KTH Electrical Engineering

Linear Precoding in MISO Cognitive Channels with Degraded Message Sets

©2012 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

JING LV, EDUARD A. JORSWIECK, RICARDO BLASCO-SERRANO,
RAGNAR THOBABEN, AND ADRIAN KLIKS

Stockholm 2012

Communication Theory Department
School of Electrical Engineering
KTH Royal Institute of Technology

Linear Precoding in MISO Cognitive Channels with Degraded Message Sets

Jing Lv*, Eduard A. Jorswieck*, Ricardo Blasco-Serrano†, Ragnar Thobaben†, and Adrian Kliks‡

*Communications Theory, Communications Laboratory, Dresden University of Technology, Dresden, Germany

E-mail: {jing.lv, eduard.jorswieck}@tu-dresden.de

†School of Electrical Engineering and ACCESS Linnaeus Centre, KTH Royal Institute of Technology, Stockholm, Sweden

E-mail: {ricardo.blasco, ragnar.thobaben}@ee.kth.se

‡Chair of Wireless Communications, Poznan University of Technology, Poznan, Poland

E-mail: akliks@et.put.poznan.pl

Abstract—In this work, the coexistence of a single-input single-output (SISO) primary link and a multiple-input single-output (MISO) secondary link is considered, where the secondary transmitter has non-causal knowledge of primary message and transmits both primary and secondary messages. The optimal beamforming vectors and power allocation at the secondary transmitter are derived to maximize the achievable secondary rate while satisfying the primary rate requirement. Moreover, the optimal linear precoding is obtained by semidefinite relaxation and rank-one decomposition, when the number of antennas at the secondary transmitter is larger than two. Finally, the performance of the proposed scheme is evaluated through numerical simulations.

I. INTRODUCTION

Consider the coexistence of primary and secondary links in the context of an overlay cognitive radio (CR) system. Assuming that the secondary transmitter has access to the message sent by the primary transmitter in a non-causal fashion, this channel model is known as the cognitive radio channel or the interference channel (IFC) with degraded message sets [1]. The additional knowledge allows for a form of asymmetric cooperation between the two transmitters. For example, one possible strategy is to have the secondary transmitter employ part of its resources to help the communication between the primary users, so that their communication is not disturbed or is even improved (e.g. in terms of rates). The remaining part of the resources are used for private communication to the secondary receiver. With accurate channel state information, the secondary transmitter can use its knowledge of the interference experienced by its receiver to pre-cancel it, for example using dirty paper (DPC) coding [2]. This strategy was shown to achieve capacity in the weak interference regime in [3] and [4]. In contrast, the capacity of part of the strong interference regime was obtained in [5] using superposition coding and interference decoding. Moreover, multiple antennas can be deployed at the secondary transmitter to enhance the desired signal and/or guide away the interference at the primary receiver [6].

The tolerable quality of service (QoS) degradation of the primary link in the underlay CR scenario is usually characterized by interference temperature constraint (ITC) [7] [8]. The coexistence of a single-input single-output (SISO) primary link and a multiple-input single-output (MISO) secondary link is considered in [9], where the achievable secondary rate is optimized under the primary rate constraint, assuming the primary link does not fully load its rate to Shannon capacity with equality, but there is room that can be exploited by spatial shaping (beamforming) at the secondary transmitter. Moreover, when rate splitting and successive decoding is viable for the secondary link, higher rate can be achieved compared to single-user decoding at the secondary receiver. With the same antenna configuration and by exploiting the knowledge of the primary message at the secondary transmitter, an achievable rate region is characterized in [10] based on previous results for the Gaussian MISO broadcast channel [11], by combining selfless relaying to maintain the rate supported by the primary link with DPC to pre-cancel the interference experienced by secondary receiver due to primary message. The optimal beamforming vectors and power allocation between the two operating modes at the secondary transmitter are derived to maximize the achievable secondary rate while satisfying the primary rate requirement. The associated rate gains over non-cooperative transmission could serve as a motivation for having the primary transmitter share its codebook and message with the secondary transmitter [12].

In this work, we resort to linear precoding at the secondary transmitter given the high complexity of DPC, and formulate the corresponding optimization problem, where the optimal beamforming vectors and power allocation are derived. Moreover, we solve the optimization problem by semidefinite relaxation and rank-one decomposition [13], when the number of antennas at the secondary transmitter is larger than two. Finally, we evaluate the performance of the proposed scheme through numerical simulations.

