide values either above or below the "real" mean trend value, as shown in Figure 15. Another important advantage can be gained from the knowledge about δ where the safety of the mechanical system is governed by the average property value rather than its point property value. The variability of the local averaged data (average value over δ) about the mean trend will be of more interest than the point-to-point variability. Significant reduction in the inherent variability can therefore be achieved by introducing the concept of spatial variability in the reliability analyses, as shown in Figure 15-c. Vanmarcke (1977) quantified the magnitude of the reduction in the point variance by introducing a parameter called the variance reduction factor (Γ^2).

The second term (systematic error) on the other hand is also called *epistemic* uncertainty. This source covers all sources of uncertainty that are related to knowledge, due to lack of data and lack of information and understanding of the physical real behaviour of the materials. Systematic error can be divided into two sub-divisions: statistical uncertainty, i.e. related to the amount of data, and bias in the measurement process, i.e. related to an idealized model of reality. This type of uncertainty, however, can be reduced by increased knowledge (e.g. increasing the number of tests and the number of test methods for the evaluation) and knowledge (e.g. direct measurement of the real property) about the geotechnical soil property in question.

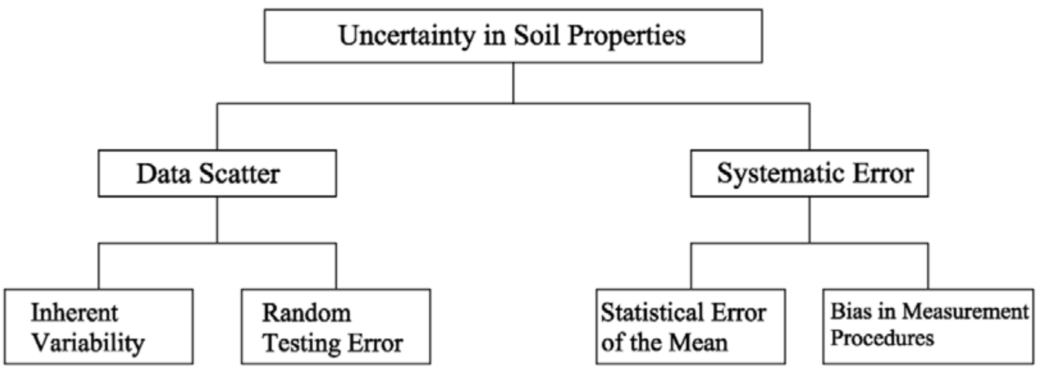

Figure 14. Categories of uncertainty in soil properties (after Christian et al. 1994)

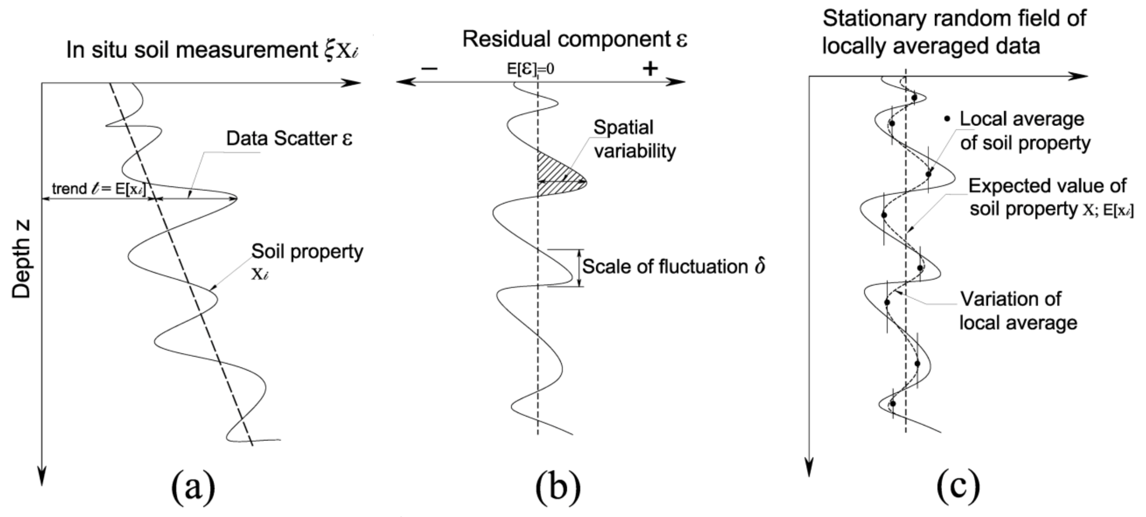

Figure 15. a) One-dimensional model of data scatter according to random field theory, showing the decomposition of in-situ soil measurements into a trend t and random residual ε components, b) concept of spatial variability parameters, and c) reduction in the inherent variability due to locally averaged data

Due to the existence of these uncertainties, the life cycle and the estimated performance of the embankments will be uncertain. However, the effect of these sources of uncertainty on the design of DM cannot be assessed by using deterministic design as they are not incorporated in the design procedure. In the previous section (2.2.2), it was shown that the concepts of variability and uncertainty can be incorporated in the design by performing RBD.

