On the Use of Wind Power for Transient Stability Enhancement of Power Systems

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Abstract

This report deals with the impact of doubly fed induction generators on the stability of a power system. The impact was quantified by means of detailed numerical simulations. The report contains a full description of the simulation, and details of the small signal analysis performed to analyse the system.

Before the simulation results are presented, a foundation is laid, explaining the theory required to understand the models used and the calculations performed in the simulation.

The derivation of a model of a doubly fed induction generator is presented, along with a description of the model of a synchronous generator. These are used in the simulation and analysis of a multi-machine power system, consisting of both of these types of generators.

An explanation of how dynamic simulations of power systems can be performed is also put forward. This is useful, not only for understanding the simulation performed for this report, but as a guide to performing simulations of this type. This is true also for a description of linearisation and small signal analysis contained in this report.

The software package MATLAB is used to perform the simulations, and the small signal analysis. Since the method described in this report is very general, it can be used to perform similar power system simulations for other power systems, and with other software.

Numerical simulations reveal that the addition of doubly fed generators, such as those in wind parks, to a power system improves the response of the system after small disturbances, but can worsen it after larger disturbances.
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Chapter 1

Introduction

As time progresses, wind power is becoming an ever more significant source of energy. The general community is looking more towards wind power to provide a renewable source of energy, and the role of wind power in power systems is becoming increasingly important.

Many of the newer, larger turbines being produced are variable speed turbines, which use doubly fed induction generators (DFIG). These are induction generators which have their stator and rotor independently excited. Turbines of this type are becoming increasingly popular, because the power converters required to control them are cheaper and less subject to power losses than those required for wound-rotor induction generators. Additionally DFIGs have certain reactive power control capabilities.

The fact that these turbines are being so commonly used makes it important to understand them. It is known that DFIGs are able to maintain their voltage at or near its steady state value when subjected to small perturbations. Because of this, it is thought that implementing controls to stabilise a power system through these generators may be useful. Synchronous generators are currently employed to do this, but it is thought that using a DFIG to do so may add additional functionality. Considering the number of DFIGs in the power system, this assumption should be investigated.

Power systems consist of a large number of elements. In this report we will consider only a very simple system as an example, and will investigate the effect of a wind park on a power system.

It can be shown that a number of generators which are close together, and behave similarly can be modelled as a single generator [2]. Then it is valid to assume that an aggregate of DFIGs, such as a wind park, can be modelled as a single DFIG, and a similar assumption can be made for synchronous generators, which usually make up the bulk of power systems.

To perform this investigation, it is necessary to develop an appropriate model for a DFIG,
and find an appropriate model for a system in which to place it, so that simulations can be run.

Control systems affecting the behaviour of DFIGs should be implemented at a later stage. These include the controls which are present in wind turbines, and external controls used to damp oscillations in the electric system. However in this report we only consider the effect of an uncontrolled DFIG, which emulates the behaviour of a wind park, on a power system, which is modelled as a single synchronous generator.
Chapter 2

Modelling

When modelling a power system, it is assumed that a wind park can be modelled as a single DFIG, while a typical power system can be modelled as a single synchronous generator. Then in order to investigate the effect of a wind park on a power system, we shall investigate the effect a DFIG has on a synchronous generator.

2.1 Doubly Fed Induction Generator

In order to begin our investigation, we must first develop the model we will use for the DFIG.

Let us look at the electrical equations for a machine. The stator and rotor voltages each contain components which correspond to ohmic losses, and the rate of change of flux-linkage. The fundamental Kirchhoff’s, Faraday’s, and Newton’s laws give [15]

\[ v_{abc} = R_s i_{abc} + \frac{d\lambda_{abc}}{dt} \] (2.1)

\[ v_{abcr} = R_r i_{abcr} + \frac{d\lambda_{abcr}}{dt} \] (2.2)

where the subscripts \( s \) and \( r \) denote the stator and rotor quantities of voltages \( v \), resistances \( R \), currents \( i \) and flux linkages \( \lambda \), and

\[ f_{abc} = \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \] (2.3)

are quantities in the \( a \), \( b \) and \( c \) axes, the axes perpendicular to the \( a \), \( b \) and \( c \) windings as shown in the schematic in Figure 2.1.
These equations can be transformed to a \( dq \) coordinate system as shown in Figure 2.2 using the following transformation matrix \([8]\)

\[
T_{dq0} = \frac{2}{3} \begin{bmatrix}
\cos(\beta) & \cos(\beta - \frac{2\pi}{3}) & \cos(\beta + \frac{2\pi}{3}) \\
-\sin(\beta) & -\sin(\beta - \frac{2\pi}{3}) & -\sin(\beta + \frac{2\pi}{3}) \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{2}
\end{bmatrix}
\] \( (2.4) \)

Then

\[
f_{dq0} = \begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix} = T_{dq0} f_{abc}. \] \( (2.5) \)

The inverse transformation is given by \( T_{qd0}^{-1} \).

The quantity \( \beta \) is the angle between the reference frame and the circuit frame. We define the speed of the reference frame to \( \omega_s \), the synchronous speed. Thus for the stator circuit, which is stationary, the relative speed is \( \omega_s \) and the angle \( \beta = \beta_s \) is equal to \( \omega_s t \) for all stator quantities. For the rotor circuit, \( \beta = \beta_r = (\omega_s - \omega_r)t \), where \( \omega_r \) is the electrical speed

\[
\omega_r = p\omega_m
\]

with \( p \) the number of pole pairs of the machine, and \( \omega_m \) being the mechanical speed of the rotor. The references are shown in Figure 2.2.
Under balanced conditions, such as in a symmetrical machine, we may neglect the zero sequence components \( f_0 \) and examine only the quantities

\[
f_{dq} = \begin{bmatrix} f_d \\ f_q \end{bmatrix},
\]

whereupon we can used the simplified transform [4]

\[
T_{dq} = \frac{2}{3} \begin{bmatrix}
\cos(\beta) & \cos(\beta - \frac{2\pi}{3}) & \cos(\beta + \frac{2\pi}{3}) \\
-\sin(\beta) & -\sin(\beta - \frac{2\pi}{3}) & -\sin(\beta + \frac{2\pi}{3})
\end{bmatrix}.
\]

(2.6)

Then

\[
f_{dq} = \begin{bmatrix} f_d \\ f_q \end{bmatrix} = T_{dq} f_{abc}
\]

(2.7)

where the inverse transformation is given by

\[
T_{dq}^{-1} = \begin{bmatrix}
\cos(\beta) & -\sin(\beta) \\
\cos(\beta - \frac{2\pi}{3}) & -\sin(\beta - \frac{2\pi}{3}) \\
\cos(\beta + \frac{2\pi}{3}) & -\sin(\beta + \frac{2\pi}{3})
\end{bmatrix}
\]

(2.8)

such that

\[
T_{dq} T_{dq}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\]

(2.9)
Then
\[ \mathbf{f}_{abc} = \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = T_{dq}^{-1} \mathbf{f}_{dq}. \] (2.10)

We will be considering a symmetrical DFIG in this report.

In a reference frame rotating at the synchronous speed \( \omega_s \), which we shall call the synchronous reference frame, \( \beta = \omega_s t \) and we get the following equations

\[ \mathbf{v}_{dqs} = R_s \mathbf{i}_{dqs} + \frac{d \mathbf{\lambda}_{dqs}}{dt} + \mathbf{I} \omega_s \mathbf{\lambda}_{dqs} \]  (2.11)

\[ \mathbf{v}_{dqr} = R_r \mathbf{i}_{dqr} + \frac{d \mathbf{\lambda}_{dqr}}{dt} + \mathbf{I} (\omega_s - \omega) \mathbf{\lambda}_{dqr}. \]  (2.12)

Here we have renamed \( \omega_r \) as \( \omega \), the electrical speed of the rotor, and

\[ \mathbf{I} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \]  (2.13)

The currents are given by the flux linkage-current relations

\[ \mathbf{\lambda}_{dqs} = L_s \mathbf{i}_{dqs} + L_m \mathbf{i}_{dqr} \]  (2.14)

\[ \mathbf{\lambda}_{dqr} = L_r \mathbf{i}_{dqr} + L_m \mathbf{i}_{dqs} \]  (2.15)

where the relationships between the inductances \( L \) are given by

\[ L_s = L_{ls} + L_m \]  (2.16)

\[ L_r = L_{lr} + L_m \]  (2.17)

and subscripts \( l \) and \( m \) denote leakage and magnetising quantities respectively.

It is convenient to express these equations in a standard per unit system as is shown in Appendix B, which is used throughout this report.

It is also convenient to consider the \( d \) and \( q \) axis components of all quantities as real and imaginary components of phasors, defined

\[ \overline{\mathbf{f}} = \mathbf{f}_d + j \mathbf{f}_q. \]

Equations (2.11) and (2.11) are then reformed as

\[ \overline{\mathbf{v}}_s = R_s \overline{\mathbf{i}}_s + \frac{1}{\omega_s} \frac{d \overline{\mathbf{\lambda}}_s}{dt} + j \overline{\mathbf{\lambda}}_s \]  (2.18)

\[ \overline{\mathbf{v}}_r = R_r \overline{\mathbf{i}}_r + \frac{1}{\omega_s} \frac{d \overline{\mathbf{\lambda}}_r}{dt} + j \frac{(\omega_s - \omega)}{\omega_s} \overline{\mathbf{\lambda}}_r. \]  (2.19)
2.1. **DOUBLY FED INDUCTION GENERATOR**

As reactances are usually given as machine parameters instead of inductances, we will express the previous equations in terms of reactances. Here the reference frame is travelling at synchronous speed \( \omega_s \), and using this as the base speed in a per unit system we have

\[
\overline{\lambda} = \overline{\psi}, \text{ flux} \\
L = X, \text{ reactance.}
\]

We would like to model the machine as a generator, so all stator currents are negated.

So the voltage equations become

\[
\overline{v}_s = -R_s \overline{t}_s + \frac{1}{\omega_s} \frac{d\overline{\psi}_s}{dt} + j \overline{\psi}_s \\
\overline{v}_r = R_r \overline{t}_r + \frac{1}{\omega_s} \frac{d\overline{\psi}_r}{dt} + j \left( \frac{\omega_s - \omega}{\omega_s} \right) \overline{\psi}_r
\]

(2.20, 2.21)

and the flux-reactance equations are now

\[
\overline{\psi}_s = -X_s \overline{t}_s + X_m \overline{t}_r \\
\overline{\psi}_r = X_r \overline{t}_r - X_m \overline{t}_s.
\]

(2.22, 2.23)

The mechanical system is described by the following equation quantities [5]

\[
\frac{d\omega}{dt} = \frac{1}{M} \left( P_m \frac{\omega_s}{\omega} - P_e \right)
\]

(2.24)

where

\[
M = \frac{2H}{\omega_s}
\]

and \( H \) is the inertia constant in seconds.