II. NOTATION

Column vectors and matrices are represented in lowercase and uppercase boldface letters, as $\mathbf{a}$ and $\mathbf{A}$ respectively. $\mathbb{C}$ and $\mathbb{C}^N$ represent the set of complex numbers and the set

Part of this work has been performed in the framework of Network of Excellence ACROPOLIS, which is partly funded by the European Union under its FP7 ICT Objective 1.1 - The Network of the Future.

of N -dimensional vectors of complex numbers, respectively. $|\cdot|$ is the absolute value of a scalar. $\|\cdot\|$ and $(\cdot)^H$ are respectively the Frobenius norm and Hermitian transpose of a vector or matrix. $\text{Re}(\cdot)$ stands for real part of a complex number. $\text{tr}(\cdot)$ and $\text{rank}(\cdot)$ respectively stand for trace and rank of a matrix. $\mathbf{A} \succeq \mathbf{0}$ means $\mathbf{A}$ is positive semidefinite. $\Pi_{\mathbf{a}} \triangleq \mathbf{a}(\mathbf{a}^H \mathbf{a})^{-1} \mathbf{a}^H$ denotes the orthogonal projection onto the column space of $\mathbf{a}$, and $\Pi_{\mathbf{a}}^\perp \triangleq \mathbf{I} - \Pi_{\mathbf{a}}$, where $\mathbf{I}$ is the identity matrix, denotes the orthogonal projection onto the orthogonal complement of the column space of $\mathbf{a}$.

III. SYSTEM MODEL

The system considered is depicted in Figure 1 and consists of a SISO primary link and a MISO secondary link with N antennas at the transmitter. The primary transmitter conveys a message d_1 with rate R_1 to its intended receiver by encoding it into a codeword with an average power constraint P_1 . At every time instant the message broadcasted by the primary transmitter is

$$x_1 = \sqrt{P_1} u_1(d_1) \quad (1)$$

where u_1 is a symbol with unit average power from the capacity achieving code for a SISO Gaussian channel with a given signal-to-noise ratio (SNR). The primary receiver has knowledge of the codebook for d_1 .

Fig. 1. System model of primary SISO link and secondary MISO link.

Similarly, the secondary transmitter wants to convey a message d_2 with rate R_2 to its receiver. Assuming it has non-causal knowledge of the message d_1 , the transmitted codeword depends in general on both messages. The secondary transmitter has a total power of P_2 . A fraction p_a of it is spent in a selfless manner to help the primary-user pair achieve its rate requirement and the rest available power p_b (where $p_a + p_b \leq P_2$) is used for the transmission of its own message. Furthermore, each of the components has an independent beamforming vector $\mathbf{w}_1$ and $\mathbf{w}_2$, with $\|\mathbf{w}_1\| = \|\mathbf{w}_2\| = 1$. The signal broadcasted by the secondary transmitter at a given time instant is therefore

$$x_2 = \sqrt{p_a} \mathbf{w}_1 u_1(d_1) + \sqrt{p_b} \mathbf{w}_2 u_2(d_2) \quad (2)$$

where the secondary transmitter uses the same codebook for d_1 as the primary transmitter, and encodes d_2 into u_2 by a different Gaussian codebook. The secondary receiver has knowledge of the codebook for d_2 .

The channels are quasi-static block flat fading, and together with the noise components they are modeled as independent and identically distributed complex Gaussian random variables. The channels from the primary transmitter to the primary receiver and the secondary receiver are denoted as h_{11} and h_{12} respectively, with $h_{11} \sim \mathcal{CN}(0, 1)$ and $h_{12} \sim \mathcal{CN}(0, \alpha)$. Similarly, the channels from the secondary transmitter to the primary receiver and the secondary receiver are denoted as h_{21} and h_{22} , with $h_{21} \in \mathbb{C}^N$ and $h_{21} \sim \mathcal{CN}(\mathbf{0}, \beta \mathbf{I}_N)$, $h_{22} \in \mathbb{C}^N$ and $h_{22} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$, where $\mathbf{0}$ and $\mathbf{I}_N$ denote the all-zero vector and the $N \times N$ identity matrix, respectively. The noise components at the primary and secondary receivers are denoted as $n_1 \sim \mathcal{CN}(0, 1)$ and $n_2 \sim \mathcal{CN}(0, 1)$, respectively.

IV. PROBLEM STATEMENT

The following rates are achievable by the primary and secondary users respectively

$$R_1 \leq \log_2 \left(1 + \frac{|\sqrt{P_1} h_{11} + \sqrt{p_a} h_{21}^H \mathbf{w}_1|^2}{1 + p_b |h_{21}^H \mathbf{w}_2|^2} \right) \quad (3a)$$

$$R_2 \leq \log_2 \left(1 + \frac{p_b |h_{22}^H \mathbf{w}_2|^2}{1 + |\sqrt{P_1} h_{12} + \sqrt{p_a} h_{22}^H \mathbf{w}_1|^2} \right) \quad (3b)$$

for any feasible choice of precoding vectors $\mathbf{w}_1$, $\mathbf{w}_2$ and powers P_1 , p_a , p_b .

