Naval Architecture

Modelling of Elastic Ship in Waves

MASTER OF SCIENCE TESIS
BY

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ABSTRACT

Ships in the seas undergo different distortions like bending, twisting or combined bending and twisting. The demand for increase in length and speed of the ships is increasing due to this the study for flexibility of structures is becoming important. Usually the structural excitations or natural frequencies of structure are given less priority since the natural frequencies of the structure are higher than encounter frequencies but since the length and speeds are increasing the encounter frequencies are closer to the fundamental frequencies. The structural behavior can be analyzed by semi-empirical formulation developed by classification societies but the flexibility of its application is limited.

The present thesis work deals with the vertical motions of the ship structure. The thesis work is divided in to two parts the first part deals with the literature review of the global loads and Hydroelasticity and the second part deals with the modeling of the structural dynamic problem. In the modeling part Hydro-elasticity theory proposed by Bishop and Price on Euler Bernoulli beam is used for solving the structural dynamic problem and the illustration of springing and whipping is presented. An attempt has also been made to study the dynamic structural responses for a particular hull with the hydrodynamic forces and added mass from high speed strip program developed at KTH and the study has been performed to analyze the influence of various parameters like added mass, damping and stiffness.

ACKNOWLEDGEMENTS

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Manohara Ranganath Draksharam, December 2012.

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CHAPTER 1: INTRODUCTION

Ships are the greatest means of trading for a long time in the world. Because of its necessity and requirement sizes of ships varies from tens to hundreds of meters. Each ship carries thousands of tons of cargo worth much depending on its length. Damage to ship incurs huge loss to the companies involved, to the environment depending on type of cargo, and in the worst case to the passengers and crew. The damage to ship can incur in many ways like fire, structure failure, bad weather etc. Therefore proper structural design is of utmost importance.

The structures in sea undergo bending, twisting or combined bending and twisting due to the various loading conditions and the loading conditions vary with time. The structural failure due to fluctuating load is known as fatigue failure. The fatigue life of the material depends on stress range and the number of load cycles. The average number of wave bending cycles for a structure in sea is approximated as 10^8 for a period of 20 years. The fluctuating load initiates the cracks which gradually grow with the fluctuating loads and finally causes the structure to fail. The number of stress cycles the ship experiences depends on operating condition, for a selected operating condition the structure may experience springing and whipping which increases the number of stress cycles and stress magnitude the ship experiences.

The ship structures are designed according to the semi empirical formulations developed by classification societies and also using the numerical procedures. For evaluating the number of stress cycles the ship experiences it is recommended to study with the help of numerical procedures. In this thesis work an attempt has been made to review the literature available for studying the dynamic structural behaviour of ships and also an attempt has been made to implement a method for studying the dynamic structural behaviour of high speed boats according to operating condition.

The report has been divided into six chapters where Chapter 1 give the introduction about the project, Chapter 2 gives the outline of the loads the ship experiences in sea and the literature review of some methods for studying the dynamic structural behaviour, Chapter 3 gives the introduction of the dynamic beam formulation and illustration of some phenomena according to formulation, Chapter 4 gives the outline of the design procedures used for studying the structural behaviour of ships, Chapter 5 gives the outline of tentative method used for illustrating the structural behaviour of high speed crafts, and Chapter 6 gives the information about the discussions, conclusions and future work proposals of the thesis.

CHAPTER 2: GLOBAL LOADS & RESPONSES

2.1 LOADS

The forces on the ship due to waves are called hydrodynamic forces and the motions of the structure due to hydrodynamic forces are known as structural responses. The response of the structure depends on inertia, damping and stiffness of the structure.

Depending on time the loading on structure can be classified as static, quasi-static and dynamic. In static, loading is constant with time. In quasi-static, loading varies very slowly with time so the inertia effects are ignored therefore the quasi-static loading is similar to static loading, and in dynamic loading the loading varies with time so the time varying effects are considered i.e. inertia and damping. The examples for static loads are calm water loads, for quasi-static loads are ice breaking loads, and for dynamic loads are wave induced loads including transient effects such as slamming (Owen F. Hughes et al, 2010). The loading on the structure will influence hull girder, hull module, principal member, and local levels of the structure. The structural hierarchy is shown in figure 2. The external pressure on the hull has influence at all the four levels of the structure (Owen F. Hughes et al, 2010). The present study is emphasised at the hull girder level.

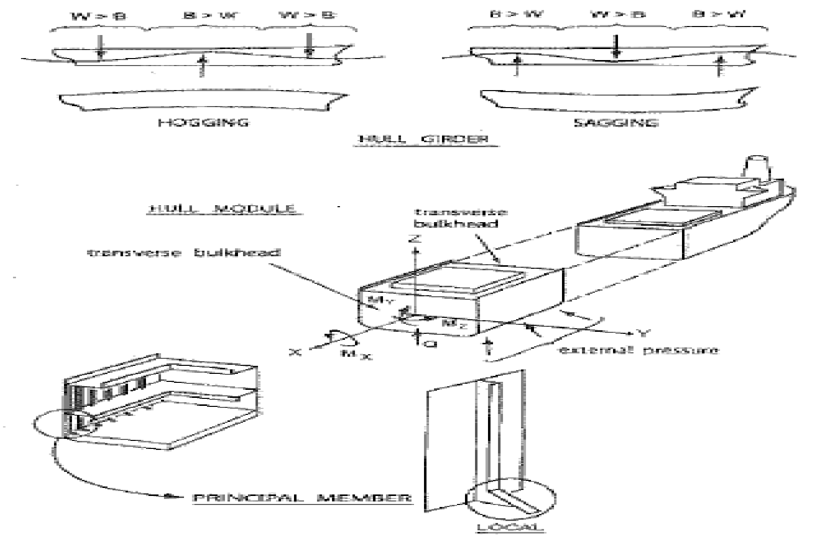

Figure 1 : Ship structural hierarchy

2.3 MOTIONS

The seas are highly irregular in nature and motions of the structure can be either linear or non-linear. For the smaller motions the linearity condition satisfies and for larger motions the non-linearity condition is applied. Frequency is the independent variable in linear motions therefore the results are studied in frequency domain. In dynamic loading the time is the independent variable. The non linear motions are studied in time domain.

2.4 DYNAMIC RESPONSES OF STRUCTURE

The dynamic properties of the structure include natural frequency, modal shapes, and modal damping factors (). The dynamic behaviour includes vibration of the structure. The dynamic behaviour leads the structure to fatigue failure. The dynamic behaviour of the structure is controlled by reducing the speed of the vessel (). The dynamic behaviour of the structure can be observed in the steady state condition and in transient condition. The steady state occurrence is called springing. The dynamic behaviour in the transient condition is called whipping.

2.4.1 Springing

The springing occurs when the encounter frequency is equal to the natural frequency of the structure. The springing phenomenon is observed in the linear excitation and non linear excitation. The results of springing in linear excitation are studied in the frequency domain

Usually the fundamental frequencies of the structure are far away from the encounter frequencies. But the demand for larger ship lengths and high speeds is increasing, due to this the fundamental frequencies of the structure are closer to the encounter frequencies which causes the resonance effects in the structure.

2.4.2 Whipping

The slamming can occur at three places, at the bottom, at the bow and at the stern. The bottom slamming occurs due to heaving and pitching . The bow slamming occurs at the flared portion of the bow. The bow slamming is a gradual phenomena compared to bottom slamming. Both the slamming occurs at different timings. But the basic principal for both the slamming is sudden change in the volume. The bow and bottom slamming increases the vertical acceleration, deflection and excite hull girder vibration. The excitation of the hull girder is called whipping. The vibration in the structures can be seen at the local level and in the global level. The influence on the structure depends on the impulse of the force. If the force is acting for a short time it influence in the local level, if the force acts for longer time it influences in the global level. The whipping also depends on the angle between the impacting surface and wave surface. Larger angle has lesser influence and the smaller angles have higher influence.

Figure 2 : Slamming in ships

In longer ships both springing and whipping phenomena are observed. The springing and whipping can occur simultaneously. In high speed ships the whipping phenomena is more commonly observed.

2.5 LITERATURE REVIEW

Hydroelasticty is a branch of science deals with the interaction between the seas and elastic structures in seas. At fluid structure interaction level there may be a significant difference between the hydrodynamic, inertial and elastic forces experienced by the elastic structures i.e. the fluid pressure acting on the structure modifies the motion and distortion of structure and which in return alters the fluid pressure acting on the structure. Hydro elasticity is a counter part of the aero elasticity and was first introduced by Heller and Abramson (S. E. Hirdaris et al, 2009). Hydroelasticity received the attention in naval architecture community through the work of Bishop and Price in 1970s. The main aim of hydroelasticty is to increase the accuracy of the fluid structure modelling by accurately explaining the physics of the engineering system (S. E. Hirdaris et al, 2009). Within the assumptions of the potential flow analysis and linear structural beam dynamics, hydro elasticity provides the possibility of assessing the wave induced loads due to symmetric (i.e. vertical bending), antisymmetric (i.e. coupled horizontal bending and twisting), and unsymmetric (coupled vertical and horizontal bending and twisting) distortions in regular and irregular waves. It also provides the means for evaluating the global wave induced loads due to transient bottom impact forces in head regular and irregular waves using convolution integral techniques (S. E. Hirdaris et al, 2009).

While studying the ships in seas as a structural dynamic problem it is natural to employ the modal analysis. Two distinct types of modal analysis are dry mode and wet mode analysis. In wet mode analysis the frequencies and modal shapes are determined by not only considering the mechanical properties of the structure but also the fluid forces i.e. the hydrostatic forces exerted by the sea on the structure are treated as stiffness forces. In dry mode analysis the frequencies and modal shapes are determines assuming hull as a free-free beam in the absence of fluid, it is assumed all the fluid forces are applied (R.E.D. Bishop et al 1976). Both the analyses are compatible and have their advantages and disadvantages. On the whole the balance lies in favour of dry mode analysis (R.E.D. Bishop et al 1976).

Hydroelastic theories can be classified into 2D linear, 2D nonlinear, 3D linear, 3D nonlinear theories. The 2D and 3D linear theories in frequency domain are well matured. The 3D non linear theories and the hydro elasticity considering non linear structural behaviour are under development (Xu-un Chen et al, 2005). At smaller motions it is assumed the linear conditions satisfy therefore linear approximations are used in linear theories and at larger motions the structures may exhibit non linear dynamic responses. The unified 3D hydroelastic theories are used in the applications of non beam like structures such as multi hull vessels. The 3D hydro elasticity is a combination of 2D beam or 3D finite element method with 3D potential flow analysis (S. E. Hirdaris et al, 2009). 1D FEM structural model and 3D potential flow analysis model is an effective combination for performing hydroelastic analysis (Ivo Senjanovic et al, 2009). When the hydroelastaic analysis is performed on a 7800TEU container the results have shown that the transfer functions of hull sectional forces at resonant vibration (springing) are much higher than at resonant ship motions (Ivo Senjanovic et al, 2009). The vertical bending moment and shear forces of a beam and 3D structural model of a dry mode analysis for the first modes are comparable but the differences increases with the model complexity and also the differences exists in the antisymmetric modal shapes. The differences in the antisymmeric model shapes is expected due to the prismatic nature of the structure (S. E. Hirdaris et al, 2003). Another aspect of hydroelasticity is slamming and whipping. A comparative study made between the recorded data and the theoretical prediction of slamming using hydroelastic approximation is in agreement (S. Aksu et al, 1993).

Some of the conclusions from literature review are, 2D and 3D linear theories in frequency domain are well matured. The 3D non linear theories and the hydro elasticity considering non linear structural behaviour are under development (Xu-un Chen et al, 2005). The ultra large container ships are quite flexible and stretch the bounds of the classification society rules therefore the hydro elasticity analysis has to be performed. The classical way of determining the ship motions are wave loads is by assuming the ships as rigid structure and the obtained wave loads are applied on the 3D FEM for analysing the longitudinal and transverse loads but the approach may not be reliable due to number of influences of wave loads and ship responses. For the reliable analysis the hydroelastic analysis has to be solved as a coupled problem (Ivo Senjanovic et al, 2009). The elastic model test data available for validation are very few so there is a immediate need for validating of results (S. E. Hirdaris et al, 2003).

CHAPTER 3: DYNAMIC BEAM FORMULATIONS

3.1 INTRODUCTION

The dynamic behaviour of the structure includes vibration of structure along length, width and rotation. The vibrating structure excites at certain frequencies known as natural frequencies or Eigen frequencies. The Eigen frequencies are associated with Eigen shapes or mode shapes. There are infinite numbers of Eigen frequencies for a structure, but the excitation in the first few Eigen frequencies is much higher than the excitation in later frequencies. Since dealing with the complete dynamic behaviour i.e. considering vibration along the width and rotation is too complex therefore in the present thesis work vertical vibrations along the length is only taken into consideration which are also known as symmetric responses.