The four electrical Equations (2.20), (2.21), (2.22), (2.23) and the mechanical Equation (2.24) describe the fifth order model for the DFIG.

It is useful for this study to represent the DFIG as a voltage \( E' \) behind a transient reactance \( X' \), such that

\[
\overline{v}_s = E' - jX' \overline{t}_s.
\]

(2.25)

In this way the model for the DFIG then becomes easier to implement in power system simulations.

Expressing the stator fluxes in terms of stator currents and rotor fluxes we get

\[
\overline{\psi}_s = \frac{X_m}{X_r} \overline{\psi}_r - \left( X_s - \frac{X_m^2}{X_r} \right) \overline{t}_s.
\]

(2.26)
For the purposes of this investigation we assume that the quantities $R_s$ and $\frac{1}{\omega_s} \frac{d\phi_s}{dt}$ are negligible, based on engineering insights, and have little impact on the system dynamics. In this case the order of the model is reduced by two, and the Equation (2.20) becomes

$$\overline{v}_s = j\overline{\psi}_s.$$  \hspace{1cm} (2.27)

Multiplying Equation (2.26) by $j$ we get

$$\overline{v}_s = j\frac{X_m}{X_r}\overline{\psi}_r - j\left(X_s - \frac{X_m^2}{X_r}\right)\overline{i}_s$$  \hspace{1cm} (2.28)

and comparing this with Equation (2.25) we get

$$X' = X_s - \frac{X_m^2}{X_r}$$  \hspace{1cm} (2.29)

$$\overline{E}' = j\frac{X_m}{X_r}\overline{\psi}_r.$$  \hspace{1cm} (2.30)

Equation (2.21) now becomes

$$\frac{d\overline{E}'}{dt} = \frac{1}{T_0} \left(jT_0\omega_s \frac{X_m}{X_r}\overline{v}_r - jT_0(\omega_s - \omega)\overline{E}' - \frac{X}{X'}\overline{E}' + \frac{X - X'}{X'}\overline{v}_s\right)$$  \hspace{1cm} (2.31)

where $X = X_s$ and $T_0$ is the transient open-circuit time constant

$$T_0 = \frac{X_r}{\omega_s R_r}.$$  \hspace{1cm} (2.32)

We can make the substitution

$$\overline{V}_r = \frac{X_m}{X_r}\overline{\psi}_r$$  \hspace{1cm} (2.33)

$$\overline{V} = \overline{v}_s$$  \hspace{1cm} (2.34)

since the linear relation between $\overline{V}_r$ and $\overline{\psi}_r$ will not affect any simulations. Dropping the subscript $s$ simplifies the expression, which becomes

$$\frac{d\overline{E}'}{dt} = \frac{1}{T_0} \left(jT_0\omega_s \overline{V}_r - jT_0(\omega_s - \omega)\overline{E}' - \frac{X}{X'}\overline{E}' + \frac{X - X'}{X'}\overline{V}\right).$$  \hspace{1cm} (2.35)

The magnitude and the angle of the voltage $E'$ are

$$E' = \sqrt{E'^2_d + E'^2_q}$$  \hspace{1cm} (2.36)

$$\delta = \tan^{-1}\left(\frac{E'_q}{E'_d}\right)$$  \hspace{1cm} (2.37)
such that
\[ E' = E'_d + jE'_q = E' e^{j\delta}. \] (2.38)
and similarly
\[ V_r = V_{dr} + jV_{qr} = V_r e^{j\theta_r}. \] (2.39)
\[ V = V_d + jV_q = V e^{j\theta}. \] (2.40)

Now
\[ \frac{dE'}{dt} = \frac{dE' e^{j\delta}}{dt} \] (2.41)
\[ = e^{j\delta} \left( \frac{dE'}{dt} + jE' \frac{d\delta}{dt} \right) \] (2.42)
\[ = \frac{e^{j\delta}}{T_0} \left( jT_0 \omega_s V_r e^{j(\theta_r - \delta)} - jT_0 (\omega_s - \omega) E' - \frac{X - X'}{X'} V e^{j(\delta - \theta)} \right). \] (2.43)

Comparing the real and imaginary parts of the bracketed part of this expression yields
\[ \frac{dE'}{dt} = \frac{1}{T_0} \left( -\frac{X}{X'} E' + \frac{X - X'}{X'} V \cos(\delta - \theta) + T_0 \omega_s V_r \sin(\delta - \theta_r) \right) \] (2.44)
\[ \frac{d\delta}{dt} = \frac{1}{E'T_0} \left( -T_0 (\omega_s - \omega) E' - \frac{X - X'}{X'} V \sin(\delta - \theta) + T_0 \omega_s V_r \cos(\delta - \theta_r) \right) \] (2.45)
and we can rewrite Equation (2.24) as [5]
\[ \frac{d\omega}{dt} = \frac{1}{M} \left( P_m \omega_s - \frac{E' V}{X'} \sin(\delta - \theta) \right). \] (2.46)

Equations (2.44), (2.45) and (2.46) describe a third-order model for the DFIG, and this model will be used in this study.

Then the model rewritten in matrix form is
\[ \begin{bmatrix} \dot{\delta} \\ \dot{\omega} \\ \dot{E'} \end{bmatrix} = \begin{bmatrix} \frac{1}{E'T_0} \left( -T_0 (\omega_s - \omega) E' - \frac{X - X'}{X'} V \sin(\delta - \theta) + T_0 \omega_s V_r \cos(\delta - \theta_r) \right) \\ \frac{1}{M} \left( P_m \omega_s - \frac{E' V}{X'} \sin(\delta - \theta) \right) \\ \frac{1}{T_0} \left( -\frac{X}{X'} E' + \frac{X - X'}{X'} V \cos(\delta - \theta) + T_0 \omega_s V_r \sin(\delta - \theta_r) \right) \end{bmatrix}. \] (2.47)
2.2 Synchronous Generator

To study the impact of the DFIG on the dynamics of a synchronous generator, we must also have equations governing the dynamics of the synchronous generator. There are many well documented models which exist to varying levels of complexity, but the simplest one to choose is one with the same order of equations as the model we have developed for the DFIG. Higher order models add little extra information.

The mechanics of the synchronous generator are modelled by the Swing Equation [2]

\[ M \frac{d^2 \delta}{dt^2} = P_m - P_e - D\omega \]

which can be decomposed to a set of two differential equations,

\[ \frac{d\delta}{dt} = \omega - \omega_s \]

\[ \frac{d\omega}{dt} = \frac{1}{M} \left( P_m - \frac{E'V}{X'} \sin(\delta - \theta) \right) \]

when \( D = 0 \). These are easier to work with.

The transient emf \( E' \) is derived somewhat differently to that in the previous section. The quantity \( \delta \) for the synchronous generator describes the rotor angle here, rather than the orientation of the internal emf as for the DFIG. In order to reach a simplified first order model of this quantity, we assume that

- the stator resistance \( R_s \) and stator transients \( \frac{1}{\omega_s} \frac{d\psi_s}{dt} \) are negligible
- the \( d \) and \( q \) axes contain no damper windings

In a per unit system, the fundamental Equations (2.1), (2.2) can be written

\[ \overline{v}_s = -R_s \overline{i}_s + \frac{1}{\omega_s} \frac{d\overline{\psi}_s}{dt} + j\overline{\psi}_s \]

\[ \overline{v}_r = R_r \overline{i}_r + \frac{1}{\omega_s} \frac{d\overline{\psi}_r}{dt} \]

where

\[ \overline{v}_r = \begin{bmatrix} V_F \\ 0 \end{bmatrix} \]

which only has a component \( V_F \), the voltage associated with the field winding, in the \( d \) axis direction.
2.2. SYNCHRONOUS GENERATOR

Following now an identical procedure as described for the DFIG yields

\[
\frac{dE'}{dt} = \frac{1}{T_0} \left( jT_0 \omega_s \nabla_r - \frac{X}{X'} E' + \frac{X - X'}{X'} V \right)
\]  

(2.55)

and since the \( q \)-axis damper winding is neglected it follows that

\[
E' = \begin{bmatrix} 0 \\ E'_q \end{bmatrix}.
\]  

(2.56)

So Equation (2.55) becomes the real valued equation

\[
\frac{dE'}{dt} = \frac{1}{T_0} \left( E_F - \frac{X}{X'} E' + \frac{X - X'}{X'} V \cos(\theta - \delta) \right)
\]  

(2.57)

where \( E_F \) is the field voltage

\[
E_F = \frac{X_r}{R_r} V_F.
\]  

(2.58)

This model of an uncontrolled synchronous generator is called the One-Axis Model, and is of the same order as our DFIG dynamic equations. The set of equations in matrix form is then

\[
\begin{bmatrix}
\dot{\delta} \\
\dot{\omega} \\
\dot{E'}
\end{bmatrix} =
\begin{bmatrix}
\frac{\omega - \omega_s}{M}
\frac{1}{M} \left( P_m - \frac{E' V}{X'} \sin(\delta - \theta) \right)
\frac{1}{T_0} \left( E_F - \frac{X}{X'} E' + \frac{X - X'}{X'} V \cos(\delta - \theta) \right)
\end{bmatrix}.
\]  

(2.59)

The state \( E' \) for the synchronous generator models only the quadrature component of the electromotive force, emf, and is commonly referred to as \( E'_q \) as in [2]. In this report we shall only refer to it as \( E' \), as its meaning for the DFIG is not the same as that for the synchronous generator, and is not important for this analysis of stability.

Excitation System of the Synchronous Generator

An uncontrolled synchronous generator has a constant field voltage, \( E_F \), exciting its field winding. However, in general, a generator is controlled in such a way that its terminal voltage is relatively constant, and the generator is in a stable state.

Automatic Voltage Regulator (AVR)

The automatic voltage regulator is a simple feedback system which compares the terminal voltage to some reference, and scales the difference, called the ‘error signal’ to a level suitable to feed the field winding. A schematic of the AVR is shown in Figure 2.4.

The symbol \( s \) in this schematic is the Laplace operator. From this we can describe, \( E_F \), the signal produced. Directly from the schematic we have

\[
(V_{REF} + V_{PSS} - V) \frac{K_A}{1 + T_e s} = E_F
\]  

(2.60)

and, rearranging and replacing the Laplace operator with the differential operator gives

\[
\dot{E}_F = \frac{1}{T_e} \left(-E_F + K_A(V_{REF} + V_{PSS} - V)\right).
\]  

(2.61)

**Power System Stabiliser (PSS)**

A small signal analysis, described in Chapter 5, reveals the well known instability property of the AVR when subject to a small disturbance. The field winding needs to be fed with an AVR, but this can cause the generator to become unstable under a small disturbance\(^1\).