The problem of maximizing the transmission rate R_2 for the secondary users while satisfying the rate requirement for the primary link R_1 has the following mathematical formulation:

$$\max_{\mathbf{w}_1, p_a, \mathbf{w}_2, p_b} \frac{p_b |h_{22}^H \mathbf{w}_2|^2}{1 + |\sqrt{P_1} h_{12} + \sqrt{p_a} h_{22}^H \mathbf{w}_1|^2} \quad (4a)$$

$$\text{s.t.} \quad \frac{|\sqrt{P_1} h_{11} + \sqrt{p_a} h_{21}^H \mathbf{w}_1|^2}{1 + p_b |h_{21}^H \mathbf{w}_2|^2} \geq 2^{R_1} - 1 \quad (4b)$$

$$\|\mathbf{w}_1\| = \|\mathbf{w}_2\| = 1 \quad (4c)$$

$$p_a + p_b \leq P_2, p_a \geq 0, p_b \geq 0. \quad (4d)$$

V. MAIN RESULTS

The denominator of the objective function in (4a) should be minimized and the numerator of the left-hand-side in (4b) should be maximized with respect to $\mathbf{w}_1$; the numerator of the objective function in (4a) should be maximized and the denominator of the left-hand-side in (4b) should be minimized with respect to $\mathbf{w}_2$.

Lemma 1. To solve the multi-objective optimization problem

$$\left(\min_{\mathbf{v}} |a + \mathbf{g}_1^H \mathbf{v}|^2, \max_{\mathbf{v}} |b + \mathbf{g}_2^H \mathbf{v}|^2 \right) \quad (5)$$

where $\|\mathbf{v}\|^2 \leq \gamma$, $\mathbf{v}, \mathbf{g}_1, \mathbf{g}_2 \in \mathbb{C}^N$, and $a, b \in \mathbb{C}$, $\mathbf{v}$ can be parametrized as

$$\mathbf{v}(\rho_1, \rho_2) = \rho_1 \frac{\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2\|} + \rho_2 \frac{\Pi_{\mathbf{g}_1} \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1} \mathbf{g}_2\|} \quad (6)$$

where $|\rho_1|^2 + |\rho_2|^2 \leq \gamma$, and $\rho_1, \rho_2 \in \mathbb{C}$.

Proof: Without loss of generality, assume that $\mathbf{v}$ is parametrized as

$$\mathbf{v} = \xi_1 \frac{\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2\|} + \xi_2 \frac{\Pi_{\mathbf{g}_1} \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1} \mathbf{g}_2\|} + \sum_{l=3}^N \xi_l \mathbf{u}_l \quad (7)$$

where the orthonormal set $\{\frac{\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2\|}, \frac{\Pi_{\mathbf{g}_1} \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1} \mathbf{g}_2\|}, \mathbf{u}_3, \dots, \mathbf{u}_N\}$ forms the orthonormal basis of space $\mathbb{C}^N$, and $\sum_{l=1}^N |\xi_l|^2 \leq \gamma$, $\xi_l \in \mathbb{C}$, $l = 1, \dots, N$.

Redistribute the power $\delta = \sum_{l=3}^N |\xi_l|^2$ spent on span $\{\mathbf{u}_3, \dots, \mathbf{u}_N\}$ to $\frac{\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2\|}$ as such

$$\mathbf{v} = (\xi_1 + e^{j\angle\phi} \sqrt{\delta}) \frac{\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2\|} + \xi_2 \frac{\Pi_{\mathbf{g}_1} \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1} \mathbf{g}_2\|} \quad (8)$$

with

$$\phi = b + \xi_1 \mathbf{g}_2^H \frac{\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1}^\perp \mathbf{g}_2\|} + \xi_2 \mathbf{g}_2^H \frac{\Pi_{\mathbf{g}_1} \mathbf{g}_2}{\|\Pi_{\mathbf{g}_1} \mathbf{g}_2\|} \quad (9)$$

that $|b + \mathbf{g}_2^H \mathbf{v}|^2$ is increased while $|a + \mathbf{g}_1^H \mathbf{v}|^2$ is unchanged, which means (8) gives a better solution to (5) than (7). Let $\rho_1 = \xi_1 + e^{j\angle\phi} \sqrt{\delta}$ and $\rho_2 = \xi_2$, then $|\rho_1|^2 + |\rho_2|^2 \leq \gamma$, and $\rho_1, \rho_2 \in \mathbb{C}$. ■

