A Cavity-Wall Element for the Statistical Energy Analysis of the Sound Transmission through Double Walls

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Abstract
The wave motion within a cavity between two flexible walls is first investigated numerically. The results then form the basis for a new SEA formulation in which each element describe one kind of coupled cavity-wall wave motion. This formulation obsoletes the non-resonant transmission path commonly used in SEA of sound transmission and compared to classical formulations it improves results at frequencies around and a bit above the double wall resonance. The new formulation is compared to three sets of measurements found in the literature showing fair agreements.

Introduction
Double walls are frequently used in buildings, vehicles and aircrafts and continue to be of interest for research, see, e.g., Refs [1-10], and the reviews on different computational methods in Refs. [11-12].

This work concerns Statistical Energy Analysis (SEA), which is a convenient approach for the prediction of the frequency- and space-averaged vibroacoustic energy within elements [13-14]. The governing equation in SEA expresses the conservation of energy based on the coupling power proportionality (CPP) law. This asserts that the rate of energy flow from one element to another is proportional to the difference in their modal power, which is the vibroacoustic energy in a frequency band times the expected difference between natural frequencies. It is presumed that the structure is a member of an ensemble of similar structures having random properties and SEA attempts to predict the averaged energy across this ensemble. It is furthermore presumed that the responses in the elements, at least in principle, may be synthesized by sets of similar local modes having resonances in the frequency band of interest, or by free waves in diffuse fields. These archetypical responses, or templates, form the basis for the evaluation of parameters in the energy equations and are
sometimes useful for evaluating response variables such as the mean square velocity and mean square sound pressure.

An SEA requires the division of the system under study into a number of SEA elements. An element may represent the total response of a substructure or the response of one kind of motion in a substructure; a beam is typically represented by four SEA elements that describe the motion of two orthogonal transverse waves, a rotational wave and a quasi-longitudinal wave [13, Sect 7.1]. In accordance, earlier works on double walls and fluid filled cylinders described the out of plane structural waves in each substructure by one element and the acoustic waves in the contained fluid as another [1-2, 15-17]. In a waveguide approach, Finnveden first observed the wave motion along fluid filled cylinders and then proposed that the elements should attribute the vibroacoustic energy of the different waves that could travel in the composite structure [18]. Thus, in a lower frequency regime the predominantly fluid element should attribute the flexibility of the pipe wall and the structural element should attribute the associated fluid inertia so that the elements represented different kinds of waves in the composite structure. This waveguide approach is used in the present work where, first, the waves that travels in a plate-fluid-plate structure are investigated and, then, new SEA elements are formulated that represent the energy of the different kinds of waves in the composite.

The original SEA model for double walls was presented by Price and Crocker [15], based on their earlier model for the sound transmission through single walls [19]. This single-wall SEA model consists of three elements: two rooms in which there are diffuse sound fields and in between them a plate performing resonant vibrations. The constant of proportionality in the CPP law is defined by Smith’s wonderful result that the radiation of sound from a mode in a structure to an infinite acoustic space is reciprocal to the mode’s reception of vibroacoustic energy from a diffuse sound field [20]. This defines the resonant path: free waves in a room to resonant vibration of plate to free waves in the receiving room. Additionally, Crocker and Price introduced indirect coupling into SEA. In a lower frequency range where the flexural waves in the plate are subsonic, its response to an acoustic forcing is defined partly by the resonant modes, and partly by modes that have a length scale comparable to the acoustic wave length and that have resonances at frequencies much below the considered frequency. The response of these latter modes is impeded by their modal mass and they radiate well into the other room. This non-resonant
response is not modelled by an SEA element but the corresponding flow of energy is described by an indirect coupling from room to room.

Crocker and Price’s double-wall SEA model adds to the single-wall model a wall and a cavity in between the walls; the radiation of sound into the cavity is described equally as the radiation into a semi-infinite space. It appears as if the Crocker and Price’s double-wall is still the basis for SEA of double walls; structural couplings and other complicating effects need of course be added when handling engineering structures. Also, one commercial SEA program adds an indirect coupling between the rooms and two indirect couplings describing energy flow from a room to the plate adjacent to the other room.

Craik and Smith systematically investigate the Crocker and Price’s model comparing calculation results to measurements of different partitions [1-2]. They also include structural coupling between the walls. In Ref. [5] Craik further improves the SEA prediction upon the argument that in a lower frequency regime where the cavity waves are plane, the forced wall motion should be sonic and therefore the radiation efficiency associated with the indirect coupling exceeds the value of unity that was presumed in earlier works.

Similar arguments are used in the present work where it is noticed that the wall-cavity-wall wave that resembles the plane acoustic cavity wave is supersonic, decreasing to sonic speed as frequency increases. The new approach differs from the one in earlier works in two respects: 1) The elements and their modal density and modal energy are defined by the waves that can travel in the plane of the composite partition; 2) The wall-cavity-wall waves are directly coupled to the rooms so there are no indirect couplings in the new model.

After a review of the SEA of double walls in Section 2, the wave motion of double wall structures is investigated with the help of a waveguide finite element model (WFEM) in Section 3. This analysis leads to a new formulation of an SEA element for the coupled cavity-wall motion. Section 4 defines couplings in the employed SEA model. Section 5 compares the results of this model to measurements of building construction walls [1], aluminium double walls [4] and a double glass window [3] showing a fair agreement. Finally, Section 6 concludes the work.
2. Statistical energy analysis of double walls, a review

This section reviews the classical SEA of double walls as defined by Crocker and Price [15] and Craik and Smith [1]. Much of the procedures and definitions are used also in the new SEA model presented in Sect. 4 and 5.

