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Model-free robot manipulation of doors and drawers
by means of fixed-grasps

Yiannis Karayiannidis, Christian Smith, Francisco E. Viña, Petter Ögren, and Danica Kragic

Abstract—This paper addresses the problem of robot interaction with objects attached to the environment through joints such as doors or drawers. We propose a methodology that requires no prior knowledge of the objects’ kinematics, including the type of joint - either prismatic or revolute. The method consists of a velocity controller which relies on force/torque measurements and estimation of the motion direction, rotational axis and the distance from the center of rotation. The method is suitable for any velocity controlled manipulator with a force/torque sensor at the end-effector. The force/torque control regulates the applied forces and torques within given constraints, while the velocity controller ensures that the end-effector moves with a task-related desired tangential velocity. The paper also provides a proof that the estimates converge to the actual values. The method is evaluated in different scenarios typically met in a household environment.

I. INTRODUCTION

A robot operating in a domestic environment needs to interact with different types of doors, drawers and cupboards: objects having handles attached to them but also being attached to the other parts of the environment through joints. That is, the robot cannot perform a free manipulation on these but needs to take into account the external joint constraints. The variation in size, orientation and type of joints makes it intractable to provide a robot with predefined kinematic models of all doors it may encounter, and it is difficult - at times impossible - to observe and estimate the kinematic structure of a door or drawer before it is opened. This means that the performance of the task of opening different types of mechanisms can be significantly improved if the need to have prior knowledge of the mechanism is removed. In this paper, we propose a method for smooth, online opening of doors, drawers, or cupboards, without any need of prior knowledge of the mechanism.

Work on the door opening problem was formally initiated in the ’90s with experimental work on door opening [1] and theoretically grounded work [2] proposing velocity-based estimation for the motion direction in door opening. Velocity-based estimation has inspired some of the recent works in position-based estimation techniques [5]–[8] with optimization algorithms working on end-effector position have also been used to estimate the geometry of the mechanism, rather than the motion direction. As estimation does not guarantee identification in each control step, those methods have been coupled with controllers that give the system the proper compliance to absorb inaccuracies of the planned trajectories. On the other hand, off-line probabilistic methods that do not consider interaction forces have been used for more advanced estimation tasks: in [9], the kinematic characteristics of complicated mechanisms with more DoFs are learned, while in [10] state estimation is studied given a detailed model of the door. Other literature on the door opening problem exploits advanced hardware capabilities. In [11], [12], robot compliant behavior has been used for a manipulation task without estimating the direction of motion or the kinematics of the mechanism in hand. In [13], slowly pulling and pushing in a prior phase is used to estimate the kinematics of the mechanism using measurements of the forces exerted on the fingertips from tactile sensors, while in [14], the objective is to exert an impulsive force on a swinging door.

In Table I, we summarize some defining characteristics of the door opening methods found in the literature and compare to the present paper. In the table, the term force control designates work that explicitly controls or limits the interaction forces, online, real-time implies that the method can be used to open a door — at human-like velocities — without any prior learning step, moderate hardware requirements means that the method can be used on a simple manipulator with velocity control and a force/torque sensor, and revolute doors and sliding doors describe what types of door kinematics that can be handled by the method. Estimate of constraints indicate methods that produce an estimate of the current kinematic constraints of a mechanism, while estimate of geometry indicate methods that produce an explicit estimate of the geometry of the door mechanics themselves. Unknown model indicates methods that will work properly even if the model (type of mechanism, i.e. revolute or prismatic joint) is not known á priori, and unknown parameters indicate methods that will work if the parameters of the mechanism (i.e. hinge position or motion axis of prismatic joint) are not known á priori. Proven parameter identification indicates whether proofs are provided for the convergence of estimates.

In previous work, we have proposed a control algorithm that produces estimates of the center of rotation for a revolute door, exploits the estimates in the proposed velocity reference and which is proved to identify the constraint direction as well as achieve velocity/force tracking for smooth door opening [15], [16]. The control scheme assumes a revolute model
for the door, but the center of rotation is considered uncertain. Furthermore, the estimator uses a projection operator to guarantee well-defined updated estimates. In contrast to previous work, we now propose a control scheme that can deal with both revolute-joint doors and sliding doors/drawers by constructively utilizing the fixed-grasp assumption. Furthermore, the design of the estimator does not require a projector operator; the estimator produces inherently well-defined estimates that converge to the actual values. Summarizing our contribution as compared to existing literature, our method can be applied to open both rotational and sliding doors, without requiring ill-defined normalization. It is not based on unusual hardware capabilities and can be implemented in any velocity controlled manipulator with a force/torque sensor at the wrist. Finally, it can be proved theoretically to achieve identification of the kinematic characteristics of the mechanism simultaneously with force/velocity convergence by explicitly considering adaptive estimates in the controller design.

The remainder of this paper is organised as follows: Section II describes the notation, system kinematics, and problem formulation. The proposed solution and the corresponding stability analysis are given in Section III followed by the evaluation via simulations and an experiment in Section IV. In Section VI the final outcome of this work is briefly discussed.

