VULNERABILITY ANALYSIS OF PUBLIC TRANSPORT NETWORKS: A DYNAMIC APPROACH AND CASE STUDY FOR STOCKHOLM

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ABSTRACT

In this paper, a dynamic and stochastic notion of public transport network vulnerability is develoepd. While previous studies have considered only the network topology, the non-continuous availability of services requires a more refined model for supply and demand interactions in order to evaluate the impacts of disruptions. We extend the measures of betweenness centrality (often used to identify potentially important links) and link importance to a dynamic-stochastic setting from the perspectives of both operators and passengers. We also formalize the value of real-time information (RTI) provision for reducing disruption impacts. The developed measures are applied in a case study for the high-frequency public transport network of Stockholm, Sweden. The importance ranking of the links varies depending on the RTI provision scheme. This suggests that RTI may have significant positive but also negative influence on disruption impacts, and that betweenness centrality (passenger/vehicle flows) may not be a good indicator of link importance.

1.   INTRODUCTION

1.1   Vulnerability analysis of transport systems

Public transport is a vital component of urban transport systems. In tackling the challenges of increasing congestion and negative environmental impacts, shifting trips from personal cars to public transport options is generally seen as one of the most important actions. For public transport to be an attractive option for travellers, the system needs to be efficient as well as robust. Efficiency means that travel should be fast, convenient, affordable and comfortable under normal operating conditions. Robustness means that the system should be able to withstand or quickly recover from disturbances such as infrastructural and vehicular malfunctions. In order to ensure that the system is robust, it is first necessary to analyse the system-wide impacts of potential disruption scenarios for travellers and operators. This enables the identification of problematic scenarios, for example expressed as a set of important network links where disruptions would be the most severe. When the scenarios have been identified, appropriate actions can be taken to reduce the problems and improve the robustness.

The research field concerned with the risk of severe transport network disruptions for the society is commonly called vulnerability analysis (Berdica, 2002). Until now, most work in transport network vulnerability analysis has focused on degradations of the physical infrastructure and major incidents, in particular for the road network and personal vehicle travel (Jenelius and Mattsson, 2012; Taylor and Susilawati, 2012). Both theoretical analysis and numerical applications have increased the understanding of how supply and demand together determine vulnerability, through the redundancy of the network and the travel patterns of the users.

Much less is known about the vulnerability of public transport networks, where services are superimposed on the roads or railways. Some studies have looked at how degradations of physical infrastructure links in a particular modal network affect connectivity and distances between stations (Angeloudis and Fisk, 2006; Criado *et al.*, 2007). Public transport network configuration plays a key role in determining the impacts of prospective service disruptions. Graph theory provides alternative measures of link importance that were applied on public transport networks worldwide (Scott et al. 2006; von Ferber *et al.*, 2009). They considered the number of immediate connections (node degree) and the betweenness centrality measure which corresponds to the share of shortest paths between nodes which go via a certain node. They concluded that network vulnerability in terms of the size of the largest connected subset is more sensitive to betweenness centrality than to node degree. The same conclusion was reached when vulnerability was defined in terms of the speed in which the system becomes fragmented (Colak *et al.*, 2010). Notwithstanding, public transport networks varied with respect to the impact of various attack scenarios (Berche *et al.*, 2009). It was suggested that robustness of metro systems corresponds to the number of cyclic paths available in the network (Derrible & Kennedy, 2010).While some general conclusions can be drawn, such analyses cannot capture many features of public transport networks that we believe are essential in order to describe their vulnerability properly.

The analysis of public transport network vulnerability considers disruptions that imply a substantial reduction in the capacity of network elements and hence their incapability to fulfil the purpose of the system. Disruptions of public transport networks need not be caused by degradations of the underlying physical infrastructure, but can also arise from degradations of the services, for example crew strike or limited infrastructure capacity (stops or tracks).

Public transport networks are characterized by greater complexity due to limited connectivity and the importance of transfers, multi-modality and the intermediate walking links. These network characteristics imply that public transport networks are made up of links that belong to distinguished sets. Public transport networks are less dense than road networks and are hence more vulnerable and are more dependent on few critical network elements. At the same time, the multimodality of public transport networks can potentially allow alternative modes to provide redundant capacity.

1.2   The dynamic approach towards public transport vulnerability

The aim of this paper is to develop a dynamic and stochastic notion of public transport network vulnerability. The non-continuous availability of public transport services requires a more refined model for emulating system supply. Supply dynamics imply that the travel costs associated with alternative paths is time-dependent and thereof adaptive passengers' path decision. The spatial distribution of network vulnerability may therefore vary over time due to variations in service capacity and reliability. Hence, the dynamics of both public transport supply and demand are modelled as well as their interactions in order to evaluate the impacts of disruptions. None of the evaluations carried out in previous studies took into consideration these underlying system dynamics.

A dynamic notion of public transport service disruption takes into account its accumulated effect on system performance. A service disruption implies that transit vehicles trains can neither progress along nor enter a disrupted network element. The disruption has a certain duration after which the system is expected to gradually recover back to normal conditions. These conditions could be defined in terms of the flow of supply (vehicles) or demand (passengers). Compared with the case of road networks, service disruption in the public transport network has wider direct implications. While service disruption is associated with the immediate effect on the disrupted network element, the dynamic nature of public transport supply results in escalating impacts on service availability and capacity further downstream. Depends on its duration, it may also impact service availability upstream and even on other lines due to its impact on vehicle scheduling.

The impacts of service disruptions depend on how the demand reacts to changes in supply. Although previous studies have not stated their behavioural assumptions, they all share the assumption that all passengers have perfect knowledge on system conditions and that they always choose the shortest path available. We relaxed these assumptions by adopting a more realistic behavioural representation. A probabilistic path choice process is used in order to model passenger decisions. The evaluation of alternative paths depends on passenger's preferences and perceptions. The latter is determined by prior-knowledge and traveller's access of real-time information on system conditions.

In this study, a dynamic public transport operations and assignment model is used as the evaluation tool. The model represents the interactions between traffic dynamics, public transport operations and traveller decisions. The different sources of public transport operations uncertainty including traffic conditions, vehicle capacities, dwell times, vehicle schedules and service disruptions are modelled explicitly. A dynamic path choice model considers each traveller as an adaptive decision maker. Travellers' progress in the public transport system consists of successive decisions based on anticipated downstream attributes. Factors such as timetables, transfers and walking distances are used for the predictions of passenger loads under various scenarios (Cats, 2011).

Advanced public transport systems (APTS) have the potential to improve system robustness. This includes real-time control and management strategies. The public transport system may also become more robust by informing passengers on downstream conditions. Previous studies demonstrated the effects of real-time information (RTI) on passenger decisions under service disruption scenarios. However, the analysis also highlighted that as passengers are more informed, passenger loads are subject to more fluctuation and therefore may counteract system robustness (Cats et al., 2011).

2.   METHODOLOGY

2.1   Public transport network definition

The physical public transport network is defined by a directed graph $G(S, E)$, where the node set S represents stops and rail stations, and the link set $E \subseteq V \times V$ represents direct connections between stops. The number of stops and links are denoted $|S|$ and $|E|$, respectively.