2.1 SEA fundamentals

Consider an element \( i \) excited by an external source providing a steady state vibroacoustic power \( P_{\text{in}}^{(i)} \) in a frequency band of width \( \Delta \omega \). A statistical energy analysis of its response is built upon the energy conservation principle, stating that the power injected into element \( i \) by external sources equals the power dissipated within this element, \( P_{\text{d}}^{(i)} \), plus the net power transmitted to other elements \( P_{c}^{(i,j)} \):

\[
P_{\text{in}}^{(i)} = P_{\text{d}}^{(i)} + \sum_{j \neq i} P_{c}^{(i,j)}.
\]

A common engineering approximation for the dissipated power accounting for linear losses, proportional to the vibroacoustic energy, is:

\[
P_{\text{d}}^{(i)} = \eta_i \omega E_i,
\]

where \( \omega \) is the frequency, \( \eta_i \) is the damping loss factor and \( E_i \) is the vibroacoustic energy of element \( i \) in the considered frequency band.

In SEA, the net power between two directly connected elements is often called coupling power and is proportional to the difference in the energy per mode of the two elements. This coupling power proportionality is herein formulated as

\[
P_{c}^{(i,j)} = C^{(i,j)} \left( \hat{e}_i - \hat{e}_j \right),
\]

where the modal power is given by

\[
\hat{e}_i = E_i / n_i; \quad n_i = \Delta N_i / \Delta \omega,
\]

\( n_i \) is the modal density, \( \Delta N_i \) is the number of modes in the considered frequency band and \( C^{(i,j)} \) is the modal power conductivity, or in any given context, simply the conductivity. The conductivity is a non-dimensional parameter defined by Eq. (3), being related to the coupling loss factor \( \eta_{c}^{(i,j)} \) as:

\[
C^{(i,j)} = \eta_{c}^{(i,j)} \omega n_i.
\]

In possibly all practical applications of SEA, the conductivity is equated to its travelling wave estimate, evaluated for one junction at a time, disregarding the rest of
the system by extending the receiving element(s) to infinity. Thus, the conductivity is
given by

\[ C^{(i,j)} = \left( P_c^{(i,j)}/\hat{e}_j \right) \delta^{(i,j)}/M_j \rightarrow 0 \]  \hspace{1cm} (6)

Moreover, the CPP in Eq. (3) describes a potential flow and consequently:

\[ C^{(i,j)} = C^{(j,i)} \]  \hspace{1cm} (7)

There are numerous applications showing that the conductivity defined by Eqs (3) and (6) obeys the law (7) including Refs [20-26].

Given the definition of the modal power in Eq. (4), the dissipated power is rewritten:

\[ P_{d}^{(i)} = M_i \hat{\xi}_i \]  \hspace{1cm} (8)

where \( M \) is the non-dimensional modal overlap factor defined as

\[ M_i = \eta_i \omega n_i, \]  \hspace{1cm} (9)

Prior to the identification of conductivities, modal overlap factors and input powers, an SEA requires the identification of the elements describing the response of the system under study. In the present work, the elements are defined by the wave types that are supported by the double wall structure, as is further discussed in Section 3.

The next subsection presents the classical SEA for double walls, where each SEA element corresponds to a physical substructure.

2.2 SEA of a double wall

Consider a partition of size \( S = a \times b \), consisting of two thin-walled plates separated by a distance \( d \). For both plates the coincidence frequency, at which the flexural wavelength of the plate \( w \) equals the acoustic wavelength, occurs at a rather high frequency

\[ f_c = \frac{c^2}{2\pi \sqrt{m_w/B_w}}, \]  \hspace{1cm} (10)

where \( c \) is the speed of sound in air, \( m_w \) is the plate’s mass per unit area and \( B_w \) is its bending stiffness. In a lower frequency regime, much below the coincidence frequency, the impedances of the plates to acoustic forcing are predominantly of mass character. At such low frequencies, the double-wall resonance is mainly determined by the masses of the walls and by the stiffness of the air within the cavity. London
defines the double-wall resonance for walls that are heavy compared to the fluid mass in the cavity as \[27\]:

\[ f_{dw} = \frac{c}{2\pi d} \sqrt{\frac{1}{\mu_1} + \frac{1}{\mu_2}}, \] (11)

where \( \mu_w = \frac{m_w}{\rho_0 d} \) and \( \rho_0 \) is the density of air. At frequencies lower than \( f_{dw} \) the plates move in phase and the Sound Reduction Index (SRI) is approximately given by the field-incidence mass law ([14], pg 101):

\[ R_{ml} = 10 \log \left( 1 + \left( \frac{m_{tot} \omega}{2 \rho_0 c} \right)^2 \right) - 5; \quad m_{tot} = m_1 + m_2 + \rho_0 d. \] (12)

At frequencies above the double wall resonance, Price and Crocker define in Ref. [15] five SEA elements, each describing the response in one of the substructures: 1) sending room; 2) plate 1; 3) cavity; 4) plate 2; 5) receiving room. The sound transmission from a room through a plate to the cavity, or vice versa, has two contributions. One is the direct resonant transmission: the sound field excites the resonances of the plate that in turn radiates into the cavity. The other is the indirect non-resonant transmission of the mass-impeded modes of the plate, described by the mass law; this second transmission path no longer exists above the coincidence frequency.

Craik and Smith, in addition to the SEA elements employed by Price and Crocker, add structural coupling. Measurements and calculations of the SRI of double wall partitions are presented in References [1-2] where the plates are made of plasterboards or chipboard; they are joined by wooden frames, modelled as Euler beams that are erected in the vertical direction at a given distance; the plates are nailed to the frames.

Fig. 1 illustrate results from Craik and Smith’s model [1] where the measured and calculated SRI for a double wall with a cavity depth of 100 mm is shown; the double wall resonance occurs at 87 Hz, while the cavity depth equals half an acoustic wavelength at 1.7 kHz and the coincidence frequency is in the 3.15 kHz third-octave band. (The construction is defined by the data in Table 1 and should be the same as in Ref. [1]). As it may be seen, Price and Crocker’s model, as interpreted by Craik and Smith, performs well at lower frequencies, where the partition is modelled as one single plate, even though the mode count is rather low at these frequencies. The model is also good at higher frequencies where the acoustic wavelength is not very
much longer than the cavity depth. At intermediate frequencies there are consistent errors that are large just above the double wall resonance and decrease as frequency increases. Most disturbing is the trend of the SRI at the intermediate frequencies where the measurements show an increase of perhaps 12 dB per octave and the model an increase of a bit more than 6 dB. This indicates that the SEA model doesn’t capture the physics of the sound transmission and might, therefore, not only give incorrect results but might also indicate incorrect trends for measures applied to improve a construction.