II. SYSTEM AND PROBLEM DESCRIPTION

Generally, doors and drawers can be opened by grasping the handle and moving it along its intended trajectory of motion; a circular path for hinged mechanisms, or a linear path for sliding doors and drawers. In this section we formally define the problem of door/drawer opening where the position of hinges, or direction of possible sliding motion is not known á priori.

A. Notation and Preliminaries

We introduce the following notation:

- Bold small letters denote vectors while bold capitals denote matrices. Underline _ denotes vectors of unit magnitude, estimates and errors between control variables and their corresponding desired values/vectors respectively. \( ^\top \) denotes the transpose of a vector or a matrix.

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- Proven Parameter Identification
- Unknown Parameters
- Unknown Model
- Estimate of Constraints
- Estimate of Geometry
- Revolute Doors
- Sliding Doors
- Moderate H/W Spec.
- Online, real-time
- Force control

The remainder of this paper is organised as follows: Section II describes the notation, system kinematics, and problem formulation. The proposed solution and the corresponding stability analysis are given in Section III followed by the evaluation via simulations and an experiment in Sections IV and V respectively. In Section VI the final outcome of this work is briefly discussed.

B. Kinematic model of robot door/drawer opening

We consider a setting of a robot manipulator in which its end-effector has achieved a fixed grasp of the handle of a mechanism kinematically modeled as revolute or prismatic joint e.g. a door or a drawer in a domestic environment. The term fixed grasp describes that there is neither relative translational or rotational motion between the handle and the end-effector.
Let \{\mathcal{e}\} and \{\mathcal{h}\} be end-effector and handle frame respectively. The fixed grasp assumption can then formalized as:

\[ \mathcal{e}\dot{P}_h = 0, \quad \mathcal{e}\mathbf{R}_h = 0 \]  \quad (1)

The two frames are attached on a kinematically known position e.g. a known point of the end-effector denoted by \(\mathbf{p}_e\). However, the end-effector and handle frames are described by different rotation matrices since they are strictly connected to the robot kinematics and the door/drawer kinematics respectively. In case of a rotating door (revolute joint) we also consider a frame \{\mathcal{h}\} attached at the center of the circular trajectory of the end-effector while opening the rotating door. The axis \(\mathbf{z}_o\) can be arbitrarily chosen. We state a convention to define the frame \{\mathcal{h}\} for revolute joints (hinged doors) and prismatic joints (sliding doors, drawers):

**Revolute joints**
- Axis \(\mathbf{z}_h\) is equivalent to \(\mathbf{z}_o\), i.e. \(\mathbf{z}_o \equiv \mathbf{z}_h\)
- Axis \(\mathbf{y}_h\) is the unit vector which is perpendicular to both \(\mathbf{z}_h\) and \(\mathbf{z}_o\) with direction towards the hinge.
- Axis \(\mathbf{x}_h\) can be regarded as the allowed motion axis; it can be formed as follows: \(\mathbf{x}_h = \mathbf{y}_h \times \mathbf{z}_h\).

**Prismatic joints**
Vector \(\mathbf{z}_h\) denotes the allowed motion axis. Axes \(\mathbf{z}_h\) and \(\mathbf{y}_h\) can be arbitrarily chosen to span the two-dimensional surface to which \(\mathbf{z}_h\) is perpendicular.

Examples of Fig. 1 illustrate the definition of the \{\mathcal{h}\} axes. Note that the \{\mathcal{h}\} axes definition is based on door/drawer kinematics that are uncertain in an unknown environment where the robot can identify and grasp handles of doors and drawers but doesn’t know the kinematics of their mechanism.

![Fig. 1: Illustrative examples of different types of rotating/sliding doors and drawers that can be modeled as revolute and prismatic joints](image)

In case of a door kinematically modeled as a revolute joint, we can define the radial vector—which is parallel to \(\mathbf{z}_h\)—as the relative position of the frames \{\mathcal{e}\} and \{\mathcal{h}\}:

\[ \mathbf{r} \triangleq \mathbf{p}_o - \mathbf{p}_e \]  \quad (2)

By expressing \(\mathbf{r}\) with respect to the end effector frame and differentiating the resultant equation we get:

\[ \dot{\mathbf{R}}_e \mathcal{e}\mathbf{r} + \mathcal{e}\mathbf{R}_e \dot{\mathbf{r}} = \dot{\mathbf{p}}_o - \dot{\mathbf{p}}_e \]  \quad (3)