2.2.3.1 Spatial variability

Soils are by nature non-homogeneous and anisotropic materials. Non-homogeneity means that a soil property varies from point to point, i.e. spatial variability. Anisotropy means that a soil property varies with direction at certain points. Spatial variability can be evaluated by having information about how a soil property (e.g. c_u) varies from point to point, or in other words how it is correlated with neighbouring points along the same direction (e.g. variation of c_u with depth as shown in Figure 8 to Figure 10). Vanmarcke (1977) showed that spatial variability can be evaluated in any direction by having information about the variability parameters, i.e. μ, σ^2 and δ, in the direction in question. Spatial variability in *x*, *y* and *z* directions is denoted in this study by δ_x, δ_y and δ_z respectively. Materials with properties characterized by $\delta_x = \delta_y = \delta_z$ are isotropic materials and materials with properties characterized by $\delta_x \neq \delta_y \neq \delta_z$ are anisotropic materials. Materials' properties with $\delta_x = \delta_y \neq \delta_z$ are considered to be isotropic within the x, y plane only, or it is called *omnidirectional* isotropy $\delta_{x,y}$ (Deutsch, 2002).

Evaluation of δ in general requires continuous and equidistance measurements of the property in question. Measurements of q_c evaluated from CPT and KPS tests will therefore be consistent with these requirements for the evaluation of δ_z of $c_{u,col}$ (Fenton, 1999; Jacksa et al., 1999). However, the technique to be applied for evaluation of δ_x and δ_y is similar, but for randomly located data, the methods tend to be computationally intensive. Nonetheless, as Fenton and Vanmarcke (2003) noted, the technique is in practice still applicable.

In this study, δ of $c_{u,col}$ was evaluated according to random field theory by means of the autocorrelation function (ACF), according to the measurements of q_c evaluated from CPT and KPS tests. According to Vanmarcke (1983), ACF is defined as the variation of the autocorrelation coefficient $\rho(k)$ with lag distance k. This method is fully described by autocovariance (c_k), as follows:

$$c_k = Cov\left(q_c\left(z_i\right), q_c\left(z_{i+j}\right)\right) = \mathrm{E}\left[\left(q_c\left(z_i\right) - \overline{q_c}\right)\left(q_c\left(z_{i+j}\right) - \overline{q_c}\right)\right] \qquad 35$$

where k is the lag distance $k = j.z$, $j = 0,1,......,n-1$, z is the distance between adjacent measurements, z_i is the value of property q_c at location i, z_{i+j} is the value of property q_c at location $i+j$, $\mathrm{E}[...]$ is the expected value and c_0 = autocovariance at lag zero (0). The sample autocorrelation function ρ'_k is obtained for different lags by the equation

$$\rho'_k = c_k / c_0 \qquad 36$$

where c_0 = autocovariance at lag 0, which is equal to the sample's variance. As Vanmarcke (1977, 1983) suggested, the correlation distance (the scale of fluctuation) can be assessed by fitting a theoretical model of ACF to sample ACF; five of the theoretical models are widely used in the analysis of geotechnical data as shown in Table 2 (Jaksa et al., 1999, Phoon et al., 2003).

For the vertical analysis of CPT data, a single exponential model was mainly used to be fitted to sample ACF by an equation of the form

$$\rho(k) = e^{-m|k|} \qquad 37$$

Table 2. Five common autocorrelation models used for analysis of geotechnical observations (Vanmarcke 1983; Jaksa et al., 1999; Phoon et al., 2003).