Proposition 1. The optimization problem (4) is solved by

$$\mathbf{w}_1(\psi_1, \psi_2, \lambda_1) = e^{j\psi_1} \sqrt{\lambda_1} \frac{\Pi_{\mathbf{h}_{22}}^\perp \mathbf{h}_{21}}{\|\Pi_{\mathbf{h}_{22}}^\perp \mathbf{h}_{21}\|} + e^{j\psi_2} \sqrt{1 - \lambda_1} \frac{\Pi_{\mathbf{h}_{22}} \mathbf{h}_{21}}{\|\Pi_{\mathbf{h}_{22}} \mathbf{h}_{21}\|}, \quad (10)$$

where

$$\psi_1 = \angle\varphi, \quad (11)$$

$$\varphi = \sqrt{P_1} h_{11} + \sqrt{p_a} e^{j\psi_2} \sqrt{1 - \lambda_1} \mathbf{h}_{21}^H \frac{\Pi_{\mathbf{h}_{22}} \mathbf{h}_{21}}{\|\Pi_{\mathbf{h}_{22}} \mathbf{h}_{21}\|}, \quad (12)$$

and by

$$\mathbf{w}_2(\lambda_2) = \sqrt{\lambda_2} \frac{\Pi_{\mathbf{h}_{21}} \mathbf{h}_{22}}{\|\Pi_{\mathbf{h}_{21}} \mathbf{h}_{22}\|} + \sqrt{1 - \lambda_2} \frac{\Pi_{\mathbf{h}_{21}}^\perp \mathbf{h}_{22}}{\|\Pi_{\mathbf{h}_{21}}^\perp \mathbf{h}_{22}\|}, \quad (13)$$

$$p_b = P_2 - p_a, \quad (14)$$

where

$$\lambda_2 = \begin{cases} \lambda_{MRT} & \text{if } \lambda_{MRT} \leq \frac{z}{\|\mathbf{h}_{21}\|^2 (P_2 - p_a)} \\ \frac{z}{\|\mathbf{h}_{21}\|^2 (P_2 - p_a)} & \text{otherwise} \end{cases} \quad (15)$$

$$\lambda_{MRT} = \frac{\|\Pi_{\mathbf{h}_{21}} \mathbf{h}_{22}\|^2}{\|\mathbf{h}_{22}\|^2}, \quad (16)$$

$$z = \frac{|\sqrt{P_1} h_{11} + \sqrt{p_a} \mathbf{h}_{21}^H \mathbf{w}_1(\psi_1, \psi_2, \lambda_1)|^2}{2R_1 - 1} - 1 \geq 0, \quad (17)$$

for certain $p_a \in [0, P_2]$, $\psi_2 \in [0, 2\pi]$ and $\lambda_1 \in [0, 1]$ to get the optimal objective value in (4a) while satisfying (17).

Proof: From Lemma 1, for the optimization problem $(\min_{\mathbf{w}_1} |\sqrt{P_1} h_{12} + \sqrt{p_a} \mathbf{h}_{22}^H \mathbf{w}_1|^2, \max_{\mathbf{w}_1} |\sqrt{P_1} h_{11} + \sqrt{p_a} \mathbf{h}_{21}^H \mathbf{w}_1|^2)$, $\mathbf{w}_1$ can be parametrized as in (10). By substitution of $\mathbf{w}_1$ with (10), the numerator of the left-hand-side in (4b) becomes

$|\varphi + \sqrt{p_a} e^{j\psi_1} \sqrt{\lambda_1} \mathbf{h}_{21}^H \frac{\Pi_{\mathbf{h}_{22}} \mathbf{h}_{21}}{\|\Pi_{\mathbf{h}_{22}} \mathbf{h}_{21}\|}|^2$, while the denominator of the objective function in (4a) becomes $1 + |\sqrt{P_1} h_{12} + \sqrt{p_a} e^{j\psi_2} \sqrt{1 - \lambda_1} \mathbf{h}_{22}^H \frac{\Pi_{\mathbf{h}_{22}} \mathbf{h}_{21}}{\|\Pi_{\mathbf{h}_{22}} \mathbf{h}_{21}\|}|^2$, which is independent of ψ_1 . Given p_a , ψ_2 and λ_1 , the optimal ψ_1 is given as in (11) to maximize the numerator of the left-hand-side in (4b). And similar to the proof of Proposition 1 in [9], given p_a , ψ_2 and λ_1 , the optimal $\mathbf{w}_2$ can be parametrized as in (13) and the optimal p_b is given as in (14). For (4a) to be feasible z should be nonnegative. ■