Notice that \(\dot{\mathbf{r}} = \mathcal{e}\mathbf{R}_e \mathcal{h}\mathbf{r} + \mathcal{e}\mathbf{R}_e \mathcal{h}\dot{\mathbf{r}} = 0\) since \(\mathcal{e}\mathbf{R}_e = 0\) and \(\mathcal{h}\dot{\mathbf{r}} = 0\) are implied by the fixed grasp assumption. By substituting \(\dot{\mathbf{r}} = \dot{\mathbf{p}}_o - \dot{\mathbf{p}}_e = 0\) as well as \(\mathcal{e}\mathbf{R}_e \mathcal{e}\mathbf{R}_e^\top = \mathbf{S}(\mathbf{\omega}_e)\) with \(\mathbf{\omega}_e\) being the rotational velocity of the end-effector, we get:

\[ \dot{\mathbf{p}}_e = \mathbf{S}(\mathbf{r}) \mathbf{\omega}_e \]  \quad (4)

which describes the first-order differential kinematics of the door opening in case of a revolute hinge. By multiplying both sides of (4) with \(\mathbf{R}_e^\top\) and using the similarity transformation for the skew-symmetric matrix \(\mathbf{S}(\mathbf{r})\) we can express the constraint in the end-effect frame as follows:

\[ \mathbf{v} = \mathbf{S}(\mathbf{r}) \mathbf{\omega}_e \]  \quad (5)

with \(\mathbf{v} = \mathbf{R}_e^\top \dot{\mathbf{p}}_e\) and \(\mathbf{\omega}_e = \mathbf{R}_e^\top \mathbf{\omega}_e\) are the translational and rotational end-effector velocities expressed in the local frame of the end-effector. By denoting with \(\ell\) the distance between the end-effector frame and the center of rotation, the radial vector \(\mathcal{e}\mathbf{r}\) can be written as follows:

\[ \mathcal{e}\mathbf{r} = \mathcal{e}(\mathbf{R}_e^\top \mathbf{y}_h) = \ell \mathbf{y}_h \]  \quad (6)

The inner product of (5) with \(\mathcal{e}\mathbf{x}_h\) and using (6) gives us:

\[ \mathcal{e}\mathbf{x}_h^\top \mathbf{v} = \ell \mathbf{z}_h^\top \mathbf{\omega}_e \]  \quad (7)

The remaining constraints, that stem from the fixed grasp assumption and are imposed to the end-effector translational and rotational velocities, are the following:

\[ \mathbf{P}(\mathcal{e}\mathbf{x}_h)\mathbf{v} = 0 \quad \mathbf{P}(\mathcal{e}\mathbf{z}_h)\mathbf{\omega} = 0 \]  \quad (8)

which can be reformulated as:

\[ \mathcal{e}\mathbf{y}_h^\top \mathbf{v} = 0 \quad \mathcal{e}\mathbf{z}_h^\top \mathbf{v} = 0 \quad \mathcal{e}\mathbf{x}_h^\top \mathbf{\omega} = 0 \quad \mathcal{e}\mathbf{z}_h^\top \mathbf{\omega} = 0 \]  \quad (9)

For sliding doors or drawers, the fixed grasp assumption implies that \(\mathbf{\omega}_e = \mathbf{\omega}_h = 0\), and the constraint (7) related to the rotational motion of the end-effector can be replaced by:

\[ \mathcal{e}\mathbf{z}_h^\top \mathbf{\omega} = 0 \]  \quad (11)

The remaining constraints [(8) or (9),(10)] remain valid for the prismatic case as well.

C. Robot kinematic model

For velocity controlled manipulators, the robot joint velocity is controlled directly by the reference velocity \(\mathbf{u}_{\text{ref}}\). In particular, the reference velocity \(\mathbf{u}_{\text{ref}} \triangleq [\mathbf{v}_{\text{ref}} \mathbf{\omega}_{\text{ref}}]^\top \in \mathbb{R}^6\) (\(\mathbf{v}_{\text{ref}} \in \mathbb{R}^3\) and \(\mathbf{\omega}_{\text{ref}} \in \mathbb{R}^3\) denote the translational and rotational parts) expressed at the end-effector frame can be considered as a kinematic controller which is mapped to the joint space to be applied at the joint velocity level as:

\[ \mathbf{q} = \mathbf{J}^\top(q) \Gamma(q) \mathbf{u}_{\text{ref}} \]  \quad (12)
• $q, \dot{q} \in \mathbb{R}^n$ is the joint positions and velocities.
• $J(q)^{-1} = J(q)^T \left[ J(q) J(q)^T \right]^{-1}$ is the pseudo-inverse of the manipulator Jacobian $J(q) \in \mathbb{R}^{6 \times n}$ which relates joint velocities to the end-effector velocities
• $\Gamma(q) = \text{diag}_{\text{block}}(R_e, R_d)$ is a transformation for mapping the velocity from end-effector frame to the global inertial frame.

If we consider the typical Euler-Lagrange robot dynamic model, the velocity error at the joint level drive the torque
If we assume a high frequency current control loop with external forces’ compensators and weak inertial dynamics, then the kinematic model is valid. This assumption is also supported by the experimental results (Section V).

D. Stability of a simple non-autonomous system using a logarithmic barrier function

In this section we will state and prove a lemma to be used in the proof of the main result of the paper.

Lemma 1: Consider the state domain $D = (-\frac{\gamma}{c}, \frac{\gamma}{c})$ and the non-autonomous system (dependent on time variable and state $\theta$) defined in $D$ and described by the following differential equation:

$$\dot{\theta} = -\gamma c(t) \tan \theta$$

with $\gamma$ and $c(t)$ being a strictly positive constant and a non-negative function of $t$ respectively.