Each link $e \in E$ may be operated by one or several public transport lines. A line l is defined by a sequence of stops or stations $l = (s_{l,1}, s_{l,2}, \ldots, s_{l,|l|})$, where $o_l = s_{l,1}$ is the origin terminal and $d_l = s_{l,|l|}$ is the destination terminal. The set of all origin-destination (OD) terminals is denoted $S_T \subseteq S$; the set of lines between origin terminal $o \in S_T$ and destination terminal $d \in S_T$ is denoted L_{od} and the set of all lines is denoted L. We let $e \in l$ mean that link e is in line l, i.e., that $e = (s_{l,i}, s_{l,i+1})$ for some i. Thus, each link e is associated with a set of lines $L_e = \{l \in L | e \in l\}$ traversing the link, and each stop is associated with a corresponding set of lines $L_s = \{l \in L | s \in l\}$.

Each link e is associated with a *riding time*, which is the time from the departure from the upstream stop to the arrival at the subsequent downstream stop. The riding time may vary systematically between different lines and also between different trips and between days depending on the current traffic conditions. The riding time on link e at time-of-day t is denoted $RT_e(t)$. If day-to-day variability in traffic conditions is considered, $RT_e(t)$ may be considered a stochastic variable. Similarly, each stop is associated with a *dwell time*, which is the time required for a vehicle to stop for boarding and alighting. Like the riding times, the dwell times may vary between lines, trips and days, depending on the current number of passengers, vehicle type, etc. The dwell time at stop s for line l at time t is denoted $DT_{s,l}(t)$ and may be considered stochastic.

Each line l is operated with a set of vehicle trips according to a schedule. The departure time of trip k of line l from the origin terminal is in general a function of a scheduled departure time and the arrival time of the previous trip, which may be stochastic due to the stochastic riding and dwell times. The set of departures on line l during a certain time interval $(t, t + \tau)$ is denoted $K_l(t, \tau)$. The number of departures may be stochastic since it is an outcome of underlying stochastic variables.

Travel demand is connected to the network through a subset of origin-destination (OD) nodes, $S_{od} \subseteq S$. The set of travellers from origin $o \in S_{od}$ to downstream destination $d \in S_{od}$ during time interval $(t, t + \tau)$ is denoted $N_{od}(t, \tau)$; the set of travellers between all OD pairs is denoted $N(t, \tau)$. The demand is assumed here to be inelastic, that is, not affected by changes in travel times etc. However, the number of travellers may be stochastic to represent day-to-day variations.

Similarly to the vehicle lines, the physical path of a traveller is defined by a sequence of stops from the origin o to the destination, that is, $j = (s_{j,1}, s_{j,2}, \ldots, s_{j,|j|})$, where $o_j = s_{j,1}$ is the origin stop and $d_j = s_{j,|j|}$ is the destination stop. The set of all physical traveller paths between origin o and destination d is denoted J_{od}, the set of all paths is denoted J. In general, which physical path a given traveller chooses a given day and time-of-day will depend on the properties of the different public transport lines and on the conditions that day, according to the preferences of the individual. Considering the dynamics and the stochasticity of the system, the probability that traveller n uses physical path j is denoted $p_n(j)$. In Section 2.3 we present a dynamic public transport route choice model that is used in this paper to calculate $p_n(j)$.

2.2   Network centrality

The ability of a transport network to withstand degradations has clear connections to the structure of the network. It has long been recognized that central links, in the sense that many paths between pairs of nodes must cross those links, are often also critical with respect to degradations. This kind of network centrality is commonly referred to as *betweenness* centrality (Freeman *et al.*, 1991; Crucitti *et al.*, 2007). For a link e, the betweenness centrality is the fraction of shortest paths, where path length is measured in number of nodes, between all pairs of nodes in the network that contain the link; if there are multiple shortest paths between a pair of nodes, the fraction of those paths that contain e is calculated.

In our case, if $f_{s_1,s_2}(e)$ denotes the fraction of shortest paths between stop s_1 and stop s_2 that contain link e, the traditional betweenness centrality of link e is

$$BC(e) = \frac{1}{|S|(|S|-1)} \sum_{s_1 \in S} \sum_{s_2 \in S \setminus s_1} f_{s_1,s_2}(e) \tag{1}$$

This simple network measure has a number of limitations which may reduce its relevance for identifying central links in real-world public transport networks. First, it assumes that all node pairs are equally important for the centrality of a link. Second, the only relevant paths between a pair of links are the shortest paths in terms of the number of intermediate nodes; this implicitly defines the path choice model for the network. In the following we develop the betweenness centrality measure for public transport networks by taking into account some of the features highlighted in the introduction:

- Dynamic system – the demand and supply between nodes may change with time
- Probabilistic path choice – not all passengers choose same path between two nodes
- Stochastic system – there is inherent variability in demand and supply between days

Furthermore, different centrality measures may be relevant depending on the perspective from which the system is viewed. In particular, one may focus on the operations of the vehicle fleet, or on the passengers. Hence, we extend the traditional betweenness centrality measure with focuses on vehicles and passengers, respectively. In each case, we show that the extended betweenness measure corresponds to a measure of flow across the link.

2.2.1 Centrality for vehicles

From an operator's perspective, a network link should be considered central if a large number of lines and vehicle trips traverse the link. Therefore, it is relevant to weight each pair of stops with the number of lines and, for each line, the number of vehicle trips, between the stops. To avoid double-counting the same trip passing several stops along the path, only the origin terminal o_l and destination terminal d_l of each line l should be considered in the aggregation.

The fact that the line schedules may vary with time means that the betweenness centrality of a link may vary depending on which time interval is considered. Hence, the betweenness centrality measure becomes dynamic. Furthermore, stochastic vehicle departure times means that the number of vehicle departures during the time interval, $|K_l(t,\tau)|$, is stochastic. To obtain a deterministic betweenness centrality measure we consider the *expected* number of departures during the time interval, $E[|K_l(t,\tau)|]$. With $\delta(l,e)$ being equal to 1 if $e \in l$ and 0 otherwise, the betweenness centrality measure from the operator's perspective is

$$OBC(e|t,\tau) = \frac{\sum_{o \in S_T} \sum_{d \in S_T} \sum_{l \in L_{od}} E[|K_l(t,\tau)|]\delta(l,e)}{\sum_{o \in S_T} \sum_{d \in S_T} \sum_{l \in L_{od}} E[|K_l(t,\tau)|]} \tag{2}$$

We may avoid the summation across origin and destination terminals by simply aggregating the measure across all public transport lines L. An equivalent formulation is thus

$$OBC(e|t,\tau) = \frac{\sum_{l \in L} E[|K_l(t,\tau)|]\delta(l,e)}{\sum_{l \in L} E[|K_l(t,\tau)|]} \tag{3}$$

Note that the numerator is simply the total number of vehicle trips traversing link e during the time interval, while the denominator is the total number of vehicle trips in the system during the same period. By dividing both the numerator and the denominator by the interval duration τ, the measure can be expressed in terms of vehicle flows instead of number of vehicles.