In order to compensate for this problem, a power system stabiliser is required. The stabiliser adds a certain signal to the input of the AVR, based on how far the speed of the generator deviates from the reference speed \( \omega_s \). A simplified schematic of the PSS is shown in Figure 2.5.

As above, \( s \) is the Laplace operator. Directly from the schematic we have

\[
(\omega - \omega_s)K_{PSS} \frac{1 + T_1 s}{1 + T_2 s} = V_{PSS}
\]  

(2.62)

and, rearranging and replacing the Laplace operator with the differential operator gives

\[
\dot{V}_{PSS} = \frac{1}{T_2} \left((\omega - \omega_s)K_{PSS} + T_1 K_{PSS} \dot{\omega} - V_{PSS}\right).
\]  

(2.63)

---

\(^1\)When subject to a large disturbance, the AVR contributes to enhancing stability.
2.2. SYNCHRONOUS GENERATOR

\[ (\omega - \omega_s) \rightarrow K_{PSS} \rightarrow \frac{1 + T_1 s}{1 + T_2 s} \rightarrow V_{PSS} \]

Figure 2.5: Power System Stabiliser

where we can replace \( \dot{\omega} \) with the expression from the decomposed swing equation

\[ \dot{V}_{PSS} = \frac{1}{T_2} \left( (\omega - \omega_s)K_{PSS} + T_1 K_{PSS} \left( \frac{1}{M} \left( P_m - \frac{E'V}{X'} \sin(\delta - \theta) \right) \right) - V_{PSS} \right). \tag{2.64} \]

Now the extended set of equations describing a controlled synchronous generator in matrix form is

\[
\begin{bmatrix}
\dot{\delta} \\
\dot{\omega} \\
\dot{E'} \\
\dot{E}_F \\
\dot{V}_{PSS}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{M} \left( P_m - \frac{E'V}{X'} \sin(\delta - \theta) \right) \\
\frac{1}{T_0} \left( E_F - \frac{X'}{X'} E' + \frac{X - X'}{X'} V \cos(\delta - \theta) \right) \\
\frac{1}{T_e} \left( -E_F + K_A (V_{REF} + V_{PSS} - V) \right) \\
\frac{1}{T_2} \left( (\omega - \omega_s)K_{PSS} + T_1 K_{PSS} \left( \frac{1}{M} \left( P_m - \frac{E'V}{X'} \sin(\delta - \theta) \right) \right) - V_{PSS} \right)
\end{bmatrix}.
\tag{2.65}
\]

We shall be using both the controlled and uncontrolled models for comparison in the simulations to follow.
Chapter 3

Case Study

In a power system there are many elements, including different types of generators and loads with many interrelations between them. Finding these interrelations involves using network matrices to make calculations [3]. As a first step towards considering general power systems, we look at a simple example as shown in Figure 3.1.

3.1 System

![System configuration](image)

Figure 3.1: System configuration
A synchronous generator, $G_S$ and a second generator, $G_A$, are connected by lines to a bus, which is, in turn, connected to an infinite bus by means of two longer lines. The infinite bus has a constant voltage $V_1$ with a constant angle $\theta_1$.

It is easier to consider system with equivalent voltages and impedances replacing all components in the system. Remembering that our generators are modelled as a transient emf $E'$ behind a transient reactance $X'$, the network can be redrawn as shown in Figure 3.2.

![Figure 3.2: Single Line Diagram](image)

Here single subscripts $k$ denote quantities measured at bus $k$. Double subscripts $ki$ denote quantities measured from $k$ to $i$.

Now if the transformers are modelled as reactances, they are taken into account within the definition of $X'$ defined in Equation (2.29) as

$$X' = X_s - \frac{X_m^2}{X_r}$$

(3.1)

where the stator reactance $X_s$ is the sum of the stator reactance of the generator and the reactance of the transformer.
3.2 Configurations

In order to examine how the presence of a DFIG impacts the stability of the synchronous generator $G_S$, it is necessary to know how $G_S$ would behave in both the presence and absence of a DFIG. This can be seen by changing the generator type of the second generator $G_A$. Having $G_A$ as a DFIG shows us how a DFIG affects $G_S$, and using a synchronous generator instead of the DFIG provides an ideal system for comparison.

It is also interesting to note how the $G_S$ behaves in each case with and without a control system.

The systems we shall study contain

- $G_S$ uncontrolled with $G_A$ synchronous
- $G_S$ uncontrolled with $G_A$ a DFIG
- $G_S$ controlled with $G_A$ synchronous
- $G_S$ controlled with $G_A$ a DFIG

We shall examine these four configurations of the system.

3.3 Disturbances

We would like to see how the system behaves when it is disturbed. A selection of typical disturbances has been chosen for this study.

- Disturbance 1: Small Disturbance
  A small disturbance should exhibit the behaviour predicted by a linearised model. The eigenvalues of a linearised model predict the frequencies present in a system after a small disturbance, and the extent to which they are dampened. A small disturbance is put into effect by reducing mechanical power of $G_S$ by 10% for a short period of time $t = 0.1s$.

- Disturbance 2: Transient Disturbance
  As a comparison to the small disturbance, we look at how the system behaves during a large disturbance. The large disturbance modelled is the disconnection of one of the lines connected to the infinite bus for a short period of time $t = 0.1s$. This effectively doubles the reactance of the connection to the infinite bus. The post disturbance conditions for this are the same as the previous example, which allows for a comparison between them.
Disturbance 3: Large Disturbance

Finally we look at a very large disturbance, where one of the lines connected to the infinite bus is disconnected permanently. This demonstrates extreme behaviour by the system.
Chapter 4

Simulation

Now that we have descriptions of the behaviour of the generators, and a system in which to view this behaviour, a simulation must be run in order to examine the behaviour of a generator.

We start by looking at the steady state conditions, which are required for initialising the simulation.

4.1 Load Flow

In order to find the initial conditions for the simulation, we must first perform load flow analysis.

The technique of determining all bus voltages in a network is called load flow. The system state is described completely by these voltages, allowing us to calculate the initial conditions for the system, which are required to perform a simulation. A non linear system of equations exists describing power flow between each of the buses in the system, which we must solve in order to find the complex values of the voltages at each of the buses in steady state.

A matrix equation describing the relations between the voltages, $V$, at the buses and currents injected into buses, $I$, in a system can be formulated using Kirchhoff’s current laws

$$\begin{bmatrix}
I_1 \\
\vdots \\
I_n
\end{bmatrix} =
\begin{bmatrix}
Y_{11} & \cdots & Y_{1N} \\
\vdots & \ddots & \vdots \\
Y_{N1} & \cdots & Y_{NN}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
\vdots \\
V_n
\end{bmatrix}$$

(4.1)

where the subscripts refer to bus numbers, and $N$ is the number of buses in the system.
This can be rewritten as
\[ I = Y_{bus} V \]
where \( Y_{bus} \) is called the Y-bus matrix. The diagonal elements \( Y_{kk} \) are the sum of all admittances connected to Bus \( k \), and the off-diagonal elements \( Y_{ki} \) are the negatives of the admittances between Buses \( k \) and \( i \). The ground is not included as a bus.

The injected current into Bus \( k \) is given by
\[ I_k = \sum_{i=1}^{N} Y_{ki} V_i \] (4.2)
where \( Y_{ki} \) is the element of the Y-bus matrix in the \( k \)th row and \( i \)th column. This subscript system will be used for other matrices of this type.

The injected complex power into Bus \( k \) is
\[ S_k = V_k I_k^* \] (4.3)
\[ = V_k \sum_{i=1}^{N} Y_{ki}^* V_i^* \] (4.4)
\[ = \sum_{i=1}^{N} V_k V_i Y_{ki} (\cos(\theta_{ki} - \psi_{ki}) + j \sin(\theta_{ki} - \psi_{ki})) \] (4.5)
\[ = \sum_{i=1}^{N} (P_{ki} + jQ_{ki}) \] (4.6)
\[ = P_k + jQ_k \] (4.7)

where \( P_k, Q_k \) are the real and reactive parts of \( S_k \), \( P_{ki} \) and \( Q_{ki} \) are the real and reactive parts of power transmitted from Bus \( k \) to Bus \( i \), \( \theta_{ki} \) is the difference between voltage phase angles, \( \theta_k - \theta_i \), and
\[ Y_{ki} = Y_{ki} e^{j\psi_{ki}}. \] (4.8)

Starting with an initial guess for the solution to all the unknown voltage magnitudes and angles, better estimates can be made in an iterative manner using the Newton-Raphson Method, hopefully converging to the solution.

**Newton-Raphson Method**

The Newton-Raphson method is a root-finding algorithm which uses a first order Taylor expansion of a function to find its solution in the vicinity of a suspected root [20]. A detailed algorithm for applying the Newton-Raphson Method to load flow can be found in [17].
4.1. LOAD FLOW

Initial Values

There are three kinds of buses in a power system:

- Slack buses, where the voltage magnitudes and angles are defined
- PV buses, where the real power and voltage magnitude is defined
- PQ buses, where real and reactive powers $P_G$ and $Q_G$ are defined.

There should be only one slack bus in a system. The net productions at each bus

\[ P_{GD} = P_G - P_D \text{ for PV and PQ buses} \quad (4.9) \]
\[ Q_{GD} = Q_G - Q_D \text{ for PQ buses} \quad (4.10) \]

where the subscripts $G$ and $D$ denoted the generated and demanded powers respectively.

In the system we have here, however, there will be no loads. In any case, it is the net power production which is important in these load flow studies.

The Y-bus matrix is also required to perform further calculations.

In order to start the load flow, we must start with an initial estimate for the values sought for each bus, $V$ and $\theta$. A standard estimate sets all the unknown voltages and angles to those of the slack bus.

Iterative Calculations

The following calculations should be performed iteratively until a solution is reached. The injected power into each bus is calculated, given the voltage estimates. The deviation of these values from those known to be true are used to update the voltage magnitudes and angles in such a way that the deviation becomes smaller.

We can calculate the power injected to each bus by using the equations above, explicitly

\[ P_k = V_k \sum_{i=1}^{N} V_i \cos(\theta_{ki} - \psi_{ki}) \quad (4.11) \]
\[ Q_k = V_k \sum_{i=1}^{N} V_i \sin(\theta_{ki} - \psi_{ki}) \quad (4.12) \]

Then the difference between the net production and the injected powers, $\Delta P_k$ and $\Delta Q_k$ are calculated for those buses where these quantities are known.

\[ \Delta P_k = P_{GDk} - P_k \quad k \neq \text{slack bus} \quad (4.13) \]
\[ \Delta Q_k = Q_{GDk} - Q_k \quad k \neq \text{slack bus or PV bus} \quad (4.14) \]
If the magnitude of all entries of $\Delta P$ and $\Delta Q$ are within a small positive constant $\epsilon$, then the current values evaluated for the bus voltages and angles are acceptable. The load flow algorithm can be terminated since the voltage magnitudes and angles have been found. Otherwise, we should continue with further calculations to update our last estimate.