Autocorrelation model	Equation
Binary noise	$\rho(k)=\begin{cases}1-c\lvert k\rvert & k\le 1/c\\ 0 & otherwise\end{cases}$
Single exponential Squared exponential	$\rho(k)=\exp(-m\lvert k\rvert)$ $\rho(k)=\exp(-bk)^2$
Cosine exponential	$\rho(k)=\exp(-d\lvert k\rvert)\cos(d\lvert k\rvert)$
Second-order Markov	$\rho(k)=(1+a\lvert k\rvert)\exp(-a\lvert k\rvert)$

Note: $\rho(k)$ is the autocorrelation function, k is the lag distance, and c, m, b, d and a are model constants (decay factors)

where m is the decay factor.

In the horizontal plane, the spatial correlation structure was assumed to be equal in all directions, i.e. omnidirectional. Due to the relatively limited number of data and the high scatter in the results, the binary noise (linear model) was mainly used to be fitted to sample ACF, as in equation

$$\rho(k) = \begin{cases}1-c\lvert k\rvert & k\le 1/c\\ 0 & otherwise\end{cases} \qquad 38$$

For the aforementioned reasons, the effect of the ACF nugget g_0 should be taken into account in evaluating the lateral scale of fluctuation. The ACF nugget comes from the combination of three phenomena: *i*) microvariability of the geological material due to the inherent variability, *ii*) sampling error due to the limited number of test data, and *iii*) random measurement error (Rendu, 1981). The linear model in Equation 38 is thus modified as

$$\rho(k)=\begin{cases}(1-g_o)-c\lvert k\rvert & k\le 1/c\\ 0 & otherwise\end{cases} \qquad 39$$

where c is the model parameter (decay factor) and g_0 is the nugget effect at lag $k=0$. For the linear model, when the effect of the nugget is ignored, the intersection between the correlation function $\rho(k)$ and the lag-distance's k axis will represent the value of δ. The one-dimensional scale of fluctuation δ can be determined through the relation (Vanmarcke, 1983)

$$\delta = 2\int_0^\infty \rho(k)\ dk \qquad 40$$

The spatial variability of $\bar{c}_{u,col}$ ($COV_{spt(\bar{c}_{u,col})}$) evaluated from CPT data within the volume of lime-cement columns can be calculated as follows:

$$COV_{spt(\bar{c}_{u,col})} = \sqrt{\Gamma^2_{xyz} COV^2_{inh(q_c)}} \qquad 41$$

$$\Gamma^2_{xyz} = \frac{\delta_x}{L_x}\frac{\delta_y}{L_y}\frac{\delta_z}{L_z} \qquad 42$$

where Γ^2_{xyz} is the variance reduction factor, δ_x, δ_y, δ_z and L_x, L_y, L_z are the scale of fluctuations and the failure domain in *x*, *y*, and *z* directions, respectively.

2.2.3.2 Uncertainty due to number of test samples

Uncertainty due to sampling is also called statistical uncertainty (COV_{stat}), which reflects the degree of error associated with the evaluation of μ based on a limited number of test samples (N_{samp}). This source of uncertainty can, however, be reduced by taking more samples. Statistical uncertainty associated with $\bar{c}_{u,col}$ can be evaluated as follows:

$$COV_{stat} = \sqrt{\frac{COV^2_{inh(q_c)}}{N_{samp}}} \qquad 43$$

2.2.3.3 Measurement and transformation uncertainties

Measurement error (COV_{meas}) arises from sampling procedure, equipment and random testing errors (Phoon and Kulhawy, 1999b). However, evaluation of COV_{meas} is difficult since its magnitude is already included within the magnitude COV evaluated from data scatter (see Figure 14). Transformation errors (COV_{trs}) arise from the indirect measurement of the actual property, e.g. evaluation of $c_{u,col}$ from q_c measurements of CPT and KPS tests according to Eqs. (1 and 2) respectively.

Information about the magnitude of COV_{meas} and COV_{trs} associated with $c_{u,col}$ evaluated from CPT measurements is not available. However, Jaksa et al. (1997) conducted an extensive study for the evaluation of COV_{meas} for the CPT test conducted on over-consolidated clays. Jaksa et al. (1997) suggest values ranging from 0.07 to 0.05 for COV_{meas} of CPT tests. Kulhawy et al. (1992) suggest a value of COV_{trs} =0.29 associated with CPT measurements calibrated with unconsolidated undrained triaxial compression tests on samples taken from natural soils. The values COV_{meas} = 0.05 and COV_{trs} =0.29 were adopted as reasonable values in this study.