According to the Rayleigh theorem (R.G.D. Bishop et al, 1978) any distortion in a structure can be expressed as an aggregate of distortions in its Eigen modes. The pictorial representation of Rayleigh theorem is shown in figure 4 where the first two modes represent the rigid body modes and the later represents distortion modes. The dynamic behaviour of the structure is illustrated in terms of inertia, damping and stiffness. The ships are considered as free-free beams in seas.

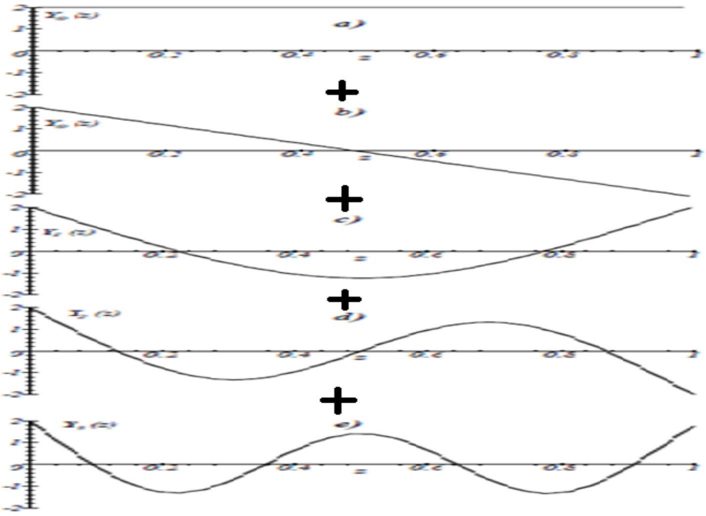

Figure 3 : Different modes of vibration

The deflections in the beam can be either linear or non linear depending on the excitation force. If the deflections are smaller in relation to length, for the corresponding excitation forces, then linearity is taken into consideration. Usually for the linearity condition to be applied the deflections should be few percent of the length. In the thesis work it is considered that the deflections are small so linear beam theory is taken into consideration. The two different linear beam theories are Euler Bernoulli beam theory and Timoshenko beam theory. In Euler Bernoulli beam theory the shear deformations and rotary inertia are neglected and in Timoshenko beam theory the shear deformation and rotary inertia are taken in to consideration. The shear deformations are important if there is influence of higher modes. In the present thesis work Euler Bernoulli beam theory is taken into consideration to simplify the analysis.

3.2 DERIVATION

The following section gives the outline of dynamic beam derivation, step by step procedure of derivation is included in appendix A or can be found in Bishop and price (1978). Figure 1 represents the slice of the beam and the forces acting on the beam element where M and V are the bending moment and the shear forces and Z(x, t) represents the buoyancy force, weight and all other fluid forces acting on the beam element.

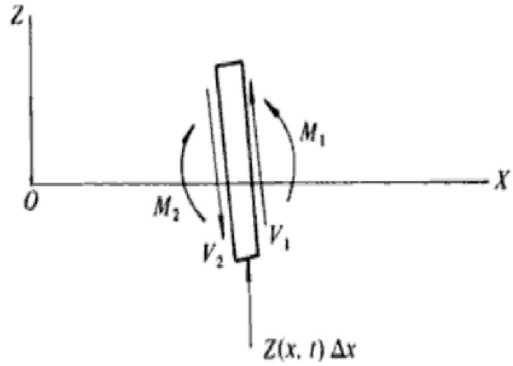

Figure 4: Beam element

The motion of a slice of the beam in the vertical direction is governed by the following equation

$$V_1 - V_2 + Z(x,t)\Delta x = \mu(x)\,\Delta x\,\ddot{w}(x,t)$$

$$\frac{\partial V}{\partial x} + Z(x,t) = \mu(x)\ddot{w}(x,t) \tag{3.1}$$

If the rotator inertia is neglected

$$M_1 - M_2 + V\Delta x = 0$$

$$V = -\frac{\partial M}{\partial x} \tag{3.2}$$

From elementary beam theory

$$M = EI(\mathrm{x})\frac{\partial^2 w(x,t)}{\partial x^2} + \beta(x)\frac{\partial^3 w(x,t)}{\partial x^2 \partial t} \tag{3.3}$$

$$V = -\frac{\partial}{\partial x}\left[EI(\mathrm{x})\frac{\partial^2 w(x,t)}{\partial x^2}\right] - \frac{\partial}{\partial x}\left[\beta(x)\frac{\partial^3 w(x,t)}{\partial x^2 \partial t}\right] \tag{3.4}$$

Substituting equation 3.4 in 3.1, the equation 3.1 can be written as

$$\mu(x)\ddot{w}(x,t) + [\beta(x)\dot{w}''(x,t)]'' + [EI(x)w''(x,t)]'' = Z(x,t) \tag{3.5}$$

Equation 3.5 is known as Euler-Bernoulli beam equation, where $\mu(x)$ is mass per unit length and $\beta(x)$ is damping per unit length and $EI(x)$ the stiffness per unit length. The first term in equation 3.5 represents the inertia effect, second term the damping and third term the stiffness of the beam.

The deflection is a function of space and time, applying separation of variables the deflection can be written as

$$w(x,t) = \sum_{r=0}^{\infty} p_r(t) w_r(x) \tag{3.6}$$

Boundary conditions for a free-free beam are the bending moment and shear forces at the ends are zero i.e. $f''(x) = f'''(x) = 0 \; at \; x = 0, l.$ The General solution for finding the natural modes is given as

$$f(x) = AF(x) + BG(x) + CH(x) + DJ(x) \tag{3.7}$$

where A, B, C, D are variables and $F(x)$, $G(x)$, $H(x)$, $J(x)$ are coefficients.

The boundary conditions are applied on the general solution equation 3.7 which gives four equations. The equations are rearranged into coefficient matrix and variable vector. For the non trivial solution the determinant of the coefficient matrix should be zero, which is equation in ω and its roots are the natural frequencies of the beam. Corresponding to the rth root the variables A, B, C and D can be calculated which are all not zero. Substituting the corresponding variables and frequencies in general solution equation 3.7 gives the natural mode shapes (Appendix A).

As mentioned in the section 3.1, according to the Rayleigh theorem any distortion in a beam can be expressed as an aggregate of distortions in its principal modes. The principle modes should obey the orthogonality condition. The orthogonality condition states that the product of the different modes and integration over the length should be equal to zero.

Applying the displacement equation 3.6 and orthogonality condition in equation 3.5 the final equation can be written as (Appendix A)

$$\sum_{r=0}^{\infty} a_{rs}\delta_{rs}\ddot{p}_r + \sum_{r=0}^{\infty} \dot{p}_r \int_0^l \beta(x) w_r''(x)\, w_s''(x) dx + \sum_{r=0}^{\infty} a_{rs}\delta_{rs}\omega_r^2 p_r = \int_0^l Z(x,t) w_s(x)\, dx \tag{3.8}$$

where

$$\begin{gathered} \int_0^l \mu(x) w_r(x) w_s(x) dx = a_{rs}\delta_{rs} \\ \delta_{rs} = 0 \; for \; r \neq s \\ \delta_{rs} = 1 \; for \; r = s \end{gathered} \tag{3.9}$$

The equation 3.8 in the matrix form can be written as

$$\boldsymbol{A\ddot{P}} + \boldsymbol{B\dot{P}} + \boldsymbol{CP} = \boldsymbol{Z(t)} \tag{3.10}$$

where **A**, **B**, and **C** are the inertia, damping and stiffness matrices. **P** and **Z** are the column vectors where **P** representing the generalised coordinates and **Z** the input loading with time. The size of **A**, **B**, **C, Z** and **P** depends on the number of modes chosen for analysis. The equation 3.10 is a second order differential equation. The general procedure for solving the higher order differential equation is to make it into the respective number of first order differential equations. Therefore it is assumed as $P = P_1,\ \dot{P} = P_2$. Substituting the assumption in equation 3.10, equation 3.10 can be written as

$$\begin{bmatrix} \dot{P}_1 \\ \dot{P}_2 \end{bmatrix} = \begin{bmatrix} P_2 \\ \dfrac{\boldsymbol{Z - BP_2 - CP_1}}{A} \end{bmatrix} \tag{3.11}$$

The generalised coordinates for each time step are obtained by solving equation 3.11 using Rung-Kutta method providing the initial values. After obtaining the generalised coordinates the deflection, bending moment, and shear force can be calculated according to equations 3.12, 3.13, and 3.14

$$w(x,t) = \sum_{s=0}^{\infty} p_r(t) w_r(x) \tag{3.12}$$

$$M(x,t) = \sum_{s=0}^{\infty} p_r(t) M_r(x) \tag{3.13}$$

$$V(x,t) = \sum_{s=0}^{\infty} p_r(t) V_r(x) \tag{3.14}$$

By neglecting the dynamic terms from equation 3.8 the static response in modal form can be written as in equation 3.15, and by rearranging equation 3.15 the generalised coordinates of static response can be written as in equation 3.16. The static deflection, bending moment, and shear force can be written similarly as in equations 3.12, 3.13, and 3.14

$$a_{ss}\omega_s^2 \overline{p_s} = \int_0^l Z(x,t) w_s(x)\, dx \tag{3.15}$$

$$\overline{p_s} = \frac{1}{a_{ss}\omega_s^2} \int_0^l Z(x,t) w_s(x) dx \tag{3.16}$$

3.3 Implementation and Demonstration

3.3.1 Introduction to coding

A beam of length 73m, width 8m, height 7m, Young's Modulus(E) 70GPa, and area moment of inertia(I) 2.94m^4 is chosen for demonstration. The code is developed using MATLAB program, the code consists of a main and 5 functions (DNV_HS, naturalmodes, coefficients, beamODE, and post processing). The DNV_HS function calculates the approximate moment of inertia (I) required for the length assumed according to DNV regulations for high speed boats. The naturalmodes function calculates the natural frequencies and mode shapes of the beam with I value calculated according to DNV_HS function. The coefficients function calculates inertia, damping, and stiffness coefficients with mode shapes and sectional information. The beamODE function solves the equilibrium equation with the coefficents and load distribution as input. MATLAB inbuilt solver ODE45 is developed using the Rung-Kutta method. The ODE45 solver is used to solve the beamODE function which consists of the equilibrium equation by providing the initial values and time range for analysis to be performed. The solver returns the generalised amplitudes and velocities for all time steps. The program structure is shown in figure 6.

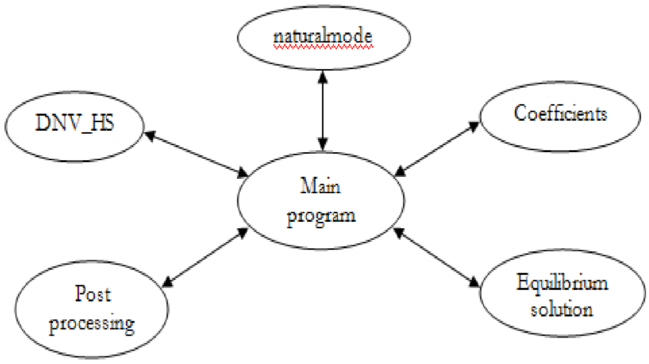

Figure 5 : Program outline

3.3.2 Static condition

An attempt has been made to compare the analytical solution with modal solution in static condition, for this purpose a uniform loading of 500 kN is applied on a simply supported beam (figure 6). From figure 7 it can be seen that the modal solution and analytical solution are in agreement with each other.