2.2.2 Centrality for passengers

When focus is on the travellers, a network link may be considered central if a large number of passengers traverse the link. It is therefore relevant to weight each pair of stops with the number of travellers between the stops. To avoid double-counting the same traveller passing several stops along the path, only the origin and destination of each traveller should be considered in the aggregation. With $\delta(j, e)$ being equal to 1 if $e \in j$ and 0 otherwise, the betweenness centrality measure from the travellers' perspective is

$$PBC(e|t, \tau) = \frac{\sum_{o \in S_{od}} \sum_{d \in S_{od}} \sum_{j \in J_{od}} E\left[\sum_{n \in N_{od}(t,\tau)} p_n(j)\delta(j,e)\right]}{\sum_{o \in S_{od}} \sum_{d \in S_{od}} E[|N_{od}(t,\tau)|]} \tag{4}$$

Note that not only $N_{od}(t, \tau)$ but also $p_n(j)$ for the same individual may be stochastic, since the probability of choosing a certain path may depend on the traffic conditions at the particular day. Again, we may avoid the summation across origins and destinations by simply aggregating the measure across all paths J. An equivalent formulation is thus

$$PBC(e|t, \tau) = \frac{\sum_{j \in J} E\left[\sum_{n \in N(t,\tau)} p_n(j)\right]\delta(j,e)}{E[|N(t,\tau)|]} \tag{5}$$

Note that the numerator is simply the expected total number of travellers traversing link e during the time interval, while the denominator is the expected total number of travellers in the system during the same period. By dividing both the numerator and the denominator by the interval duration τ, the measure can be expressed in terms of traveller flows instead of number of vehicles.

2.3 Disruption scenarios and impacts

To evaluate any kind of change to a system, a general approach is to describe both the new state and the baseline, reference state in terms of *scenarios*, and to evaluate the difference in some system performance measures between the two states. The scenario definitions must include all relevant dimensions of the system that are different between the new state and the old state, but does not need to include any dimensions that are unchanged. The characterization of a scenario therefore differs depending on the focus of the analysis. In our case, we are interested in the impacts of network disruptions, which means that the scenarios should contain a representation of such events. Furthermore, some appropriate performance measures with which to evaluate the scenarios need to be defined.

Previous vulnerability studies of public transport networks have considered the system as static and deterministic (Angeloudis and Fisk, 2006; Berche *et al.*, 2009; Derrible and Kennedy, 2010). Also, very simple measures of network performance have been used. In particular, the scenarios considered have been complete removals of nodes or links from the network, and the performance of the system has been evaluated as the number of interconnected nodes in the largest network component, and the mean distance (in terms of number of intermediate nodes along the shortest path) between all node pairs in the network . Hence, the performance of other links is independent of the disruption, and only node pairs with the disrupted link in their shortest paths are affected by the disruption. The implication that the remaining network functions as normal, even the disconnected parts of disrupted lines, is highly unrealistic as a model for unplanned network disruptions.

In Section 2.1 above we have already adapted the analysis of link centrality to account for dynamic and stochastic demand and supply and probabilistic path choice. Here we will adapt the analysis of disruptions in the same direction. Like the centrality measures, different impact measures can be defined depending on the perspective of the analysis.

2.3.1 Impacts for vehicles

From a private operator's perspective, disruption impacts are intuitively linked to the *costs* associated with the disruption. The causes and sizes of the costs may depend on the operator's fleet management, contract etc. One important component may be the total travel time of the vehicles, since that affects fuel and labour costs and potentially on-time arrival performance penalties. Given that riding times and dwell times are stochastic, the operator's cost in a scenario is also stochastic. We assume here that the total operating cost $OC(\sigma|t,\tau)$ in scenario σ during some time interval $(t, t + \tau)$ can be expressed as the sum of the operating cost of each vehicle trip during the interval. To have a deterministic value for the evaluation we consider the expected operating cost, that is,

$$OC(\sigma|t,\tau) = E\left[\sum_{l \in L} \sum_{k \in K_l(t,\tau)} OC_k(\sigma)\right] \tag{6}$$

where $OC_k(\sigma)$ is the operating cost associated with vehicle trip k in scenario σ.

In a dynamic setting, an important aspect of a disruption is the *recovery time*, that is, the time from the beginning of the disruption at t_D until the system has recovered to operating normally again at some time $t_D + \tau_R$. The recovery time τ_R will be different for different scenarios and is determined by the dynamic interactions of supply and demand. Before the disruption and after the recovery, the expected operating cost is by definition the same in the disruption scenario and in the baseline scenario. To evaluate the impacts of a disruption scenario σ, it is therefore sufficient to compare the operating cost with that in the baseline scenario σ_0 during the recovery time. For simplicity, we write the operating cost during the recovery time as $OC(\sigma)$. The impact of disruption scenario σ from the operator's perspective is then

$$\Delta OC(\sigma) = OC(\sigma) - OC(\sigma_0) \tag{7}$$

2.3.2 Impacts for passengers

From the perspective of the travellers, evaluating the impacts of network disruptions involves comparing and summing the various aspects of the impacts for different travellers. The impacts must therefore be expressed in units such that interpersonal comparisons and summations are meaningful. For many reasons, not least in cost-benefit analyses of robustness-improving investments, it is desirable to express the disruption impacts in economic terms. This allows prevention, repair and restoration costs to be added and compared to other impacts such as late arrivals (Jenelius, 2010).

With these aims, it is reasonable to express the impacts for passengers in terms of changes in *welfare*, essentially the total utility of the passengers expressed in monetary terms. With $W_n(\sigma)$ denoting the welfare of passenger n in scenario σ, the total welfare during time interval $(t, t + \tau)$ in scenario σ is

$$W(\sigma|t,\tau) = E\left[\sum_{o \in S_{od}} \sum_{d \in S_{od}} \sum_{n \in N_{od}(t,\tau)} W_n(\sigma)\right] \tag{8}$$

Again for simplicity, we write the total welfare for all passengers during the recovery time as $W(\sigma)$. The impact of disruptions scenario σ from the passengers' perspective is thus the total change in welfare from the baseline scenario during the recovery time, i.e.,

$$\Delta W(\sigma) = W(\sigma) - W(\sigma_0) \tag{9}$$

In the case study of this paper, we use travel time as an indicator of welfare – the shorter the travel times, the higher the welfare. The impact of a disruption is thus evaluated by the change in total travel time, or average passenger travel time.

2.3.3 Element importance and the value of real-time information

In this paper we are primarily interested in two dimensions of the disruption scenarios: the network element that is disrupted, i.e., the set of disrupted links and nodes, and the type of real-time information provided to the passengers (different information provision schemes are described in more detail in Section 3). Other factors such as the start time and duration of the disruption are held constant in all scenarios. A disruption scenario σ involving network element d and real-time information scheme r can then be summarized as the pair $\sigma = (d, r)$. We let $d = 0$ denote a scenario with no disruption and $r = 0$ a scenario with no real-time information.

Following Nicholson & Du (1994) and Jenelius et al. (2006), the *importance* of a network element is defined as the impact of a disruption of the element. Many other terms have been used in different fields for the same concept, including "criticality" (Taylor and Susiwalati, 2012), "vitality" (Ratliff *et al.*, 1975; Ball *et al.*, 1989), "vulnerability" (Murray-Tuite and Mahmassani, 2004), "significance" (Sohn, 2006), "delta centrality" and "information centrality" (Latora and Marchiori, 2007). The main purpose behind the importance measure is to compare and rank different elements. This allows, for example, the identification of locations in the transport system where disruptions would be particularly severe. Disruptions of such elements represent worst-case scenarios and the elements can also be considered potential targets for antagonistic attacks on the system.

Identifying important elements means that targeted measures can be taken to reduce the risk (i.e., the probability and/or consequences) of disruptions in those locations. More generally, the importance of each element combined with the probability of the element being disrupted is useful when allocating resources to reduce the overall vulnerability of the society.