The double wall resonance is not observed in the measured SRI, whereas there is a plateau extending approximately one octave above this frequency. The dip in the 160 Hz band, seen also in the measurements where the cavity depth is 50 mm and 150 mm, might be caused by an increase in the plate mobility. (The distance between the beams is a bit more than half a plate wavelength which would correspond to the first set of resonance if the boundary conditions were in between simply supported and clamped.)

2.3 Indirect coupling for sonic cavity-wall motion
In Reference [5] Craik improves the non-resonant coupling loss factor between a room and the cavity upon the argument that the wall-vibrations forced by a plane acoustic wave in the cavity radiates as if the considered frequency was the coincidence frequency, since the acoustic waves in the cavity are sonic. As seen in Fig. 1, this significantly improves the agreement between model and measurements at intermediate frequencies.

The SEA for double walls in References [1, 15] is herein the basis for the analysis, except that a cavity-wall element replaces the old cavity element. The next section first investigates the wave motion in a double wall and then identifies the elements of the new SEA model.

3. Waves propagating in a double wall and the associated SEA elements
The structure of interest consists of an air cavity in between two plates. The plates are parallel to the $x$-$y$ plane and are separated by a distance $d$ in the $z$-direction. The motion is harmonic of the form of

$$p(x, y, z, t) = \text{Re} \left( \sqrt{2} \tilde{p}(x, y, z, \omega) e^{-i\omega t} \right)$$

$$u_w(x, y, t) = \text{Re} \left( \sqrt{2} \tilde{u}_w(x, y, \omega) e^{-i\omega t} \right)$$

(13)
where \( p \) is the sound pressure in the fluid and \( u_w \) is the displacement in the \( z \)-direction of plate \( w \). The equations of motion for this structure are

\[
\nabla^2 \tilde{p} + k_w^2 \tilde{p} = 0; \quad k_w = \omega/c,
\]

\( (14) \)

\[
B_w \nabla^2 \tilde{u}_w - m_w \omega^2 \tilde{u}_w = \nu_w \tilde{p}_w; \quad \nu_1 = -1; \quad \nu_2 = 1.
\]

\( (15) \)

The plate velocities equal the fluid velocity at their interfaces and it follows that

\[
\rho_o \omega^2 \tilde{u}_w = \frac{\partial \tilde{p}(x, y, z = z_w, \omega)}{\partial z}; \quad z_i = 0; \quad z_2 = d.
\]

\( (16) \)

The set of equations (14)-(16) are first solved numerically. Then, based on the knowledge gained, the equations are approximated and employed to define the SEA elements used for the analysis of the transport of vibroacoustic energy through double wall partitions.

### 3.1 Numerical analysis of the coupled cavity wall motion

The waveguide finite element method is a versatile tool for investigations of wave motion in structures that have constant material and geometrical properties along one direction [28-30]. Using the WFEM, the motion’s dependence on the cross sectional coordinates is approximated with FE polynomial shape functions. Upon the application of standard FE procedures follows a set of coupled ordinary differential equations describing wave propagation and decay along the structure.

Fig. 2 shows the FE mesh in the \( x-y \) plane used to model the 100 mm building construction double wall in Ref. [1], the data are defined in Table 1. The plasterboards are described by three-node quadratic deep shell elements and the fluid by nine-node Lagrange type elements [29-30]. All motion in the \( y \) direction is blocked so the model is, in effect, two-dimensional. Fig. 2 also shows the mesh used to depict the wave motion in the \( x-z \) plane.

Fig. 3 shows the dispersion curves for this double wall. The two straight curves at the bottom are the uncoupled quasi-longitudinal waves in the two plates; these have slightly different wavenumbers as the plates have slightly different characteristics. The two straight lines at the top are the \textit{in vacuo} flexural waves in the plates. The calculated wavenumbers for the flexural waves are close to those for the uncoupled plates, also at frequencies below the double wall resonance (87 Hz), which is perhaps surprising. The waveforms, however, exhibit coupling between the plates and between the plates and the fluid, as Fig. 4 shows. At low frequencies one of the coupled flexural waves is anti-symmetric and the other is symmetric, whereas the
plate amplitudes are in both cases roughly equal. Above the double wall resonance each wave is localised to either of the plates while the amplitude of the other plate decreases rapidly with frequency.

The waves that cut-on, or start propagating, at 87 Hz and 1.7 kHz are predominantly fluid waves that have supersonic wave speeds along the structure decreasing towards sonic speed as frequency increases. The waveforms of the predominantly two fluid wave types are shown in Fig. 5 and Fig. 6. In the former, the fluid motion is almost plane with some motion of the walls, which decreases with frequency. In the latter, the fluid moves while there is almost no motion of the walls. The behaviour of these two fluid waves is evidently different. It is significant for an SEA that the fluid waves do not exist below their cut-on frequencies and that the almost plane wave is supersonic, or near sonic, in the plane of the plates, so the associated wall motion radiates well.

3.2 Identification of SEA elements

The WFEM has revealed the waves that carry energy along the double wall. It has shown that the plate’s flexural and quasi-longitudinal waves are almost as in vacuo. It has shown the existence of supersonic, almost plane, waves having large motion both in the plates and in the fluid; these cavity-wall waves might have been neglected in previous SEA of double walls. Finally, it has shown oblique fluid waves involving very little wall motion.

In an infinite homogenous structure, these waves would propagate independently. In a finite structure that is not perfectly homogenous, the waves are coupled. If, however, the structure is large, the waves might be weakly coupled and may characterize SEA elements. The SEA model defined upon these arguments is illustrated in Fig. 7; the elements are defined below and the couplings are defined in Section 4.