Remark 1. Given p_a , to maximize the desired signal power received at the two receivers, the generalized maximal ratio transmission (MRT) solution is

$$\mathbf{w}_1^{MRT} = \frac{\mathbf{h}_{21}}{\|\mathbf{h}_{21}\|} \frac{h_{11}}{|h_{11}|}, \quad (18)$$

$$\mathbf{w}_2^{MRT} = \frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|}, \quad (19)$$

$$p_b = \min \left(P_2 - p_a, \frac{z}{|\mathbf{h}_{21}^H \frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|}|^2} \right) \quad (20)$$

where

$$z = \frac{(\sqrt{P_1} |h_{11}| + \sqrt{p_a} |\mathbf{h}_{21}|)^2}{2R_1 - 1} - 1. \quad (21)$$

To get the maximal achievable secondary rate, a line search is carried out over p_a .

Remark 2. Given p_a , to minimize the interfering signal power received at the two receivers, the generalized zero-forcing (ZF) solution is

$$\mathbf{w}_1^{ZF} = -\frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|} \frac{h_{12}}{|h_{12}|}, \quad (22)$$

$$\mathbf{w}_2^{ZF} = \frac{\Pi_{\mathbf{h}_{21}}^\perp \mathbf{h}_{22}}{\|\Pi_{\mathbf{h}_{21}}^\perp \mathbf{h}_{22}\|}, \quad (23)$$

$$p_b = P_2 - p_a \quad (24)$$

where

$$z = \frac{|\sqrt{P_1} h_{11} - \sqrt{p_a} \mathbf{h}_{21}^H \frac{\mathbf{h}_{22}}{\|\mathbf{h}_{22}\|} \frac{h_{12}}{|h_{12}|}|^2}{2R_1 - 1} - 1 \geq 0. \quad (25)$$

To get the maximal achievable secondary rate, a line search is carried out over p_a while satisfying (25).

VI. SEMIDEFINITE RELAXATION AND RANK-ONE DECOMPOSITION

In this section, the optimization problem (4) is solved by another approach when the number of antennas at the secondary transmitter is larger than two. By incorporating the power into the beamforming vector (i.e. $\mathbf{w}_1 := \sqrt{p_a} \mathbf{w}_1$, $\mathbf{w}_2 := \sqrt{p_b} \mathbf{w}_2$), (4) can be rewritten as (26). After homogenization [14], (26) can be reformulated as (27). (4) and (27) have the same optimal value, and $(\mathbf{w}_1^*/t^*, \mathbf{w}_2^*/t^*)$ is optimal to (4) if $(\mathbf{w}_1^*, \mathbf{w}_2^*, t^*)$ solves (27).

$$\max_{\mathbf{w}_1, \mathbf{w}_2} \frac{\mathbf{w}_2^H \mathbf{h}_{22} \mathbf{h}_{22}^H \mathbf{w}_2}{\mathbf{w}_1^H \mathbf{h}_{22} \mathbf{h}_{22}^H \mathbf{w}_1 + 2\text{Re}(\sqrt{P_1} h_{12} \mathbf{w}_1^H \mathbf{h}_{22}) + P_1 |h_{12}|^2 + 1} \quad (26a)$$

$$\text{s.t.} \frac{\mathbf{w}_1^H \mathbf{h}_{21} \mathbf{h}_{21}^H \mathbf{w}_1 + 2\text{Re}(\sqrt{P_1} h_{11} \mathbf{w}_1^H \mathbf{h}_{21}) + P_1 |h_{11}|^2}{\mathbf{w}_2^H \mathbf{h}_{21} \mathbf{h}_{21}^H \mathbf{w}_2 + 1} \geq 2^{R_1} - 1 \quad (26b)$$

$$\mathbf{w}_1^H \mathbf{w}_1 + \mathbf{w}_2^H \mathbf{w}_2 \leq P_2. \quad (26c)$$

$$\max_{\mathbf{w}_1, \mathbf{w}_2, t \in \mathbb{C}} \frac{\text{tr}(\mathbf{h}_{22} \mathbf{h}_{22}^H \mathbf{w}_2 \mathbf{w}_2^H)}{\text{tr}\left(\begin{bmatrix} \mathbf{h}_{22} \mathbf{h}_{22}^H & \sqrt{P_1} h_{12} \mathbf{h}_{22} \\ \sqrt{P_1} h_{12}^* \mathbf{h}_{22}^H & P_1 |h_{12}|^2 + 1 \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \mathbf{w}_1^H & t^* \mathbf{w}_1 \\ t \mathbf{w}_1^H & |t|^2 \end{bmatrix}\right)} \quad (27a)$$

