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Low-Speed Sensorless Control with Reduced Copper Losses for Saturated PMSynRel Machines

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Abstract—Permanent-magnet assisted synchronous reluctance (PMSynRel) machines are generally well suited for sensorless operation at all speeds since the rotor topology possesses a magnetic saliency. However, magnetic saturation can result in a vanishing differential saliency which renders sensorless control at certain operating points difficult (or even impossible) at low-speed. In this paper, an optimization procedure, based on results from finite-element (FEM) based simulations, is proposed. As output, current reference trajectories are obtained in which copper losses are kept at minimum while the capability for sensorless control is still maintained. The results from the FEM-based simulations are in good agreement with corresponding experimental results. For the experimental prototype in consideration, the torque limit when operating sensorless at low-speed is increased substantially from below 45% to around 95% of its rated value with only slightly increased copper losses. Additionally, the impact of position dependent harmonics on the magnetic cross saturation (affecting the steady-state position estimation error) are found to be substantial. This highlights that this spatial variation should be taken into consideration for accurate prediction of performance during sensorless operation even if the winding of the machine is of the conventional distributed type.

Index Terms—Electric drive, magnetic saturation, permanent-magnet assisted synchronous reluctance machine, position estimation, sensorless control, signal injection.

NOMENCLATURE

i_d, i_q	d - and q -axes current components.
v_d, v_q	d - and q -axes voltage components.
L'_d, L'_q	d - and q -axes differential inductances.
L'_{dq}, L'_{qd}	d - and q -axes differential cross inductances.
$\hat{P}$	Estimate of P .
$\tilde{P}$	Estimation error of P , $\tilde{P} = P - \hat{P}$.
T_e	Shaft torque.
V_c	Injected carrier signal amplitude.
$\mathbf{v}, \mathbf{i}$	Vector containing d - and q -axes voltage/current components.
θ	Rotor angular position.
ψ_d, ψ_q	d - and q -axes flux linkages.
ω_c	Injected carrier signal angular frequency.
$\Delta L'$	$\Delta L' = L'_q - L'_d$.
LPF	Low-pass filter.

I. INTRODUCTION

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PERMANENT-magnet assisted synchronous reluctance (PMSynRel) machines are attractive in automotive applications due to the high torque densities that can be reached with relatively little amount of expensive rare-earth permanent magnet material [1]–[3]. This fact has also been underlined by the significant increase in rare-earth metal prices during recent years [4].

The PMSynRel rotor topology possesses an inherent magnetic saliency which enables sensorless control (operation without using a position sensor) at all speeds. Low- and zero speed operation is achieved using carrier signal injection methods [5]–[8]. These methods (and more recent variants thereof) rely on tracking a rotor position dependent magnetic saliency which, unfortunately, can be significantly affected by magnetic saturation arising during loaded conditions [9]–[12]. However, these techniques have matured to a stage that they today can be found in certain industrial products available on the market [13].

Recently, attention has been put on analyzing different rotor topologies with respect to their capability for low-speed sensorless control as well as developing design methodologies to enable low-speed sensorless control without affecting other machine performance characteristics negatively [14]–[18]. This field of research is presently very active and it is the authors opinion that significant output can be expected during the coming few years.

In this paper, an existing PMSynRel design, developed to maximize the torque density without taking into consideration sensorless operation, is first analyzed using finite-element (FEM) based simulations together with corresponding experimental verifications. In order to minimize the copper losses, which represent the major part of the total machine losses at low speeds, the maximum torque-per-ampere (MTPA) current reference trajectory should be followed. However, it is found that operation along the MTPA trajectory is not possible during sensorless operation due to the fact that the effective magnetic saliency vanishes for larger torques. This significantly limits the available torque for low-speed sensorless operation with minimized copper losses (in line with the results in [19]) to below 45% of its rated value.