If we have not reached a suitable solution to these equations, then we must update our current evaluation of the bus voltage magnitudes and angles in such a way that they approach the true solution. This requires the use of the Jacobian matrix of the function which describes the linear relation between the inputs $\frac{\Delta V}{V}$ and $\theta$ and the outputs $P$ and $Q$.

The Jacobian matrix $J_{LF}$ is formulated such that

$$
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = J_{LF} \begin{bmatrix}
\Delta \theta \\
\frac{\Delta V}{V}
\end{bmatrix} = \begin{bmatrix}
H & N \\
J & L
\end{bmatrix} \begin{bmatrix}
\Delta \theta \\
\frac{\Delta V}{V}
\end{bmatrix}
$$

(4.15)

where

$$
\begin{align*}
P &= [P_1 \ldots P_N]^T \\
Q &= [Q_1 \ldots Q_N]^T
\end{align*}
$$

(4.16)

(4.17)

and the entries of the matrix are as follows

$$
\begin{align*}
H_{ki} &= \frac{\partial P_k}{\partial \theta_i} = Q_{ki} & j \neq k, \text{ slack bus} \\
N_{ki} &= V_i \frac{\partial P_k}{\partial V_i} = P_{ki} & j \neq k, \text{ slack bus, PV-bus} \\
J_{ki} &= \frac{\partial Q_k}{\partial \theta_i} = -P_{ki} & j \neq k, \text{ slack bus} \\
L_{ki} &= V_i \frac{\partial Q_k}{\partial V_i} = Q_{ki} & j \neq k, \text{ slack bus, PV-bus} \\
H_{kk} &= \frac{\partial P_k}{\partial \theta_k} = Q_{ki} - Q_k & k \neq \text{ slack bus} \\
N_{kk} &= V_k \frac{\partial P_k}{\partial V_k} = P_{ki} + P_k & k \neq \text{ slack bus, PV-bus} \\
J_{kk} &= \frac{\partial Q_k}{\partial \theta_k} = -P_{ki} + P_k & k \neq \text{ slack bus} \\
L_{kk} &= V_k \frac{\partial Q_k}{\partial V_k} = Q_{ki} + Q_k & k \neq \text{ slack bus, PV-bus}
\end{align*}
$$

and from Equation (4.5)

$$
\begin{align*}
P_{ki} &= V_k V_i Y_{ki} (\cos(\theta_{ki} - \psi_{ki})) \\
Q_{ki} &= V_k V_i Y_{ki} (\sin(\theta_{ki} - \psi_{ki})).
\end{align*}
$$

(4.18)

(4.19)

The amounts by which our estimates should be updated are then

$$
\begin{bmatrix}
\Delta \theta \\
\frac{\Delta V}{V}
\end{bmatrix} = \begin{bmatrix}
H & N \\
J & L
\end{bmatrix}^{-1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
$$

(4.20)

and the updated values are

$$
\begin{align*}
\theta_k &= \theta_k + \Delta \theta_k & k \neq \text{ slack bus} \\
V_k &= V_k \left(1 + \frac{\Delta V_k}{V_k}\right) & k \neq \text{ slack bus, PV-bus}
\end{align*}
$$

(4.21)

(4.22)
4.2 Initial Conditions

Now initial conditions for the power system have been solved for, the initial conditions of the generators and control system can be calculated.

**Synchronous Generator**

From load flow analysis, we can determine the voltage magnitudes and angles at all buses in steady state. Let us look at a bus \( k \) supplied by a generator. We know that

\[
S_k = V_k I_k^* \tag{4.23}
\]

\[
P_k + jQ_k = V_k e^{j\theta_k} I_k^* \tag{4.24}
\]

giving the current \( I_k^* \) through the line as follows

\[
I_k = \left( \frac{P_k + jQ_k}{V_k e^{j\theta_k}} \right)^* \tag{4.25}
\]

from which can find the initial complex value of the transient emf of the generator at bus \( k \)

\[
E_k' = V_k e^{j\theta_k} + X' I \tag{4.26}
\]

\[
= E_k e^{j\delta_k}. \tag{4.27}
\]

The mechanical dynamics of the synchronous generator can be described by

\[
\frac{d\omega}{dt} = \frac{1}{M} (P_m - P_e). \tag{4.28}
\]

Setting this derivative to zero, the mechanical power in steady state is solved as

\[
P_m = P_e. \tag{4.29}
\]

The mechanical power, \( P_m \), remains constant during normal operation.

Also in steady state all time derivatives are zero. So

\[
\dot{E}' = 0 = \frac{1}{T_0} \left( E_F - \frac{X}{X'} E' + \frac{X - X'}{X'} V \cos(\delta - \theta) \right) \tag{4.30}
\]

\[
\dot{E}_F = 0 = \frac{1}{T_e} (-E_F + K_A (V_{REF} + V_{PSS} - V)) \tag{4.31}
\]

and to satisfy this requirement we must have

\[
E_F = \frac{X}{X'} E' + \frac{X - X'}{X'} V \cos(\delta - \theta) \tag{4.32}
\]

\[
V_{REF} = \frac{E_F}{K_A} + V. \tag{4.33}
\]
In steady state also
\[ V_{PSS} = 0 \] (4.34)
since no stabilisation should be required.

**Doubly Fed Induction Generator**

Many of the initial conditions for a synchronous generator in steady state apply to DFIGs in steady state. The ones which differ in our model are described here.

Recall that the mechanical dynamics of the DFIG was described by
\[
\frac{d\omega}{dt} = \frac{1}{M} \left( P_m \frac{\omega_s}{\omega} - P_e \right) \tag{4.35}
\]
The steady state electrical speed is usually given in terms of slip \( s \) defined by
\[
s = \frac{\omega_s - \omega}{\omega_s} \tag{4.36}
\]
Setting the derivative in Equation (4.35) to zero, and using the definition of slip, this can be solved to give the mechanical power in steady state as
\[
P_m = \frac{\omega}{\omega_s} P_e = P_e(1 - s) \tag{4.37}
\]
which remains constant during normal operation.

We must now find the value that \( V_r = V_r e^{j\theta_r} \) takes in steady state. All time derivatives are zero so
\[
\dot{\delta} = 0 = \frac{1}{E'T_0} \left( -T_0(\omega_s - \omega)E' - \frac{X - X'}{X'} V \sin(\delta - \theta) + T_0 \omega_s V_r \cos(\delta - \theta_r) \right)
\]
\[
\dot{E'} = 0 = \frac{1}{T_0} \left( -\frac{X}{X'} E' + \frac{X - X'}{X'} V \cos(\delta - \theta) + T_0 \omega_s V_r \sin(\delta - \theta_r) \right).
\]
Solving these equations we get
\[
V_r \sin(\delta - \theta_r) = \frac{1}{T_0 \omega_s} \left( \frac{X}{X'} E' - \frac{X - X'}{X'} V \cos(\delta - \theta) \right) = \alpha
\]
\[
V_r \cos(\delta - \theta_r) = \frac{1}{T_0 \omega_s} \left( T_0(\omega_s - \omega)E' - \frac{X - X'}{X'} V \sin(\delta - \theta) \right) = \beta
\]
\[
V_r = \sqrt{\alpha^2 + \beta^2}
\]
\[
\theta_r = \delta - \tan^{-1} \left( \frac{\alpha}{\beta} \right).
\]
The quantity \( V_r \) remains constant throughout simulations.
4.3 Dynamics

Control Constants

Recall Figures 2.4 and 2.5. There are five selectable constants $K_A$, $T_e$, $K_{PSS}$, $T_1$ and $T_2$, which must be chosen for the generator controllers.

In the simulation, we have chosen to use typical values for the controllers $K_A$ and $T_e$, which define the behaviour of the AVR. Then, in order to tune the PSS, we have chosen a typical value for $T_1$, and from there, optimal values of $K_{PSS}$ and $T_2$ can be found. A method for choosing these will be described in Chapter 5.

The values chosen for $K_A$, $T_e$ and $T_1$ can be found in Appendix C.

4.3 Dynamics

Once the steady state conditions have been determined, we are ready to see what happens after the system is disturbed. For this, we need to set up a framework describing the system dynamics.

A set of differential equations governs the rotation of the generators against their buses. A set of algebraic equations describing the power flow out of buses in the system can be used to find the complex voltages at the buses. The differential and algebraic equations can solved simultaneously using the MATLAB function ode23t [19].

Differential Equations

Recall that the synchronous generator dynamics were described in Equations (2.65). In a slightly different form for convenience, the dynamic equations for one generator are

$$
\begin{align*}
\begin{bmatrix}
\frac{\omega - \omega_s}{M} \\
\frac{1}{T_0} \left( E_F - \frac{X}{X'} E' + \frac{X - X'}{X'} V \cos(\delta - \theta) \right) \\
\frac{1}{T_e} \left( K_A (V_{ref} + V_{PSS} - V) \right) \\
\frac{1}{T_2} \left( (\omega - \omega_s) K_{PSS} + T_1 K_{PSS} \left( \frac{1}{M} \left( P_m - \frac{E' V}{X'} \sin(\delta - \theta) \right) \right) - V_{PSS} \right)
\end{bmatrix}
\end{align*}
$$

where $\mathbf{x}$ is the set of states

$$
\mathbf{x} = \begin{bmatrix} \delta & \omega & E' & E_F & V_{PSS} \end{bmatrix}^T,
$$
and \( y \) is the set of algebraic variables

\[
x = [\theta \ V]^T.
\]

These equations describe a synchronous generator controlled with an exciter. For generators in the power system without control, the quantities \( \frac{dE_F}{dt} \) and \( \frac{dV_{PSS}}{dt} \) are set to zero.