2.2.3.4 Total uncertainty

According to (Ang and Tang, 1990; Phoon and Kulhawy, 1999b), the total uncertainty associated with evaluation of $c_{u,col}$ along the failure surface, $COV_{Total(\bar{c}_{u,col})}$, can be found by summing the individual sources of uncertainty as follows:

$$COV_{Total(\bar{c}_{u,col})} = \sqrt{COV^2_{spt(\bar{c}_{u,col})} + COV^2_{Stat} + COV^2_{meas} + COV^2_{trs}} \qquad 44$$

It can be shown from Eqs. 20-44 that the spatial variability and different sources of uncertainty can be incorporated into the RBD procedure directly.

Chapter 3: Summary of Appended Papers

The following section is a summary of the appended papers. The papers are presented in chronological order. The summary of each paper is presented separately. However, the reader is advised to read the section continuously. The emphasis in the summary is on describing the objective of each paper, followed by the methodology used and the important findings, and finally the major conclusions. At the end of each summary, the connection to the next paper is described.

3.1 Paper I

Strength variability in lime-cement columns based on cone penetration test data

Mohammed Salim Al-Naqshabandy, Niclas Bergman and Stefan Larsson

Ground improvement (2012), **165** (1), 15-30

In this paper the statistical evaluation of CPT data was comprehensively described. The objective was to make a contribution to the empirical knowledge with regard to strength variability evaluation within the volume of lime-cement columns. The methodologies used were motivated. This study was based on the field test, in which 30 CPT soundings were performed in lime-cement columns to evaluate the variability parameters within the volume of lime-cement columns from the measurements of the cone tip resistance (q_c).

The variability parameters of q_c (i.e. μ , σ^2 and δ) were evaluated for the test site within the volume of LC columns. The PDF of q_c measurements was detected using simple statistical approaches. The type of PDF has been tested by Kolmogorov-Smirnov against the normal and log-normal distribution. The methodology according to the random field theory, as described by Vanmarcke (1977), was used to evaluate δ in *x*, *y* and *z* directions. This method can be fully described by evaluating the sample autocorrelation function ACF with varying lag distances. A single exponential ACF model was found to be the best fit for δ_z , while due to the limited amount of data in the horizontal direction, linear function was the best fit for the horizontal δ_{xy} . The study discusses the concept of the design and the characteristic values with respect to the design of LC columns. Parameter study was conducted to find the effect on the design value of the type of PDF, the number of test samples, N, the sensitivity factor, α, and the target reliability index, β_t. An equation has also been derived for the calculation of the design value for the log-normally distributed variables, which implicitly takes account of the spatial variability and statistical uncertainties.

The results showed that the samples' *COV* of the tested columns with respect to q_c range from 22-67%. This wide range in the *COV* indicates the high variability. The variability is highly volume-dependent and the high *COV* evaluated from CPT data is probably due to the small size of the CPT probe with respect to the column segment, and it is also too small to represent the strength of the whole column cross-section. The distribution of PDF for q_c followed log-normal distribution. This means that in RBD, the strength property of the columns evaluated from q_c measurements can be modelled as a log-normally distributed random variable. The evaluated δ_z ranged from 20-70 cm, and δ_{xy} 2-3 m. The variance reduction (Γ^2) within the volume tested was evaluated. The study demonstrates the significant effect of many factors, for example the type of PDF, N, α, δ and β_t, on the evaluation of the design value.

Some important recommendations were drawn up based on this study:

- Due to the high variability in the columns, it is recommended that the type of PDF and the spatial variability should be considered in the evaluation of the design value.
- The spacing between samples should be greater than δ to guarantee the independency between test samples.

The study also highlights further research which needs to be addressed in the future to verify the following questions:

- We have attributed the high *COV* in this site to the small size of the CPT probe with respect to the size of the LC columns. This hypothesis can be verified by conducting the test with the aid of a bigger probe by using for example a KPS probe in parallel with the CPT probe to find the differences in the variability measurements.
- The present study only considers the effect of PDF, δ and N on the evaluation of the design value of the individual parameter. However, in order to find the effect of PDF, δ and N on the embankment's stability, the analyses should be conducted in such a way that other variables in the system can be considered.

- It is important that the effect of other sources of uncertainty, i.e. model transformation and measurement errors, on the reliability of embankments is shown.
- The sensitivity factor influences the design value evaluation. However, information about α for the property parameters of lime-cement columns and for the other parameters involved in the system is unknown, and it is important that it be studied.