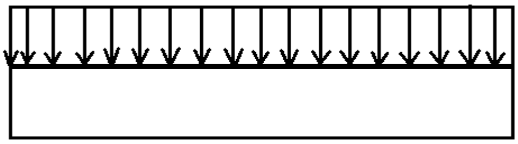

Figure 6 : uniformly loaded beam

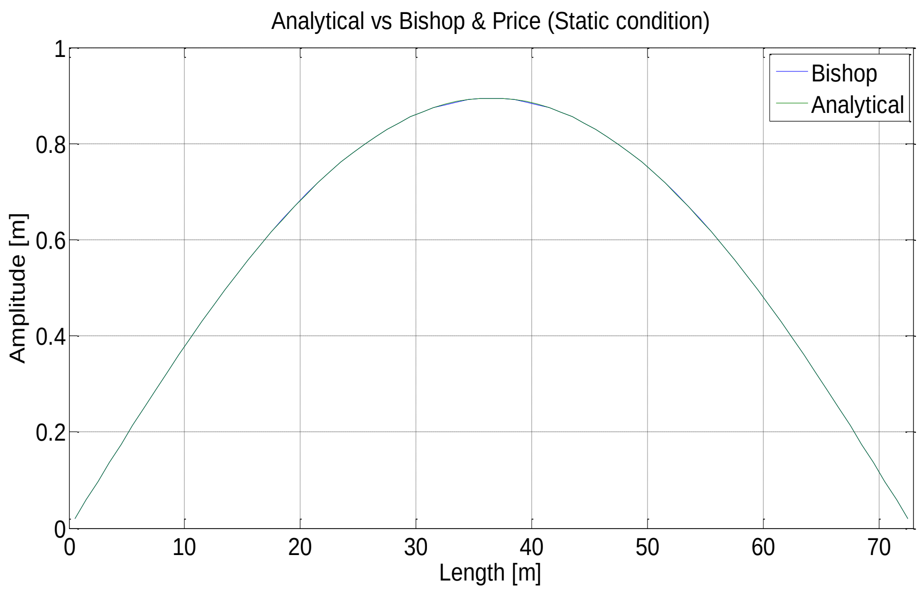

Figure 7: comparison between analytical and bishop & price for uniformly distributed load case at static condition

3.3.3 Dynamic condition

An attempt has been made to illustrate the dynamic behaviour of the structure i.e. structural responses with respect to time, for this uniform load of 500 kN is applied on a simply supported beam (figure 6). The load is varied at a frequency of 1rad/s which is away from the natural frequency of the structure (quasi-static). Figure 10 illustrates the amplitude at mid section with respect to time. From figure 7 and figure 8 the static and quasi-static conditions are in agreement.

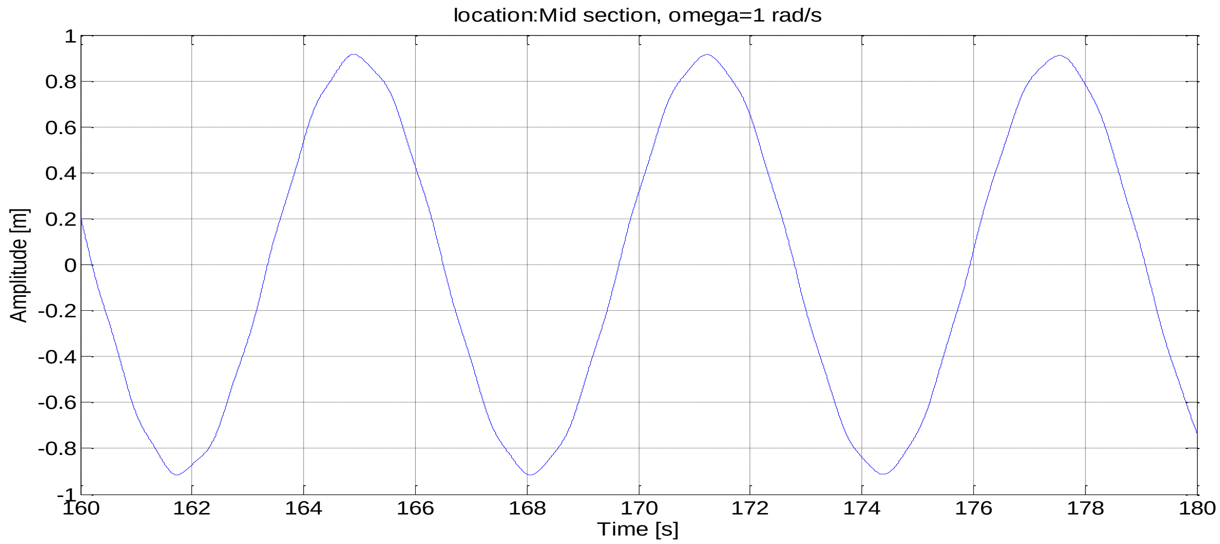

Figure 8: Amplitudes at mid section when frequency is away from natural frequency i.e. 1 rad/s

<u>Springing</u>

For illustrating the springing phenomena loading frequency is made equal to natural frequency of the structure i.e. 8.5 rad/s. Figure 9 illustrates the spring effect at amidships section. From figure 9 it can be seen that the structure reaches steady state around 7s and by comparing figure 8 and figure9, it can be observed that the amplitude in figure 9 are higher than in figure 8.

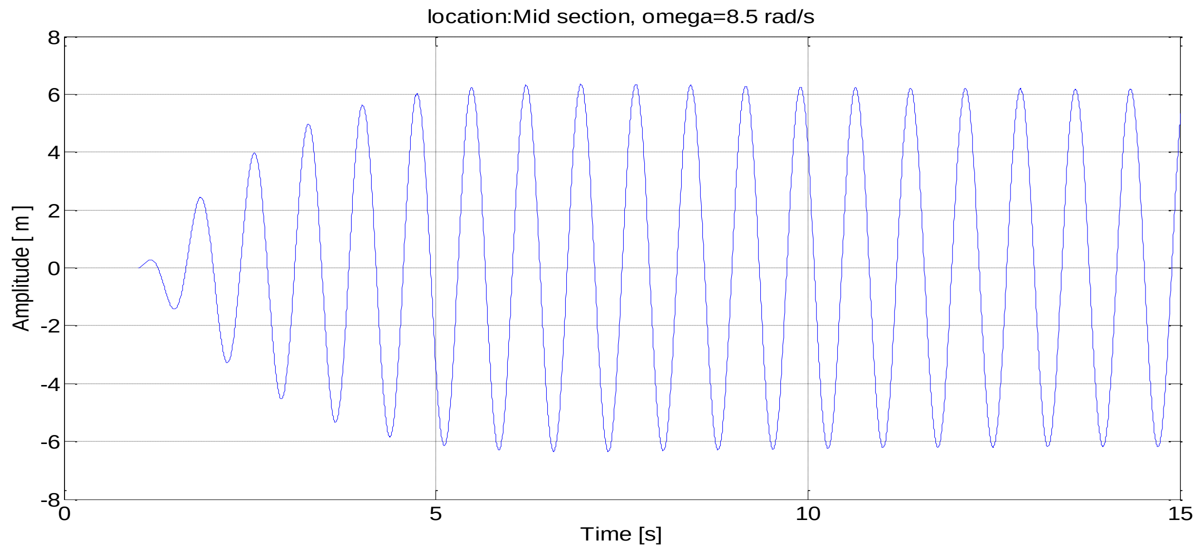

Figure 9: Amplitudes at mid section when frequency is close to natural frequency i.e. 8.5 rad/s

<u>Whipping</u>

For illustrating the whipping phenomena loading on the structure for the first 20s is taken as zero and a sudden of 100 kN is applied at 20th s for 2 seconds and the load is brought to zero after 2 seconds. Figure 10 shows the whipping effect at amidships section. From figure 10 it can be seen that, at 20th s the amplitude increases suddenly and also the number of stress cycles the structure experiences before it comes to rest.

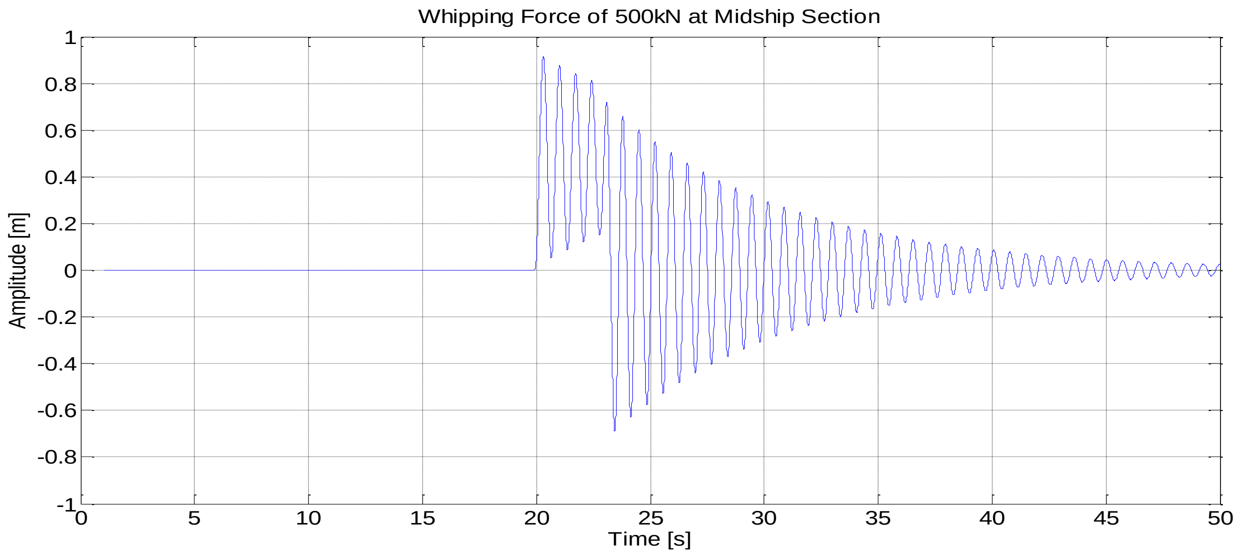

Figure 10: Response of the structure due to the sudden load at 20th second.

CHAPTER 4

4.1 MODELLING

The common ways of designing the ship structures are using the semi empirical formulation developed by the classification societies or using the numerical procedures. Designing the structure using classification societies is simpler but are based on simple quasi static procedures, which may not be suitable for all operating conditions or may not meet all the ship designers requirements. An example of how the bending moment formulation of classification societies is developed is illustrated in section 4.2 On the other side using numerical procedures require lot of computation time but as mentioned earlier since the length and speed of the ships are increasing it is required to have the good understanding of structural behaviour of the ships.

4.2 DNV FORMULATIONS

The bending moment formulation taken from DNV is shown in equations 4.1 and 4.2. As it can be seen from equations 4.1 and 4.2 that length and breadth and block coefficient are taken into consideration. The wave is defined in terms of wave coefficient. The moment is calculated according to force times the perpendicular distance.

$$Mw = 0.19 \cdot Cw \cdot B \cdot L^2 \cdot C_B \text{ hogging moment in kN-m} \tag{4.1}$$

$$Mw = 0.110 \cdot Cw \cdot B \cdot L^2 \cdot (C_B + 0.7) \text{ sagging moment in kN-m} \tag{4.2}$$

Where L: ship length in m
B: ship breadth in m
C_B: block coefficient
Cw: wave height coefficient

$$Cw = 10.75 - \left[\frac{300 - L}{100}\right]^{3/2} \quad for\ 90m < L < 300m$$

The wave bending moment shown in equation 4.1 is derived as follows, the following section is taken from [].Figure 12 is considered for illustrating the wave bending moment derivation.

The sinusoidal wave is applied to a block shaped ship which produces added buoyancy at the midship and reduced buoyancy at the ends. The forward part of ship is considered, due to symmetry the upward force is located at the centroid of added buoyancy and downward force is located at the centroid of lost buoyancy.