3.2.1 Plate elements

The flexural plate motion is, a bit above the double wall resonance, almost as if in vacuo, and, accordingly, for each plate one SEA element can be defined that is weakly coupled to the rest of the structure. The mode count and modal density for plate \( w \) are given by [13, p140]

\[
N_w = \frac{S k_w^2}{4\pi} ;
\]  

\( (17) \)
\[ n_w = \frac{\partial N}{\partial k_w} \frac{\partial k_w}{\partial \omega} = \frac{S k_w}{2\pi c_{g,w}}, \]  
\[ (18) \]

where
\[ k_w = \sqrt{\omega |m_w / B_w|}; \quad c_{g,w} = \partial \omega / \partial k_w; \]  
\[ (19) \]

\( k_w \) is the flexural wavenumber and \( c_{g,w} \) is the group velocity. Eq (18), together with the damping loss factor, defines the elements’ modal overlap factors. The conductivities, quantifying the energy flow to the rooms and to the cavity-wall element, are defined by the radiation efficiencies, calculated by Leppington et al [31], as is discussed in Section 4.1.

3.2.2 Room elements and beam elements
The SEA model mimics a measurement in a transmission loss suit: there is a source in the source room and sound is eventually transferred to the receiving room. These large 3-D fluid cavities have modal densities and losses that are so large that their actual values do not influence the calculated SRI.

The vertical beams that support the double wall structure are modelled as point-connected Euler beams, the details are found in Ref. [1].

3.2.3 Cavity-wall element below the coincidence frequency
The cavity-wall waves are supersonic and therefore at frequencies below the coincidence frequency, the plates’ bending stiffness, the first term in Eq. (15), is not significant. Eqs (15) and (16) then define mixed Robin boundary conditions for the fluid:
\[ \mu_w d \frac{\partial \tilde{p}}{\partial z} = -v_w \tilde{p}, \quad z = z_w. \]  
\[ (20) \]

A solution to Eqs. (14) that describes the wave motion in the \( x-y \) plane and fulfils the boundary conditions (20) is given by
\[ \tilde{p} = \tilde{p}_0 (\sin \gamma_r z + \mu_2 \gamma_r d \cos \gamma_r z) e^{i\kappa r (\cos \theta + \sin \theta)}, \]  
\[ (21) \]

where any value of the angle \( \theta \) is admissible,
\[ \kappa_r^2 = k_w^2 - \gamma_r^2, \]  
\[ (22) \]

and \( \gamma_r \) is one of the numerable solutions to the transcendental eigenvalue problem
\[ (1 - \mu_1 \mu_2 \gamma_r^2 d^2) \sin (\gamma_r d) + (\mu_1 + \mu_2) \gamma_r d \cos (\gamma_r d) = 0. \]  
\[ (23) \]
It is straightforward to solve Eq. (23) numerically; however, analytical solutions are more descriptive, which helps the analyst. If $\mu_1 \gg 1$, $\mu_2 \gg 1$, London’s formula (11) for the double wall resonance is useful, this is the case for all structures studied in this work. The eigenvalues $\gamma_r$ are then well approximated by

$$\gamma_0 = \frac{2\pi f_{dw}}{c} = \frac{1}{d} \sqrt{\frac{1}{\mu_1} + \frac{1}{\mu_2}},$$

$$\gamma_r = 2\pi f_r / c = r\pi / d; \ r = 1, 2, \ldots.$$  (25)

Consequently, for each eigenvalue $\gamma_r$ there is a two-dimensional wave field in the $x$-$y$ plane with a given mode shape or wave form in the $z$-direction, as defined by Eq. (21). Each of these wave fields are here used to define an SEA element. These elements have mode counts that, in analogy with Eq. (17), are given by

$$N_r = S \kappa_r^2 / 4\pi$$  (26)

where $\kappa_r$ is defined by Eq. (22) and it is implicit that the mode count is zero if $\kappa_r^2 < 0$. At the cut on frequencies, the group velocity is zero and the modal density defined by Eq. (18) is infinite [28]. This equation is therefore not useful and instead the modal density, defined for a frequency band of width $\Delta \omega = \omega_n - \omega_l$, is given by

$$n_r = \left(N_r(\omega_r) - N_r(\omega_l)\right) / \Delta \omega = S \left(\kappa_r^2(\omega_r) - \kappa_r^2(\omega_l)\right) / 4\pi \Delta \omega.$$  (27)

3.2.3 Cavity-wall element at and above the coincidence frequency

Eq. (23) is derived upon the assumption flexural motion of the walls is subsonic, which is not true at and above the coincidence frequency. At such high frequencies, it is seen in Figs. 5 and 6 that there is very little wall motion and, therefore, the cavity-wall element becomes a conventional SEA cavity element defined for rigid walls. The approximate solution (25) applies also above coincidence as it equals the solution for blocked wall motion. With rigid walls, instead of the eigen solution (21), we have $\gamma_0 = 0$. If, however, the double wall frequency is much lower than the coincidence frequency, this difference won’t affect the modal density (27) much. The transition of the cavity-wall element into a cavity element will, however, affect the conductivities. In particular, when the element is defined for rigid walls there is no direct transfer of energy between the element and the rooms, as is discussed in the next section.

4. Couplings in the SEA double wall model

The SEA elements used in the analysis of double walls are identified in the previous section. The structural coupling between the walls via the beams (coupling path G in
Fig. 7) is described as in Ref [1]. The indirect mass impeded coupling path through the entire structure, which exists below and around the double wall resonance, coupling path C, is defined by the mass law (12). All other conductivities are defined by the power radiated from a vibrating surface to an acoustic volume, as is described in what follows.