$$\text{s.t.} \text{tr}\left(\begin{bmatrix} \mathbf{h}_{21} \mathbf{h}_{21}^H & \sqrt{P_1} h_{11} \mathbf{h}_{21} \\ \sqrt{P_1} h_{11}^* \mathbf{h}_{21}^H & P_1 |h_{11}|^2 \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \mathbf{w}_1^H & t^* \mathbf{w}_1 \\ t \mathbf{w}_1^H & |t|^2 \end{bmatrix}\right) \geq (2^{R_1} - 1) (1 + \text{tr}(\mathbf{h}_{21} \mathbf{h}_{21}^H \mathbf{w}_2 \mathbf{w}_2^H)) \quad (27b)$$

$$\text{tr}\left(\begin{bmatrix} \mathbf{I}_N & \mathbf{0}^H \\ \mathbf{0} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \mathbf{w}_1^H & t^* \mathbf{w}_1 \\ t \mathbf{w}_1^H & |t|^2 \end{bmatrix}\right) + \text{tr}(\mathbf{w}_2 \mathbf{w}_2^H) \leq P_2 \quad (27c)$$

$$\text{tr}\left(\begin{bmatrix} \mathbf{0}_N & \mathbf{0}^H \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{w}_1 \mathbf{w}_1^H & t^* \mathbf{w}_1 \\ t \mathbf{w}_1^H & |t|^2 \end{bmatrix}\right) = 1. \quad (27d)$$

After semi-definite relaxation (SDR) [14], (27) becomes

$$\max_{\mathbf{W}_1, \mathbf{W}_2} \frac{\text{tr}(\mathbf{H}_2 \mathbf{W}_2)}{\text{tr}(\mathbf{H}_1 \mathbf{W}_1)} \quad (28a)$$

$$\text{s.t.} \text{tr}(\mathbf{H}_3 \mathbf{W}_1) \geq (2^{R_1} - 1) (1 + \text{tr}(\mathbf{H}_4 \mathbf{W}_2)) \quad (28b)$$

$$\text{tr}(\mathbf{H}_5 \mathbf{W}_1) + \text{tr}(\mathbf{W}_2) \leq P_2 \quad (28c)$$

$$\text{tr}(\mathbf{H}_6 \mathbf{W}_1) = 1 \quad (28d)$$

$$\mathbf{W}_1 \succeq 0, \mathbf{W}_2 \succeq 0 \quad (28e)$$

where

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{h}_{22} \mathbf{h}_{22}^H & \sqrt{P_1} h_{12} \mathbf{h}_{22} \\ \sqrt{P_1} h_{12}^* \mathbf{h}_{22}^H & P_1 |h_{12}|^2 + 1 \end{bmatrix}, \mathbf{H}_2 = \mathbf{h}_{22} \mathbf{h}_{22}^H \quad (29)$$

$$\mathbf{H}_3 = \begin{bmatrix} \mathbf{h}_{21} \mathbf{h}_{21}^H & \sqrt{P_1} h_{11} \mathbf{h}_{21} \\ \sqrt{P_1} h_{11}^* \mathbf{h}_{21}^H & P_1 |h_{11}|^2 \end{bmatrix}, \mathbf{H}_4 = \mathbf{h}_{21} \mathbf{h}_{21}^H \quad (30)$$

$$\mathbf{H}_5 = \begin{bmatrix} \mathbf{I}_N & \mathbf{0}^H \\ \mathbf{0} & 0 \end{bmatrix}, \mathbf{H}_6 = \begin{bmatrix} \mathbf{0}_N & \mathbf{0}^H \\ \mathbf{0} & 1 \end{bmatrix} \quad (31)$$

and $\mathbf{0}_N$ denotes the $N \times N$ all-zero matrix. By Charnes-Cooper variable transformation [15], and definition of the transformed variables $\mathbf{X}_1 = s \mathbf{W}_1$ and $\mathbf{X}_2 = s \mathbf{W}_2$, where $s \geq 0$ complies with $\text{tr}(\mathbf{H}_1 (s \mathbf{W}_1)) = 1$, (28) is equivalent to [14]

$$\max_{\mathbf{X}_1, \mathbf{X}_2, s} \text{tr}(\mathbf{H}_2 \mathbf{X}_2) \quad (32a)$$

$$\text{s.t.} \text{tr}(\mathbf{H}_1 \mathbf{X}_1) = 1 \quad (32b)$$

$$\text{tr}(\mathbf{H}_3 \mathbf{X}_1) \geq (2^{R_1} - 1) (s + \text{tr}(\mathbf{H}_4 \mathbf{X}_2)) \quad (32c)$$