In the present framework, the importance of an element is evaluated conditional on a certain value for the other dimension of the scenario, i.e., a certain real-time information provision scheme. Given scheme r, the importance of network element d is then, from the operator's perspective,

$$OI(d|r) = OC(d, r) - OC(0, r) \quad \forall r, d \neq 0 \tag{10}$$

and from the passengers' perspective,

$$PI(d|r) = W(d, r) - W(0, r) \quad \forall r, d \neq 0 \tag{11}$$

One of the possible ways to reduce the impacts of network disruptions is to provide real-time information to the passengers. This provides information about the expected arrival times of different lines, and may be provided at various levels of completeness. Real-time information can allow passengers to choose alternative paths to their destinations, avoiding the lines that are negatively affected by the disruption. For a given disruption scenario d, the value of real-time information scheme r is

$$VRTI(r|d) = W(d, r) - W(d, 0) \quad \forall r \neq 0, d \tag{12}$$

2.4   Public transport system model

In this section we present a dynamic public transport model that can be used to evaluate the centrality measures and disruption impacts defined above. An implementation of the model is used in the case study described in Section 3.

2.4.1   Supply

The supply of the public transport system consists of network configuration and service availability with their respective attributes. The supply is represented in terms of individual vehicle runs, where each run has a corresponding timetable that is used for control (e.g. dispatching, holding at time

points) and the calculation of measures of performance (e.g. on-time performance). In addition, public transport vehicles follow a schedule that consists of a sequence of trips. It is important to model the chain of trips that a vehicle undertakes in order to capture the dependency between successive trips through the propagation of delays from trip to trip.

The departure time of trip k of line l from the origin terminal is calculated as the scheduled exit time, or the time the vehicle is available to depart after it completed its previous trip and some stochastic recovery time, if that occurs later. Public transport trip travel times then consist of two parts: riding times between departure time from stop s and arrival time at stop $s + 1$, denoted $RT_{s,l}^k$, and dwell times at stops, denoted $DT_{s,l}^k$. The exit time from stop s is thus

$$ET_{s,l}^k = \sum_{s'=s_1}^{s-1} RT_{s',l}^k + \sum_{s'=s_1}^{s} DT_{s',l}^k \tag{13}$$

Dwell times depend on the number of passengers boarding and alighting and are also stochastic. Riding times are composed of running times on links and delays at intersections. In this paper the effects of background traffic are modelled implicitly by representing link travel times as random variables with distributions that are derived from empirical travel times of public transport vehicles. Delays at intersections are determined by individual stochastic queue server that generates service times that follow a pre-defined distribution.

Supply and demand of public transport systems interact dynamically. The effects of demand on supply are primarily manifested through the dwell time. In addition, passenger decisions are influenced by how the public transport system evolves as manifested through the path choice process presented in the following section.

2.4.2   Demand

In this study, travel demand is represented as an OD matrix at the stop level. Thus, trips are initiated at an origin stop and passengers have to choose their path to a pre-defined destination stop. The level of representation models individual passengers that undertake successive decisions and makes it possible to study the interaction of passenger decisions and public transport performance.

Each traveller undertakes successive path choice decisions that are triggered by the evolving public transport system conditions. The evaluation of alternative actions depends on travellers' preferences and expectations. The latter are determined by prior knowledge, experience (e.g. elapsed waiting time) and the availability of real-time information. It is assumed that in the context of high-frequency urban public transport systems, travellers have a prior knowledge of network topology, timetable travel times and planned headways. The information that is available to a traveller when making a certain decision is determined by the dissemination means and their locations, and by individual characteristics. Travellers' ability to carry out a decision is also subject to vehicle capacity constraints.

A path alternative $a_i^{od} \in A^{od}$ is a member of the path set for origin o to a destination d and is defined by an ordered set of stops ($S_{a_i^{od}} \subseteq S$), lines ($L_{a_i^{od}} \subseteq L$) and connection links ($C_{a_i^{od}} \subseteq C$). Connection links are access, egress and transfer links that can be carried on by various non-public transport travel modes (e.g. walking, cycling, park and ride). S, L and C are the sets of all the stops, lines and connection links in the network, respectively.

A graphical representation of the path definition consists of nodes that correspond to stops and origin and destination locations, while links are either segments of public transport lines or connection links. In order to maintain a uniform and standard path definition, an artificial looping connection link is introduced for each node. The artificial connection links reflect the alternative of staying at the stop, have a null travel time and hence have no influence on the actual decision process. Note that each element in the path alternative is a set. This definition enables to group several public transport lines

that provide an equivalent connection between a pair of public transport stops or several public transport stops which are connected by the same public transport lines. Hence, alternatives that imply that passengers are indifferent towards them are defined as a single path.

The dynamic path choice model includes three decision models: *connection*, *boarding* and *alighting*. A connection decision takes place when the traveller chooses at which public transport stop to initiate the trip, and also each time the traveller alights from a public transport vehicle. A boarding decision is made for each arriving public transport vehicle when the traveller waits at a stop. Once on-board a vehicle, a traveller makes an alighting decision immediately upon boarding and may reconsider this decision in light of new information.

Making a decision corresponds to restricting the initial path set to a subset of paths that are still feasible for the remaining trip. Let us consider the general decision case where individual decision maker n is at certain location o with path set A^{od}. The individual has to choose an action c from the set of alternative actions C. The path set associated with action c is denoted $A^c \subset A^{od}$.

The utility of passenger n associated with path i is denoted $v_{i,n}$. The utility attached to action c is then given by the logsum over the path set A^c,

$$v_{c,n} = ln \sum_{i \in A^c} e^{v_{i,n}} \tag{14}$$

In the context of the proposed path choice model, the logsum term expresses the utility of an action as a function of the utilities of the respective path alternatives. Hence, it reflects the joint utility for a bundle of alternatives. Note that since A^{od} is divided into mutually exclusive and collectively exhaustive subsets for the respective actions, each alternative path in A^{od} is taken into consideration exactly once in the choice probability.

All traveller decisions are represented with multinomial logit models. Hence, the probability to choose action c is

$$p_n(c) = \frac{e^{v_{c,n}}}{\sum_{c \in C} e^{v_{c,n}}} \tag{15}$$

The probability that individual n follows a certain physical network path alternative j, $p_n(j)$, can be formulated as the joint conditional probability of intermediate decisions $c_1, c_2, \ldots, c_m$ that lead to the composition of this specific path,

$$p_n(j) = p_n(c_1) \cdot p_n(c_2|c_1) \cdot p_n(c_3|c_1, c_2) \cdot \ldots \cdot p_n(c_m|c_1, \ldots, c_{m-1}) \tag{16}$$