The equations describing the dynamics of the DFIG were described in Equation (2.47), which can be rewritten for convenience as

\[
f(x, y) = \begin{bmatrix}
\frac{1}{E'T_0} \left( -T_0(\omega_s - \omega)E' - \frac{X - X'}{X'} V \sin(\delta - \theta) + T_0\omega_s V_r \cos(\delta - \theta_r) \right) \\
\frac{1}{M} \left( P_m \frac{\omega_s}{\omega} - \frac{E'V}{X'} \sin(\delta - \theta) \right) \\
\frac{1}{T_0} \left( -\frac{X'}{X}E' + \frac{X - X'}{X'} V \cos(\delta - \theta) + T_0\omega_s V_r \sin(\delta - \theta_r) \right) \\
0 \\
0
\end{bmatrix}.
\]

(4.39)

The complete set of equations describing the system is formed by a compilation of these states. We rename \( x \) ‘the states of the system’, and redefine it so

\[
\delta = [\delta_1 \ldots \delta_{N'}]^T
\]

(4.40)

\[
\omega = [\omega_1 \ldots \omega_{N'}]^T
\]

(4.41)

\[
E' = [E'_1 \ldots E'_{N'}]^T
\]

(4.42)

\[
E_F = [E_{F1} \ldots E_{F{N'}}]^T
\]

(4.43)

\[
V_{PSS} = [V_{PSS1} \ldots V_{PSS{N'}}]^T
\]

(4.44)

\[
x = \begin{bmatrix}
\delta \\
\omega \\
E' \\
E_F \\
V_{PSS}
\end{bmatrix}
\]

(4.45)

where \( N' \) is the number of generators, and the subscript of each state denotes the generator the state belongs to.

We also rename \( y \) ‘the algebraic variables of the system’, and redefine it so

\[
\theta = [\theta_1 \ldots \theta_N]^T
\]

(4.46)

\[
V = [V_1 \ldots V_N]^T
\]

(4.47)

\[
y = \begin{bmatrix}
\theta \\
V
\end{bmatrix}
\]

(4.48)
where \( N \) is the number of buses, and the subscript of each algebraic variable denotes the bus it belongs to.

### Algebraic Equations

The state differential equations depend on the values \( V \) and \( \theta \) at the buses in the system. For a system of \( N \) buses, we have \( 2N \) additional variables, which require a further set of \( 2N \) equations to solve for. A readily available set of equations are those describing the power flow out of each bus.

The injected complex power into Bus \( k \) is given by

\[
S_k = \sum_{i=1}^{N} (P_{ki} + jQ_{ki})
\]  

which can be decomposed to

\[
P_k = \sum_{i=1}^{N} P_{ki} = \sum_{i=1}^{N} V_k V_i Y_{ki} \cos(\theta_{ki} - \psi_{ki})
\]

\[
Q_k = \sum_{i=1}^{N} P_{ki} = \sum_{i=1}^{N} V_k V_i Y_{ki} \sin(\theta_{ki} - \psi_{ki}).
\]

These describe the power flow within the network. When the entire system is considered, the power flowing between the generator and its local bus must also be taken into account. The same is true for loads, but they are not being considered in this report.

For every generator bus, the power flow out of a node is

\[
0 = \frac{E_k' V_k}{X_k'} \sin(\theta_k - \delta_k) + P_k
\]

\[
0 = \frac{V_k^2}{X_k'} - \frac{E_k' V_k}{X_k'} \cos(\theta_k - \delta_k) + Q_k...
\]

For every other bus, the power flow out of a node is

\[
0 = P_k
\]

\[
0 = Q_k.
\]

This is a complete set of algebraic equations

\[
0 = g(x, y)
\]

describing the power flow in the system.

Finally, the complete system dynamics are described by

\[
\begin{cases}
\dot{x} = f(x, y) \\
0 = g(x, y)
\end{cases}
\]
CHAPTER 4. SIMULATION

Disturbance

The disturbance is simulated by dividing time into three parts. For each part, we run the integrating function \texttt{ode23t}, which requires initial conditions to start.

- **Pre disturbance**
  The system is in steady state from when we start the simulation \( t_0 \) to the time the disturbance starts \( t_f \). The set of algebraic and differential equations are integrated with the initial conditions of the simulation until \( t_f \).

- **Disturbance**
  The system is in a disturbed state from the time \( t_f \) when the disturbance starts until the disturbance is over \( t_c \). We simulate a disturbance by changing some system input to the function \texttt{ode23t}, before running the simulation again from \( t_f \) to \( t_c \). The initial conditions here are the final conditions of the previous integration.

- **Post disturbance**
  After the disturbance is over, the system is in a post disturbance state from the time \( t_c \) when the disturbance is over to till the end of the simulation \( t_{fin} \). The system inputs are set to their post disturbance state before the function is integrated again, and the initial conditions are the final conditions of the previous integration.
Chapter 5

Linearised System

A simple control system involves the use of linear control. There are more advanced non-linear techniques involving the use of energy and Lyapunov functions, but here we consider only a simple case, to demonstrate the capability of a DFIG to influence the stability of a synchronous generator.

5.1 Linearisation

In order to see the effects of a disturbance in a linear system, we can look at its eigenvalues. Since we intend only to operate closely about a stable operating point, we can linearise the system about this point, which gives us a close approximation to our system.

The behaviour of a dynamic system can be described by a set of first order differential equations [8]

$$\dot{x} = f(x, y).$$  

(5.1)

where $f$ is a function of $x$, the state variables, and $y$, the algebraic variables which influence the states.

Similarly, there are outputs, which are described algebraically, and can be written

$$0 = g(x, y).$$  

(5.2)

The equilibrium of these points occurs when all time derivatives of the states are zero, when all the states are constant in time. Equilibrium points satisfy the equation

$$\dot{x} = f(x_0, y_0) = 0$$  

(5.3)

where $x_0$ is the state variable vector at the equilibrium point, and $y_0$ the corresponding set of algebraic variables.
If we consider a nearby solution to the equilibrium point

\[ x = x_0 + \Delta x \]  
\[ y = y_0 + \Delta y \]  

where \( \Delta x \) and \( \Delta y \) are small, then

\[ \dot{x} = \dot{x}_0 + \Delta \dot{x} \]  
\[ = f(x_0 + \Delta x, y_0 + \Delta y). \]  

A Taylor expansion yields

\[ x_0 + \Delta x = f(x_0, y_0) + A\Delta x + B\Delta y + O(\|\Delta x, \Delta y\|^2) \]  
\[ y_0 + \Delta y = g(x_0, y_0) + C\Delta x + D\Delta y + O(\|\Delta x, \Delta y\|^2) \]

where

\[
A = \left. \frac{\partial f(x, y)}{\partial x} \right|_{x_0, y_0}, \quad \text{dim}(A) = N' \times N' \\
B = \left. \frac{\partial f(x, y)}{\partial y} \right|_{x_0, y_0}, \quad \text{dim}(B) = N' \times N \\
C = \left. \frac{\partial g(x, y)}{\partial x} \right|_{x_0, y_0}, \quad \text{dim}(C) = N \times N' \\
D = \left. \frac{\partial g(x, y)}{\partial y} \right|_{x_0, y_0}, \quad \text{dim}(D) = N \times N
\]

are Jacobian matrices evaluated at the equilibrium point.

Since the deviations denoted by \( \Delta \) are small, a first order approximation can be taken without too much loss of information. Also the algebraic equations, \( g \), can be written in such a way that

\[ 0 = g(x, y). \]  

so

\[
\Delta \dot{x} = A\Delta x + B\Delta y \]  
\[ 0 = C\Delta x + D\Delta y. \]

Provided \( D \) is non-singular, we find the relation

\[
\Delta \dot{x} = (A - BD^{-1}C)\Delta x \\
= \Lambda \Delta x
\]

where \( \Lambda \) is the state matrix describing the linear relationships between the states \( x \).
5.2 Building the Linearised Matrix

To find the linear matrix describing our system, we must find the matrices $A$, $B$, $C$, and $D$ defined above.

The states of the system were defined in Equation (4.45), and the algebraic variables in Equation (4.48). Again, these are

$$ x = \begin{bmatrix} \delta \\ \omega' \\ E' \\ E_F \\ V_{PSS} \end{bmatrix}, $$

$$ y = \begin{bmatrix} \theta \\ V \end{bmatrix}. $$

Recall the synchronous generator dynamics were described by Equations (4.38) and (4.39).

The matrices $A$ and $B$ are straightforward to calculate.

The matrices $C$ and $D$ can be formed by looking at the algebraic equations $g$. Recall these from Equations (4.50) to (4.53).

It is only the algebraic variables $y$ which determine the power flow within the network. The powers flowing from a bus $k$ towards its generator are

$$ P_{gk} = \frac{E'_k V_k}{X'_k} \sin(\theta_k - \delta_k) $$

$$ Q_{gk} = \frac{V^2_k}{X'_k} - \frac{E'_k V_k}{X'_k} \cos(\theta_k - \delta_k). $$

Then the algebraic equations can be rewritten

$$ 0 = P_{gk}(x, y) + P_k(y) \text{ for } k = \text{generator bus} $$

$$ 0 = Q_{gk}(x, y) + Q_k(y) \text{ for } k = \text{generator bus} $$

$$ 0 = P_{gk} = Q_{gk} \text{ for } k \neq \text{generator bus}. $$

Then the matrix $C$ is then the Jacobian of these equations with respect to $x$.

The matrix $J_{LF}$ describing the relationship between the powers injected to the network, from Equation (4.7),

$$ S_k = V_k I_k^* = V_k \sum_{i=1}^N Y_{ki}^* V_i^* $$
and the algebraic variables $y$ was previously calculated in Section 4.1. The Jacobian $D$ of the equations $g$ with respect to $y$ is almost the same as was calculated in the load flow procedure.

The Jacobian was previously defined by

$$
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = J_{LF} \begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix} = \begin{bmatrix}
H & N \\
J & L
\end{bmatrix} \begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix}
\tag{5.21}
$$

where the entries of $J_{LF}$ were defined in Section 4.1. The power flowing from each generator to the system is also function of $y$. We must take this into account when examining the Jacobian matrix.

We require the matrix $D$ which satisfies

$$
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} = D \begin{bmatrix}
\Delta \theta \\
\Delta V
\end{bmatrix}.
\tag{5.22}
$$

Assuming $D$ takes the form

$$
D = \begin{bmatrix}
H_D & N_D \\
J_D & L_D
\end{bmatrix}\bigg|_{x_0,y_0}
\tag{5.23}
$$

we perform the following manipulations to derive $D$:

Form the matrices $N''$ and $L''$ such that

$$
N''_{ki} = \frac{N_{ki}}{V_i} \quad J''_{ki} = \frac{N_{ki}}{V_i}.
\tag{5.24}
$$

Then, by considering the power from the generators, we have

$$
H_D = H + \frac{\partial P_g}{\partial \theta} \quad \tag{5.26}
$$

$$
N_D = N'' + \frac{\partial P_g}{\partial V} \quad \tag{5.27}
$$

$$
J_D = J + \frac{\partial Q_g}{\partial \theta} \quad \tag{5.28}
$$

$$
L_D = L'' + \frac{\partial Q_g}{\partial V}. \quad \tag{5.29}
$$

where

$$
P_g = \begin{bmatrix}
P_{g1} & \cdots & P_{gN}
\end{bmatrix}^T \quad \tag{5.30}
$$

$$
Q_g = \begin{bmatrix}
Q_{g1} & \cdots & Q_{gN}
\end{bmatrix}^T \quad \tag{5.31}
$$

where $N$ is the number of buses, and the subscript of each algebraic variable denotes the bus it belongs to.
Our system contains an infinite bus, which is unchanging. Once all references to this have been removed, we have the linearised system

\[
\Delta \dot{x} = (A - BD^{-1}C)\Delta x \quad (5.32)
\]

\[
= \Lambda \Delta x \quad (5.33)
\]

where \( \Lambda \) is the Jacobian matrix of the linearised system.