4.1 Radiation efficiency
Leppington et al [31] calculate the sound radiation into an infinite acoustic space from a rectangular simply supported thin walled plate, which is inserted into an infinite baffle and has its vibration velocity defined by the modes having resonances in a frequency band while these have, on average, equal vibration amplitude. The radiated sound power is given by

\[ P_{\text{rad}} = \rho_c c \sigma S \langle \tilde{v}^2 \rangle, \]  

(28)

where \( \sigma \) is the radiation efficiency and \( \langle \tilde{v}^2 \rangle \) is the plate’s temporal and spatial mean square vibration velocity in the transverse direction. The radiation efficiency is often expressed as a function of frequency. The analysis in Ref. [31] is, however, made in the wavenumber domain considering that the radiating surface’s vibrational waves have wavenumber \( k_f \) and the fluid waves \( k_f \). Thus, Leppington’s radiation efficiency [31, Eqs. (7.6), (7.7) and (7.11)] is given by

\[
\sigma(v) = \begin{cases} 
\frac{c(a + b)}{2 \pi^2 v k_f a b (v^2 - 1)^{1/2}} \left[ \ln \left( \frac{v + 1}{v - 1} \right) + \frac{2v}{v^2 - 1} \right], & v > 1 \\
0.5 - 0.15 \frac{a}{b} \sqrt{k_f a}, & v = 1 \\
(1 - v^2)^{-1/2}, & v < 1 
\end{cases}
\]

(29)

where \( a \) and \( b \) are the side lengths, so that \( S = a b, a \leq b \), and

\[ v = k_f / k_f. \]

(30)

4.2 Wall to room and wall to cavity-wall coupling
The conductivity is equated as in Eq. (6) based on the radiated power (28):

\[
C^{(i,j)} = \frac{c \sigma^{(i,j)} m_i \langle \tilde{v}^2_i \rangle}{\mu_i d \hat{e}_i},
\]

(31)
where $\hat{e}_i$ is the modal power associated with the wave motion considered and the quantity $m_i \langle \hat{v}_i^2 \rangle / \hat{e}_i$ is defined by the wave form associated with these waves. For the flexural vibration of plate $w$, radiating into an acoustic volume, $R$,

$$\sigma^{(w,R)} = \sigma \left( k_w / k_a \right)$$

and

$$m_w \langle \hat{v}_w^2 \rangle / \hat{e}_w = n_w$$

where $n_w$ is the modal density of plate $w$. Thus, the conductivity for the plate - room coupling (coupling path D in Fig 7) is defined.

The current work follows the practice in Refs. [15, 1] and models the radiation into the cavity as if the plate was radiating into an infinite space. The conductivity for the coupling between a plate and the plane cavity-wall element (coupling path E in Fig. 7) at frequencies below the cut-on of the first oblique cavity mode is therefore also defined by Eqs. (31) and (33). At frequencies above the cut-on, the sum of the conductivities for path E and F is likewise defined by Eqs. (31) and (33) while they are here set proportional to the modal density of the plane and oblique cavity-wall modes respectively. (Thus, it is assumed that all mode to mode coupling loss factors are equal.) Consequently, the conductivity between plate $w$ and cavity wall element $r$ is defined as:

$$C^{(w,r)} = \frac{c \sigma^{(w,r)} n_w n_r}{\mu_w d} n_{rot} ; \quad n_{rot} = \sum_r n_r .$$

The radiation efficiency $\sigma^{(w,r)}$ from the plate to the cavity is given by the radiation efficiency $\sigma \left( k_w / k_a \right)$ in Eq. (29) times the multipliers suggested by Craik and Smith in Fig. 5 of Ref. [1].

4.3 Cavity-wall to room coupling

To evaluate the conductivity between a cavity-wall element and a room, first the mean square vibration velocity of the wall is related to the modal power of a cavity-wall element; then, the radiation efficiency from the cavity-wall element to the room is defined.

4.3.1 Wall vibration in cavity-wall waves

In the cavity-wall element at frequencies below coincidence all strain energy is in the fluid, whereas the kinetic energy is both in the walls and in the fluid. The strain
energy density in an acoustic volume is $e_p = |\vec{p}|^2 / 2\rho_0 c^2$. For resonant modes, the averaged kinetic energy equals the averaged strain energy density, and the total acoustic energy in the cavity-wall element is $2\left\langle e_p \right\rangle_v V$, where $\left\langle e_p \right\rangle_v$ is the strain energy averaged over the volume $V$. Thus, the modal power potential for a cavity-wall element $r$ is given by

$$\hat{e}_r = \frac{S |\vec{p}_0|^2}{4n_r \rho_0 c^2} \int_0^d \left( \sin \gamma z + \mu_2 \gamma d \cos \gamma z \right)^2 d z,$$

if wave type $r$ is propagating and zero otherwise; the factor $1/4$ comes from averaging the strain energy in a diffuse wave field in the $x$-$y$ plane. The integral in Eq. (35) could be solved analytically, while it is here evaluated by quadrature at no numerical cost. From Eqs. (20) and (21) it follows that the spatially and temporally squared averaged vibration velocity of plate $w$ associated with cavity-wall element $r$ is:

$$\left\langle \vec{v}_{w,r}^2 \right\rangle = \frac{|\vec{p}_0|^2}{4 \left( \omega \rho_0 d \mu_w \right)^2} \left( \sin \gamma_z + \mu_2 \gamma d \cos \gamma_z \right)^2,$$

It follows that

$$\frac{m_w \left\langle \vec{v}_{w,r}^2 \right\rangle}{\hat{e}_r} = \frac{n_r c^2}{\omega^2 S d \mu_w} \int_0^d \left( \sin \gamma z + \mu_2 \gamma d \cos \gamma z \right)^2 d z.$$

For heavy walls, London’s formula applies and the quantity $\gamma_0 d$ is small. A series expansion then provides an approximate plane wave cavity-wall to room conductivity, while this is still work in progress.