3.   APPLICATION

3.1   Network description

The supply and demand representations presented in Section 2.4 were implemented as a dynamic transit operations and assignment model called BusMezzo (Cats, 2011). The framework and details of the supply representation in BusMezzo are described in Toledo et al. (2010). The model was applied to the Stockholm inner-city rapid transit system. The system consists of the seven Metro lines, four high-demand trunk bus lines and one light rail train line. The complete network of these lines was coded in BusMezzo with the real-world timetables, vehicle schedules and walking distances between platforms and stops. The network is shown in Figure 1. During the morning peak-period (6:00-9:00) there are approximately 700 service trips carried out by over 200 vehicles. Since the peak-period headway of each of the transit lines is shorter than 8 minutes, all travellers are assumed to arrive randomly at stops and have prior knowledge of planned headways and timetable in-vehicle times. The three different

transit modes have different vehicle types, operating speeds, travel time variability, dwell time functions and are operated with different holding control strategies.
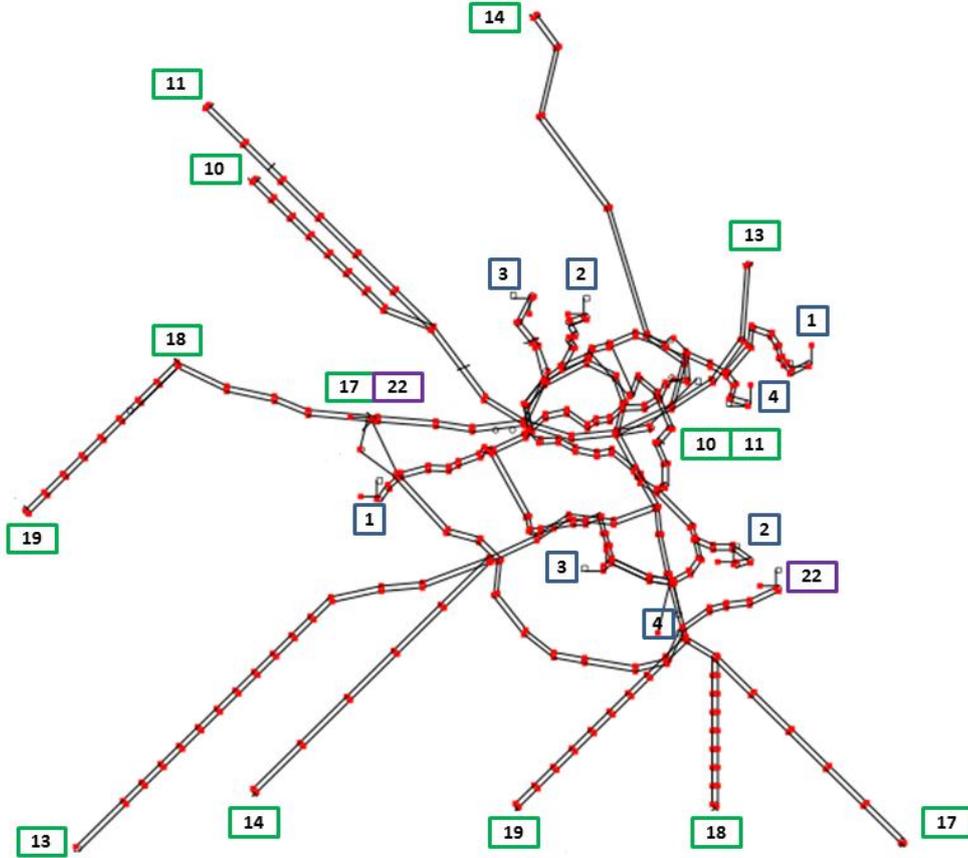

Figure 1: Stockholm's inner city rapid network as displayed by BusMezzo.

To allow a warming up period for the transit supply, passenger demand was simulated only for the peak hour (7:00-8:00). Approximately 125,000 passenger trips are initiated during this hour. The passenger demand was extracted from data collected at entrance barriers at Metro stations, passenger counts at transfer locations and LRT stations (SL, 2009) and automatic passenger counts on trunk buses. The passenger stop-to-stop OD matrix was obtained by applying a proportional trip distribution procedure.

A rule-based choice-set generation algorithm was used as a pre-process step to the simulation runs. It resulted in 99,270 alternative paths for the entire network. This master-set was used in the construction of action-dependent choice-sets throughout the simulation. The parameters of the choice-set generation model and the dynamic path choice model were estimated based on a stated-preferences survey on transit route choice decisions (Cats, 2011). The utility associated with path $i \in A^c$ is defined as

$$v_{i,n} = \beta_i^{wait} t_{i,n}^{wait}(t) + \beta_i^{ivt} t_{i,n}^{ivt}(t) + \beta_i^{walk} t_{i,n}^{walk} + \beta_i^{trans} trans_i , \qquad (17)$$

where $t_{i,n}^{wait}(t)$ and $t_{i,n}^{ivt}(t)$ are the time-dependent expected waiting time and in-vehicle time, respectively. $t_{i,n}^{walk}$ is the expected walking time and $trans_i$ is the number of transfers involved with the path alternative. β_i are the corresponding coefficients. The average values assigned to the coefficients were derived from the stated-preferences survey. Each individual is assigned with coefficients sampled from a normal distribution to account for the heterogeneity of preferences in the population. The trip fare is fixed for the entire network and hence does not affect passenger path decisions.

3.2 Applying the centrality betweenness measures for the Stockholm rapid transit network

The case study considered the case of normal operations and disruptions on critical links. The passenger betweenness centrality (PBC) measure (5) was used for identifying the candidate critical links. The base case scenario of normal operations and information conditions for the Stockholm network was simulated to allow the calculation of the PBC measure across the network. Table 1 presents the five network segments with the highest ranking which were selected for further analysis. The average passenger load during the rush hour is equivalent to the numerator of the PBC measure for t=7:00AM and τ equals to an hour. Similarly, the average number of vehicle departures per corresponds to the nominator of the operator's betweenness centrality (OBC) measure (3). Note that the OBC ranking differs from the PBC ranking as supply is not perfectly adjusted to passenger flows. Hence, the selected disruption scenarios do not reflect the candidate critical links for the public transport rolling stock. All of these segments are in the core of the network where there are rapid transit alternatives. Furthermore, all of them are metro segments. Figure 2 illustrates the main network and allows identifying the five segments as well as the availability of the number and complexity of alternative paths. The two busiest segments are the two metro lines that enter the inner-city from the south. This reflects the distribution patterns of population and employment in the Stockholm area.

Table 1: Network segments selected for vulnerability analysis.

PBC Ranking	Metro line	Direction	Segment	Average passenger load during the rush hour $PBC(e\|t,\tau)$ $\cdot E[\|N(t,\tau)\|]$	Average number of vehicle departures during the rush hour $OBC(e\|t,\tau)\cdot$ $\sum_{l\in L}E[\|K_l(t,\tau)\|]$
1	Green (17,18,19)	Northbound	Gullmarsplan – Hötorget	27,186	36.1
2	Red (13,14)	Northbound	Liljeholmen – Centralen	18,363	24.0
3	Green (17,18,19)	Southbound	Alvik – Centralen	14,785	32.8
4	Blue (10,11)	Southbound	Fridhemsplan – Centralen	11,510	16.9
5	Red (13,14)	Southbound	Centralen – Hornstull	10,508	24.0