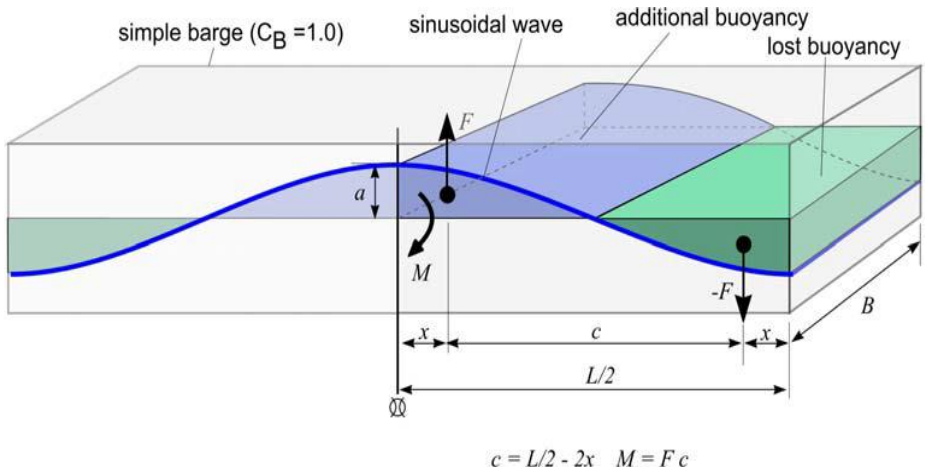

Figure 11

The force is found as

$$F = 9.8 \cdot 1.025 \cdot B \cdot a \cdot \int_0^{L/4} cos\left(\frac{2\pi x}{L}\right) dx = 1.60 \cdot B \cdot a \cdot L \qquad (4.3)$$

The centroid location is

$$x = \frac{\int_0^{L/4} x \cdot cos\left(\frac{2\pi x}{L}\right) dx}{\int_0^{L/4} cos\left(\frac{2\pi x}{L}\right) dx} = \frac{L(\pi - 2)}{4\pi} \qquad (4.4)$$

c is

$$c = {}^{L}\!/_{2} - 2\frac{L(\pi - 2)}{4\pi} = 0.318 \cdot L \qquad (4.5)$$

The bending moment is

$$Mw = 0.510 \cdot B \cdot a \cdot L^2 \qquad (4.6)$$

The effect of block coefficient is taken into account by assuming all the lost volume is at the ends of the ship so the effective length is reduced by C_B thus the C_B^2 term is introduced.

$$Mw = 0.510 \cdot B \cdot a \cdot L^2 \cdot C_B^2 \qquad (4.7)$$

It is assumed Cw is the design wave height, amplitude a is $Cw/2$. C_B^2 is replaced by $0.88 \cdot C_B$ (0.88 being the possible block coefficient for low speed vessels), the wave bending moment becomes

$$Mw = 0.224 \cdot Cw \cdot B \cdot L^2 \cdot C_B \qquad (4.8)$$

Equation 4.8 is higher than equation 4.1 and is noted that Cw is larger than recommended wave height i.e. $0.0607 \cdot L^{0.5}$. For ships of 150m the $0.0607 \cdot L^{0.5}$ wave is 83.5% of the Cw value and is included by setting up the amplitude a to $0.417 \cdot Cw$ which gives the wave bending moment as in equation 4.1

$$Mw = 0.19 \cdot Cw \cdot B \cdot L^2 \cdot C_B \qquad (4.9)$$

4.3 Numerical Methods

Strip methods are commonly used numerical methods for evaluating the responses of the ship in waves, responses includes motion responses (surge sway, roll etc.) and structural loads (bending moment, shear forces). In strip method 3D hydro mechanical problem is divided into series of 2D problems, the hydrodynamic coefficients i.e. added mass, damping, and wave excitation forces for 2D problems are calculate and the 2D coefficients are integrated to get the 3D coefficients[4].

Strip methods can be classified into rigid body and elastic strip methods. In rigid body strip methods the rigid body mode shapes are only considered in evaluating the structural loads and in elastic strip methods the distortion mode shapes are also taken into consideration. The strip methods can further be sub categorised into linear and non linear strip methods. As introduced in section 2.3 in linear strip methods it is assumed that the linearity condition satisfies i.e. motion of the structure is proportional to the excitation force. In linear strip methods the results are studied in frequency domain. In non linear strip methods any of the linear condition is replaced by non linearity, such as at larger motions the excitation force is not proportional to the amplitude. In non linear strip methods the results are studied in time domain. The non linear strip methods require more computation resources. The linear strip methods are commonly used because of the simplicity included in the modelling, and the results are also in fairly in agreement with the experimental results.

4.4 Hydro Elasticity

Numerical methods are helpful in understanding the structural behaviour for various sea conditions. Modelling of elastic ships in waves is known as Hydroelasticity of ships. The hydroelastic theory includes the mutual interaction between the inertial, hydrodynamic and elastic forces of the structure and sea. Hydroelasticity is schematically illustrated in figure 14. For a chosen sea condition the hydrodynamic forces on the hull are calculated and the obtained hydrodynamic forces are applied on structural dynamics problem (direction 1 in the figure below) and structural deformations are calculated. Taking the kinematic effect of the structure into consideration the new set of hydrodynamic forces are calculated (direction 2 in figure) and obtained hydrodynamic forces are again provided to the structural dynamic problem (repeat step1)and the new set of structural deformations are calculated. The procedure is repeated for a given set of time or sea condition. Finally the structure is designed according to the obtained results.

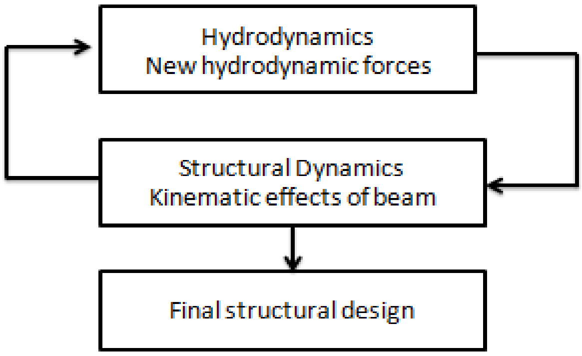

Figure 12 : Hydroelastic interaction

Chapter 5: Tentative modelling study

5.1 Introduction

An attempt has been made to study the dynamic structural responses for a particular hull with the hydrodynamic forces and added mass from high speed strip program developed at KTH Garme and Rosen (2003). Dimensions and geometry of hull used for analysis are given in figure 13, and the hull is divided into 127 sections. Since the structures of high speed boats are stiff and for simplicity it is assumed that the influence of structural deformation on the hydrodynamic forces can be neglected therefore the coupling effect is not taken into consideration. While applying loads on the structure the hydrodynamic loads are only taken into consideration while the inertia loads are excluded for simplicity.

The vessel speed of 35 knots and three different wave conditions of significant wave height of 3m and wave periods of 5, 7 and 9 s are chosen for analysis. The hydro dynamic simulations are performed for a time span of 125s. Two different approaches are used to study the responses of structure, one w.r.t. time and other w.r.t. length. An example for w.r.t. time is shown in figure 14 and w.r.t. length is shown in figure 15. Figure 14 illustrates the force distribution along the fore part (section-96) of hull for a wave height of 3m and wave period of 5s. Figure 15 illustrates the force distribution, amplitude, bending moment, shear force along the length at particular time steps.

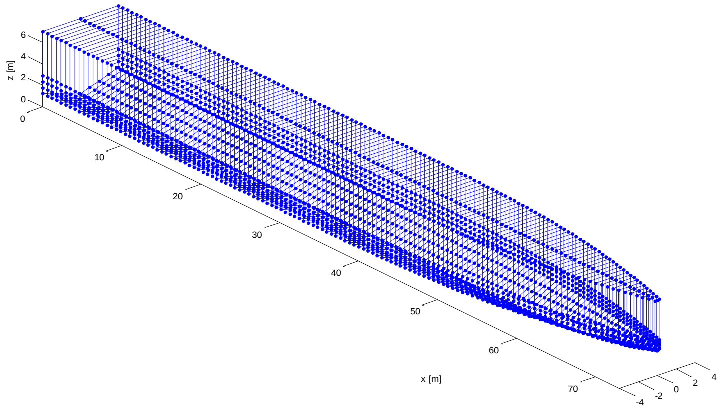

Figure 13: Tentative model hull Geometry

Length	73m
Breadth	8m
Height	7m
Displacement	640tonnes
Speed	35 knots
Wave height	3m
Wave periods	5, 7, 9s
Young's Modulus	70, 200 GPa
Bending Moment (DNV)	$5.9*10^7$ Nm

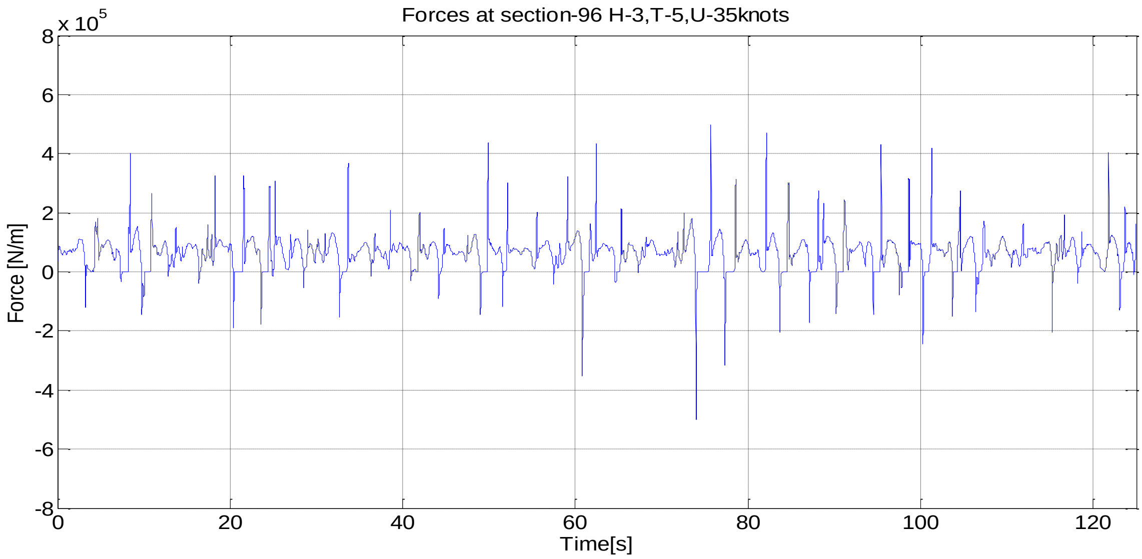

Figure 14: Forces acting on section 96

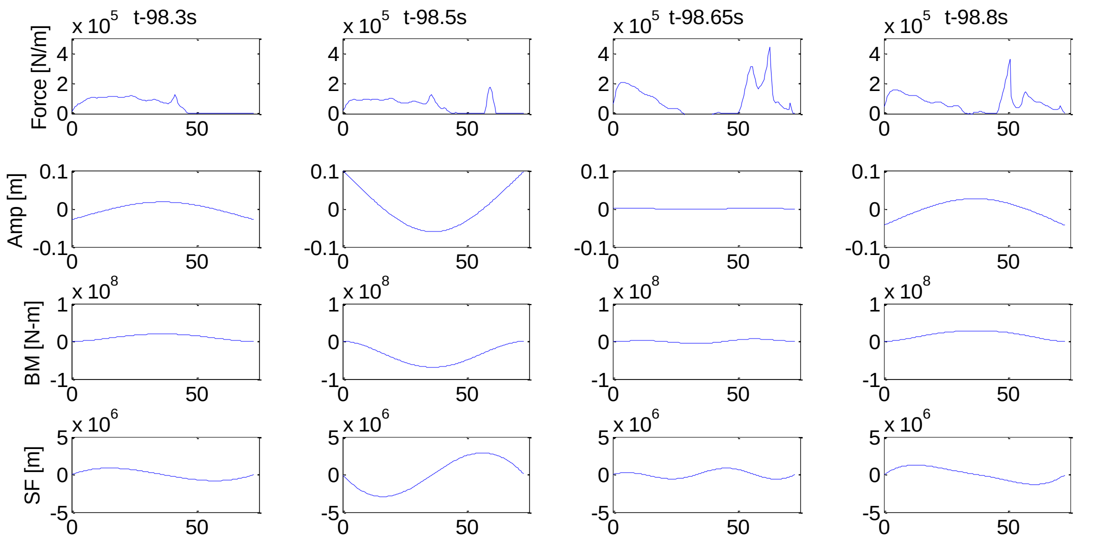

Figure 15: Forces, Amplitudes, Bending Moments, Shear forces along the length of the ship at particular time steps

5.2 STUDY PARAMETERS

Attempts have been made for studying the influence of added mass, damping and stiffness on the structural responses. The influence of added mass on the structure depends on the structural mode shape therefore added mass values from the hydro dynamic analysis are to be separated according to different modes shapes, but for simplicity it is assumed that the influence of added mass on all mode shapes to be the same. Information about separation of added mass according to modal shapes is available in Bishop & Price (1979). Two different added mass values, one the total value from the hydrodynamic analysis and the other the half the values from hydrodynamic analysis are chosen for analysis.

Damping in structure according to Bishop and Price (1979) can be calculated using different methods, of which Johnson Ayling & Couchman (1962) is chosen in the thesis work. Usually evaluating the damping in structures is critical, the normal procedure for evaluating the damping is to obtain the total damping of the system and then the hydrodynamic damping is deducted from the total damping for obtaining the structural damping (R E D Bishop et al (1978)). Two different values, one the values calculated according to Johnson Ayling & Couchman (1962) and other by increasing the values of Johnson Ayling & Couchman (1962) by 10 times.

For verifying the influence of stiffness two different moment of inertias (I), 1 and 3m^4 are chosen. The natural frequencies corresponding to 3m^4 are 20.12, 55.42, and 108.83 and for 1m^4 are 11.7, 32.25, 63.32.