If $c(t)$ satisfies the persistent excitation condition i.e. $\int_{t_0}^{t+T_0} c(\tau) d\tau \geq \alpha_0 T_0$ for some $\alpha_0$ and $T_0$, then $\theta(t)$ converges to zero exponentially, and

$$\theta(t) = \arcsin \left[ e^{-\gamma} \int_{0}^{t} c(\tau) d\tau \theta(0) \right],$$

Note that a non-negative sinusoidal or step-function satisfies the aforementioned condition.

Proof: Consider the function $U(\theta) : D \to \mathbb{R}^+$, given by:

$$U(\theta) = -\ln(\cos \theta)$$

Differentiating $U(\theta)$ with respect to time and substituting (13) we get:

$$\dot{U}(\theta, t) = -\gamma c(t) \tan^2 \theta$$

Since $c(t) \geq 0, \dot{U}(\theta, t) \leq 0$ which in turn implies $U(\theta(t)) \leq U(\theta(0))$ and $\theta(t) \in D, \forall t$ for $\theta(t) \in D$, we can express the system using the variable $\sigma = \sin \theta$ as follows: $\dot{\sigma} = -\gamma c(t) \sigma$ and subsequently calculate the analytic solution which is given by:

$$\theta(t) = \arcsin \left[ e^{-\gamma} \int_{0}^{t} c(\tau) d\tau \theta(0) \right]$$

which implies the convergence stated above.

E. Control Objective

Our target is to control the motion of the robot to manipulate and open an external mechanism, such as a door or drawer, irrespective of its kinematic structure. In applications for dynamic unstructured — e.g. domestic — environments, it is difficult to design a priori the desired velocity satisfying the constraints imposed by the mechanism. This is due to the difficulties of identifying the kinematic characteristics of the mechanism. The execution of a trajectory incompatible with system constraints gives rise to high interaction forces which may be harmful to both the manipulated mechanism and the robot, and does not lead to a successful task accomplishment.

The task can be naturally described in the handle frame, but the desired variables should be defined at the end-effector frame to be executable by the robot. Let $f_d, \tau_d$ and $v_d(t)$ be the desired force, torque and velocity expressed at the end-effector frame respectively. Let $v_d(t)$ be the desired velocity along the motion axis of frame $\{h\}$. Then the desired velocity at the end-effector frame is defined along $\hat{\mathbf{x}}_h$, i.e. $\mathbf{v}_d = \hat{\mathbf{x}}_h v_d(t)$, and the force control objective can be achieved by projecting the desired force on the orthogonal complement space of $\hat{\mathbf{x}}_h$ (constrained directions) i.e. $\mathbf{P}(\hat{\mathbf{x}}_h) f_d$; a small valued or zero vector $f_d$ corresponds to small forces along the constraint directions. On the other hand, the desired rotational velocity can be defined using $v_d(t)$ along the axis $d = d\hat{\mathbf{z}}_h$ , i.e. $\mathbf{w}_d(t) = d v_d(t)$, with:

$$d = \left\{ \begin{array}{ll} 1/\ell, & \text{for rotational mechanisms} \\ 0, & \text{for sliding mechanisms} \end{array} \right.$$ (17)

Note that prismatic kinematics can be approximated by rotational kinematics using large values of $\ell$.

If we denote the total interaction force and torque expressed at the end-effector frame with $f \in \mathbb{R}^3$ and $\tau \in \mathbb{R}^3$ respectively the control objective can be formulated as:

Problem 1 (Door/Drawer Opening Problem): Design a velocity control $u_{\text{ref}}$ such that $\mathbf{P}(\hat{\mathbf{x}}_h) f_d \to \mathbf{P}(\hat{\mathbf{x}}_h) f_d$, $\tau \to \tau_d$, $v \to \hat{\mathbf{x}}_h v_d(t)$, $\omega \to d\hat{\mathbf{z}}_h v_d(t)$, without knowing accurately the motion axis $\hat{\mathbf{x}}_h$, the corresponding constraint directions $\mathbf{P}(\hat{\mathbf{x}}_h)$, or the axis of rotation $\hat{\mathbf{x}}_h$ and the variable $d$. From a high level perspective, we consider that the opening task is accomplished when the observed end-effector trajectory (equivalent with the handle trajectory) has progressed far enough to enable the robot to perform a subsequent task, like passing through a door. Hence, some perception system observing the progress of the opening of the mechanism is additionally required to provide the robot with the command to halt the opening procedure.

III. CONTROL DESIGN

In this section, we will propose a solution to Problem 1 from Section II-E. The proposed controller is analytically proved to achieve the control objectives.