To increase the torque at low speeds while still maintaining a high efficiency, an optimization procedure is proposed in this paper which obtains a current reference trajectory in which copper losses are kept at minimum while still enabling stable sensorless control. Stable sensorless control is guaranteed since the amplitude of the error signal containing position information always has a predefined lower limit. For the PMSynRel design in consideration, the resulting current

reference trajectory results in a substantial torque increase (experimentally demonstrated) to around 95% of its rated value.

The experimental results presented in this paper are in good agreement with the results from the FEM-based simulations. However, it is also found that accurate prediction of the steady-state position estimation error, caused by cross-coupled inductances [10]–[12], requires that the computed flux linkages ψ_d and ψ_q are averaged for each rotor position. This highlights that this spatial variation should be taken into consideration for accurate prediction of performance during sensorless control even if the winding of the machine is a conventional distributed winding (as is the case for the PMSynRel machine in consideration in this paper). Finally, operation in the inverse saliency region, where the difference between the differential inductance along the d - and q -axes changes sign, is also analyzed and limits in terms of output torque and impact of spatial harmonics are investigated.

The paper is organized as follows. In Section II, the general theory of sensorless operation using pulsating carrier injection and its limitations are presented. Section III presents the proposed optimization procedure for determining suitable operating points for low-speed sensorless operation. Experimental results are compared with corresponding simulations in Section IV and conclusions are reported in Section V.

II. LOW-SPEED SENSORLESS CONTROL: THEORY AND OPERATING LIMITS

The geometry and main parameters of the PMSynRel machine used for experimental verification in this paper are shown in Fig. 1 and Table I, respectively. As seen in Fig. 1, the (rotor-fixed) dq -coordinate system is defined such that the d -axis is aligned with a magnetic north pole of the rotor. The PMSynRel machine in consideration is developed to operate in both traction and charging mode for a hybrid electric vehicle (HEV) application in [20]. The coil pitch 4/6 is selected so that third-order harmonics are eliminated, enabling the winding to be delta connected without additional losses. In this paper, however, the windings are Y-connected. The combined permanent-magnet and reluctance torque components enable a high torque density and a high efficiency.

Fig. 1. Cross section of the prototype PMSynRel machine.

TABLE I
MACHINE PARAMETERS

	Value	Unit
Winding type	Y-connection	-
Coil pitch	4/6	-
Number of slots	24	-
Number of phases	3	-
Number of pole pairs	2	-
Outer stator diameter	180	mm
Active length	210	mm
Rotor outer diameter	112	mm
Magnet material	NdFeB	-
Rated torque	114	Nm

A. Carrier Signal Injection

As is well known, by adding a high-frequency carrier voltage to the voltage references sent to the modulator, position information can be detected in the high-frequency carrier current provided that the machine possesses rotor saliency. In this paper, the pulsating voltage vector injection method proposed in [6] is considered. At low speeds, the back-electromotive force (EMF) is small in magnitude and the resistive voltage drop due to the resulting high frequency current is considerably smaller than the corresponding inductive voltage drop. Hence, the carrier current $\mathbf{i}_c$ and the carrier voltage $\mathbf{v}_c$ are related as

$$\mathbf{v}_c = \mathbf{L}_{dq} \frac{d\mathbf{i}_c}{dt} \quad (1)$$

where the content of the inductance matrix $\mathbf{L}_{dq}$ are detailed below. When the pulsating voltage vector injection method is applied, the carrier voltage can be expressed as $\hat{\mathbf{v}}_c = [V_c \cos \omega_c t \ 0]^T$. The resulting current in the *estimated* rotor reference frame $\hat{\mathbf{i}}_c = [\hat{i}_{cd} \ \hat{i}_{cq}]^T$ is then found as

$$\hat{\mathbf{i}}_c = \int \mathbf{T}(\tilde{\theta}) \mathbf{L}_{dq}^{-1} \mathbf{T}(\tilde{\theta})^{-1} \hat{\mathbf{v}}_c dt \quad (2)$$