Details of all particular entries of matrices can be found in Appendix D.

5.3 Eigenvalues

The eigenvalues of the Jacobian \( \Lambda \) describing the linearised system tell us something about how the system behaves. If the system is disturbed slightly, so as not to move the trajectory of the system too far from the linearised path, the eigenvalues \( \lambda \) take the form

\[
\lambda = \sigma \pm j\omega_p \quad (5.34)
\]

where \( -\sigma \) is the damping of the frequency \( \omega_p \) at which the system oscillates.

Extracting information contained in the eigenvalues of a linearised system is sometimes referred to as ‘small signal analysis’.

Control Constants

Recall the discussion in Section 4.2 on choosing control constants. There are five constants which required choosing to complete the control design. These were \( K_A, T_e, K_{PSS}, T_1 \) and \( T_2 \).

The constants \( K_A, T_e \) and \( T_1 \) are chosen based on typical values. The following strategy for choosing \( K_{PSS} \) and \( T_2 \) is used, which utilises the eigenvalues of the state matrix \( \Lambda \).

The damping ratio of an eigenvalue, \( \zeta \), is defined as [8]

\[
\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega_p^2}}. \quad (5.35)
\]

A two dimensional array of least damping constants \( \zeta(K_A, T_2) \) can be computed numerically for different values of \( K_A \) and \( T_2 \). The maximum \( \zeta \) can be found in this array, and the corresponding constants \( K_A \) and \( T_2 \) chosen for use in the model. The constants used for this study can be found in Appendix C.
Stability

The eigenvalues of the system also tell us something of the stability of the equilibrium point of the system. From the expressions for $\lambda$ above, we can see that the eigenvalues which are further towards $-\infty$ the eigenvalues are from the imaginary axis, the higher the damping ratios and the damping of the system. On the other hand, if any eigenvalue of the system crosses the imaginary axis, and falls into the right half plane, the system contains negative damping, and hence is unstable. In Section 6 we shall examine the eigenvalues in our system.
Chapter 6

Results

Here we present the results of simulations, which show how the dynamics of generator $G_S$ are affected by generator $G_A$, in the system described in Chapter 3.

6.1 Eigenvalues

In Chapter 5 we described how to find the eigenvalues $\lambda$ of a system at an equilibrium point. We now look at the eigenvalues for each configuration of the system in this case study, evaluated at its post disturbance equilibrium point. These are described in Section 3.2.

For the four configurations mentioned, when the post disturbance state the same as the pre disturbance state described in Section 4.2, the eigenvalues are found in Tables 6.1-6.4, together with their corresponding damping ratios $\zeta$ and frequencies of oscillation $f_p$ in Hz.

When the disturbance in the system is Disturbance 3 in Section 3.3, as a result of one of the lines to the infinite bus being removed permanently, the post disturbance equilibrium

<table>
<thead>
<tr>
<th>eigenvalues $\lambda$</th>
<th>damping ratio $\zeta$</th>
<th>frequency $f_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.0181183 \pm j11.0153$</td>
<td>0.0016448</td>
<td>1.7531</td>
</tr>
<tr>
<td>$-0.069114 \pm j7.0375$</td>
<td>0.0098203</td>
<td>1.1201</td>
</tr>
<tr>
<td>$-7.674$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-0.3856$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6.2: $G_S$ uncontrolled, $G_A$ a DFIG

<table>
<thead>
<tr>
<th>eigenvalues $\lambda$</th>
<th>damping ratio $\zeta$</th>
<th>frequency $f_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-6.21793 \pm j12.9711$</td>
<td>0.43227</td>
<td>2.0644</td>
</tr>
<tr>
<td>$-0.47839 \pm j8.5667$</td>
<td>0.055756</td>
<td>1.3634</td>
</tr>
<tr>
<td>$-2.3171$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-0.33883$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.3: $G_S$ controlled, $G_A$ synchronous

<table>
<thead>
<tr>
<th>eigenvalues $\lambda$</th>
<th>damping ratio $\zeta$</th>
<th>frequency $f_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-92.9199$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-8.3926$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-3.03133 \pm j10.2534$</td>
<td>0.28351</td>
<td>1.6319</td>
</tr>
<tr>
<td>$-2.8332 \pm j9.5339$</td>
<td>0.28486</td>
<td>1.5174</td>
</tr>
<tr>
<td>$-2.2474 \pm j6.5071$</td>
<td>0.32645</td>
<td>1.0356</td>
</tr>
</tbody>
</table>

Table 6.4: $G_S$ controlled, $G_A$ a DFIG

<table>
<thead>
<tr>
<th>eigenvalues $\lambda$</th>
<th>damping ratio $\zeta$</th>
<th>frequency $f_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-93.8358$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-7.12353 \pm j16.246$</td>
<td>0.40157</td>
<td>2.5856</td>
</tr>
<tr>
<td>$-6.57185 \pm j9.03675$</td>
<td>0.58815</td>
<td>1.4382</td>
</tr>
<tr>
<td>$-2.6471 \pm j6.0353$</td>
<td>0.40167</td>
<td>0.96055</td>
</tr>
<tr>
<td>$-3.0963$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

point is moved from the pre disturbance equilibrium point. The altered set of eigenvalues are found in Tables 6.5-6.8.

**Eigenvalue Movement**

It is interesting to note certain values as functions of the reactance of the long line $X_{14}$. Here we have looked at the voltage at the common connection point of the two generators, and the position of eigenvalues.

Figure 6.1 shows the decreasing voltage at the common connection point, Bus 4, with
Table 6.5: $G_S$ uncontrolled, $G_A$ synchronous, Single Line between Buses 1 and 4

<table>
<thead>
<tr>
<th>eigenvalues $\lambda$</th>
<th>damping ratio $\zeta$</th>
<th>frequency $f_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.0153817 \pm j11.0526$</td>
<td>0.0013917</td>
<td>1.7591</td>
</tr>
<tr>
<td>$-0.1177 \pm j5.1696$</td>
<td>0.022763</td>
<td>0.82276</td>
</tr>
<tr>
<td>$-6.9964$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-0.29906$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.6: $G_S$ uncontrolled $G_A$ a DFIG, Single Line between Buses 1 and 4

<table>
<thead>
<tr>
<th>eigenvalues $\lambda$</th>
<th>damping ratio $\zeta$</th>
<th>frequency $f_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-5.6897 \pm j12.7917$</td>
<td>0.40641</td>
<td>2.0359</td>
</tr>
<tr>
<td>$-1.0275 \pm j7.6406$</td>
<td>0.13327</td>
<td>1.216</td>
</tr>
<tr>
<td>$-1.2492$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-0.049981$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.7: $G_S$ controlled, $G_A$ synchronous, Single Line between Buses 1 and 4

<table>
<thead>
<tr>
<th>eigenvalues $\lambda$</th>
<th>damping ratio $\zeta$</th>
<th>frequency $f_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-91.947$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-2.13762 \pm j10.7399$</td>
<td>0.19521</td>
<td>1.7093</td>
</tr>
<tr>
<td>$-5.42989 \pm j9.56449$</td>
<td>0.4937</td>
<td>1.5222</td>
</tr>
<tr>
<td>$-8.2968$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-0.74256 \pm j4.7548$</td>
<td>0.1543</td>
<td>0.75675</td>
</tr>
</tbody>
</table>

Table 6.8: $G_S$ controlled, $G_A$ a DFIG, Single Line between Buses 1 and 4

<table>
<thead>
<tr>
<th>eigenvalues $\lambda$</th>
<th>damping ratio $\zeta$</th>
<th>frequency $f_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-92.8752$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$-7.32386 \pm j16.8017$</td>
<td>0.39959</td>
<td>2.6741</td>
</tr>
<tr>
<td>$-7.07331 \pm j7.48385$</td>
<td>0.68689</td>
<td>1.1911</td>
</tr>
<tr>
<td>$-2.2811 \pm j5.7935$</td>
<td>0.36636</td>
<td>0.92206</td>
</tr>
<tr>
<td>$-2.0703$</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
the increase of the reactance of the long line $X_{14}$ at the equilibrium point. This implies that the systems should stay stable up until $X_{14} = 0.78$. These solutions are found using Load Flow.

This solution, however, does not take into account the dynamics of the system, described by the complete set of eigenvalues. The Figure 6.2 shows the paths traced by the eigenvalues in the vicinity of the imaginary axis. When the reactance of the long line is increased past $X_{\text{crit}}$, an eigenvalue travels into the right half plane. These critical reactances are lower than expected when looking at the static system.

Looking at both of these confirms that this investigation deals with problems related to the dynamics of the system.

### 6.2 Behaviour

The linearised behaviour gives an idea of what happens in a system through the examination of its eigenvalues. However the true system may behave somewhat differently. It is therefore important to examine the behaviour of the system described by the non-linear sets of differential equations, especially when the disturbances to the system become large. Linearised sets of equations only hold when the disturbances are small.

The system is subject to the disturbances described in Section 3.3. The figures showing the angular behaviour and power flow of $G_S$ due to these disturbances are shown in Figures 6.3 to 6.10. Recall that the subscript 2 denotes quantities at Bus 2, belonging to $G_S$. 

![Figure 6.1: Partial Nose Curve](image)

---

CHAPTER 6. RESULTS
6.3 Conclusion

The system is said to be locally stable about an equilibrium point if it remains within a small region surrounding the equilibrium point when subjected to a small perturbation. If the system returns to the equilibrium point, the system is asymptotically stable [8]. Given this, we can say the following.

The figures show that in after Disturbances 1 and 2, which are relatively small disturbances, $G_S$ has its angular swings damped more when $G_A$ is the DFIG we have looked at. However, after Disturbance 3, which is a rather more severe disturbance, the DFIG increases the instability of the system.

This is not clear when looking at the eigenvalues in Section 6.1. The damping ratios $\zeta$...
of the system are higher in the case where $G_A$ is a DFIG than when it is a synchronous generator. When $G_S$ is uncontrolled, these $\zeta$ are much higher, and even when $G_S$ is controlled, the DFIG contributes significantly to increasing these $\zeta$.

Although the damping ratios are increased with the presence of the DFIG, some eigenvalues along the real axis are moved closer towards the imaginary axis.