3.2 Paper II

Effect of spatial variability of the strength properties in lime-cement columns on embankment stability

Mohammed Salim Al-Naqshabandy, Niclas Bergman and Stefan Larsson

Proc. of the 4th International Conference on Grouting and Deep Mixing (2012) Marriott New Orleans, ASCE, Geotechnical Special Publication No. 228, ***Vol. 1****, 231-242*

The main objective of this paper was to investigate the effect of the spatial variability of $c_{u,col}$ on the stability of embankments. The study was based on field tests, in which 30 CPT soundings were performed in a random manner in the columns to evaluate $c_{u,col}$. One single CPT sounding was performed on the soft soil to evaluate $c_{u,soil}$. The test site was located in Kista and was part of a major ground improvement project for construction of a new highway 10 km north of Stockholm. The tested area was 16 m x 16 m. In this area, a total of 236 lime-cement columns were manufactured individually with an area ratio of 28%, which was designed for an embankment of 4 m in height.

The variability parameters of $c_{u,col}$ were evaluated within the volume of the improved area. The strength property of the whole improved area was evaluated based on the weighted average method. Level-2 reliability analysis was used according to the FOSM method to evaluate the safety of the embankment and to find the effect of the spatial variability and statistical uncertainty of the improved soil property on the reliability of the embankment. The distribution parameters and type of PDF for $c_{u,col}$ and $c_{u,soil}$ were detected. The statistical analyses for the CPT data described in the previous paper were used to evaluate the variability parameters. Knowledge about the spatial variability along the failure surface is required in order to perform RBD, hence the factor Γ^2 was evaluated along the failure surface. The FS of the embankment was assessed according to Fellenius's method of slices, and FOSM analyses were performed on the most critical failure surface. In the analyses, only the shear failure mode was considered for LC columns and the surrounding soil. The two parameters $c_{u,col}$ and $c_{u,soil}$ were considered random variables. Due to the lack of information and for the sake of simplicity, the correlation between the random variable was ignored.

Results from statistical analyses indicate that values of $\overline{c}_{u,col}$ and $\overline{c}_{u,soil}$ were of the order of 110 kPa and 10 kPa, respectively. The samples' *COVs* were 10% and 27% for the soil and columns, respectively. The PDFs of both $q_{c,col}$ and $q_{c,soil}$ were found to be normally distributed. The evaluated δ_z and $\delta_{x,y}$ parameters from the regional average $q_{c,col}$ measurements were 40 cm and 4 m, respectively. The regional average δ_z evaluated from $q_{c,soil}$ was 0.2 cm.

The evaluated FS for the embankment was 1.63. The corresponding RBD analyses were carried out in two cases; first when the effect of spatial variability of the random variables is not considered (i.e. $\Gamma^2_{i,surf} = 1$), and second when $\Gamma^2_{i,surf}$ is evaluated along the failure surface and considered in the analyses. The evaluated β were 2.18 and 5.7 for the two cases respectively. This simple example shows that the spatial variability has a significant influence on the reliability of embankment. The deterministic design practice cannot address the impact of the statistical uncertainty and spatial variability, since FS did not change in both cases.

Some recommendations have been drawn up based on this study:

- The analysis suggests that reliability of the embankment can be significantly underestimated if the effect of the spatial variability is ignored.
- Current deterministic design method of LC columns cannot capture the significant influence of the variability on the reliability of the design. It is therefore recommended that an RBD analysis is performed in parallel with FS design methods.

Further research which needs to be conducted was discussed in the present study as follows:

- The current study only considered the resistance parameters as uncorrelated random variables. There is a need for reliability evaluation for the embankment by considering the other parameters, i.e. from the loads, in order to better understand their effect on the embankment's performance function.

- There is a need for a study to evaluate the correlation between $c_{u,col}$ and $c_{u,soil}$, since information about the correlation is not available.
- Further studies should consider the effect of the other sources of uncertainty (transformation and statistical uncertainties) on the reliability of embankments.

3.3 Paper III

Effect of uncertainties of improved soil shear strength on the reliability of embankments

Mohammed Salim Al-Naqshabandy and Stefan Larsson

Journal of Geotechnical and Geoenvironmental Engineering; posted ahead of print August 1, 2012. doi:10.1061/(ASCE) GT. 1943-5606.0000798

The work in this study is closely related to the previous study, where the effect of uncertainties associated with the evaluation of $c_{u,col}$ on the reliability of embankments is studied in a broader sense. In RBD analyses, five of the parameters that are involved in the mechanical system were considered random variables, i.e. $c_{u,col}$, $c_{u,soil}$, γ_{col}, γ_{soil} and γ_{emb}. Reliability analyses were performed according to levels 2 and 3, based on FORM and MC methods respectively. The analyses were performed on embankments of different heights, i.e. 4, 6 and 8 m. The variables were assumed to be uncorrelated and to follow log-normal PDFs. The analyses were performed on the most critical slip surface found by the deterministic FS method. The embankments were designed for different area ratio (A).