**4.3.2 Sound radiation from wall vibrations in cavity-wall waves**

The ratio of ms vibration velocity of the wall to the modal power of the cavity-wall waves is calculated above; to evaluate the conductivity (31), the radiation efficiency for these waves is also required. It is typically assumed that the radiation efficiency of a limp wall forced by a diffuse sound field equals unity [1, 15, 19]. Craik, however, argues in Ref. [5] that plane acoustic waves between two walls have sonic speed. Consequently he defines the radiation efficiency by its value at coincidence, and, as seen in Fig 1, the SEA prediction is improved. The results for fully coupled fluid-structure motion in Fig. 3 show that Craik’s surmise is almost correct: the cavity-wall waves are supersonic, asymptotically approaching sonic speed as frequency increase.
The radiation efficiency is thus given by $\nu < 1$ in Eq. (29), while it should not exceed the maximum value for sonic wave speed given by Eq. (29) for $\nu = 1$.

At and above coincidence, the cavity pressure no longer excites the mass impeded wall motion, but the resonant modes only. At these higher frequencies, the importance of the resonant wall vibrations increase, the cavity-wall waves turn into cavity waves with blocked walls and the room to cavity-wall coupling vanishes. A dodge to account for the two latter effects is to multiply the radiation efficiency associated with this coupling with the factor $(1 - \sigma^{w,R})$ or zero, whichever is larger. Consequently, the radiation efficiency from cavity wall element $r$ to room $R$ is given by

$$\sigma_r^{(r,R)} = \min \left( \sigma(k_r / k_a), \sigma(1) \right) \cdot \max \left( 1 - \sigma(k_w / k_a), 0 \right).$$  \hspace{1cm} (38)

Upon this result, the conductivity for the coupling of a cavity wall element and a room is

$$C^{(r,R)}_w = \rho_0 c S \sigma_r^{(r,R)} \frac{m_w \left< \tilde{v}_{w,r}^2 \right>}{\hat{e}_r},$$  \hspace{1cm} (39)

where the factor $m_w \left< \tilde{v}_{w,r}^2 \right> / \hat{e}_r$ is defined in Eq (37).

All the parameters of the cavity-wall element needed for the SEA of a double wall are now detailed: its scheme is illustrated in Fig. 7.

4.3.3 Comment on the radiation efficiency

The radiation efficiencies for a plate and a cavity-wall element, both radiating into a room, are different. The waves defining the cavity-wall element are supersonic, approaching sonic speed as frequency increases. The radiation efficiency for a plate, on the other hand, approach as frequency increases its maximum value at coincidence and decreases towards unity at even higher frequencies. Fig. 8 describes this peculiar behaviour.

5. Comparison to measurements

The performance of the proposed SEA model is evaluated for three kinds of double wall structures found in the literature: $i$) three building constructions [1], $ii$) two aluminium structures [4] and $iii$) a double glass window [3]. Tables 1-3 detail the properties of the investigated structures.

The quantity of interest is the SRI which for large rooms with sufficient damping is given by
\[ R = 10 \log \left( \frac{1}{\tau} \right); \quad \tau = \frac{P_{\text{inc}}}{P_{\text{trans}}} = \frac{(M \hat{e})_{\text{receiver room}}}{\left( \frac{k^2 S}{8 \pi^2} \right) \hat{e}_{\text{source room}}} = \frac{A \hat{e}_{\text{receiver room}}}{S \hat{e}_{\text{source room}}}. \] (40)

\( P_{\text{inc}} \) is the power coming onto the partition of area \( S \), \( P_{\text{trans}} \) is the power transmitted to the receiving room where the equivalent absorption area is \( A \).

5.1 Estimates of damping

The damping loss factors are required to calculate the modal overlap factors in Eq. (9) and have a strong influence on the results. It is, nevertheless, often difficult to retrieve damping values from the quoted articles. Some considerations are needed.

The damping loss factors of the rooms are based on their equivalent absorption areas, which are so large that their precise values are irrelevant for the calculated SRI.

In Reference [1], measurements of the reverberation time for the 100 mm cavity are reported. In the absence of any other information, it is here assumed that the damping loss factors for the plane and oblique cavity-wall elements are equally defined by this reverberation time. It is, furthermore, assumed that the damping loss factors for the 50 mm and the 150 mm cavities are given by the one for the 100 mm cavity in inverse proportion to the cavity depth. The arguments for this assumption are that most of the damping arises from viscous forces and heat conduction at the walls and frames and from air pumping at the interface of these structural components. Therefore, the losses might be independent of cavity depth in which case the loss factor is inversely proportional to cavity depth. The reverberation time is measured in situ and is therefore defined by the total losses including both damping and coupling losses. The latter might have been estimated by the SEA model and subtracted from the total losses though this was not made here. There is no information about the cavity damping in Refs. [3-4].

Wall-loss measurements are provided in Refs. [1, 4]; Ref. [1] presents these measurements for one of the plasterboard walls forming a part of the 50 mm double wall. The information on damping loss is consequently incomplete and, furthermore, do not distinguish between total and internal damping losses. The SEA predictions that correspond to the measurements in references must therefore be based on some speculations; Table 1-3 defines the current authors’ estimates.
5.2.1 Building constructions

Fig. and Figs. 9-10 present the sound reduction indexes of the three building construction double walls; the results from the new SEA model are compared to those from the old model and to the measurements in Ref. [1].

The agreement in the low frequency region, below, around and an octave above the double wall resonance, is good but somewhat erratic, probably so as the mode count is rather low for most of the elements. There is, however, a consistent overestimation of the SRI in the 160 Hz band, which might be caused by the increased mobility of the plates because of resonances. The proposed SEA model performs quite well at the coincidence frequency in the 3.15 kHz band, in particular for the 100 mm cavity. The calculation results presented here and those by Craik and Smith [1] should be identical in the high frequency range. This is not exactly so, which might be caused by difficulties in reading the data. In the frequency region between the double wall resonance and coincidence, the agreements between the new model and the measurements are excellent for the 50 mm and the 100 mm cavities and quite good for the 150 mm cavity. The increase with frequency of the calculated SRI is generally correct and, given the expected uncertainty of an SEA in general, and of the loss factors in particular, the results are better than warranted.

