EXTERNAL DAMPING OF STAY CABLES USING ADAPTIVE AND SEMI-ACTIVE VIBRATION CONTROL

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Abstract
In this paper, the performances of different external damping systems for stay cables are studied based on numerical simulations. Two types of dampers have been analysed; a near anchorage viscous damper and a tuned mass damper (TMD) mounted near the midspan of the stay cable. For the passive case, both dampers are tuned to the fundamental mode of vibration of the cable. The optimal viscous damping for the near anchorage damper is determined based on well-known equations for a taut string. For the TMD, parametrical studies have been performed to determine the optimal damping ratio as function of the damper mass. The resulting vibration mitigation from the two systems are also studied for higher modes of vibration and the potential increase in performance using an adaptive or semi-active vibration control system is studied.

Keywords: Stay-cable; external damping system, TMD, semi-active control, finite element analysis.

1. Introduction
Stay cables for cable-stayed bridges often have a very low inherent structural damping, making them prone to vibrations. Wind or wind-rain induced vibrations may cause excessive vibrations, that without countermeasures may risk to decrease the fatigue service life of the cable. Forming of longitudinal water rivulets during rain may initiate so-called wet-galloping. One of the first cases of rain-induced cable vibrations was observed on the Meiko-Nishi Bridge in 1986. Large amplitude vibrations were observed at relatively low wind velocities, 5 – 15 m/s, and for higher cable modes in the range of 1 – 3 Hz. On the Great Belt Bridge, cable vibrations with amplitudes of 2 m have been observed, caused by icing. The forming of ice on the cables resulted in a significant change in the aerodynamic force coefficients, causing aerodynamic instability and making them more prone to wind induced vibrations. [1]

Different countermeasures for mitigate cable vibrations are available. The cable surface can be modified with longitudinal channels, dimples or bumps that can disrupt the formation of longitudinal water rivulets and will significantly decrease the risk of galloping and vortex-shedding. Cross ties with stabilising ropes can be used to couple several cables with each other and to change the dynamic system. Mechanical damping systems can be installed, e.g. near anchorage viscous dampers or TMD. Near anchorage dampers can consist of a chock absorber with one end mounted to the bridge deck and the other end connected to the stay cable. The primary vibration mitigation is due to increased structural damping of the stay cable, produced by the chock absorber. Due to geometrical implications, the damper is most often installed relatively close to the anchorage, resulting in a decreased efficiency for longer cables. A TMD works on the principle of disrupting a structural mode of vibration by means of a small suspended mass. For the case of no inherent damping of the TMD, two adjacent modes of vibrations will be obtained with similar magnitude as the undamped structure. Introducing a damping in the TMD will however contribute to the damping of the primary structure. For lightly damped structures as stay cables, a relatively small mass may produce significant vibration mitigation. External dampers are most often designed based on optimal parameters for the fundamental mode of vibration. This may however result in a poor performance for higher modes. Advanced dampers with a variable range of stiffness and/or damping can in combination with a control system be used to improve the vibration mitigation for higher modes.
2. Models of a stay cable with an external damper

The dynamic behaviour of a stay cable is analysed with a 2D FE-model, using the commercial software SOLVIA03 [2]. The cable is modelled with beam elements with negligible flexural stiffness. The inclination and the sag of the cable are neglected, as well as the out-of-plane motion.

Data for the cable is taken from [3], also studied in [4]. The cable has a length \( L = 93 \) m, mass \( m = 114 \) kg/m, axial stiffness \( EA = 1615 \) MN and an axial pre-stress of 5017 kN. The first natural frequency of the undamped cable is \( f_1 = 1.127 \) Hz.

The model of the near anchorage damper is illustrated in Figure 1, the distance \( x_c = 0.02L \). The model with a TMD is illustrated in Figure 2, in which \( x_c = 0.4L \). The position of the TMD is chosen to be able to mitigate vibrations from higher modes. The first four modes of vibration for the undamped cable are presented in Figure 3, based on an eigen-value analysis, accounting for the axial pre-stress.

3. Methods of analysis

Throughout this paper, the cable is subjected to a vertical harmonic unit load, acting at \( x = 0.4L \). The results are based on the vertical displacements at the same location. The results are presented as a dynamic amplification factor \( R_d = \frac{d_{\text{dyn}}}{d_{\text{stat}}} \), where \( d_{\text{stat}} \) is the static displacement of the unit load and \( d_{\text{dyn}} \) the corresponding peak dynamic displacement.

For the case of linear and non-varying dampers, the steady state frequency response can be calculated using Eq. (1). \( M, C, K \) are the mass, damping and stiffness matrices respectively and \( F \) is the force vector. The frequency response function (FRF) is then found by solving for the frequency dependent displacement vector \( x(\omega) \), for each prescribed circular frequency \( \omega \) [5].

\[
\left(-\omega^2 M + i\omega C + K\right)x(\omega) = F
\]
For the case of adaptive or semi-active dampers, the damping or the stiffness is incrementally updated in the analysis. A direct time integration scheme is then employed and the load consists of a harmonic force with linearly increasing frequency. The rate of change in frequency is chosen sufficiently long to approximate the steady-state response.