5.2 STUDY OF INERTIA EFFECT

Two different simulations are performed one with added mass values from the hydrodynamic analysis of wave height of 3m and wave period of 5s, and other by reducing the values by half . The young's modulus of 70GPa, area moment of inertia 2.94m^4 and the damping values calculated according to Johnson Ayling & Couchman (1962) are considered for analysis. Figure 16 is the part of the simulation at the mid ship section (section 62) between 95th and 105th s, where the upper plot with higher added mass and the lower with lesser added mass simulations. The complete simulation output at midship section (section 62) is presented in Appendix C figure 22

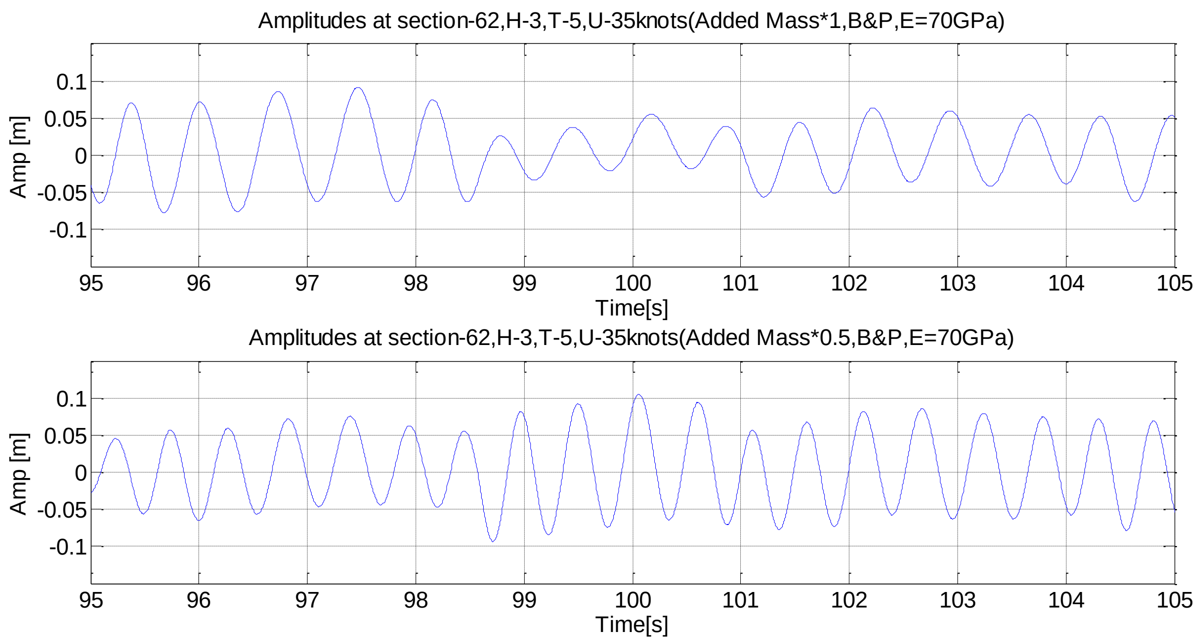

Figure 16: Amplitudes at midship section (section 62) in addedmass study

From the simulations it is observed that the amplitudes are higher for higher added mass simulation. And the other observation from the simulation is that the amplitudes at around 100ths with higher added mass are lesser than the amplitudes with smaller added mass (figure 16) for understanding a close observation is made by looking at individual time steps(figure 17) and it is observed due to the history of the motion of the structure i.e. from figure 17 it can be observed that the force acting at a particular time during higher added mass simulation is opposing the motion of the structure while in the other case it is supporting the motion of the structure.

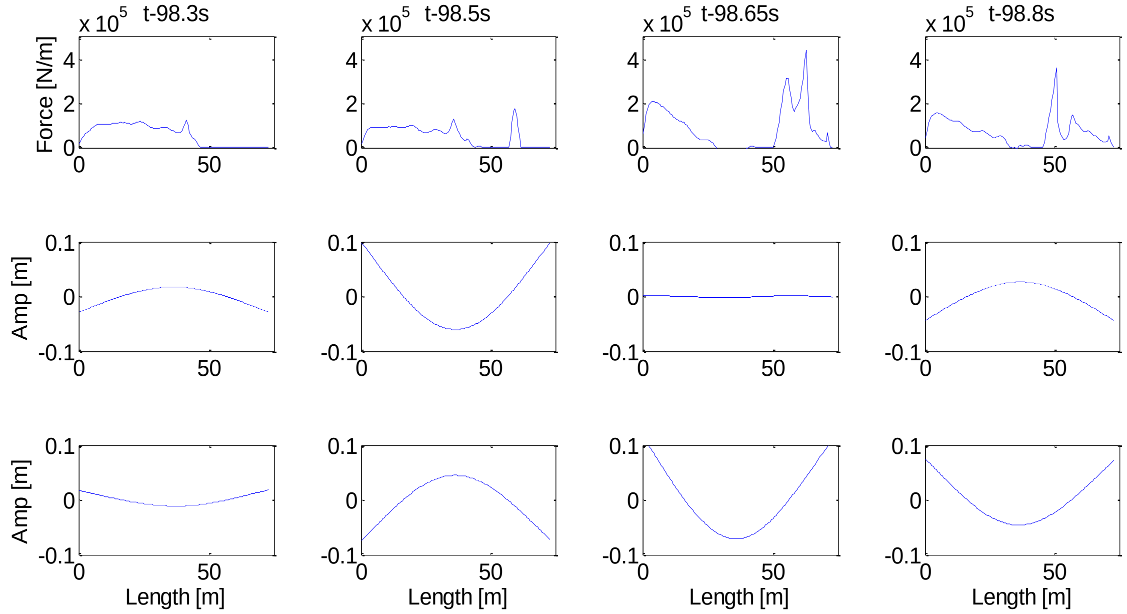

Figure 17: Forces, Amplitude (Addedmass form wave period 5s), Amplitude (Addedmass from wave period5s*0.5), along the length of the ship at particular time steps for added mass study.

From the added mass study it has been observed that on average amplitudes are higher with the higher added mass influence. As the results can be expected

5.3 Study on damping effect

For verifying the influence of damping two different simulations are performed, one with the values calculated according to Johnson Ayling & Couchman (1962) and other by increasing the values of Johnson Ayling & Couchman (1962) by 10. The young's modulus of 70 GPa, area moment of inertia 2.94m^4, and the added mass values from 5s wave period simulations are chosen for analysis. Figure 18 shows the part of the simulation output at midship section (section 62) between 70th and 100th s, where the above plot with lesser damping are over 0.1m with lesser damping and 0.05m with higher damping (figure 18). The comparison between the amplitudes at mid ship section (section 62) for the complete simulation is shown in Appendix C figure 23.

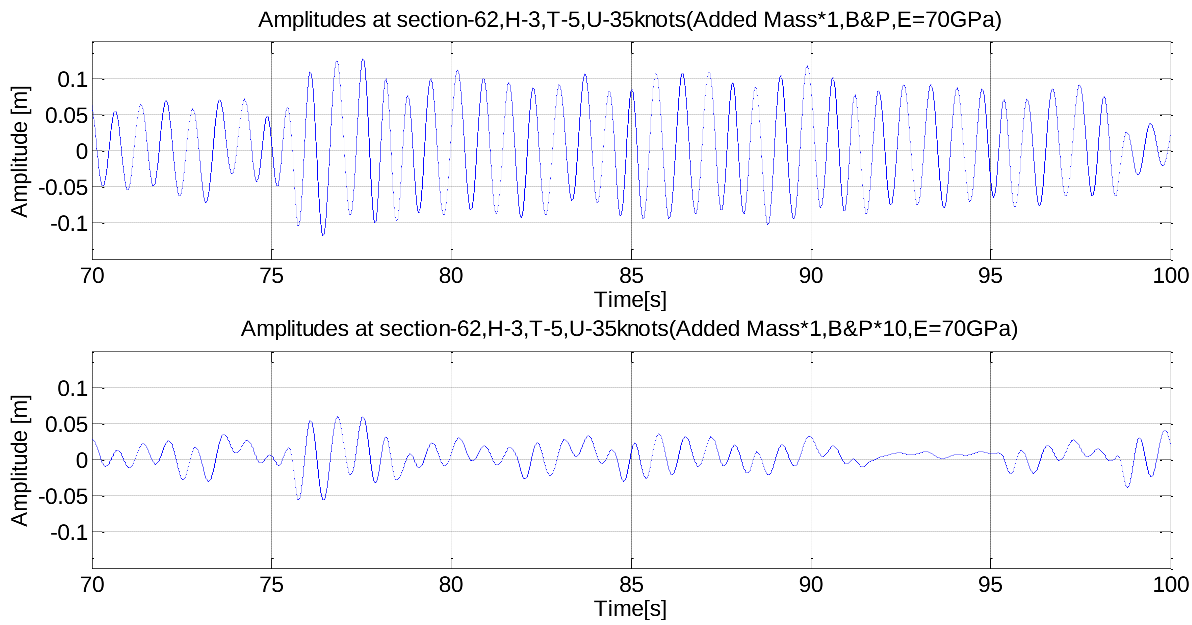

Figure 18: amplitudes at midship section (section 62) for damping study

In order to illustrate the influence of damping during whipping two different simulations are performed, one with the damping values calculated according to Johnson Ayling & Couchman (1962) and the other by increasing the values of Johnson Ayling & Couchman (1962) by 10. Since the slamming loads from the hydro dynamic simulations have not shown considerable whipping, slamming load at a particular time (around 75s) has been increased by 3 times. Figure 19 shows the simulation performed where the above plot with lesser damping and the below plot with higher damping. From figure 19 it can be seen that the lesser damping has the influence of whipping for longer time than the higher damping value. The higher damping has reached its normal operation at around 90th s.

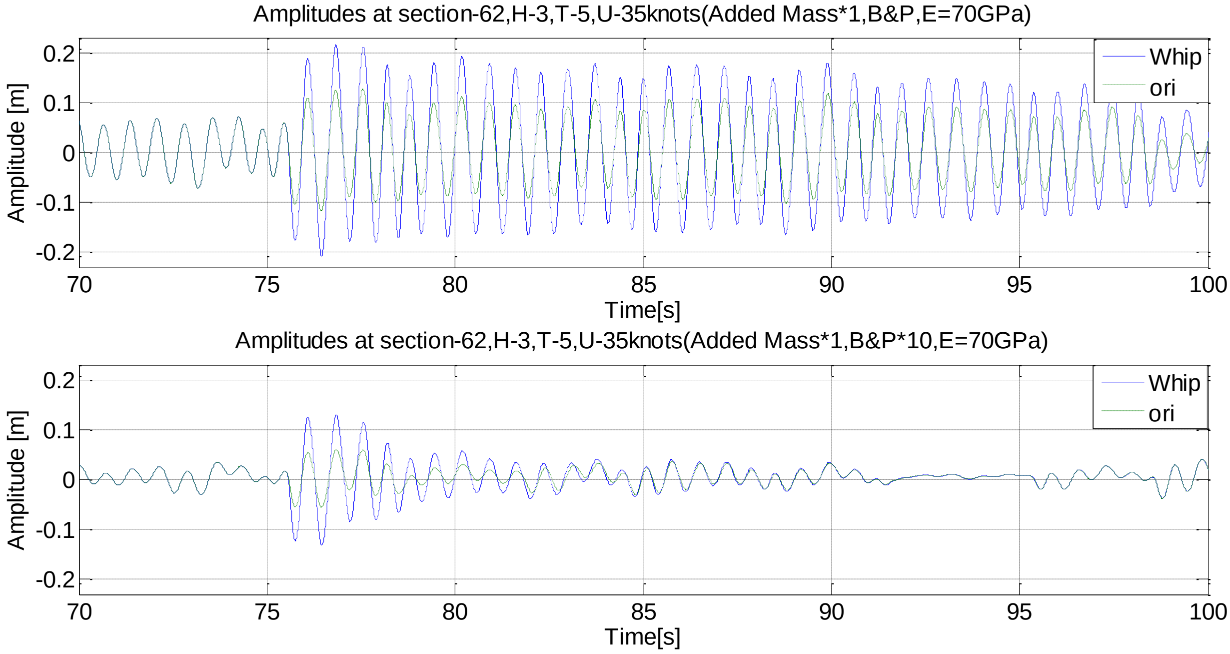

Figure 19: Amplitudes at midship section (section 62) for the damping study with whipping.

5.4 Study of structural stiffness

For verifying the influence of stiffness two different simulations are performed with moment of inertias (I), 1 and $3m^4$. Young's modulus of 70GPa, added mass values from the hydrodynamic simulation of wave period 5s, and damping values calculated according to Johnson Ayling & Couchman (1962) are chosen for analysis.