The results of the sensitivity analyses identified the most important parameters that affect the mechanical system. The main resistance parameter was found to be $c_{u,col}$ with $\alpha_{c_{u,col}}$ varying on average from 0.7 to 0.9 when $COV_{Total(\bar{c}_{u,col})}$ varied from 20% to 70%. The main load parameter was found to be γ_{emb} with $\alpha_{\gamma_{emb}}$ varying on average between 0.48 and 0.27 when $COV_{Total(\bar{c}_{u,col})}$ varied from 20% to 70%, respectively. The reliability of the tested embankments on the other hand improved significantly when the spatial variability was considered. The performance level improved from below average to good performance for all the embankments analysed according to the U.S. Army Corps of Engineers recommendations. The evaluated β, before and after spatial variability consideration, increased from 2.5 to 4.4, from 2.4 to 4.8 and from 2.2 to 4.2 at the designed A of 35% and 55% for the 4 m, 6 m and 8 m embankments, respectively. The effect of the spatial variability of the original soil on the performance function was found to be minor.

The main findings with regard to the effect of different sources of uncertainty on the reliability of embankments are listed below:

- The effect of the transformation uncertainty COV_{trs} on the ultimate limit state performance function was found to be very significant. It was found to be the most likely source of uncertainty that affects the reliability of embankments constructed on LC columns. The embankments' performance levels increased significantly from above average to high when COV_{trs} decreased from 0.4 to 0.2, corresponding to the increase in β from 3.8 to 5.6 respectively. Significant improvement in the reliability level can therefore be achieved by reducing COV_{trs}.
- The effect of the statistical uncertainty COV_{stat} on the reliability was also significant, where β increased from 3 to 4.5 when N increased from 2 to 60, respectively. However, it was found that a good performance level can be achieved with only N=10.
- The uncertainty due to measurement errors was found to have negligible influence on the reliability of embankments.

Some important issues are also highlighted based on the current study, however further research works are needed to address the following points:

- In addition to the spatial variability, COV_{trs} was found to be the major source of uncertainty that affects the reliability of the embankments. This source of uncertainty, however, has not been evaluated for LC columns. Further studies are therefore needed to determine the magnitude of COV_{trs}.
- It was also highlighted that the spatial variability should be considered and the use of reliability analyses alongside deterministic stability analyses is highly recommended for the design of the columns. This important aspect can be achieved practically by providing RBD in its simplest form (i.e. partial factor design).
- Information about the correlation between $c_{u,col}$ and $c_{u,soil}$ is unknown. However, this issue should not be ignored in the RBD analyses and their effect should be shown in future work.

3.4 Paper IV

Variability of strength and deformation properties in lime-cement columns evaluated from CPT and KPS measurements

Niclas Bergman, Mohammed Salim Al-Naqshabandy and Stefan Larsson

Georisk (2012), [Article under review]

The aim of the paper was to investigate the variability parameters of LC columns based on two test methods (i.e. CPT and KPS). The study also addressed the effect of the test type on the evaluation of the design value. Two different sites were tested. The first site was located in Kista, 10 km north of Stockholm, where 60 columns were selected randomly. 30 of the selected columns were tested by CPT and 30 by KPS. The second site was located in Lidingö, a small island to the east of Stockholm, where 12 columns were tested by CPT and KPS. A parametric study was conducted to study the effect of different sources of uncertainty, i.e. transformation uncertainty, spatial variability, measurement error and statistical uncertainty, on the design value evaluated from the test methods.

Results show that the COV from CPT data ranged from 18 to 59% with an average of 29%, while COV from KPS ranged from 19 to 47% with an average of 22%. The ranges of the evaluated δ_z from CPT and KPS were respectively 8 - 71 cm and 11 - 77 cm with an average of 30 cm and 40 cm. The results suggest that less variability can be achieved by using a larger probe, due to the local average along the bigger probe. Results of scale of fluctuation from both tests are also consistent with COV results, because the larger δ_z evaluated from KPS indicates smoother variability than the smaller δ_z evaluated from CPT which indicates rapid variability. However, Γ^2_{CPT} will be less than Γ^2_{KPS} for the same domain size, and thus the spatial variability evaluated from both tests is approximately the same. The type of test will therefore not have a significant influence on the evolution of the design value. A parametrical study conducted on the evaluation of the design value for the individual parameters shows the important influence of the spatial variability, the transformation uncertainty and the statistical uncertainty on the evaluation of the design values. However, the influence of these uncertainties on the whole system is still in question and needs to be studied further.