With small disturbances, the DFIG works to dampen the oscillations of $G_S$. The DFIG and $G_S$ have different characteristics, and one would expect them to not oscillate in phase. However, with the larger Disturbance 3, we move the post disturbance eigenvalues of the system closer to imaginary axis. The large disturbance causes the DFIG to consume reactive power, causing a voltage stability from which it cannot recover. This then leads to an angular instability, with $G_S$ both uncontrolled and controlled.
6.3. CONCLUSION

It is not the larger value of $X_{14}$ responsible for the instability, as can be seen in Figure 6.9. This Figure shows the same post disturbance system is stable after being affected with small Disturbance 1. This implies that the DFIG is less robust than its similarly rated synchronous generator.

It would appear then that DFIGs are useful for damping the initial oscillations in $G_S$ that occur after a small disturbance upsets the system, but are less useful under large disturbances. Extending this, we can say that the presence of a wind park in the vicinity of a typical power system will improve the angular behaviour of the power system under small disturbances, but may decrease voltage stability under larger disturbances.

Figure 6.4: Disturbance 1, $G_S$ controlled
Figure 6.5: Disturbance 2, $G_S$ uncontrolled
Figure 6.6: Disturbance 2, $G_S$ controlled
Figure 6.7: Disturbance 3, $G_S$ uncontrolled
6.3. CONCLUSION

Figure 6.8: Disturbance 3, $G_S$ controlled
Figure 6.9: Single Line between Buses 1 and 4: Disturbance 1, $G_S$ uncontrolled
6.3. CONCLUSION

Figure 6.10: Single Line between Buses 1 and 4: Disturbance 1, $G_S$ controlled
Chapter 7

Future Developments

In this report, we have dealt only with the influence uncontrolled DFIGs have on the stability of a power system. In order to simulate a wind turbine more fully, further modelling of the mechanisms which are likely to affect stability should be carried out. These include elements such as rotor voltage control and blade pitch control.

Ultimately, it would be useful to develop an additional control to feed into the rotor side voltage in order to damp the oscillations in a power system as quickly as possible.

7.1 Converters

A more complete model of a DFIG will include equipment required to control it. For power control, a DFIG will have converters on both the rotor and grid side of its terminals. The rotor current should be decomposed into $d$ and $q$ axis components, so that the active power $P$ and reactive power $Q$ can be controlled independently of one another, as shown in Figure 7.1 [13]. The subscript $\text{REF}$ denotes reference values. The converter shown consists of four Proportional-Integral ($PI$) controllers. The outputs will be voltages to be fed back into the rotor side in the directions of the $d$ and $q$ axis.

We can assume [1] that the grid side of the generator is able to follow its reference values at any time, and so a grid side converter need not be modelled. Only the rotor side converter needs to be considered.

7.2 Pitch Control

At high windspeeds, the mechanical input power to a turbine must be limited in order to avoid damage to the windmill. For this purpose pitch control is the most common
method used in almost all variable-speed wind turbines. This involves altering the pitch of the blades about their chord line. This line connects the tip of the blade to the base of the blade. The pitch angle is the angle between the chord line of the blade and the plane of rotation. Using pitch control, we can control the amount of power harnessed from the wind.

The turbine has a rated speed, such that when the speed of the wind goes above this, the turbine produces a nominal power. When the wind speed is below this rated speed the turbine should produce as much power as possible. Maximum energy capture is achieved by pitching the blades. When the wind speed is above rated speed, the pitch angle should be controlled so that the energy capture is at the rated power of the generator [12].

There are many different ways of modelling pitch control. It is not known exactly how different manufacturers implement pitch control, but the structure is always very similar. Below nominal wind speed, the optimal pitch angle is 0. When the wind speed is higher than the nominal value, the optimal pitch angle increases steadily with increasing wind speed. The size of the blades and blade drives limit the rate of change of the pitch angle. A reasonable model might have the pitch angle somewhere between -3° and 10° [11], and the pitch rate at 4°/s [10]. Pitch rates up and down may differ.

A proportional controller can be used to control the pitch angle. When the difference between the rotor speed $v_r$ and the maximum rotor speed $v_r^*$ is negative, we should decrease $\beta$. The error signal $v_r - v_r^*$ is scaled up, but its rate of change should be limited to simulate the time taken to pitch the blades. Since the wind is never in a steady state, a proportional controller is not necessary [16]. Nevertheless, [13] has a more complicated

$$
\begin{align*}
\text{Figure 7.1: Rotor Side Converter}
\end{align*}
$$
controller system, which can be examined if need be.

7.3 Mechanical Power Input

The mechanical power input is

\[ P_m = \frac{1}{2} \rho A_r C_p(\lambda, \beta) w^3 \]  \hspace{1cm} (7.1)

where

- \( \rho \) is the air density
- \( A_r \) is the swept rotor area
- \( w \) is the windspeed
- \( \beta \) is the pitch angle and
- \( \lambda \) is the tip speed ratio
- \( C_p(\lambda, \beta) \) is the coefficient of performance

and

\[ \lambda = \frac{v_r r}{w} \]  \hspace{1cm} (7.2)

where \( v_r \) is the rotor speed, and \( r \) is the rotor plane radius.

The wind speed should be passed through a low pass filter, as high wind speed variations even out over the rotor area. We can use the low pass filter

\[ G(s) = \frac{1}{1 + \tau s} \]  \hspace{1cm} (7.3)

where \( \tau \) depends on rotor diameter, turbulence, and average wind speed. In [16] \( \tau \) is set to 4s.

The power curves of different wind turbines are very similar [16]. Therefore a general approximation for \( C_p \) can be used for all turbines. We will use

\[ C_p(\lambda, \beta) = c_1 \left( \frac{c_2}{\lambda_i} - c_3 \beta - c_4 \right) e^{-\frac{c_5}{\lambda_i}} + c_6 \lambda \]  \hspace{1cm} (7.4)

\[ \frac{1}{\lambda_i} = \frac{1}{\lambda + c_7 \beta} - \frac{c_8}{\beta^3 + 1} \]  \hspace{1cm} (7.5)

where the constants \( c \) can be chosen to more accurately model the power curve. The wind turbine model in Simulink [18] uses the values found in [9].
7.4 Power Systems

In order to develop an additional control to damp oscillations, linear control could be used on the DFIG. However, to damp these oscillations as quickly as possible, non-linear controls may be implemented to more efficiently deal with behaviour of the system, which is itself non-linear.

Lyapunov and Energy functions could be useful for this purpose [15]. Lyapunov proposed that the stability of a non-linear system could be ascertained without numerical integration, via the use of Lyapunov functions. Although there are well-proven algorithms for classical systems consisting of multiple machines, it may be difficult to find these functions for systems which also consists of asynchronous generators [6].

In this report we have looked at with stability of power systems. It is known that asynchronous generators such as DFIGs can reduce voltage stability, by consuming much reactive power on starting [2]. It may be of interest to investigate what fraction of power from asynchronous generators such as DFIGs could be placed in a power system to optimise both angular and voltage stability.
Appendix A

Reference Changes

In Chapter 2 we introduced the concept of transformation of symmetric rotor windings to a rotating reference frame, and defined a transformation matrix. Here we go into detail on how this is used.

Take for example Equations (2.1) and (2.2).

\[
\begin{align*}
v_{abcs} &= R_s i_{abcs} + \frac{d\lambda_{abcs}}{dt} \quad \text{(A.1)} \\
v_{abcr} &= R_r i_{abcr} + \frac{d\lambda_{abcr}}{dt} \quad \text{(A.2)}
\end{align*}
\]

Premultiplying by \( T_{dq} \) we get

\[
\begin{align*}
v_{dqs} &= R_s i_{dqs} + T_{dq} \frac{d}{dt} \left( T_{dq}^{-1} \lambda_{dqs} \right) \quad \text{(A.3)} \\
&= R_s i_{dqs} + \left( T_{dq} \frac{d}{dt} T_{dq}^{-1} \right) \lambda_{dqs} + \frac{d}{dt} \lambda_{dqs} \quad \text{(A.4)} \\
v_{dqr} &= R_s i_{dqr} + T_{dq} \frac{d}{dt} \left( T_{dq}^{-1} \lambda_{dqr} \right) \quad \text{(A.5)} \\
&= R_s i_{dqr} + \left( T_{dq} \frac{d}{dt} T_{dq}^{-1} \right) \lambda_{dqr} + \frac{d}{dt} \lambda_{dqr} \quad \text{(A.6)}
\end{align*}
\]

where

\[
T_{dq} \frac{d}{dt} T_{dq}^{-1} = \frac{2}{3} \left[
\begin{array}{ccc}
\cos \beta & \cos \beta - \frac{2\pi}{3} & \cos \beta + \frac{2\pi}{3} \\
-\sin \beta & -\sin \beta - \frac{2\pi}{3} & -\sin \beta + \frac{2\pi}{3}
\end{array}
\right] \times
\frac{d\beta}{dt} \left[
\begin{array}{cc}
-\sin(\beta) & -\cos(\beta) \\
-\sin(\beta - \frac{2\pi}{3}) & -\cos(\beta - \frac{2\pi}{3}) \\
-\sin(\beta + \frac{2\pi}{3}) & -\cos(\beta + \frac{2\pi}{3})
\end{array}
\right] \quad \text{(A.7)}
\]
\[
\frac{d\beta}{dt} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{A.8}
\]
\[
= \frac{d\beta}{dt} I, \tag{A.9}
\]

the matrix \( I \) is as defined in Equation (2.13), and \( \beta \) is the angle between the reference frame and the rotating frame we wish to transform.

We have taken the synchronously rotating reference frame as the reference frame. So, for the stator, which is stationary,
\[\beta = \omega_s t\]
and thus
\[T_{dq} \frac{d}{dt} T_{dq}^{-1} = \omega_s I\]

For the rotor frame, which travels at a speed \( \omega \),
\[\beta = (\omega_s - \omega) t\]
and so
\[T_{dq} \frac{d}{dt} T_{dq}^{-1} = (\omega_s - \omega) I\]
yielding the Equations (2.11) and (2.12).
Appendix B

Per Unit System

The per unit system is a common method to express true values as normalised quantities. It is more convenient to express values using the per unit system, as it simplifies representations of power systems with several voltage levels and transformers, and allows for more easy comparisons between generators. Also having numbers of the same magnitude allows for higher accuracy in numerical calculations.

The definition of a per unit value of a quantity is

\[
\text{per unit value} = \frac{\text{true value}}{\text{base value}} \quad (B.1)
\]

For a power system, a single phase base power \(S_b\) and a base frequency \(\omega_b\) are chosen. Line to neutral voltages \(V_b\) are chosen for sections between transformers. The ratio of base voltages connected by a transformer should be equal to the transformer ratio. All other base quantities can be calculated from these. The base quantities will be denoted by the subscript \(b\).