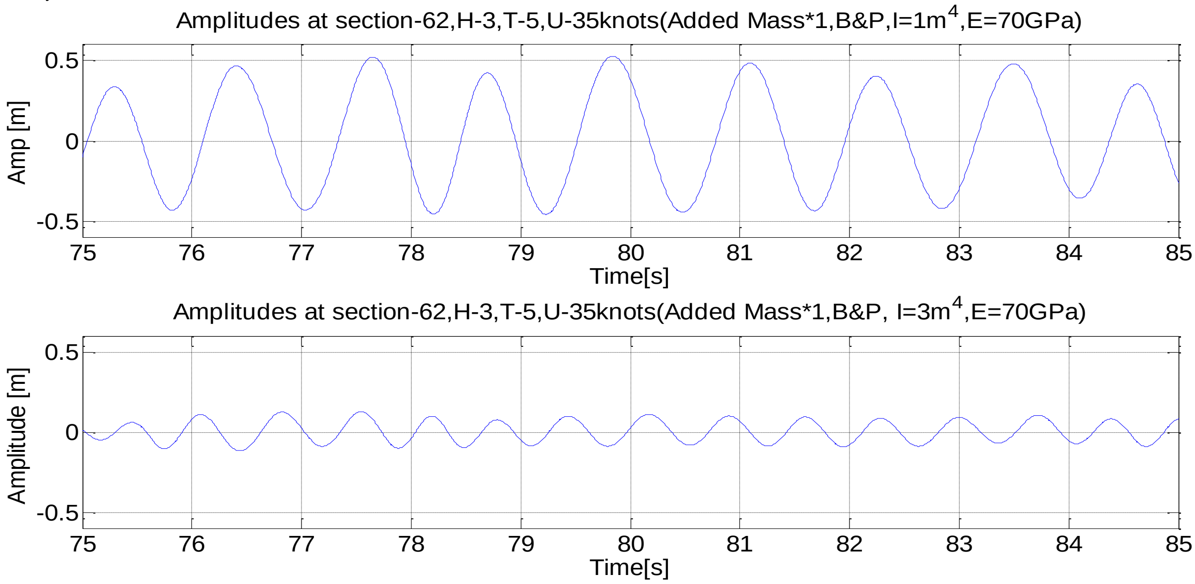

Figure 20: amplitudes at midship section (section 62) for stiffness study

Figure 20 shows the amplitudes at midship section (section 62) where the upper plot from $I=1m^4$ and lower plot from $I=3m^4$ simulations. Figure 21 shows the bending moment at midship section (section 62) where the upper plot with I=1m^4 and the lower plot with I=3m^4. From the figures 20 and 21 it can be observed that the amplitudes and bending moment are higher for lesser stiffer structure. The amplitudes for I=1m^4 are around 0.5m and for I=3m^4 are around 0.1m. The bending moment for I=1m^4 are around 2e8 N-m and for I=3m^4 are around 1.5e8 N-m.

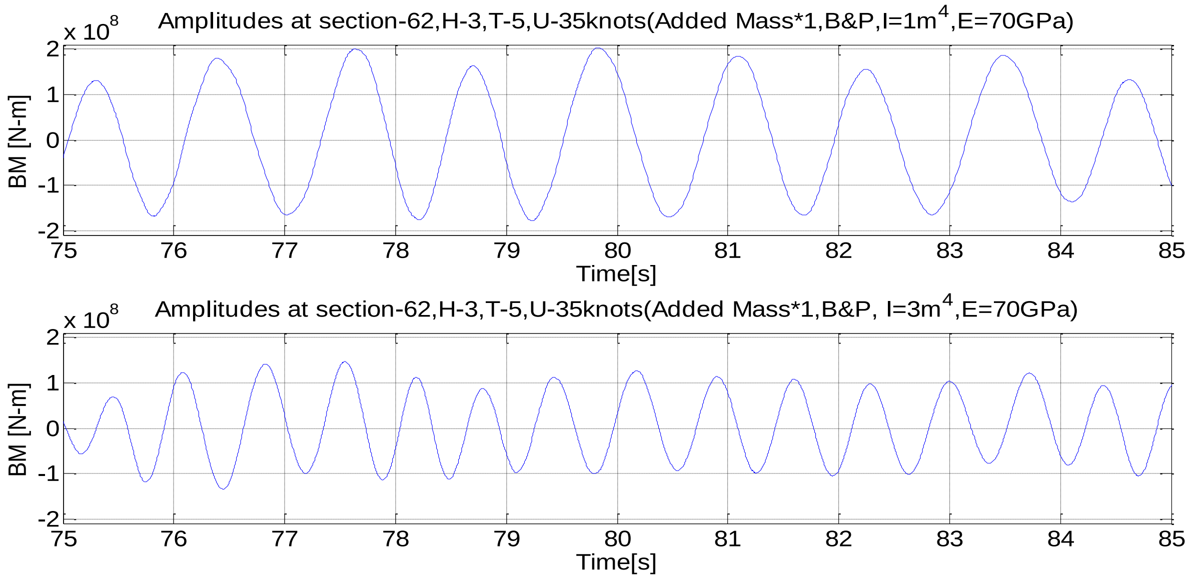

Figure 21: Bending moments at midship section (section 62) for stiffness study.

5.6 Study of various time periods

In order to verify the responses of the structure for various sea conditions, wave height of 3m and wave periods of 5, 7, and 9s are chosen for analysis. The young's modulus with 70 GPa, damping values calculated according to Johnson Ayling & Couchman (1962), and the added mass of respective sea condition are chosen. An investigation is performed to verify whether encounter time period are closer to the natural period of the structure, it is observed that the encounter time period are away from the natural time periods of the structure so no considerable effects are observed and the slamming loads are well below so no considerable whipping is observed in the simulation. The comparison between the amplitudes at mid ship section (section 62) for the complete simulations are shown in Appendix C figure 27.

Chapter 6: Discussions and Conclusions

This thesis work reviews the area of dynamic global hull structure responses and hydroelasticity. Thesis has been divided in to two parts, fist part deals with the literature review of global hydroelasticity and from the literature review section it is understood that the frequency domain theories are matured and the attempts have to be made towards non linear time domain methods. In the second part attempt has been made to formulate the Euler-Bernoulli beam in time domain using the theory proposed by Bishop and Price (1979). A comparison has been made between analytical and model solution, static and quasi-static solution of a uniformly loaded simply supported beam, and the results are in agreement. Springing and Whipping has been illustrated for a chosen loading condition.

In the tentative model chapter an attempt has been made to study the structural responses for a particular hull with the hydrodynamic forces and added mass from high speed strip program developed at KTH (Garme and Rosen (2003)). An attempt has also been made to understand the influence of various parameters like added mass, damping and stiffness and the results were as expected. In the added mass study it is observed that the average amplitudes are higher with the higher added mass influence i.e. higher the influence of inertia higher the amplitudes. In the damping study it is observed that the influence of whipping is lesser with higher damping i.e. higher the damping in the structure lesser the vibrations. In the stiffness study it is observed that the amplitudes and bending moments are higher in lesser stiffer structure i.e. less stiffer the structure higher the structural loads.

Due to the time limitations the analysis of tentative model remains incomplete. Some of the suggestions for further work in the tentative model are, only hydro dynamic forces are considered in the study attempts can be made to take the resultant force from the hydrodynamic analysis i.e. opposing force due to motion and weight of the structure has to be subtracted from the hydrodynamic forces, attempts have to be made for separating the added mass according to mode shapes, and elaborate study has to be performed while verifying with various sea conditions. In the thesis work 3 modes shapes are included, it is recommended to consider 4 or 5 mode shapes.

Some of the general suggestion are, since the same problem can be solved using energy methods attempts can be made to solve the problem using energy methods, and attempts can be made assuming the structure as Timoshenko beam. Since no realistic data is available for comparison attempts can be made for collection some experimental results.

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APPENDIX A- DERIVATION

NATURAL FREQUENCIES

General solution for finding the natural frequencies

$$f(x) = AF(x) + BG(x) + CH(x) + DJ(x)$$

Boundary conditions for free-free beam are that the bending moment and shear forces at the ends are zero.

$$f''(x) = f'''(x) = 0 \; at \; x = 0, l$$

Applying boundary condition in the general solution the

$$\begin{gathered} AF''(0) + BG''(0) + CH''(0) + DJ''(0) = 0 \\ AF''(l) + BG''(l) + CH''(l) + DJ''(l) = 0 \\ AF'''(0) + BG'''(0) + CH'''(0) + DJ'''(0) = 0 \\ AF'''(l) + BG'''(l) + CH'''(l) + DJ'''(l) = 0 \end{gathered}$$

$$\begin{bmatrix} F''(0) & G''(0) & H''(0) & J''(0) \\ F''(l) & G''(l) & H''(l) & J''(l) \\ F'''(0) & G'''(0) & H'''(0) & J'''(0) \\ F'''(l) & G'''(l) & H'''(l) & J'''(l) \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = 0$$

For Non trivial solution the determinant of the coefficients is zero

$$\begin{vmatrix} F''(0) & G''(0) & H''(0) & J''(0) \\ F''(l) & G''(l) & H''(l) & J''(l) \\ F'''(0) & G'''(0) & H'''(0) & J'''(0) \\ F'''(l) & G'''(l) & H'''(l) & J'''(l) \end{vmatrix} = 0$$

This is the equation in ω and its roots are the natural frequencies. Corresponding to the rth root the coefficients A, B, C and D can be calculated which are all not zero. Therefore the finite set of principal modes is available. The principle modes eventually obey the following equation.

$$\frac{1}{\mu(x)}[EI(x)w_r''(x)]'' = \omega_r^2 w_r(x)$$

ORTHOGONALITY

The principle modes should obey the orthogonality condition. The orthogonality condition states that the product of the different modes and integration over the length should be equal to zero which means that each mode is identical.

$$\frac{1}{\mu(x)}[EI(x)w_r''(x)]'' = \omega_r^2 w_r(x)$$

By multiplying with the sth mode and integrating over the length the above equation can be written as

$$\omega_r^2 \int_0^l \mu(x) w_r(x) w_s(x) = \int_0^l [EI(x)w_r''(x)]'' \, w_s(x)$$

From integration by parts and applying the the boundary limits the right hand side of the above equation can be shown as

$$\int_0^l [EI(x)w_r''(x)]'' w_s(x)dx = \int_0^l EI(x)w_r''(x)w_s''(x)dx$$

For simplicity of the equation the following simplified terms are assumed.

$$\int_0^l \mu(x)w_r(x)w_s(x)dx = a_{rs}\delta_{rs}$$
$$\int_0^l EI(x)w_r''w_s''dx = \omega_r^2 a_{rs}\delta_{rs}$$
$$\delta_{rs} = 0\ for\ r \neq s$$
$$\delta_{rs} = 1\ for\ r = s$$

PRINCIPAL COORDINATES

According to the Rayleigh theorem any distortion in a beam can be expressed as aggregate of distortions in its principal modes. Therefore the displacement equation can be written as fallows.

$$w(x,t) = \sum_{r=0}^{\infty} p_r(t)w_r(x)$$

Substituting the above equation in the equation of motion the equation of motion can be written as

$$\mu(x)\sum_{r=0}^{\infty}\ddot{p_r}(t)w_r(x) + \sum_{r=0}^{\infty}p_r(t)[EI(x)w_r''(x)]'' + \sum_{r=0}^{\infty}\dot{p_r}(t)[\beta(x)w_r''(x)]'' = Z(x,t)$$

Applying orthogonality condition the above equation can be written as

$$\sum_{r=0}^{\infty}\ddot{p_r}(t)\int_0^l \mu(x)w_r\, w_s(x)dx + \sum_{r=0}^{\infty}p_r(t)\int_0^l [EI(x)w_r''(x)]''\, w_s(x)$$
$$+\sum_{r=0}^{\infty}\dot{p_r}(t)\int_0^l [\beta(x)w_r''(x)]''\, w_s(x) = \int_0^l Z(x,t)\, w_s(x)dx$$

Treating the damping similar to the flexural rigidity it can be shown that

$$\int_0^l [\beta(x)w_r''(x)]''\, w_s(x)dx = \int_0^l \beta(x)w_r''(x)\, w_s''(x)dx$$

Substituting the damping simplification and the simplifications from the natural frequencies section the equation of motion can be written as

$$\textstyle\sum_{r=0}^{\infty} a_{rs}\delta_{rs}\ddot{p_r} + \sum_{r=0}^{\infty}\dot{p_r}\int_0^l \beta(x)w_r''(x)\, w_s''(x)dx + \sum_{r=0}^{\infty} a_{rs}\delta_{rs}\omega_r^2 p_r = \int_0^l Z(x,t)w_s(x) \qquad \text{xxx}$$

The equation of motion in the matrix form can be written as

$$\boldsymbol{A\ddot{P}} + \boldsymbol{B\dot{P}} + \boldsymbol{CP} = \boldsymbol{Z(t)}$$

Where **A**, **B**, **C** are the inertia, damping and stiffness matrices and **P** and **Z** are the column vectors representing the responses and input loading.