The resulting structural damping due to the dampers is estimated using the Half-Power Bandwidth method, according to Eq. (2). To improve the accuracy of the method, a curve fit using a 4th order polynomial is used, as illustrated in Figure 4.

\[ \zeta = \frac{f_2 - f_1}{f_2 + f_1}. \]  

(2)

Figure 4: Illustration of the Half-Power Bandwidth method.

4. Optimal damper parameters

For a near anchorage damper, the relation between the viscous damper \( c_d \) and the resulting cable damping \( \zeta_i \) for mode \( i \) can be presented in a normalised form according to Figure 5, [7]. The results in Figure 5 are produced based on the steady-state response of the model in Figure 1, varying the viscous damper \( c_d \). For the case study cable, an optimal viscous damping \( c_d = 376 \text{kNs/m} \) is calculated based on the first mode of vibration. For \( x_c = 0.02L \), the optimal damping is about 1%.

For the TMD, the stiffness \( k_d \) and viscous damper \( c_d \) are calculated according to Eq. (3). The mass of the TMD is often presented as a ratio of the total mass of the cable, \( \gamma = m_{\text{TMD}} / m_{\text{cable}} \).

\[ k_d = m_d \omega_d^2, \quad c_d = 2\zeta_{TMD} \sqrt{k_d m_d} \]  

(3)
A parametrical study is performed to determine the TMD parameters for the optimal cable damping $\zeta_{\text{cable}}$. A series of steady-state analyses are performed using the model in Figure 2 and varying the mass and the damping of the TMD. The results for the first mode of vibration are shown in Figure 6. For low TMD damping ratios, the FRF is characterised by two separate resonance peaks, adjacent to the structural mode of the cable. For increased TMD damping ratios, the two peaks are eventually merged and the optimal damping is found in this transition zone. For low TMD mass ratio, the transition is more distinct, causing a sudden change in estimated total damping. For the present case, $\gamma > 0.7$ results in a moderate increase in total cable damping. For further analysis, $\gamma = 0.9$ is used, corresponding to about 100 kg in TMD mass. The appertaining optimal damping is $\zeta_{\text{TMD}} = 9\%$, corresponding to $c_d = 127 \text{ Ns/m}$, about 3000 times less than for the near anchorage damper. The resulting cable damping is about $6\%$. Similar magnitude of damping has been obtained by [6].

![Figure 6: Resulting cable damping due to a TMD, influence of the damping ratio $\zeta_{\text{TMD}}$ and mass ratio $\gamma$.](image)

The performance of the near anchorage damper and the TMD is shown in Figure 7, for the first mode of vibration. An inherent damping $\zeta_{\text{cable}} = 0.1\%$ is set for the cable itself, to limit the resonant response. For the case of the undamped TMD, the fundamental mode of vibration for the cable is separated in two adjacent peaks. The response at resonance is reduced by about a factor 2. Using the optimal damping according to Figure 6, no distinct resonance peaks are obtained and the response at resonance is reduced by about a factor 50, compared to the cable with no damper. The near anchorage damper maintains a single fundamental mode of vibration of the cable, but with increased damping. The response at resonance is however about 3 times larger compared to the optimal damped TMD. The TMD performs better than the near anchorage damper mainly due to the location further up along the cable, where the magnitude of vibration is larger.

![Figure 7: Steady-state response of the first mode of vibration, comparison of a near anchorage damper and a TMD.](image)
5. Adaptive vibration control

In vibration control of damping systems, the damper has time-variant properties that can be controlled by a control system. The physical behaviour of such systems can vary, but is often based on a controllable electrical input current. In this paper, it is assumed that either the stiffness $k_d$ or viscous damping $c_d$ can be controlled independently. One sometimes distinguishes between adaptive systems that can change its property within a few cycles of vibration (seconds) and semi-active systems that can change within a few milliseconds.

A schematic of the control procedure is illustrated in Figure 8. The system of equation is solved using a direct time incrementation. The previous $t_{\text{incr}}$ of time response is modulated with a window function and zero-padded to obtain a virtually higher frequency resolution. The modulated signal is subjected to a Fast Fourier Transform and the dominant frequency $f_i$ is estimated. The procedure is often denoted Short Time Fourier Transform (STFT). For the near anchorage damper, the natural frequency closest to $f_i$ is determined and the corresponding optimal viscous damper $c_d$ is calculated according to Figure 5. For the case of the TMD, a new stiffness $k_d$ is instead calculated according to Eq. (3). The damping or stiffness matrix is updated and the analysis is forwarded $t_{\text{incr}}$.

![Figure 8: Schematics for the procedure of the adaptive control.](image_url)

6. Vibration mitigation in higher modes

The performance of the TMD and the near anchorage damper is studied for higher modes of vibration. A comparison is also performed between passive dampers with constant damping properties and adaptive/semi-active dampers that can change in stiffness or viscous damping. The analysis is performed using a direct time incrementation. A harmonic unit load is applied with a linear increase in frequency from 0.5 Hz to 5.0 Hz during a total time of 2 h in the simulation, to approximate the steady-state response.