A. Translational velocity reference with torque feedback

Let $\hat{\mathbf{x}}_h(t)$ denote the online estimate of motion direction $\hat{\mathbf{x}}_h$. Dropping the argument $t$ from $\hat{\mathbf{x}}_h(t)$ and $v_d(t)$ for notation convenience we introduce a reference velocity vector $\mathbf{v}_{\text{ref}}$ for controlling the end-effector translational velocity:

$$\mathbf{v}_{\text{ref}} = \hat{\mathbf{x}}_h v_d - \mathbf{P}(\hat{\mathbf{x}}_h) \mathbf{v}_f$$

(18)

where $\mathbf{v}_f$ is a PI force feedback input defined as follows:

$$\mathbf{v}_f = \alpha_f \hat{\mathbf{f}} + \beta_f T \left[ \mathbf{P}(\hat{\mathbf{x}}_h) \hat{\mathbf{f}} \right]$$

(19)

with $\hat{\mathbf{f}} = \mathbf{f} - f_d$ and $\alpha_f, \beta_f$ being positive control constants.
Let $\theta(t)$ denotes the angle formed between the actual vector $\xi_h$ and its online estimate $\hat{\xi}_h$ which is time-varying. Given that the estimate $\hat{\xi}_h$ is a unit vector, $\cos \theta(t)$ can be defined as follows:

$$\cos \theta(t) = \hat{\xi}_h^T \xi_h \tag{20}$$

In general, an online estimate of the vector $\xi_h$ provided by an adaptive estimator is not unit but in the following we are going to design an update law that produces estimates of unit magnitude. In the following, we drop out the argument of $t$ from $\theta(t)$ for notation convenience.

The velocity error $v = \hat{v} - v_{\text{ref}}$ can be decomposed along $\xi_h$ and the corresponding orthogonal complement space as:

$$v = \hat{P}(\hat{\xi}_h)(v + v_f) + \hat{\xi}_h(v \cos \theta(t) - v_d) \tag{21}$$

where $v$ denotes the magnitude of the velocity. In case of velocity controlled manipulators, it is assumed that $\hat{v} = 0 \ (12)$ which implies the following closed-loop system equations:

$$\hat{P}(\hat{\xi}_h)v_f = -\hat{P}(\hat{\xi}_h)^T \xi_h v \tag{22}$$

$$v = \frac{1}{\cos \theta(t)} v_d \tag{23}$$

We design the update law for $\xi_h$ as follows:

$$\xi_h = -\gamma v_d \hat{P}(\hat{\xi}_h)v_f \tag{24}$$

The use of the update law (24) is instrumental for the stability analysis and the convergence proof. Furthermore, update law (24) has two basic properties:

**Property 1:** The update law (24) ensures that the norm of $\xi_h(t)$ is invariant, i.e. starting with $||\xi_h(0)|| = 1$, $||\xi_h(t)|| = 1$, $\forall t$.

**Proof:** By projecting (24) along $\xi_h$ yields

$$\frac{d}{dt} (||\xi_h(t)||^2) = -2\gamma v_d \hat{P}(\hat{\xi}_h)^T \xi_h v_f = 0 \quad \text{(since $\hat{P}(\hat{\xi}_h)^T \xi_h = 0$).} \quad \blacksquare$$

**Property 2:** For the closed loop system (22) and (23), the update law (24) yields the scalar differential equation (13) with respect to angle $\theta$ defined in (20) with $\epsilon(t) := v_d^2 \geq 0$, i.e., the estimate converges to the true vector.

**Proof:** The second property is proven by projecting both sides of (24) along $\xi_h$: Substituting (20) in the left side of the projected (24) yields:

$$\xi_h^{T} \xi_h = \frac{d}{dt} (\cos \theta) = -\sin \theta \tag{25}$$

Substituting (22), (23) and $\xi_h^{T} \hat{P}(\hat{\xi}_h)^T \xi_h = \sin^2 \theta$ in the right side of the projected (24) yields:

$$-\gamma v_d \xi_h^{T} \hat{P}(\hat{\xi}_h)v_f = \frac{\gamma v_d^2 \xi_h^{T} \hat{P}(\hat{\xi}_h) \xi_h}{\cos \theta} \xi_h = \gamma v_d^2 \sin \theta \tan \theta \tag{26}$$

By combining (25) and (26) we get:

$$\sin \theta (\hat{\theta} + \gamma v_d \tan \theta) = 0 \tag{27}$$

Eq. (27) has a trivial solution $\theta(t) = 0$ and the solution and the solution given by Lemma 1 in equation (13) with $\epsilon(t) = v_d^2$ which includes the trivial one. \quad \blacksquare

Using the aforementioned properties of the update law (24), we can prove the following Theorem:

**Theorem 1:** Consider a velocity controlled manipulator, with first order differential kinematics described by (12), which has achieved a fixed grasp with the handle of a sliding/rotating door or a drawer.

If the robot is driven by a velocity control input $v_{\text{ref}}$ (18) that uses a PI force feedback input $v_f$ (19) as well as the update law (24) to estimate the local motion axis $\xi_h$, then a subset of Problem 1 will be solved, i.e., smooth opening of the moving mechanism will be achieved. Analytically, the following convergence results are guaranteed: $\xi_h \to \hat{\xi}_h$, $v \to \gamma v_d$, $P(\hat{\xi}_h)f \to 0$, given that $v_d$ is appropriately chosen.