APPENDIX B1- NATURAL FREQUENCIES AND MODE SHAPES OF FREE-FREE CASE

General solution

$$X = A_1 \sinh kx + A_2 \cosh kx + A_3 \sin kx + A_4 cos\ kx$$

Boundary conditions

Free –Free $\qquad X'' = X''' = 0 \quad at\ x = 0, l$

Applying boundary conditions in the general solution

$$\begin{bmatrix} 0 & k^2 & 0 & k^2 \\ -k^2 sin\ kl & -k^2 cos\ kl & k^2 sinh\ kl & k^2 cosh\ kl \\ -k^3 & 0 & k^3 & 0 \\ -k^3 cos\ kl & k^3 sin\ kl & k^3 cosh\ kl & k^3 sinh\ kl \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} = 0$$

For the non trivial solution

$$\begin{vmatrix} 0 & k^2 & 0 & -k^2 \\ k^2 sinh\ kl & k^2 cosh\ kl & -k^2 sin\ kl & -k^2 cos\ kl \\ k^3 & 0 & -k^3 & 0 \\ k^3 cosh\ kl & k^3 sinh\ kl & -k^3 cosh\ kl & k^3 sinh\ kl \end{vmatrix} = 0$$

$$Cosh\ kl * cos\ kl = 1 \qquad i.e. \qquad kl = 0,\ 4.73, \quad 7.85, \quad 11$$

Natural frequencies

$$\omega_n = k_n^2 \sqrt{\frac{EI}{\mu}}$$

Modal shapes

$$\begin{bmatrix} cosh\ kl & -sin\ kl & -cos\ kl \\ 0 & -1 & 0 \\ sinh\ kl & -cos\ kl & sin\ kl \end{bmatrix} \begin{bmatrix} A_2 \\ A_3 \\ A_4 \end{bmatrix} = -\begin{bmatrix} \sinh kl \\ 1 \\ \cosh kl \end{bmatrix} A_1$$

$$A_1 = -1$$

$$\begin{bmatrix} A_2 \\ A_3 \\ A_4 \end{bmatrix} = \begin{bmatrix} cosh\ kl & -sin\ kl & -cos\ kl \\ 0 & -1 & 0 \\ sinh\ kl & -cos\ kl & sin\ kl \end{bmatrix}^{-1} \begin{bmatrix} \sinh kl \\ 1 \\ \cosh kl \end{bmatrix}$$

Coefficients for different kl values

kl	A1	A2	A3	A4
4.73	-1	1.0178	-1	1.077
7.85	-1	0.9992	-1	0.9928
11	-1	1	-1	1.0089

Deflection

$$X_1 = -1 \cdot \sinh\left(4.73 \cdot (\frac{x}{L})\right) + 1.0178 \cdot \cosh\left(4.73 \cdot (\frac{x}{L})\right) - 1 \cdot \sin\left(4.73 \cdot (\frac{x}{L})\right) + 1.077 \cdot \cos\left(4.73 \cdot (\frac{x}{L})\right)$$

$$X_2 = -1 \cdot \sinh\left(7.85 \cdot \left(\frac{x}{L}\right)\right) + 0.9992 \cdot \cosh\left(7.85 \cdot \left(\frac{x}{L}\right)\right) - 1 \cdot \sin\left(7.85 \cdot \left(\frac{x}{L}\right)\right) + 0.9928 \cdot \cos\left(7.85 \cdot \left(\frac{x}{L}\right)\right)$$

$$X_3 = -1 \cdot \sinh\left(11 \cdot (\frac{x}{L})\right) + 1 \cdot \cosh\left(11 \cdot (\frac{x}{L})\right) - 1 \cdot \sin 11\left(\frac{x}{L}\right) + 1.0089 \cdot \cos\left(11 \cdot (\frac{x}{L})\right)$$

Bending Moment

$$M_1 = E \cdot I \cdot X_1''$$
$$= E \cdot I \cdot \left(-1 \cdot \left(\frac{4.73}{L}\right)^2 \cdot \sinh\left(4.73 \cdot (\frac{x}{L})\right) + 1.0178 \cdot \left(\frac{4.73}{L}\right)^2 \cdot \cosh\left(4.73 \cdot (\frac{x}{L})\right) + 1 \cdot \left(\frac{4.73}{L}\right)^2 \cdot \sin\left(4.73 \cdot (\frac{x}{L})\right) - 1.077 \cdot \left(\frac{4.73}{L}\right)^2 \cdot \cos\left(4.73 \cdot (\frac{x}{L})\right)\right)$$

$$M_2 = E \cdot I \cdot X_2''$$
$$= E \cdot I \cdot \left(-1 \cdot \left(\frac{7.85}{\mathrm{L}}\right)^2 \cdot \sinh\left(7.85 \cdot \left(\frac{x}{L}\right)\right) + 0.9992 \cdot \left(\frac{7.85}{\mathrm{L}}\right)^2 \cdot \cosh\left(7.85 \cdot \left(\frac{x}{L}\right)\right) + 1 \cdot \left(\frac{7.85}{\mathrm{L}}\right)^2 \cdot \sin\left(7.85 \cdot \left(\frac{x}{L}\right)\right) - 0.9928 \cdot \left(\frac{7.85}{\mathrm{L}}\right)^2 \cdot \cos\left(7.85 \cdot \left(\frac{x}{L}\right)\right)\right)$$

$$M_3 = E \cdot I \cdot X_3''$$
$$= E \cdot I \cdot \left(-1 \cdot \left(\frac{11}{\mathrm{L}}\right)^2 \cdot \sinh\left(11 \cdot (\frac{x}{L})\right) + 1 \cdot \left(\frac{11}{\mathrm{L}}\right)^2 \cdot \cosh\left(11 \cdot (\frac{x}{L})\right) + 1 \cdot \left(\frac{11}{\mathrm{L}}\right)^2 \cdot \sin\left(11 \cdot (\frac{x}{L})\right) - 1.0089 \cdot \left(\frac{11}{\mathrm{L}}\right)^2 \cdot \cos\left(11 \cdot (\frac{x}{L})\right)\right)$$

Shear force

$$V_1 = E \cdot I \cdot X_1'''$$
$$= E \cdot I \cdot \left(-1 \cdot \left(\frac{4.73}{L}\right)^3 \cdot \cosh\left(4.73 \cdot (\frac{x}{L})\right) + 1.0178 \cdot \left(\frac{4.73}{L}\right)^3 \cdot \sinh\left(4.73 \cdot (\frac{x}{L})\right) + 1 \cdot \left(\frac{4.73}{L}\right)^3 \cos\left(4.73 \cdot (\frac{x}{L})\right) + 1.077 \cdot \left(\frac{4.73}{L}\right)^3 \cdot \sin\left(4.73 \cdot (\frac{x}{L})\right)\right)$$

$$V_2 = E \cdot I \cdot X_2'''$$
$$= E \cdot I \cdot \left(-1 \cdot \left(\frac{7.85}{\mathrm{L}}\right)^3 \cdot \cosh\left(7.85 \cdot \left(\frac{x}{L}\right)\right) + 0.9992 \cdot \left(\frac{7.85}{\mathrm{L}}\right)^3 \cdot \sinh\left(7.85 \cdot \left(\frac{x}{L}\right)\right) + 1 \cdot \left(\frac{7.85}{\mathrm{L}}\right)^3 \cdot \cos\left(7.85 \cdot \left(\frac{x}{L}\right)\right) + 0.9928 \cdot \left(\frac{7.85}{\mathrm{L}}\right)^3 \cdot \sin\left(7.85 \cdot \left(\frac{x}{L}\right)\right)\right)$$

$$V_3 = E \cdot I \cdot X_3'''$$
$$= E \cdot I \cdot \left(-1 \cdot \left(\frac{11}{\mathrm{L}}\right)^3 \cdot \cosh\left(11 \cdot (\frac{x}{L})\right) + 1 \cdot \left(\frac{11}{\mathrm{L}}\right)^3 \cdot \sinh\left(11 \cdot \left(\frac{x}{L}\right)\right) + 1 \cdot \left(\frac{11}{\mathrm{L}}\right)^3 \cdot \cos\left(11 \cdot \left(\frac{x}{L}\right)\right) + 1.0089 \cdot \sin\left(11 \cdot (\frac{x}{L})\right)\right)$$

APPENDIX C: TENTATIVE MODEL PLOTS

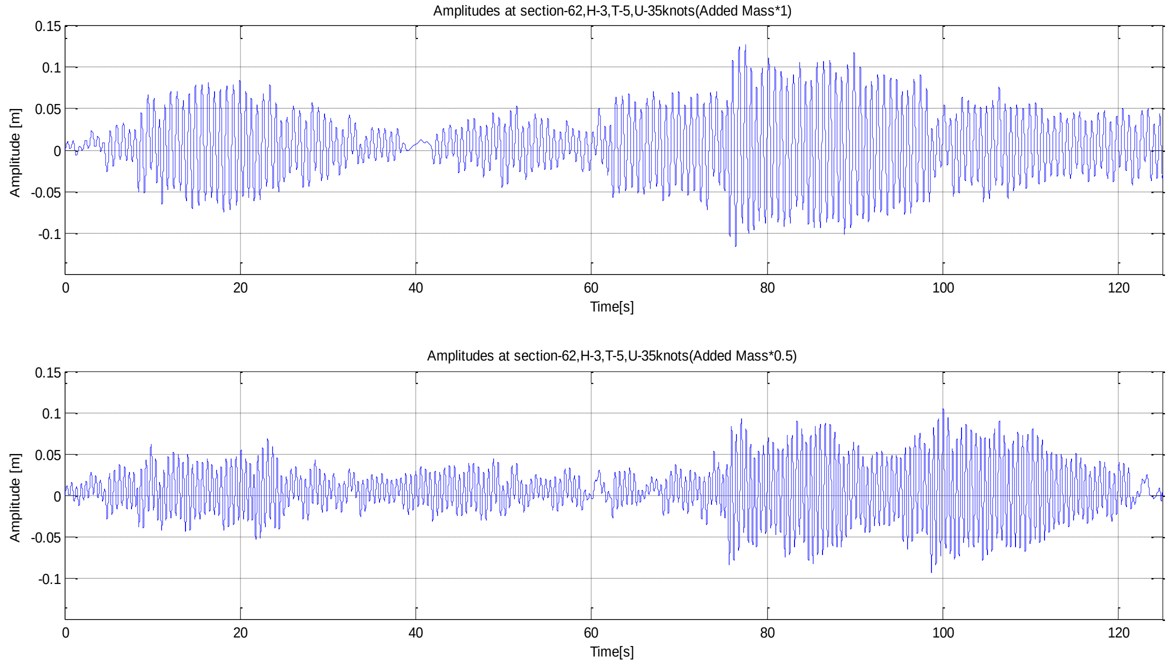

Figure 22: Study on added mass influence (Amplitudes at section 62 (around mid ship) for a sea condition of time period of 5s and E=70GPa)

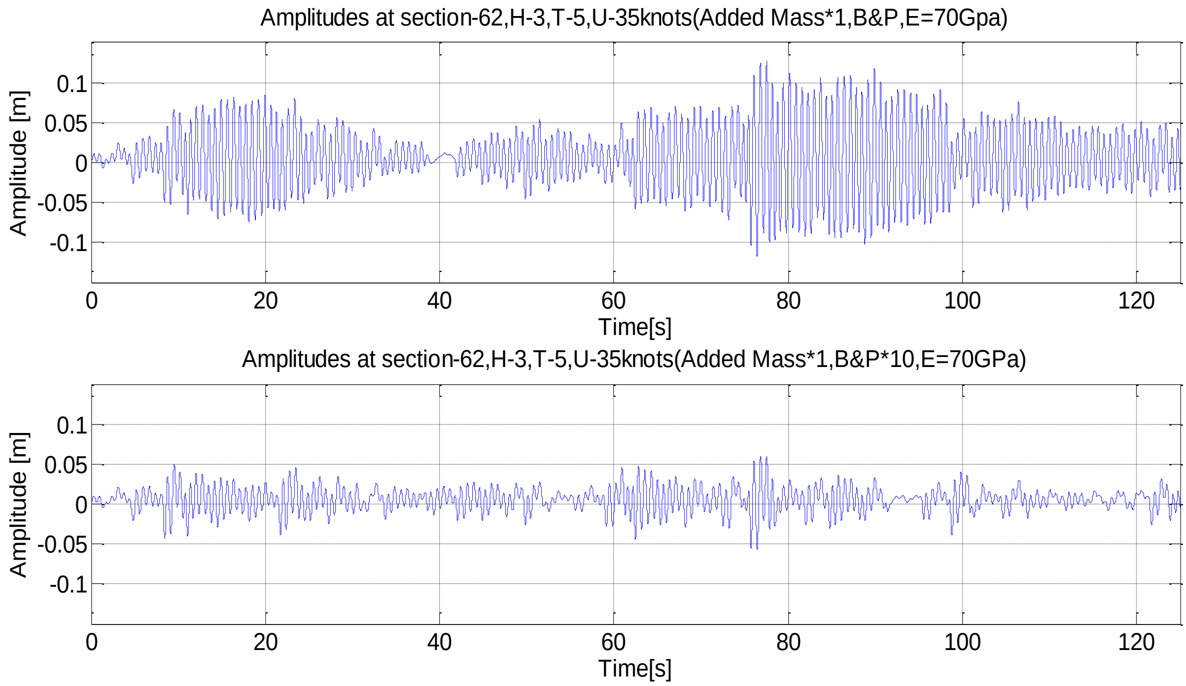

Figure 23: Study on damping influence (Amplitudes at section 62 (around mid ship) for a sea condition of time period of 5s and E=70GPa, AM*1)