3.5 Paper V

Partial factor design for a highway embankment founded on lime-cement columns

Mohammed Salim Al-Naqshabandy and Stefan Larsson

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Rational estimation of soil properties is essential for reliable and safe design, since current design methods for stability of lime-cement columns are deterministic and the uncertainties are not incorporated in the design. In order to achieve a reliable design, the uncertainty should be incorporated in the design of LC columns directly. This paper addresses the need for application of partial factor design (PFD) for safety and reliability assessment of lime-cement columns. The study was carried out on an example highway embankment of 6 m in height. Eight of the parameters involved in the mechanical system were considered random variables. The sensitivity factors for the random variables were evaluated from the first order reliability method (FORM). Partial factors were evaluated for the random variables according to the approximate location of the design values. It was shown that the PFD method satisfies both the safety and the reliability requirements of the columns. Despite the high uncertainty associated with $c_{u,col}$, PFD can be applied for the design of LC columns in a simple manner. Three random variables, i.e. $c_{u,col}$, $c_{u,soil}$ and γ_{emb}, out of eight random variables which are considered in this study were found to have the greatest effect on the mechanical system. The emphasis in the study, however, was on performing more research that takes into account different cases in order to support the drawn conclusions.

3.6 Paper VI

Evaluation of partial factors for the undrained shear strength of lime-cement columns

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This paper proposes a PFD approach for the design of ground improvement with lime-cement columns supporting highway embankments. Stability of embankments is assessed by combining both deterministic and RBD formats. Partial factors ($\gamma_{c_{u,col}}$) are assessed for the undrained shear strength of the columns ($c_{u,col}$) based on the determination of the design value by using the first order reliability method (*FORM*). The effect of the spatial variability, statistical, measurement errors and transformation uncertainties associated with $c_{u,col}$, on the assessment of $\gamma_{c_{u,col}}$ is addressed and discussed. The study highlights the effect of the correlation between $c_{u,col}$ and $c_{u,soil}$. The concept of the design and the characteristic values of the variables were also discussed. Six load and resistance parameters are considered random variables (i.e. $c_{u,col}$, $c_{u,soil}$, γ_{col}, γ_{soil}, γ_{emb} and q).

The main focus of this study was to evaluate $\gamma_{c_{u,col}}$. In order to do so, the FORM method was used to analyse the reliability of the design of three embankments 4 m, 5 m and 6 m in height. The analyses were performed in different soil conditions, where $c_{u,soil}$ varied from 10 to 20 kPa. The spatial variability and the uncertainty associated with $\bar{c}_{u,col}$ were varied in the analyses. During the analyses, the target level of reliability was set to $\beta_t = 3.8$. Accordingly, and based on the degree of uncertainty, a number of values for $\gamma_{c_{u,col}}$ were determined. The evaluated $\gamma_{c_{u,col}}$ were applied in the PFD for the design of two other embankments, 2 m and 8 m in height respectively, based on only three most influential parameters, i.e. $c_{u,col}$, $c_{u,soil}$ and γ_{emb}. The evaluated $\gamma_{c_{u,col}}$ were calibrated in accordance with $\beta_t = 3.8$ and for the three influential parameters. Based on the variability and the uncertainty associated with $c_{u,col}$, and for a specific site condition, a number of $\gamma_{c_{u,col}}$ were suggested for the application of PFD in the design of LC columns.

Some conclusions can be drawn according to the current study as follows:

- The mechanical system of LC columns supporting highway embankments is a complex system, were eight random variables are included in the ULS function. This study demonstrates that only three of the parameters, $c_{u,col}$, $c_{u,soil}$ and γ_{emb}, have a significant effect on the ULS.