$$\text{tr}(\mathbf{H}_5 \mathbf{X}_1) + \text{tr}(\mathbf{X}_2) \leq s P_2 \quad (32d)$$

$$\text{tr}(\mathbf{H}_6 \mathbf{X}_1) = s \quad (32e)$$

$$\mathbf{X}_1 \succeq 0, \mathbf{X}_2 \succeq 0, s \geq 0. \quad (32f)$$

(28) and (32) have the same optimal value, and $(\mathbf{X}_1^*/s^*, \mathbf{X}_2^*/s^*)$ is optimal to (28) if $(\mathbf{X}_1^*, \mathbf{X}_2^*, s^*)$ solves (32) which is a separable SDP problem (it can be

solved e.g. by CVX [16]). If the resulted $\mathbf{X}_1^*$ and $\mathbf{X}_2^*$ are of rank one, then the solution is optimal; otherwise, a rank-one optimal solution can be constructed by applying a recent matrix decomposition theorem to $\mathbf{X}_1^*$ and/or $\mathbf{X}_2^*$ when $N \geq 3$ (Theorem 2.3 and/or Theorem 2.2 in [17], when $N = 2$ a rank-one decomposition may not exist for $\mathbf{X}_2$ that has 3 constraint matrices), which means the SDR (28) is tight. Algorithm 1 summarizes the procedure leading to an optimal solution $(\mathbf{w}_1^*, \mathbf{w}_2^*)$ of (4).

Algorithm 1: Algorithm to solve (4) with $N \geq 3$

Input: $\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3, \mathbf{H}_4, \mathbf{H}_5, \mathbf{H}_6, \mathbf{I}_N$.

Output: An optimal solution $(\mathbf{w}_1^*, \mathbf{w}_2^*)$ of (4).

- 1 solve SDP (32) finding an optimal solution $(\mathbf{X}_1^*, \mathbf{X}_2^*, s^*)$;
 - 2 let $\mathbf{X}_1^* := \mathbf{X}_1^*/s^*$ and $\mathbf{X}_2^* := \mathbf{X}_2^*/s^*$;
 - 3 **if** $\text{rank}(\mathbf{X}_1^*) = 1$ **then**
 - 4 perform an eigen-decomposition $\mathbf{X}_1^* = \mathbf{x}_1^* (\mathbf{x}_1^*)^H$;
 - 5 **else**
 - 6 find rank-one matrix $\mathbf{x}_1^* (\mathbf{x}_1^*)^H$ by applying Algorithm 3 in [17] to $\mathbf{X}_1^*$ with constraint matrices $\mathbf{H}_1, \mathbf{H}_3, \mathbf{H}_5$ and $\mathbf{H}_6$;
 - 7 **end**
 - 8 **if** $\text{rank}(\mathbf{X}_2^*) = 1$ **then**
 - 9 perform an eigen-decomposition $\mathbf{X}_2^* = \mathbf{x}_2^* (\mathbf{x}_2^*)^H$;
 - 10 **else**
 - 11 find rank-one matrix $\mathbf{x}_2^* (\mathbf{x}_2^*)^H$ by applying Algorithm 2.1 and 2.2 in [17] to $\mathbf{X}_2^*$ with constraint matrices $\mathbf{H}_2, \mathbf{H}_4$ and $\mathbf{I}_N$;
 - 12 **end**
 - 13 let $\mathbf{x}_1^* = \begin{bmatrix} \mathbf{w}_1^* \\ t^* \end{bmatrix}$ and $\mathbf{x}_2^* = \begin{bmatrix} \mathbf{w}_2^* \\ t^* \end{bmatrix}$; output $\mathbf{w}_1^* := \mathbf{w}_1^*/t^*$ and $\mathbf{w}_2^* := \mathbf{w}_2^*/t^*$.
-

VII. NUMERICAL SIMULATION

The performance of the proposed linear precoding (LP) in overlay CR is compared with that of optimal beamforming (OB) in overlay CR [10] (which has the same system model but secondary transmitter deploys DPC to pre-cancel interference experienced by secondary receiver due to primary message) and optimal beamforming in underlay CR [9] (where secondary transmitter has no knowledge of primary message; rate splitting and successive interference cancellation is deployed when it is viable), in terms of achievable secondary rate. The load of the primary link is defined as the ratio of primary rate requirement and instantaneously achievable primary rate without secondary transmission. For simulation, we set primary power constraint $P_1 = 10\text{dB}$ and channel variance $\alpha = \beta = 0.5$.