**Proof:** Consider the following Lyapunov-like function:

$$V = \alpha f_{\beta} ||P(\hat{\xi}_h)f||^2 + \frac{1}{\gamma} U(\theta) \tag{28}$$

with $U(\theta)$ being defined in (15). By differentiating (28), adding and subtracting $\gamma^2 ||P(\hat{\xi}_h)f||^2$, $\beta^2 ||P(\hat{\xi}_h)f||^2$ and subsequently substituting (22), (23) we get:

$$\dot{V} = -\alpha f_{\beta} ||P(\hat{\xi}_h)f||^2 - \beta^2 ||P(\hat{\xi}_h)f||^2$$

$$- \frac{v_d}{\cos \theta} P(\hat{\xi}_h)\xi_h - \frac{1}{\gamma \cos \theta} \xi_h^T \xi_h \tag{29}$$

To cancel out the terms of the second line in (29) we substitute the update law (24).

Consequently, the derivative of function $V$ (28) along the system trajectories (22), (23) and (24) is given by:

$$\dot{V} = -\alpha f_{\beta} ||P(\hat{\xi}_h)f||^2 - \beta^2 ||P(\hat{\xi}_h)f||^2$$

Hence, $V(t) \leq V(0), \forall t$ which implies that $\mathcal{L}(P(\hat{\xi}_h)f)$ is bounded and that $\theta(t) \in D$ provided that $\theta(0) \in D$ (For details see Section II-D). Consequently, (22), (23) imply that $P(\hat{\xi}_h)f_v$ and $v$ are bounded. Furthermore, the boundedness $P(\hat{\xi}_h)f_v$ implies that the update law rate $\hat{\xi}_h$ is bounded. Differentiating (22), (23) and using the boundedness of $\mathcal{L}(P(\hat{\xi}_h)f_v)$, $P(\hat{\xi}_h)v_f$ and $\hat{\xi}_h$, it can be easily shown that $\frac{d}{dt} [P(\hat{\xi}_h)f_v]$ is bounded. Hence, the second derivative of $V$ is bounded allowing the use of Barbalat’s Lemma to prove that $\dot{V} \to 0$ and consequently $P(\hat{\xi}_h)f_v \to 0$, $P(\hat{\xi}_h)f \to 0$. Note that the aforementioned convergence results are referred to the estimated motion space defined by $\hat{\xi}_h$. By using the Property 2 of the update law, (24) yields (13) and subsequently (14) that implies the exponential convergence of $\theta$ to zero or $\xi_h \to \hat{\xi}_h$ for $v_d$ satisfying the persistent excitation condition. \quad \blacksquare

**B. Rotational velocity reference with torque feedback**

Since the translational velocity is strictly connected to the rotational velocity, the reference rotational velocity will be defined using the desired translational velocity $v_d$ as follows:

$$\omega_{\text{ref}} = \hat{\omega} v_d - \omega_r. \tag{30}$$
where \( \hat{d} \) is the online estimate of \( d \) and it is appropriately designed as follows:

\[
\dot{\hat{d}} = -\gamma_d v_d \omega_r \tag{31}
\]

and \( \omega_r \) is a PI torque feedback input defined as follows:

\[
\omega_r = \alpha_r \hat{\tau} + \beta_r I(\hat{\tau}) \tag{32}
\]

The design of the update law (31) is instrumental for the proof of the following theorem:

**Theorem 2:** Consider a velocity controlled manipulator, with first order differential kinematics described by (12), which has achieved a fixed grasp with the handle of a sliding/rotating door or a drawer. If the robot is driven by a velocity control input that consists of both \( v_{\text{ref}} \) (18) and \( \omega_{\text{ref}} \) (30) that uses a PI torque feedback input \( \omega_r \) (32) as well as the update law (31) to estimate the vector \( \hat{d} \), then Problem 1 will be entirely solved, i.e., the following convergence results—additionally to those of Theorem 1—are guaranteed: \( \hat{\tau} \rightarrow 0 \), \( I(\hat{\tau}) \rightarrow 0 \). Using the aforementioned boundedness results as well as those implied by Theorem 1, it can be easily proved by differentiating \( \hat{W} \) that \( \hat{W} \) is bounded. Hence, applying Barbalat’s Lemma we get that \( \hat{\tau} \rightarrow 0 \), \( I(\hat{\tau}) \rightarrow 0 \). Using the aforementioned convergence results as well as \( \theta \rightarrow 0 \), it can be shown that \( \hat{d} \rightarrow d \) provided that \( v_d \) satisfies the persistent excitation condition and hence \( \omega \rightarrow d v_d \).