5.2.2 Aluminium double wall

Reference [4] presents measurements of the SRI of aluminium double walls, with cavity depths of 125 mm and 250 mm. These measurements are compared to SEA results in Figs. 11-12. For both constructions, the calculated SRI is too large at frequencies below the coincidence whereas around this frequency the agreement is good. It is interesting to note that the cut-on of oblique cavity modes, around 700 Hz and 1.4 kHz, hardly influence the SRI.

Reference [4] claims that there is no flanking transmission in the measurement setup. It is, however, very difficult to completely avoid flanking transmission and leakage and it might be that such coupling paths restricts the SRI below 60 dB. It is also plausible that there is some direct coupling between the plates if the fluid near field extends from one plate to the other, as would be the case at low frequencies and frequencies not much below the coincidence frequency. To mimic such imperfections a minute coupling loss between the plates is added to the SEA model: \( \eta_c = 10^{-5} \). Figs 11-12 show that this minor modification increases the agreement between measurements and calculations.
5.2.3 Glass window

Ref. [3] presents measurements and calculations of the SRI of a double glass window, which are compared to results from the proposed SEA. Fig. 13 shows that at low and middle frequencies the agreement is rather satisfying, whereas a bit below and above the coincidence frequency the SEA prediction is quite wrong.

One reason for this might be that the constant, frequency independent, loss factor is not correct at all frequencies. It is also plausible that there is some direct coupling between the glasses caused by flanking transmission or fluid near field transmission. It turns out that if the damping of the glass plates is reduced and a small coupling loss is added, the SEA prediction considerably improves as Fig. 13 shows.

5.3 A reduced SEA model

In the mid-frequency region, between the double wall frequency and either the cut-on frequency for the oblique cavity waves or the coincidence frequency, whichever comes first, the sound transmission is dominated by the room to cavity-wall to room path (coupling paths A in Fig 7). This is illustrated in Fig 14 where the results from a three element SEA model are compared to those of the full model.

5.3.1 A simplified formula for the transmission coefficient

Based on the reduced SEA model, a formula for the transmission efficiency of double walls is derived upon the assumptions that the walls have equal properties. It is moreover assumed that the walls are heavy compared to the contained air and that cavity response is damping controlled. In mathematical terms it is thus assumed that

\[ \mu_1 = \mu_2 = \mu \ll 1; \quad C^{(r,R1)} = C^{(r,R2)} \ll M, \quad (41) \]

From the heaviness of the walls it follows that \( (\gamma_0 d) \ll 1 \). Now, calculating the transmission efficiency from the SEA model and evaluating the resulting expression to leading order of the small parameters in Eq. (41), it follows that

\[ \tau = \frac{0.23 c^5 a T_k}{d^6 \mu_2^2 \omega^4} = \frac{\pi a 4 \left( 0.5 - 0.15 \frac{a}{b} \right)^2}{d \mu^3 \eta (k_a d)^5} \quad (42) \]

The SRI thus increases with 15 dB per octave, with 12 dB per mass doubling and with 3 dB per loss factor doubling. The formula for the SRI of double walls taught in fundamental courses in noise and vibration control, e.g., [32-33] has the same dependence on mass but predicts an increase of the SRI with 12 dB octave and is
independent of damping. The SRI in Fig 1 and Fig 14 increases about 13 dB/octave. This is a typical characteristic shown by all the double walls herein investigated (the reverberation time $T_R$ measured by Craik and Smith decreases approximately with 1.3 dB/octave). Fig 14 shows the SRI for the 100 mm building construction double wall calculated with the full SEA model, the reduced SEA model and the simplified formula for the transmission coefficient.

The building constructions considered herein are well described by the approximations (41); however, for cavities that have response that is not controlled by damping but by coupling loss, such that $M, M_{C,C} = C_{(r,R)}^{(r,R1)} = C_{(r,R2)}^{(r,R2)}$, it is possible to derive the following approximated formula:

$$\tau = \frac{\pi \sqrt{k_a a}}{\mu^3 (k_a d)^3},$$

(43)

6. Conclusions

The present article develops the classical SEA model for double walls [1-15]. The investigated structures have wall masses that are large compared to the mass of the contained fluid, i.e., $\mu > 1$, and the coincidence, when uncoupled flexural plate waves and acoustic waves have equal wave length, is at a rather high frequencies. Ref. [1] shows that the old SEA model and measurements compare poorly in a mid-frequency range, from the double wall resonance to a bit below the coincidence. The new model improves mid-frequency results, while at frequencies below and above this region it equals the old model.

First, the waveguide FEM is used as a diagnostic tool to find the waves that travels along the fully coupled wall – cavity – wall system. It is seen that there are no waves in the cavity at frequencies below the double-wall resonance. At frequencies above this resonance there are quasi-plane fluid waves that involve quite large, mass impeded, wall motion; these are the cavity-wall waves. At even higher frequencies, when the cavity depth is larger than half the acoustic wave length, there are oblique cavity waves. The walls’ in-plane and flexural waves are almost as for the walls in isolation. At lower frequencies there are, however, fluid near fields that decay away from one wall towards the other. These near fields couple the walls’ motion and an improved low frequency model, of particular interest for shallow cavities, would describe this coupling. Similarly, as the frequency approaches the coincidence frequency, the near fields should extend further away from the walls and there might be a direct wall to wall coupling. Neither of these improvements was attempted here.
It was, however, observed that a minute *ad hoc* wall to wall coupling improved the SEA predictions at frequencies somewhat below the coincidence.

The new SEA model is depicted in Fig. 7. It contains two acoustic elements, describing the source and receiver rooms, and two plate elements, describing flexural vibrations in the walls. For the building constructions examples, it also contains a beam element describing the frames erected between the walls. These five elements and the couplings between them are modelled as in Ref. [1].

The modes that correspond to the cavity-wall waves define one SEA element for the cavity; the other element is defined by the oblique cavity waves. Below the double-wall resonance there is no transmission through the cavity but only the mass impeded room to room coupling (path C).