- Since lime-cement columns are installed in very soft soils, the reliability of the design and the evaluated $\gamma_{c_{u,col}}$ are influenced by the levels of inherent variability and uncertainty in $\overline{c}_{u,col}$. Accordingly, multiple values of $\gamma_{c_{u,col}}$ can be adopted in PFD to accommodate the uncertainty.
- The evaluated values for the influential parameters according to $\beta_t = 3.8$ were 2.5-3, 1.14 and 1.22 for $\gamma_{c_{u,col}}$, $\gamma_{c_{u,soil}}$ and $\gamma_{\gamma_{emb}}$ respectively.
- The analyses show that for relatively large slip surfaces, δ has practically no significant influence on the evaluation of $\gamma_{c_{u,col}}$.
- In order to perform a reliable design for high inherent variability ($COV_{q_c(col)}$ > 0.3), it is recommended that N > 10.
- The positive correlation, $\rho_{c_{u,col},c_{u,soil}}$ has a negative impact on the reliability. The reliability of the design can therefore be overestimated if the effect of positive correlation is ignored. However, the effect was found to be small for a weak correlation, $\rho_{c_{u,col},c_{u,soil}}$ <+0.25.

Chapter 4: Conclusions and Further research

4.1 Conclusions

In this study, statistical evaluation of the variability parameters of $c_{u,col}$ based on CPT and KPS data has been presented. One of the objectives was to study the effect of the inherent variability and the associated uncertainty on the reliability of embankments founded on lime-cement columns. Another was to develop a practical tool that can be used in the design of lime-cement columns and can consider the inherent variability and uncertainty. The study was performed on different embankment heights, and at different soils and columns conditions. The following conclusions have been drawn and suggestions made based on the findings of the current study:

- Inherent variability of $c_{u,col}$ is very high and should be taken into consideration in the design of lime-cement columns.
- The reliability of embankments at ULS increases significantly when the effect of spatial variability is considered in the RBD design procedure.
- The reliability of embankments is influenced by the number of test samples. However, it has been shown that when N is equal to or greater than 10 it has practically no significant effect on the evaluation of β. In order to perform a reliable design for high inherent variability (i.e. $COV_{q_c(col)}$ > 0.3), it is recommended that N should be at least 10.
- Positive correlation, $\rho_{c_{u,col},c_{u,soil}}$, has negative impact on the reliability of the embankments. Therefore, reliability of the design can be overestimated if the effect of positive $\rho_{c_{u,col},c_{u,soil}}$ is neglected. However, the effect of $\rho_{c_{u,col},c_{u,soil}}$ on the reliability is practically insignificant for $\rho_{c_{u,col},c_{u,soil}}$ <+0.25.
- In addition to the spatial variability, transformation uncertainty has a significant influence on the reliability of embankments and hence in the evaluation of partial factors $\gamma_{c_{u,col}}$.
- Three of the parameters included in the mechanical system where found to be the most influential, $c_{u,col}$, $c_{u,soil}$ and γ_{emb}. It is recommended that these parameters should be considered as random variables in RBD calculations.
- For the evaluation of the design value at the ULS the recommended average values of α are 0.8 and 0.6 for R and L parameters respectively.

- Results from the level 3 reliability method, i.e. MC, agreed well with that of level 2 method (FORM). RBD analysis by FORM is thus considered an effective and reliable design tool for the ULS design of lime-cement columns.
- Since some sources of uncertainty associated with $c_{u,col}$ are highly uncertain (e.g. transformation uncertainty), it is recommended that the RBD be used as a complement to the traditional deterministic design in order to take better account of the uncertainties.
- The proposed PFD method for the design of lime-cement columns is simple and compatible with both RBD and FS designs of embankments.

4.2 Further research

The conclusions drawn from the present study are based on a number of assumptions and limitations (see Chapter 1, section 1.3) and the results can therefore not be generalized for many other cases. As a consequence, there is a need for further research of this nature to address the following points:

- The results of this study were found to be affected by the degree of $\rho_{c_{u,col},c_{u,soil}}$. However, information about this parameter is not available yet and needs to be investigated.
- There is a need for further studies to assess transformation uncertainty associated with the evaluation of $c_{u,col}$ from CPT and KPS measurements.
- The proposed values of $\gamma_{c_{u,col}}$ were relatively high, due to the high variability and uncertainty associated with $c_{u,col}$. The proposed partial factors can be decreased when more information about $c_{u,col}$ from different test methods is involved in the design. This can be achieved for example by performing multivariate analyses as described by Ching et al. (2010).
- The results from this study were based on the assumption of the shear failure mode in the columns. Methodologies presented in this thesis can be used to evaluate the probability of failure (p_f) for embankments according to Bishop's method. However, it is essential to investigate the reliability for other failure modes that can occur.

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