Fig. 2. Achievable secondary rate versus SNR: 2 antennas at the secondary transmitter.

As in Figure 2, the achievable secondary rate of the proposed algorithm is almost unchanged with varying primary link load, as in the case of optimal beamforming in overlay CR. As SNR increases, the achievable secondary rate of the proposed scheme approaches that of optimal beamforming in overlay CR, and the gap between the two becomes almost constant at high SNRs.

Furthermore, the performance of the proposed scheme is compared with that when generalized MRT or ZF (as in Remark 1 and Remark 2) is deployed at the secondary transmitter (as MRT overlay and ZF overlay in Figure 3). As in Figure 3, the proposed scheme achieves a higher secondary rate than MRT overlay or ZF overlay, with 50% or 100% load and at all SNRs. At high SNR, ZF overlay has almost the same slope as the proposed scheme, while MRT overlay has a smaller one.

VIII. CONCLUSIONS

In this paper the optimal beamforming vectors and power allocation are derived to maximize the achievable secondary rate while satisfying the primary rate requirement, for a SISO

Fig. 3. Achievable secondary rate versus SNR: 2 antennas at the secondary transmitter.

primary link and a MISO secondary link to coexist, where the secondary transmitter transmits both primary and secondary messages. Moreover, semidefinite relaxation and rank-one decomposition is applied to solve the optimization problem, when the number of antennas at the secondary transmitter is larger than two.

IX. ACKNOWLEDGMENT

The second author would like to thank Gan Zheng for interesting discussions on the relaxation approach and the rank decomposition.

REFERENCES

- [1] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. on Information Theory*, vol. 52, no. 5, pp. 1813–1827, May 2006.
- [2] M. Costa, "Writing on dirty paper (corresp.)," *IEEE Trans. on Information Theory*, vol. 29, no. 3, pp. 439–441, May 1983.
- [3] W. Wu, S. Vishwanath, and A. Arapostathis, "Capacity of a class of cognitive radio channels: Interference channels with degraded message sets," *IEEE Trans. on Information Theory*, vol. 53, no. 11, pp. 4391–4399, nov. 2007.
- [4] A. Jovicic and P. Viswanath, "Cognitive radio: An information-theoretic perspective," *IEEE Trans. on Information Theory*, vol. 55, no. 9, pp. 3945–3958, sept. 2009.
- [5] I. Maric, R. Yates, and G. Kramer, "Capacity of interference channels with partial transmitter cooperation," *IEEE Trans. on Information Theory*, vol. 53, no. 10, pp. 3536–3548, oct. 2007.
- [6] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Sel. Topics in Signal Process.*, vol. 2, no. 1, pp. 88–102, Feb. 2008.
- [7] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Sel. Areas in Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.
- [8] A. Goldsmith, S. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: An information theoretic perspective," *Proc. IEEE*, vol. 97, no. 5, pp. 894–914, May 2009.
- [9] J. Lv and E. A. Jorswieck, "Spatial shaping in cognitive system with coded legacy transmission," in *Proc. of International ITG Workshop on Smart Antennas (WSA)*, Feb. 2011.
- [10] J. Lv, R. Blasco-Serrano, E. A. Jorswieck, R. Thobaben, and A. Kliks, "Optimal beamforming in MISO cognitive channels with degraded message sets," *IEEE Wireless Communications and Networking Conference (WCNC 2012)*, accepted.

- [11] G. Caire and S. Shamai (Shitz), "On the achievable throughput of a multiantenna Gaussian broadcast channel," *IEEE Trans. on Information Theory*, vol. 49, no. 7, pp. 1691–1706, Jul. 2003.
- [12] E. Hossain, L. Le, N. Devroye, and M. Vu, "Cognitive radio: from theory to practical network engineering" in *New directions in wireless communications research*. Springer, 2009, ch. 10, pp. 251–289.
- [13] Z. Luo, W. Ma, A. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20–34, May 2010.
- [14] A. De Maio, Y. Huang, D. Palomar, S. Zhang, and A. Farina, "Fractional QCQP with applications in ML steering direction estimation for radar detection," *IEEE Trans. on Signal Processing*, vol. 59, no. 1, pp. 172–185, Jan. 2011.
- [15] A. Charnes and W. W. Cooper, "Programming with linear fractional functionals," *Naval Res. Logist. Quarter.*, vol. 9, pp. 181–186, 1962.
- [16] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 1.21," <http://cvxr.com/cvx>, Apr. 2011.
- [17] W. Ai, Y. Huang, and S. Zhang, "New results on hermitian matrix rank-one decomposition," *Mathematical Programming*, vol. 128, pp. 253–283, 2011.