The new element formulation obsoletes the non-resonant transmission paths used in earlier double-wall formulations. Instead there is a direct coupling between the acoustic volumes and the resonant cavity-wall modes, which depends on the cross sectional wave form. The conductivity also depends on the radiation efficiency; it is unity at the double-wall resonance (as in Refs. [1]) and then increases towards the value at sonic wave speed (as in Ref. [5]). Similarly, the modal density is high at the resonance and then decreases towards the one for a two-dimensional cavity. These factors explain why the reduction index does not have a distinct minimum at the double-wall resonance but instead have a rather constant value in a frequency region, which extends roughly an octave above this resonance.

The new SEA model is compared to three sets of measurements found in the literature: a building construction double wall [1], an aluminium double wall [4] and a double glass window [8]. The SEA agrees with Craik and Smith’s measurements [1]; in fact, the agreement for the 50 mm and 100 mm double-walls is much better than warranted by the uncertainty in the damping loss estimates. On the other hand, the result for the 150 mm building construction is only quite good as are the results for the aluminium double walls and the glass window. This might be explained by uncertainty in the damping estimates. Also, if a minute direct coupling between the plates is introduced, predictions are improved at frequencies a bit below the coincidence, at which the SRI is very high, which either suggests in-accuracies in the measurements or, more likely, the need for further improvements of the SEA model.

Finally, this successful derivation emphasises that an SEA element is an element of response that need not be localised to one substructure of the whole structure. The
successful application of SEA to a new kind of structure, therefore, requires
diagnostic measurements and diagnostic calculations so that the proper elements can
be identified.

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References
1. R.J.M. Craik, R.S. Smith, Sound transmission through double leaf lightweight
223-245.
2. R.J.M. Craik, R.S. Smith, Sound transmission through lightweight parallel
3. M. Villot, C. Guigou and L. Gagliardini Predicting the acoustical radiation of
finite size multi-layered structures by applying spatial windowing on infinite
4. V. Hongisto, M. Lindgren, R. Helenius, Sound insulation of double walls – an
904-923.
5. R.J.M. Craik, Non-resonant sound transmission through double walls using
6. J. Brunskog, The influence of finite cavities on the sound insulation of double-
3727-3739.
7. J. Wang, T.J. Lu, J. Woodhouse, R.S. Langley, J. Evans, Sound transmission
through lightweight double-leaf partitions: theoretical modelling, Journal of
8. J-D Chazot, J-L Guayder, Prediction of transmission loss of double panels
with a patch-mobility method, Journal of the Acoustical Society of America


Figure 1. Sound reduction index of building constriction with a cavity depth of 100mm. Measurements, solid line [1]; Old model, dotted line [1]; Ref [5], dash-dot line; new model, dashed line.
Fig. 2 Waveguide FE mesh for the building construction with 100 mm cavity depth. Left, mesh used for calculations; Right, mesh used for displaying wave forms.
Fig. 3 Dispersion curves for a double wall. Calculated with WFEM (large dots); calculated for uncoupled substructures: bending waves (full curves), acoustic waves (dashed curve); longitudinal waves in plates (dotted curves). The wave that starts propagating around 87 Hz is the cavity-wall wave and the one around 1700 Hz is the first oblique cavity waves. Curves for bending and longitudinal waves are double because the plates are slightly different.
Fig. 4. Displacements of the fluid and the plates in flexural waves. Top left 2.1 Hz; top right 2.9 Hz; bottom left, 116 Hz; bottom right, 136 Hz;
Fig. 5a. Displacements of the fluid and the plates in cavity-wall wave just above the cut-on at 87 Hz.

Fig. 5b. Displacements of the fluid and the plates in cavity-wall wave at 620 Hz.
Fig. 6. Displacements of the fluid and the plates in oblique cavity wave just above the cut-on at 1700 Hz.
Fig. 7. Proposed SEA model for a double wall. The couplings are identified with arrows. A) Cavity-wall element to room, Eq. (39) for $r = 0$; B) Oblique cavity element to room, Eq. (39) for $r \geq 1$; C) Room to room coupling defined by the mass-law, Eq. (12); D) Plate to room, Eq. (32); E) Plate to cavity-wall element, Eq. (34) for $r = 0$; F) Plate to oblique cavity element, Eq. (34) for $r \geq 1$; G) Plate to frames, Ref [1].
Fig. 8. Radiation efficiency, up until the coincidence frequency, as a function of the wavenumber in the structure $k_b$ and the wavenumber in air $k_a$. The full circle indicates the cavity-wall wave and the solid line its evolution with frequency. The full triangle indicates the flexural wave in a plate and the dashed line its evolution with frequency.
Fig. 9. SRI of a 50 mm building construction double wall. Measurement (full curve) and calculation (dotted curve) both from Ref [1]; new SEA model (dashed line).
Fig. 10. SRI of a 150 mm building construction double wall. Measurement (full curve) and calculation (dotted curve) both from Ref [1]; new SEA model (dashed line).
Fig. 11. SRI of a 125 mm aluminium double wall. Measurement (full curve) [4]; new SEA model (dotted line); new SEA model with a plate to plate coupling loss factor $\eta_c = 10^{-5}$ (dashed line).
Fig. 12. SRI of a 250 mm aluminium double wall. Measurement (full curve) [4]; new SEA model (dotted line); new SEA model with a plate to plate coupling loss factor $\eta_c = 10^{-5}$ (dashed line).
Fig. 13 SRI of a 12 mm double glass window. Measurement (full curve) [8], calculation (dotted curve) [8]; new SEA model (dashed lines): A) wall loss factor $\eta_w = 0.05$ (as suggested in Ref. [8]); B) $\eta_w = 0.005$ ; C) $\eta_w = 0.005$ and with a plate to plate coupling loss factor $\eta_c = 10^{-3}$. 
Fig. 1 SRI for a 100 mm building construction double wall. Measurement (dashed line) 5, proposed SEA model (thick line), reduced SEA model (thin line) simple formula Eq. (38) (squared line). It is strange that Eq. (38) is 5dB out.
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Table 2 Aluminium
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Table 3 Glass
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