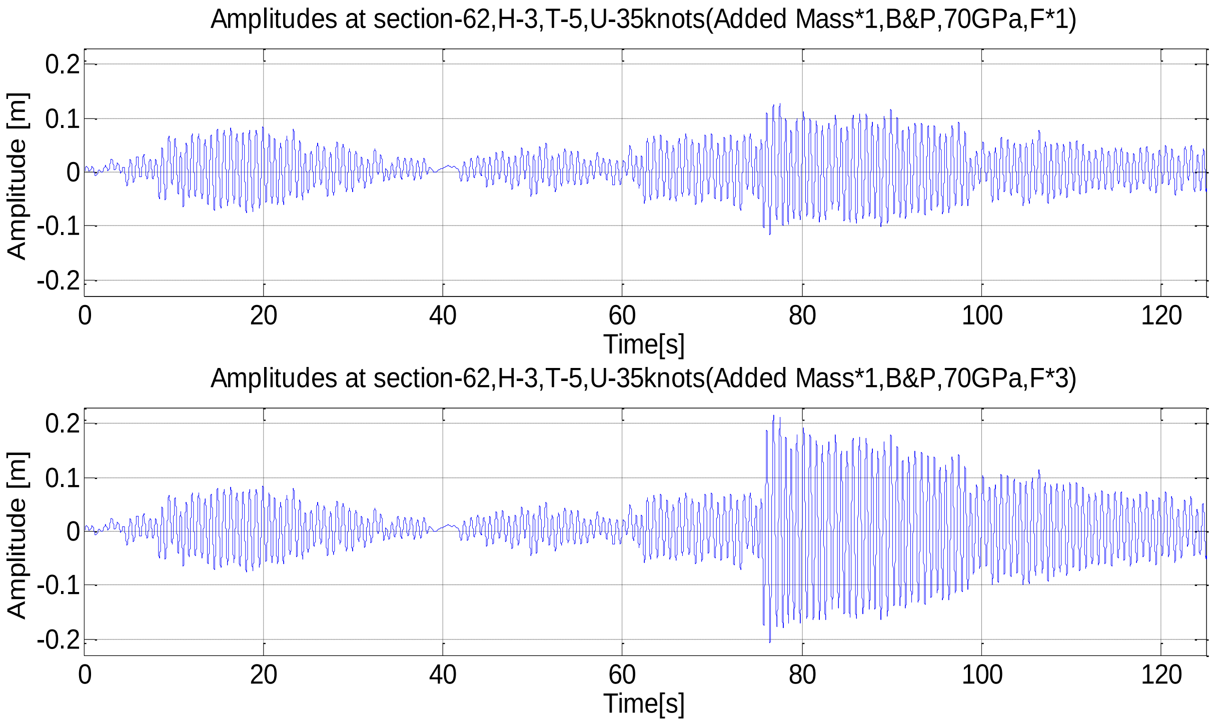

Figure 24: illustration of whipping with higher excitation force

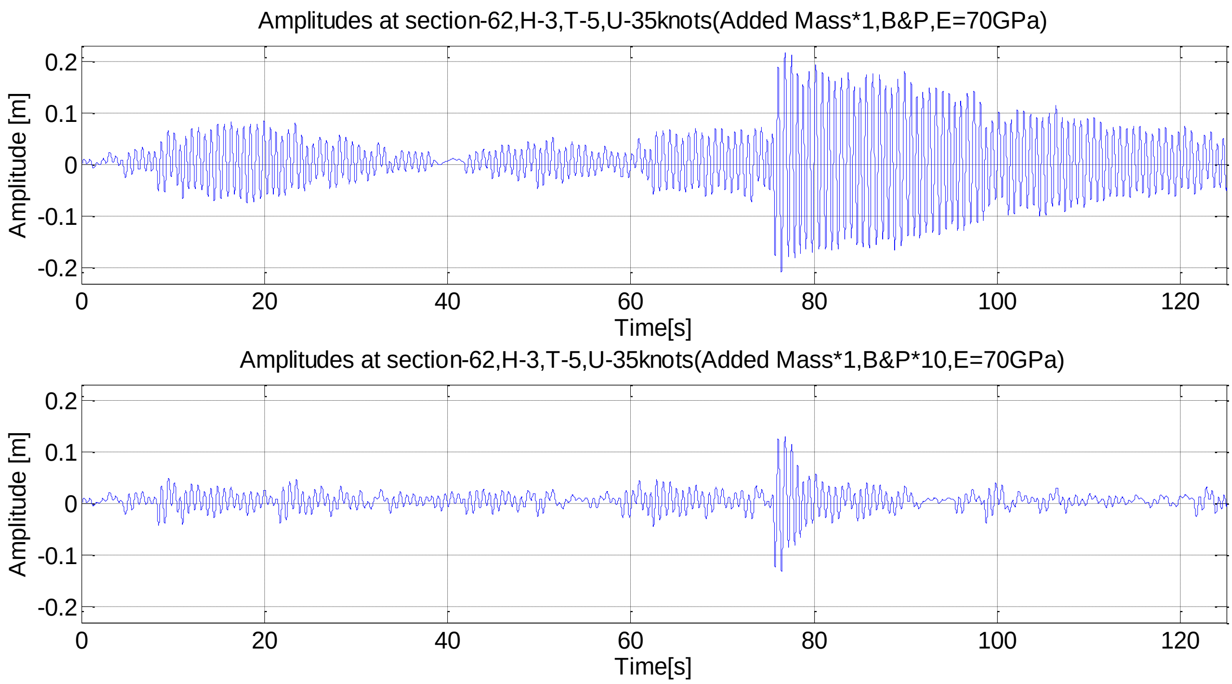

Figure 25: comparative study of whipping with different damping values

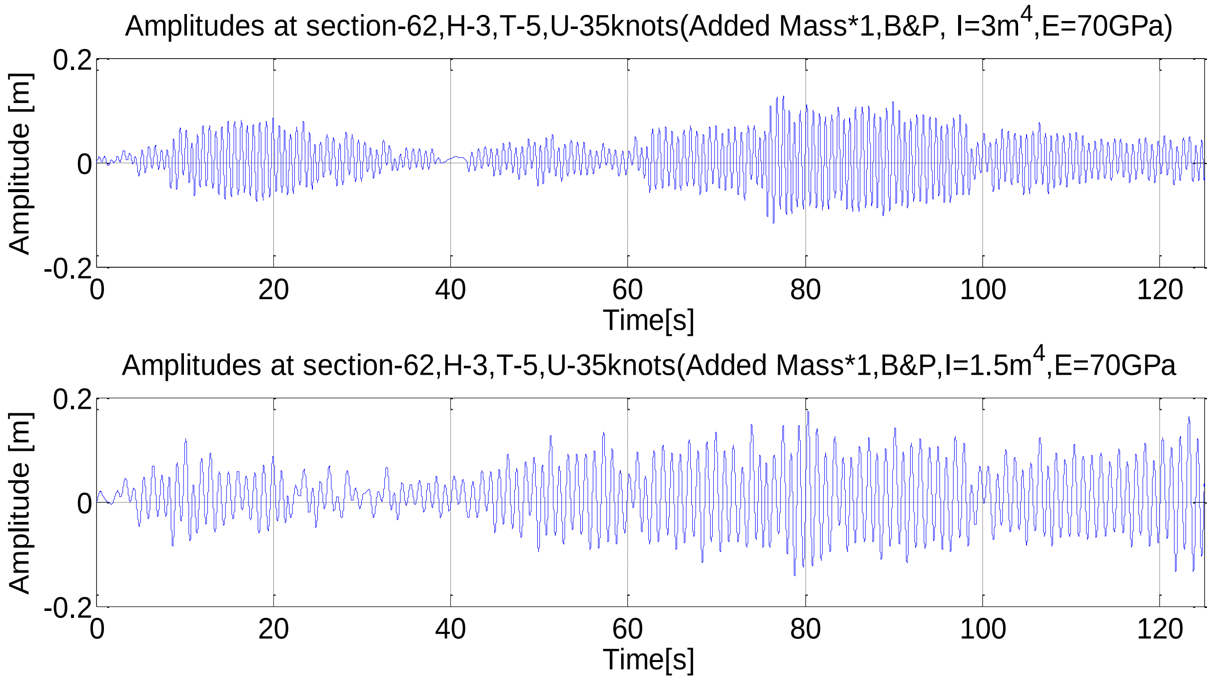

Figure 26: Study on Material influence (Amplitudes at section 62 (around mid ship) for a sea condition of time period of 5s, AM*1, damping from B&P)

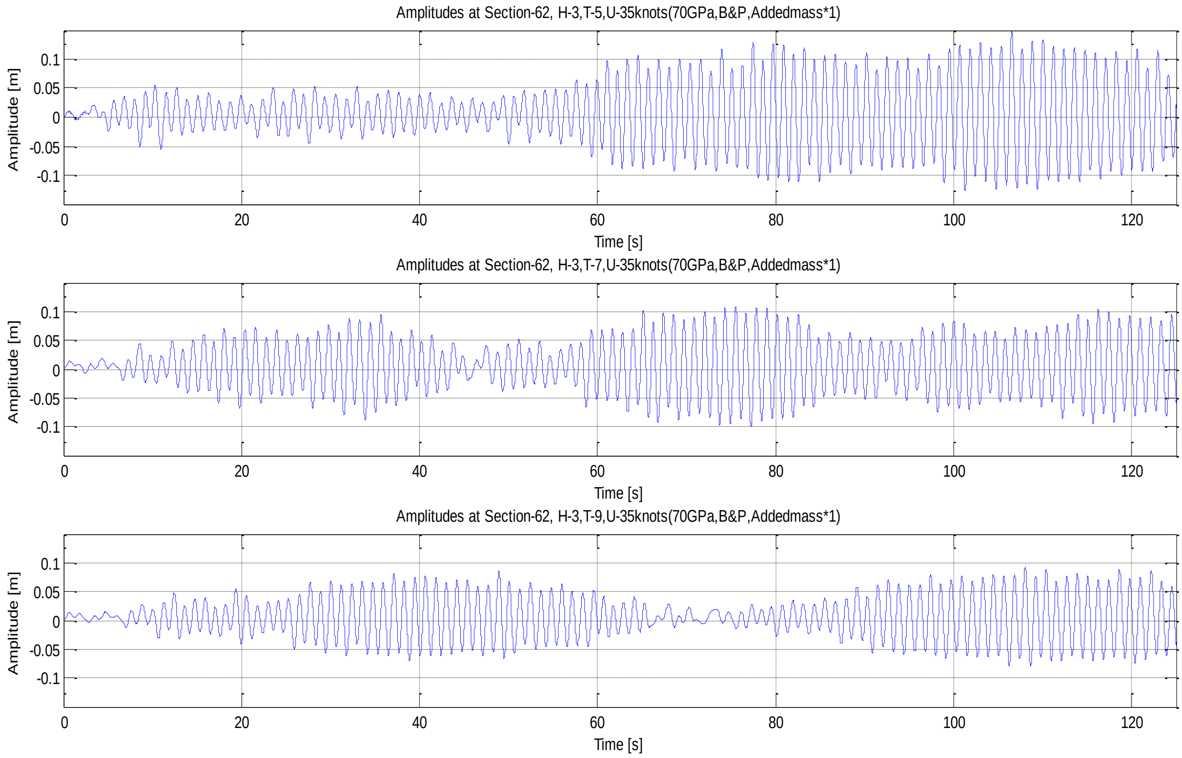

Figure 27: Amplitudes at section 62 (around mid ship) for sea condition time periods of 5, 7 and 9s (E=70GPa, AM*1, damping from B&P)