**Proof:** First, we will reform \( \omega_{\text{ref}} \) by adding/subtracting the term \( d(v - v_d) \) and using (23) as follows:

\[
\omega_{\text{ref}} = dv + \hat{d} v_d - \omega_r + d(\cos \theta - 1) v_d \tag{33}
\]

with \( \hat{d} = \hat{d} - d \). For design purposes we consider the following positive definite function:

\[
W = \alpha_r \beta_r ||I(\hat{\tau})||^2 + \frac{1}{2\gamma_d} ||\hat{d}||^2 + \xi U(\theta) \tag{34}
\]

with \( U(\theta) \) being defined in (15) and \( \xi \) being a positive constant. By differentiating (34) with respect to time and substituting \( \omega = \omega_{\text{ref}} \) given by (33), (16) (for \( c(t) = v_d^2 \)), the rotational constraints (8), (10), and (7) or (11) we get:

\[
\dot{W} = -\alpha_r^2 ||\dot{\hat{\tau}}||^2 - \beta_r^2 ||I(\hat{\tau})||^2 - \omega_r^2 d(\cos \theta - 1) v_d \\
- \xi \gamma_d v_d^2 \tan^2 \theta + \hat{d} \left( -\gamma_d \hat{d} + v_d \omega_r \right) \tag{35}
\]

To cancel the last term of the right side part of (35) we set \( \hat{d} = -\gamma_d v_d \omega_r \), which corresponds to the update law (31).

By using (31) and the inequality:

\[
\omega_r^T d \left( \frac{\cos \theta - 1}{\cos \theta} \right) v_d \leq \frac{||\omega_r||^2}{\cos \theta} + ||d||^2 v_d^2 \left( \frac{\cos \theta - 1}{\cos \theta} \right)^2 \tag{37}
\]

we can upper-bound \( \dot{W} \) (35) as follows:

\[
\dot{W} \leq -\alpha_r^2 ||\dot{\hat{\tau}}||^2 - \beta_r^2 ||I(\hat{\tau})||^2 + \frac{||\omega_r||^2}{\cos \theta} \\
- \xi \gamma_d v_d^2 \tan^2 \theta + ||d||^2 v_d^2 \left( \frac{1 - \cos \theta}{\cos \theta} \right)^2 \tag{38}
\]

By expanding \( ||\omega_r||^2 \), using (32), setting \( \xi > \gamma_d ||d||^2 / \gamma \) and after some trigonometric calculations we get:

\[
\dot{W} \leq -\alpha_r^2 ||\dot{\tau}||^2 - \beta_r^2 ||I(\hat{\tau})||^2 - ||d||^2 v_d^2 \left( \frac{1 - \cos \theta}{\cos \theta} \right)^2 \tag{39}
\]

Since \( \cos \theta \leq 1 \) and \( \theta(t) \in D \) provided that \( \theta(0) \in D \) (Theorem 1), the derivative of function \( W(34) \) can be upper-bounded as follows:

\[
W \leq -\alpha_r^2 ||\dot{\tau}||^2 - \beta_r^2 ||I(\hat{\tau})||^2
\]

Hence, \( W(t) \leq W(0), \forall t \) which implies that \( I(\tau) \) and \( d \) are bounded. Consequently, by using (33) and taking into account the constraints (8), (10), and (7) or (11), \( \hat{\tau} \) is bounded and hence \( \hat{d} \) is bounded. Using the aforementioned boundedness results as well as those implied by Theorem 1, it can be easily proved by differentiating \( \hat{W} \) that \( \hat{W} \) is bounded. Hence, applying Barbalat’s Lemma we get that \( \hat{\tau} \rightarrow 0 \), \( I(\hat{\tau}) \rightarrow 0 \). Using the aforementioned convergence results as well as \( \theta \rightarrow 0 \), it can be shown that \( \hat{d} \rightarrow d \) provided that \( v_d \) satisfies the persistent excitation condition and hence \( \omega \rightarrow d v_d \).

**C. Summary and Discussion**

The proposed controller produces local estimates of the unconstrained motion direction and rotation axis (in case of rotational door) using the update laws (24) and (31). The estimates are used within velocity references (18), (30) that enforce the robot to move with a desired velocity while controlling forces/torques along the constrained directions, guaranteeing compliant behavior. The main condition for guaranteed performance is that the initial estimate of the motion axis is not perpendicular to the true value i.e. \( \theta(0) \in D \). An example where this condition is not satisfied could be when opening a drawer with an initial estimate that it is a sliding door (c.f. Fig. 2, cases 4 and 5). This can be overcome by using a moderate deviation in the initial estimate (see Section IV). The proposed method alone can not handle the case where the initial estimate is in the opposite direction of the true value, as this would generate a closing motion. This can be handled by an external system that stops the motion and retries with a different initial estimate if measured forces are too high, similar to a human who first pushes a door, and when it doesn’t open, tries to pull it instead.

By defining the controller in the end-effector frame and estimating the motion directions locally, the proposed method is applicable to both revolute and prismatic doors/drawers. Coupling the estimator with the controller makes the method inherently on-line, and enables proofs of convergence of both estimates and force/torque errors. It is trivial to extend the method to produce explicit estimates of the physical location of the hinge of a revolute door, as estimates of both radial direction and radial distance are available. If we make the assumption that a large enough radial distance (we arbitrarily choose 10 m) implies a prismatic mechanism, the following steps will identify the hinge position:

**Step 1:** Assume a rotational mechanism if \( ||d|| > 0.1 \). For \( ||d|| < 0.1 \) we assume a sliding door or a drawer and do not proceed further.