3.3 Scenarios design

The case study considers a short-horizon and unplanned disruption. For each of above five segments, we simulated a disruption that takes place between 7:15 and 7:45. Since the disruption is short-horizon and unplanned, the system operators cannot employ any special measures to mitigate the impacts of the disruption such as providing a replacement service or operating the remaining disconnected parts of the line as independent lines. Note that segment closure on a certain line does not imply disruptions on other lines since each metro line has a distinguished infrastructure – tracks and platforms – and fleet.
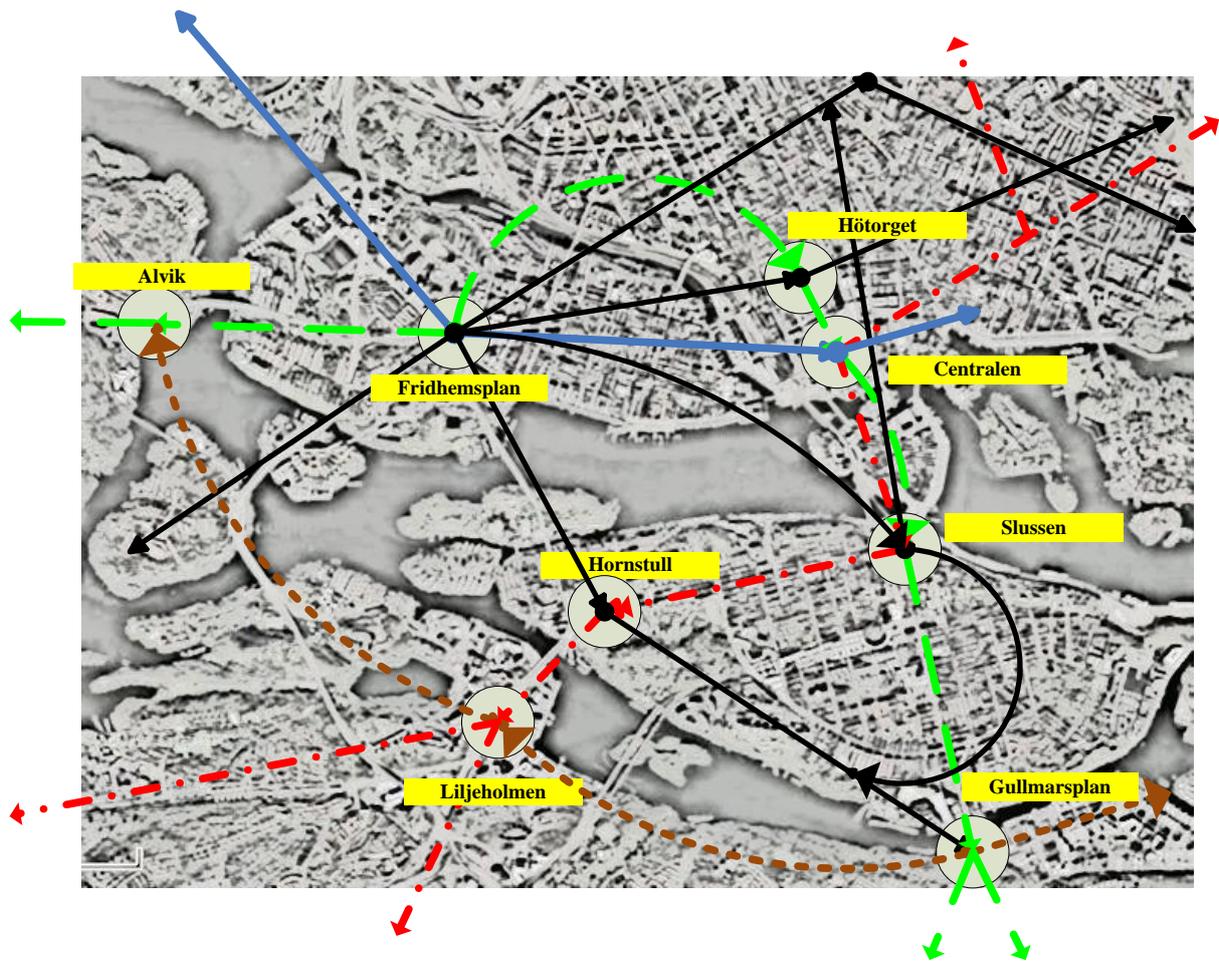
Figure 2: A schematic representation of the network core and stations of interest (The three metro corridors: Green, Blue and Red lines, the orbital light rail train in brown and the 4 trunk bus lines are in black).

The provision of real-time information (RTI) could mitigate the impacts of disruptions by allowing passengers to make more informed decisions and choose alternative paths. The case study considers passenger information systems with RTI regarding the next vehicle arrival time at various levels of coverage and comprehensiveness. The simulation of traffic dynamics and transit operations in BusMezzo emulates transit performance and the production of automated data collection (ADC) methods, such as automatic vehicle location (AVL) and automatic passenger counts (APC). This data can be processed in order to generate predictions on future transit conditions that will be disseminated to travellers. The prediction scheme used by BusMezzo is designed to replicate the method that is commonly used by transit agencies for generating real-time arrival information. The dissemination of RTI may influence travellers' decisions and ultimately passenger flows. The following RTI provision scenarios were simulated:

- No-RTI: Passengers have no access to RTI, all travel decisions rely on prior knowledge
- Stop-RTI: RTI is available at stops and rail stations regarding all transit services departing from the a specific rail platform or bus stop
- Cluster-RTI : RTI is available at stops and rail stations regarding all transit services departing from all platforms and bus stops within a single station/hub or a walking distance of up to 500 meters
- Network-RTI: RTI is available regarding all transit services in the network to all individuals through personal mobile devices

The experimental design consists of 6 network operational conditions: normal operations (D0) and D1-D5 disrupted segments corresponding to their PBC ranking), and 4 levels of RTI provision: No-RTI, Stop-RTI, Cluster-RTI and Network-RTI. The combinations of operational conditions and RTI provision defined 24 scenarios. For each scenario, 10 simulation runs were conducted for a three hours period with a uniform passenger demand during the peak-hour. This number of replications yielded a maximum allowable error of less than 5% for the average passenger travel time. The execution time for a single run was less than 1 minute on a standard PC.

3.4   Results

Table 2 presents the average passenger travel time under each combination of operational and information provision conditions. Under normal operational conditions, the average travel time in the case study network is 24 minutes which consists of waiting times, in-vehicle times and walking times at transfer locations. As expected, service disruptions result in considerable longer travel times. Note that $PI(d|r)$ is the average change and the impact varies substantially for different origin-destination pairs. An increase of 1% induces 500 additional passenger-hours during the rush hour only.

The availability of real-time information is utilized by passengers for making travel decisions that involve higher utility. For the non-disrupted network the provision of RTI leads to a reduction of 2% in the total passenger travel times. The last column in Table 2 presents the share of disruption impact that is mitigated by the RTI provision where a ratio of -1 indicates that all of that information provision compensates for the entire impact. The mitigation effect of RTI provision varies considerably across disruptions as it is constrained by the availability of viable alternatives paths for various destinations. The general trend is that more comprehensive RTI provision results in shorter travel times. However, the marginal gain from extending RTI provision can have negligible (D0), harmful (D2) or extremely positive (D5) impacts. The topology of Stockholm's transit network plays an important role in determining the opportunities to gain from more informed downstream connection decisions.

Table 2: Passenger travel time and relative changes under the different scenarios.

Operational scenario	RTI provision scenario	$W(d,r)$ [sec]	$PI(d\|r)/$ $W(0,r)$ [%]	$VRTI(r\|d)/$ $W(d,0)$ [%]	$\dfrac{VRTI(r\|d) \cdot PI(d\|r)}{W(d,0) \cdot W(d,0)}$ [%]
D0	No-RTI	1452	---	---	---
	Stop-RTI	1427	---	-1.74	---
	Cluster-RTI	1424	---	-1.92	---
	Network-RTI	1420	---	-2.22	---
D1	No-RTI	1607	10.69	---	---
	Stop-RTI	1619	13.52	0.77	5.70
	Cluster-RTI	1584	11.23	-1.45	-12.91
	Network-RTI	1576	11.00	-1.95	-17.75
D2	No-RTI	1467	1.03	---	---
	Stop-RTI	1483	3.98	1.13	28.45
	Cluster-RTI	1503	5.56	2.48	44.57
	Network-RTI	1477	4.03	0.68	16.91
D3	No-RTI	1561	7.49	---	---
	Stop-RTI	1507	8.30	-1.00	-12.08
	Cluster-RTI	1498	5.18	-4.03	-77.85
	Network-RTI	1545	6.16	-3.44	-55.84
D4	No-RTI	1558	7.35	---	---
	Stop-RTI	1520	6.52	-2.50	-38.26
	Cluster-RTI	1525	7.10	-2.15	-30.31