Some common quantities used here are

\[
I_b \triangleq \frac{S_b}{V_b} \quad (B.2)
\]

\[
Z_b \triangleq \frac{V_b}{I_b} \quad (B.3)
\]

\[
L_b \triangleq \frac{Z_b}{\omega_b} \quad (B.4)
\]

\[
\lambda_b \triangleq \frac{V_b}{\omega_b} \quad (B.5)
\]

\[
\psi_b \triangleq \omega_b \lambda_b \quad (B.6)
\]

To see how the per unit system has been used here, we consider an example. In Chapter
2 we had Equation (2.11)

\[ v_{dqs} = R_s i_{dqs} + \frac{d\lambda_{dqs}}{dt} + I\omega_s \lambda_{dqs} \]

Dividing through by

\[ Z_b = \frac{V_b}{I_b} = \omega_b L_b = \frac{\omega_b \lambda_b}{I_b} \]

we find the equation represented with per unit values

\[ \frac{v_{dqs}}{V_b} = \frac{R_s}{Z_b} \frac{i_{dqs}}{I_b} + \frac{d}{dt} \frac{\lambda_{dqs}}{\omega_b \lambda_b} + I\omega_s \frac{\lambda_{dqs}}{\omega_b \lambda_b} \]

\[ v_{dqs-pu} = R_{s-pu} i_{dqs-pu} + \frac{1}{\omega_b} \frac{d\lambda_{dqs-pu}}{dt} + I\omega_s \frac{\lambda_{dqs-pu}}{\omega_b} \]

where the subscript \(pu\) denoted per unit values. Choosing \(\omega_b = \omega_s\) yields

\[ v_{dqs} = R_s i_{dqs} + \frac{1}{\omega_s} \frac{d\lambda_{dqs}}{dt} + I\lambda_{dqs} \]

(B.7)

where all values are per unit.

A similar procedure can be used to convert all nominally valued equations to per unit.

Equation B.7, when written using phasors, is equivalent to Equation (2.18).
Appendix C

Values

All values used are per unit values, and are chosen as typical values.

Synchronous Generator

\[
\begin{align*}
X_2 &= X_{t2} + X_{d2} = 0.10 + 1 \text{ p.u.} \\
X_2' &= 0.15 \text{ p.u.} \\
T_{02} &= 6 \text{ p.u.} \\
H_2 &= 4 \text{ p.u.}
\end{align*}
\]

Doubly Fed Induction Generator

Some values used for the DFIG are taken from [14]. These same values are used for a synchronous generator as a comparison.

\[
\begin{align*}
X_3 &= X_{t3} + X_{d3} = 0.10 + 1 \text{ p.u.} \\
X_3' &= 0.10 \text{ p.u.} \\
T_{03} &= 0.4 \text{ p.u.} \\
H_3 &= 4 \text{ p.u.} \\
s &= -0.03
\end{align*}
\]

Power system

\[
P_{G2} = 0.8 \text{ p.u.}
\]
APPENDIX C. VALUES

\[ P_{G3} = 0.4 \text{ p.u.} \]
\[ X_{14} = 0.20 \text{ p.u.} \]
\[ X_{24} = 0.10 \text{ p.u.} \]
\[ X_{34} = 0.10 \text{ p.u.} \]

Control Constants

\[ K_A = 100 \]
\[ T_e = 0.01 \]
\[ T_1 = 2.5 \]

The constants chosen through eigenvalue analysis are

- when \( G_A \) is synchronous
  \[ T_2 = 0.1075 \]
  \[ K_{PSS} = 0.0076 \]

- when \( G_A \) is a DFIG
  \[ T_2 = 0.0737 \]
  \[ K_{PSS} = 0.0088 \]
Appendix D

Matrix Entries

In the MATLAB code, matrices $A$, $B$, $C$ and $D$ are initially blank matrices with values assigned to them. In the following sections, we will mention only those expressions which are non-zero. Subscripts are omitted, but here we understand each quantity to have the the same subscript. All of these should be evaluated at the steady state values calculated in Section 4.2.

The binary variable $K_{CTRL}$ is used as a switch to accommodate generators without control.

Matrix $A$

The entries in matrix $A$ for synchronous generators are

\[
\begin{align*}
\frac{\partial \dot{\delta}}{\partial \omega} &= 1 \\
\frac{\partial \dot{\omega}}{\partial \delta} &= \frac{1}{M} \left( -\frac{E'V}{X'} \cos(\delta - \theta) \right) \\
\frac{\partial \dot{\omega}}{\partial E'} &= \frac{1}{M} \left( -\frac{V}{X'} \sin(\delta - \theta) \right) \\
\frac{\partial \dot{E}'}{\partial \delta} &= \frac{1}{T_0} \left( -\frac{X - X'}{X'} V \sin(\delta - \theta) \right) \\
\frac{\partial \dot{E}'}{\partial E'} &= \frac{1}{T_0} \left( -\frac{X}{X'} \right) \\
\frac{\partial \dot{E}'}{\partial E_F} &= \frac{1}{T_0} \\
\frac{\partial \dot{E}_F}{\partial E_F} &= -\frac{K_{CTRL}}{T_e}
\end{align*}
\]
\[
\frac{\partial \dot{E}_F}{\partial V_{PSS}} = \frac{K_{CTRL}}{T_c} (K_A)
\]
\[
\frac{\partial \dot{V}_{PSS}}{\partial \delta} = \frac{K_{CTRL}}{T_2} \left( \frac{K_{PSS} T_1}{M} \left( -\frac{E'}{X'} \cos(\delta - \theta) \right) \right)
\]
\[
\frac{\partial \dot{V}_{PSS}}{\partial \omega} = \frac{K_{CTRL}}{T_2} (K_{PSS})
\]
\[
\frac{\partial \dot{V}_{PSS}}{\partial E'} = \frac{K_{CTRL}}{T_2} \left( \frac{K_{PSS} T_1}{M} \left( -\frac{V}{X'} \sin(\delta - \theta) \right) \right)
\]
\[
\frac{\partial \dot{V}_{PSS}}{\partial V_{PSS}} = -\frac{K_{CTRL}}{T_2}
\]

and differing expressions for the DFIG are

\[
\frac{\partial \dot{\delta}}{\partial \delta} = \frac{1}{E'T_0} \left( -\frac{X - X'}{X'} V \cos(\delta - \theta) - T_0 \omega_s V_r \sin(\delta - \theta_r) \right)
\]
\[
\frac{\partial \dot{\delta}}{\partial \omega} = 1
\]
\[
\frac{\partial \dot{\delta}}{\partial E'} = -\frac{1}{E'^2 T_0} \left( -\frac{X - X'}{X'} V \sin(\delta - \theta) + T_0 \omega_s V_r \cos(\delta - \theta_r) \right)
\]
\[
\frac{\partial \dot{\omega}}{\partial \omega} = \frac{1}{M} \left( -\frac{P_m \omega_s}{\omega^2} \right)
\]
\[
\frac{\partial \dot{E}'}{\partial \delta} = \frac{1}{T_0} \left( -\frac{X - X'}{X'} V \sin(\delta - \theta) + T_0 \omega_s V_r \cos(\delta - \theta_r) \right)
\]
\[
\frac{\partial \dot{E}'}{\partial \omega} = 0
\]
\[
\frac{\partial \dot{E}'}{\partial E'} = \frac{1}{T_0} \left( -\frac{X}{X'} \right)
\]

Matrix \(B\)

The entries in matrix \(B\) for synchronous generators are

\[
\frac{\partial \dot{\omega}}{\partial \theta} = \frac{1}{M} \left( \frac{E'V}{X'} \cos(\delta - \theta) \right)
\]
\[
\frac{\partial \dot{\omega}}{\partial V} = \frac{1}{M} \left( -\frac{E'}{X'} \sin(\delta - \theta) \right)
\]
\[
\frac{\partial \dot{E}'}{\partial \theta} = \frac{1}{T_0} \left( -\frac{X - X'}{X'} V \sin(\delta - \theta) \right)
\]
\[
\frac{\partial \dot{E}'}{\partial V} = \frac{1}{T_0} \left( \frac{X - X'}{X'} \cos(\delta - \theta) \right)
\]
\[
\frac{\partial \hat{E}_F}{\partial V} = \frac{K_{CTRL}}{T_e} (-K_A)
\]
\[
\frac{\partial \hat{V}_{PSS}}{\partial \theta} = \frac{K_{CTRL}}{T_2} \left( \frac{K_{PSS} T_1}{M} \left( \frac{E' V}{X'} \cos(\delta - \theta) \right) \right)
\]
\[
\frac{\partial \hat{V}_{PSS}}{\partial V} = \frac{K_{CTRL}}{T_2} \left( \frac{K_{PSS} T_1}{M} \left( -\frac{E}{X'} \sin(\delta - \theta) \right) \right)
\]

and differing expressions for the DFIG are
\[
\frac{\partial \delta}{\partial \theta} = \frac{1}{E'T_0} \left( \frac{X - X'}{X'} V \cos(\delta - \theta) \right)
\]
\[
\frac{\partial \delta}{\partial V} = \frac{1}{E'T_0} \left( \frac{X - X'}{X'} \sin(\delta - \theta) \right)
\]
\[
\frac{\partial \dot{E}'}{\partial \theta} = \frac{1}{T_0} \left( \frac{X - X'}{X'} V \sin(\delta - \theta) \right)
\]
\[
\frac{\partial \dot{E}'}{\partial V} = \frac{1}{T_0} \left( \frac{X - X'}{X'} \cos(\delta - \theta) \right)
\]

Matrix C

Matrix C is independent of the type of generators exist in the system. The entries in it are
\[
\frac{\partial P_g}{\partial \delta} = -\frac{E' V}{X'} \cos(\theta - \delta)
\]
\[
\frac{\partial P_g}{\partial E'} = \frac{V}{X'} \sin(\theta - \delta)
\]
\[
\frac{\partial Q_g}{\partial \delta} = -\frac{E' V}{X'} \sin(\theta - \delta)
\]
\[
\frac{\partial Q_g}{\partial E'} = \frac{V}{X'} \cos(\theta - \delta)
\]

Matrix D

Matrix D is independent of the type of generators exist in the system. It is built up by using the Jacobian matrix in the load flow. The additional entries needed to take account of the power flow from the generators are
\[
\frac{\partial P_g}{\partial \theta} = \frac{E' V}{X'} \cos(\theta - \delta)
\]
\[
\frac{\partial P_g}{\partial V} = \frac{E'}{X'} \sin(\theta - \delta)
\]
\[
\frac{\partial Q_g}{\partial \theta} = \frac{E'V}{X'} \sin(\theta - \delta)
\]
\[
\frac{\partial Q_g}{\partial V} = 2V - \frac{E'}{X'} \cos(\theta - \delta)
\]
Bibliography