**Step 2:** Calculate the estimated radial direction, coinciding with handle frame axis \( \vec{\omega}_h \) by the outer product of \( \vec{\xi}_d \), \( \hat{d} ||\hat{d}||^{-1} \). **Step 3:** Center of rotation \( \vec{p}_o \) is then: \( \vec{p}_o = \vec{p}_e + R_x \vec{\omega}_h ||\hat{d}||^{-1} \).

Note that the proposed control scheme can be directly implemented on any velocity-controlled robotic manipulator with a force/torque sensor in the end-effector frame. Implementation on a torque controlled robot may require the reference acceleration, i.e. time derivative of reference
velocity. Then, force/torque feedback terms \( v_f, \omega_\tau \) should only consist of the integral of the force error (projected on the constrained direction) and the torque, so that differentiation of noisy force/torque measurements is avoided. A dynamic controller can be developed from the velocity controller following the steps described in our previous work [15].

IV. SCENARIOS AND EVALUATION

To demonstrate the generality of the approach, we consider five different scenarios, common in domestic environments, see Fig. 2. All cases are treated with the same initial estimates and controller gains. Cases (i) and (ii) are typical revolute doors with vertical axis, with the hinge to the left or to the right, respectively. Case (iii) models a revolute door with axis of rotation parallel to the floor, such as is common for ovens. The radius of these doors are all 50 cm. Case (iv) models a sliding door, and case (v) a drawer. The common initial estimate used is that of a prismatic joint, assuming \( \hat{d}(0) = 0 \). The initial estimate of the unconstrained direction of motion is 30° offset from the normal direction to the plane of the door or drawer. The initial estimates are shown as red arrows, and the true direction is shown as black arrows in Fig. 2. The angular values are the initial errors of the estimates. The controller gains are chosen as follows: \( \alpha_f = \alpha_\tau = 0.05, \beta_f = \beta_\tau = 0.005, \gamma = \gamma_d = 2000 \). The desired motion velocity is 5 cm/s, given as \( v_d = 5(1 - e^{-10t}) \) cm/s to avoid sharp initial transients.

![Fig. 2: The five different simulation cases. The angular measurement indicates the initial error. Note that all scenarios are initialized with the same estimate.](image)

In Fig. 3 - upper plot, the response of the motion axis estimation errors are shown; convergence to the actual axis is achieved even for big initial errors. Figure 3 - lower plot depicts the estimate of the inverse signed distance \( \hat{d} \) between the end-effector and the hinge; note that estimate \( \hat{d} \) is not modified when the estimate coincides with the actual parameter and it converges to its actual value in all cases. Combining estimates of the modulated rotation axis with the motion axis we can calculate the center of rotation of the rotational doors in real time; simulation gives errors approximately 1.4 cm after 1.5 sec, or opening the door 7.5 cm. Given the threshold of \( \| \hat{d} \| > 0.1 \), the revolute doors are identified as such after 0.2 s. Fig. 4 shows the Euclidian norms of force and torque errors \( e_f = \| P(\varphi_{\mathbf{x}_h})\hat{f} \| \) and \( e_\tau = \| \hat{\tau} \| \) respectively. Errors converge to zero following the convergence rate of modulated rotation axis and motion axis.

![Fig. 3: Upper figure: estimation error in the orientation of motion axis, lower figure: estimates of the inverse distance between hinge and end-effector. True values are 2 for cases (i) and (iii), -2 for case (ii), and 0 for cases (iv) and (v).](image)

![Fig. 4: Force and torque responses, upper figure: norm of the projected force error, lower figure: norm of the torque error.](image)

V. EXPERIMENTS

To further demonstrate the viability of the approach, we implemented the controller on a real velocity controlled robot platform (see [15], [16] for technical details of the robot), and let it open a cupboard door. See Figure 5 for an illustration of the setup. As in simulation, the initial estimate of the unconstrained direction of motion is 30° off of the true value. Estimate convergence is shown in Figure 6, and force and torque errors are shown in Figure 7. With the real robot, the arm was close to a singular configuration at \( t = 4 \) s, which gave a small temporary error in the estimate of \( \hat{d} \), but no significant instabilities.

VI. CONCLUSIONS

We propose a unified method for manipulating different types of revolute and prismatic mechanisms. The method
is model-free and it can be used to identify the type and the geometrical characteristics of one-joint mechanisms. By coupling estimation and action the method is inherently online and can be used in real-time applications. The method consists of a generalized velocity controller using estimates of the local motion direction, the axis of rotation and update laws for the estimated vectors. The design of the overall scheme guarantees compliant behavior and convergence of the estimated vectors to their actual values.

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Fig. 5 : A cupboard door opened by a robot.

Fig. 6 : Real robot trial. Upper plot: estimation error for motion axis, lower plot: estimated inverse distance between hinge and end-effector. True value (1.82) is marked with a line.

Fig. 7 : Real robot trial. Upper figure: norm of the projected force error, lower figure: norm of the torque error.