	Network-RTI	1503	5.90	-3.54	-60.04
D5	No-RTI	1482	2.08	---	---
	Stop-RTI	1480	3.73	-0.15	-3.97
	Cluster-RTI	1453	2.03	-1.97	-96.82
	Network-RTI	1456	2.57	-1.75	-67.89

Figure 3 presents how the average number of public transport legs per trip varies under various scenarios. Upon disruption, passengers tend to deviate to alternative paths which involve higher complexity. This is especially pronounced in the case of scenario D1 as it forces a large number of passengers that travel from the south to the city center to take either the orbital trunk bus or the orbital light rail train from Gullmarsplan and then change to a radial line (see Figure 2). Interestingly, the number of boardings per trip tends to decrease with greater availability of RTI provision. In the undisrupted case (D0), this decrease is obtained already when RTI is provided at the stop level, suggesting that uninformed travellers make more unnecessary transfers instead of waiting longer for a direct service.
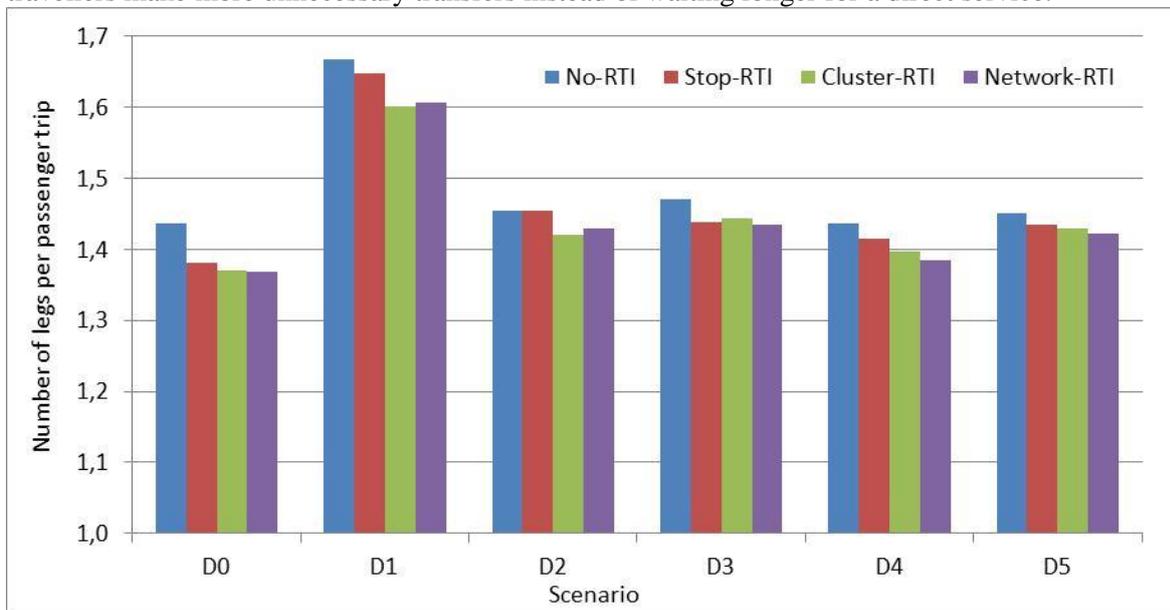

Figure 3: Number of public transport legs per passenger trip under various disruptions and information provision.

The vulnerability analysis consists of aggregate system measures which arise from the detailed dynamic representation of system supply and demand. In order to illustrate the importance of system dynamics, let us consider the temporal variations in passenger load for trunk line 1. Each line in figure 4 corresponds to a rush-hour vehicle-trip for the case of no disruption (D0) and disruption on the southbound direction of the Blue line (D4) with the highest level of information provision (RTI-Network). It is evident that even for the case of normal operations there are substantial temporal variations as a result of system dynamics and the interaction between inherent sources of supply and demand variability. This temporal variation is distorted significantly in the case of a disruption on the southbound direction of the blue line (D4) also shown in Figure 4. The Green line is the primary alternative for passengers coming with the Blue Line and heading towards the city center with trunk line 1 being a secondary alternative (see Figure 2). The vehicle capacity constraint is reached for a couple of the trips and the load is relieved at Hötorget which lies within the city center. As the system recovers from the disruption, passenger loads gradually resemble previous patterns and levels. This suggests that it is important to capture how the system evolves over time when analyzing the impacts of disruptions on redundant capacity and mitigation strategies.
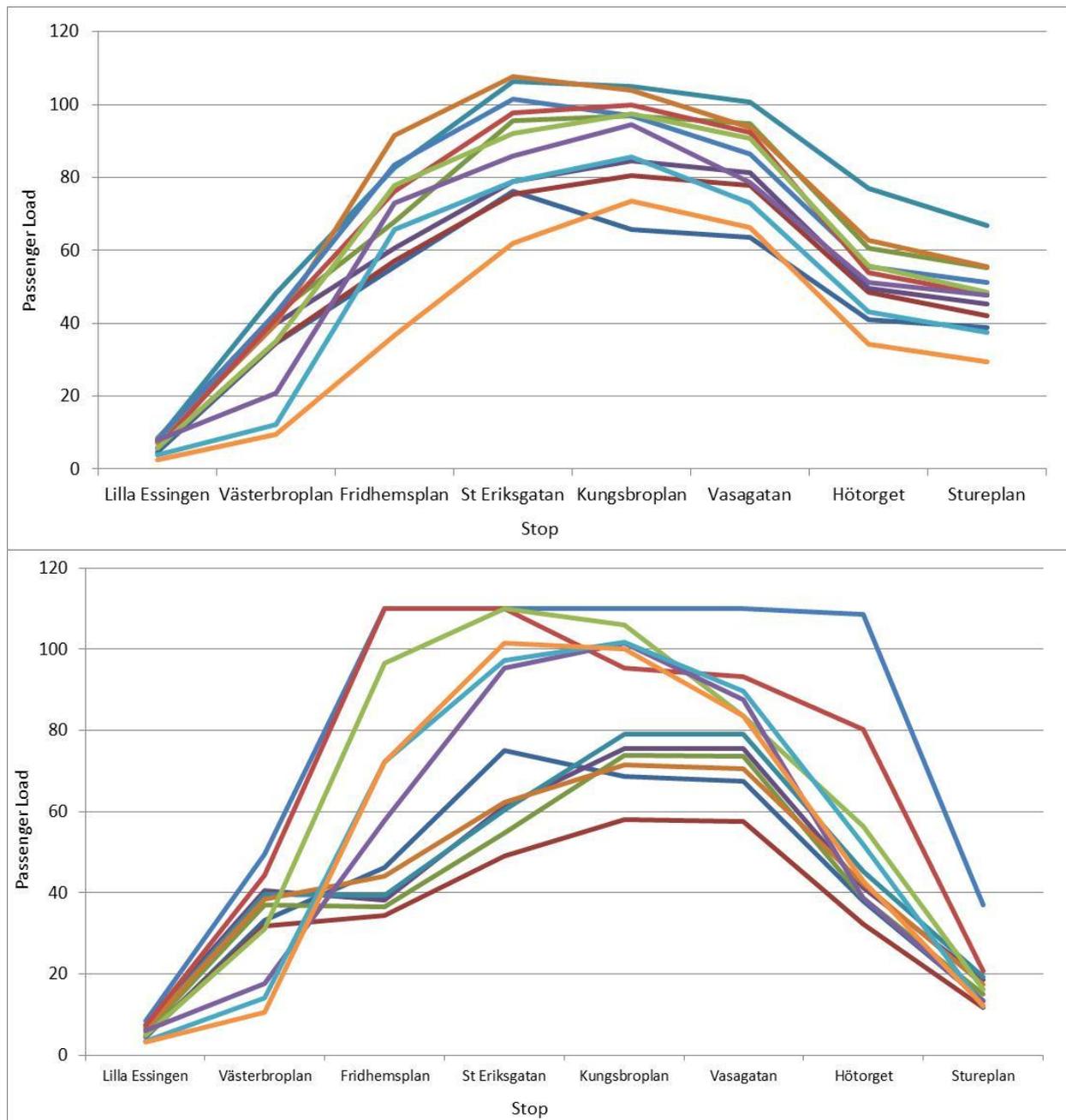
Figure 4: Passenger loads at key stops along trunk line 1eastbound during the rush hour for D0 (above) and D4 (below) scenarios, both are for RTI-network provision. Each line corresponds to a rush-hour vehicle-trip.