For the TMD, three different models are analysed; a passive damper tuned to first mode of vibration, an adaptive damper that can be retuned in increments corresponding to the first four modes and an adaptive/semi-active damper that can change frequency continuously. For all TMD-models, the viscous damping $c_d = 127 \text{Ns/m}$ is used, corresponding to the optimal damping for the first mode. For the near anchorage damper, a passive viscous damper tuned to the first mode is compared with a damper that can change in increments of the first four modes of vibration. For all the adaptive/semi-active models, the real-time dominant frequency is evaluated every 5 s, based on the previous 5 s of response.
The results from the simulations are presented in Figure 9 and Figure 10. For the first mode, the passive and the incrementally adaptive dampers yield the same results. As presented in Figure 7, the TMD is about 3 times more efficient than the near anchorage damper. Even greater vibration mitigation is obtained with the continuously variable TMD. The reason is likely that a continuous update of the tuning frequency allows mitigating the two adjacent modes produced by the TMD. As a result, the largest magnitude is obtained closer to the fundamental frequency of the undamped cable.

For the higher modes, the passive TMD experience a significant loss of performance, due to detuning. The incrementally tuned TMD shows more pronounced double peaks for higher modes. This is due to the use of constant viscous damping, corresponding to a lower damping ratio for higher frequencies. A significant improvement is found using the continuously adaptive TMD.

The passive viscous near anchorage damper does not suffer from the same detuning as the TMD. The reason is that it mainly operates on increased cable damping rather than changing the dynamic modes of vibration. The difference between the passive and adaptive viscous damper is also less than the corresponding difference for the TMD. The reason is that the optimal viscous damping is inversely proportional to the mode number, Figure 5, where as the stiffness of the TMD depends on the square of the tuning frequency, Eq. (3).

![Figure 9](image1.png)  
**Figure 9:** Steady-state response of the cable due to different damping systems, $f_1$ and $f_2$.

![Figure 10](image2.png)  
**Figure 10:** Steady-state response of the cable due to different damping systems, $f_3$ and $f_4$. 
The peak response from Figure 9 and Figure 10 are summarised in Table 1. The dynamic amplification for all modes are related to the same static displacement. Since the static mode of displacement differs from the higher modes of vibrations, $R_{d}<1$ is sometimes obtained. The results aim at comparing the relative difference between different damping systems rather than estimating a realistic absolute value of the displacement magnitude.

**Table 1:** Peak steady-state dynamic amplification factor, based on vertical displacement at $x_c/L = 0.4$, comparison of different external damping systems.

<table>
<thead>
<tr>
<th></th>
<th>$R_{d,1}$</th>
<th>$R_{d,2}$</th>
<th>$R_{d,3}$</th>
<th>$R_{d,4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMD, passive $(f_1)$</td>
<td>10.08</td>
<td>17.18</td>
<td>11.39</td>
<td>21.69</td>
</tr>
<tr>
<td>TMD, adaptive $(f_1-f_4)$</td>
<td>10.08</td>
<td>1.41</td>
<td>0.78</td>
<td>1.17</td>
</tr>
<tr>
<td>TMD, adaptive (cont.)</td>
<td>8.24</td>
<td>1.04</td>
<td>0.37</td>
<td>0.29</td>
</tr>
<tr>
<td>Viscous $(f_1)$</td>
<td>33.93</td>
<td>5.05</td>
<td>1.83</td>
<td>5.09</td>
</tr>
<tr>
<td>Viscous $(f_1-f_4)$</td>
<td>33.93</td>
<td>3.91</td>
<td>1.34</td>
<td>2.37</td>
</tr>
</tbody>
</table>

7. **Conclusion**

From the results presented in this paper, the following conclusions are made.

- Due to very low inherent damping of stay cables, very high dynamic response at resonance is obtained.

- The near anchorage viscous damper has a rather limited performance, depending on the relatively short distance between the anchorage point and the connection of the damper. This decreases the performance for longer cables.

- The adaptive viscous damper shows moderate improvement for higher modes compared to the corresponding passive damper.

- A TMD can result in significant vibration mitigation, even for a moderate damper mass.

- For the TMD, an optimal damping value exists, that increases with increased damper mass.

- The passive TMD is sensitive to detuning, resulting in a poor performance for higher modes.

- An adaptive/semi-active TMD can increase the vibration mitigation significantly for higher modes. The best performance was obtained by a continuously variable stiffness TMD.

The results presented in this paper contain several simplifications, the following should be addressed in further research.

- The influence of the cable flexural stiffness and cable sag may change the mode of vibration and the natural frequencies. The response may also be amplitude dependent.

- Further parametric studies to find optimal TMD tuning parameters for higher modes of vibration and optimal position along the cable.

- In practice, adaptive/semi-active damping devices often have a more complicated physical behaviour than a linear change in viscous damping or stiffness. The application of magnetorheological dampers for near anchorage dampers have been studied by [8], similar procedures may be applicable for a adaptive/semi-active TMD.

- The use of coupled or uncoupled multi-passive dampers may serve as an alternative to the adaptive TMD.
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References