The five disruption scenarios were selected based on scanning the network for the segments with the highest PBC measure. However, the ranking of the disruption scenarios by the additional travel time they induce does not exercise the same order. D1 has the highest $PI(d|r)$ regardless of the RTI provision scenario. Hence, $PI(D1|r)$ is used as the reference value. Figure 5 presents the relative change in $PI(d|r)/W(0, r)$ for each scenario where $PI(D1|r)$ corresponds to 100%. In addition, a normalized PBC measure for each disruption scenario is shown where the value of D1 is again used as a benchmark, as well as the corresponding measure based on the link with the maximum load for each of these segments. In line with the PBC index, D1 is clearly the most harmful disruption. In contrast, D2 was ranked second in terms of PBC but results in less significant impacts than segments that were ranked lower. In fact, it ranked lowest among the five disruption scenarios in the case that no RTI is available. Hence, link centrality does not necessarily imply link importance as measured by its impact on total travel costs. Interestingly, the ranking also depends on the availability of RTI. For example, the system

is more vulnerable to D3 than to D4 if RTI is provided at the local stop level, while this is reversed if passengers have access also to RTI regarding nearby stops. This suggests that our assumptions about passenger knowledge may have important implications on which links are to be considered critical for system performance.
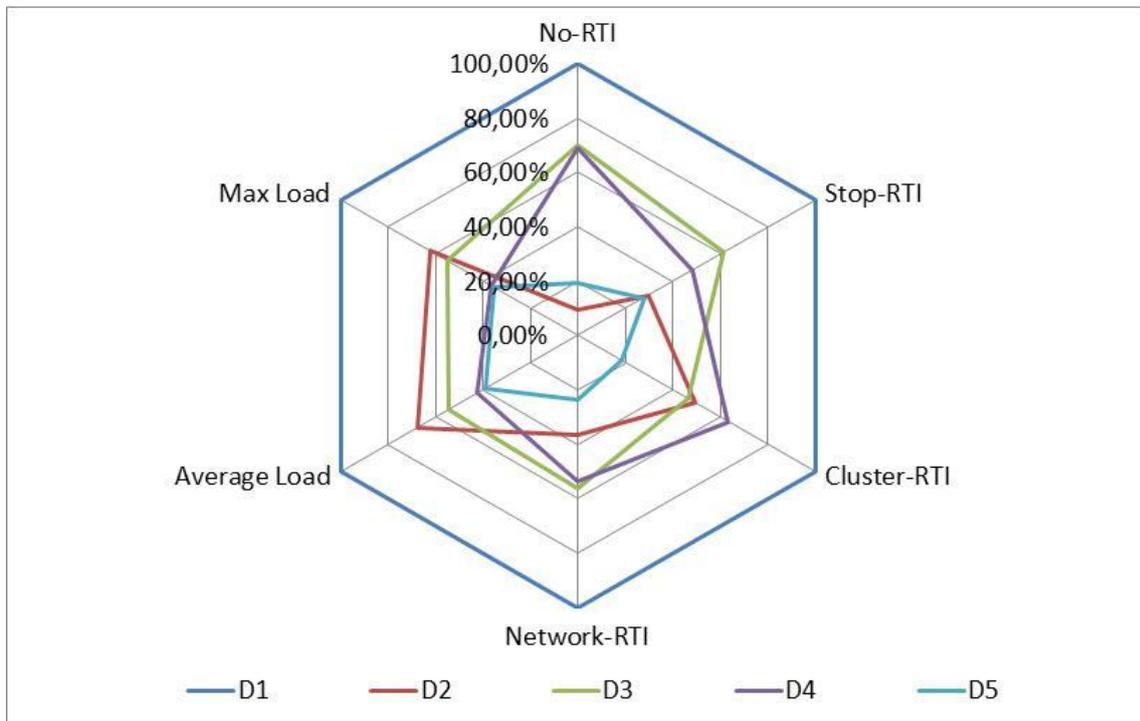

Figure 5: Comparison of the relative impact of disruption scenarios on total passenger travel time.

4. CONCLUSIONS

The performance of public transport networks is determined by the interaction between supply and demand dynamics. This interaction is manifested in the propagation of supply deterioration and passenger path choice strategies in the case of service disruptions. Hence, the vulnerability analysis of public transport networks has to consider system dynamics. In this paper, a dynamic and stochastic notion of network vulnerability was presented. This involves time-dependent service availability and passenger demand, the inherent stochastic processes in the public transport system and the accumulated effect of service disruptions. Moreover, the impacts of service disruptions depend on how the demand reacts to changes in supply. Hence, the underlying assumptions on passengers' knowledge and path choice strategies have great implications on the vulnerability analysis. A probabilistic and adaptive path choice model was integrated into the proposed methodology. Network centrality, impact measures and the importance of network elements were defined in terms of rolling stock and passenger flows. Moreover, the proposed methodology facilitates the assessment of the value of real-time information for mitigating the impacts of service disruptions.

The dynamic approach for network vulnerability analysis was applied to the rapid public transport system in Stockholm, Sweden. Candidate important links were identified based on the centrality measure obtained from the performance under normal operational conditions. The results of the vulnerability analysis suggest that link centrality does not necessarily imply link importance as measured by its impact on total travel costs. In addition, link importance depends on the level of information that is available to passengers as the value of real-time information provision varies for different disruption scenarios.

The vulnerability analysis of public transport networks will facilitate the integration of measures to improve system robustness into its planning, operations and real-time management. The identification of a critical links can provide guidelines for infrastructure investment decisions, including the design of transfer facilities. An analysis of alternative networks will allow the formulation of criteria for robust system planning. Timetable and vehicle scheduling design which considers risk distribution may reduce the likelihood of disruption, while better management and restoration strategies will reduce its impact. Real-time information can help mitigating the impacts of disruptions not only by its dissemination to passengers but also by enabling more proactive fleet management strategies.

The dynamic approach for network vulnerability allows investigating how the system restores under a longer time period. This will require the definition of measures to identify recovery patterns to pre-disruption performance. Future studies may consider different kinds of disruptions such as node closure and random network failures. It is also interesting to further analyse the spatial distribution of the impacts of service disruption for example by constructing accessibility measures.

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