where

$$\mathbf{T}(\tilde{\theta}) = \begin{bmatrix} \cos \tilde{\theta} & -\sin \tilde{\theta} \\ \sin \tilde{\theta} & \cos \tilde{\theta} \end{bmatrix} \quad (3)$$

and $\tilde{\theta} = \theta - \hat{\theta}$ is the rotor position estimation error. In general, the inductance matrix $\mathbf{L}_{dq}$ can be expressed as

$$\mathbf{L}_{dq} = \begin{bmatrix} L'_d & L'_{dq} \\ L'_{qd} & L'_q \end{bmatrix} \quad (4)$$

where

$$L'_d = \frac{\partial \psi_d(i_d, i_q)}{\partial i_d}, \quad L'_q = \frac{\partial \psi_q(i_d, i_q)}{\partial i_q} \quad (5a)$$

$$L'_{dq} = \frac{\partial \psi_d(i_d, i_q)}{\partial i_q}, \quad L'_{qd} = \frac{\partial \psi_q(i_d, i_q)}{\partial i_d}. \quad (5b)$$

If the iron loss is disregarded, the condition $L'_{qd} = L'_{dq}$ holds [21]. Based on this, from (2)–(5b), the resulting current vector in the estimated rotor reference frame can be expressed as

$$\hat{\mathbf{i}}_c = \frac{V_c \sin \omega_c t}{2\omega_c} \begin{bmatrix} 2(L'_d \sin^2 \tilde{\theta} + L'_q \cos^2 \tilde{\theta} + L'_{dq} \sin 2\tilde{\theta}) \\ \frac{L'_d L'_q - L'^2_{dq}}{\Delta L' \sin 2\tilde{\theta} - 2L'_{dq} \cos 2\tilde{\theta}} \\ \frac{L'_d L'_q - L'^2_{dq}}{\Delta L' \sin 2\tilde{\theta} - 2L'_{dq} \cos 2\tilde{\theta}} \end{bmatrix}. \quad (6)$$

Now, the error signal ε is formed by demodulating and low-pass filtering $\hat{i}_{cq}$ as

$$\varepsilon = \text{LPF} \left\{ \hat{i}_{cq} \sin \omega_c t \right\}. \quad (7)$$

From (6), ε is found as

$$\begin{aligned} \varepsilon &= \frac{V_c}{4\omega_c} \frac{\Delta L' \sin 2\tilde{\theta} - 2L'_{dq} \cos 2\tilde{\theta}}{L'_d L'_q - L'^2_{dq}} \\ &= \frac{V_c}{4\omega_c} \frac{\sqrt{\Delta L'^2 + 4L'^2_{dq}} \sin(2\tilde{\theta} - \varphi)}{L'_d L'_q - L'^2_{dq}} \end{aligned} \quad (8)$$

where $\varphi = \cos^{-1} \left[\Delta L' / \sqrt{\Delta L'^2 + 4L'^2_{dq}} \right]$ is the phase shift induced by cross-saturation (i.e., $L'_{dq} \neq 0$). The position estimate is now updated, e.g. using the phase-locked loop type estimator in [22], so that the error signal ε is forced to zero. Letting $\varepsilon \rightarrow 0$, it is evident from (8) that $\varphi \neq 0$ introduces a steady-state position estimation error $\tilde{\theta}^* = \varphi/2$ that must be compensated for. The compensation method proposed in [23] is adopted in this work. Assuming a small estimation error $\tilde{\theta}$ yields $\sin 2\tilde{\theta} \approx 2\tilde{\theta}$ and $\cos 2\tilde{\theta} \approx 1$. Eq. (8) can then be approximated as

$$\varepsilon \approx \frac{V_c}{2\omega_c} \frac{\Delta L' \tilde{\theta} - L'_{dq}}{L'_d L'_q - L'^2_{dq}} = \frac{V_c}{2\omega_c} \frac{\Delta L' \tilde{\theta}}{L'_d L'_q - L'^2_{dq}} - K_c \quad (9)$$

where $K_c = V_c L'_{dq} / (2\omega_c (L'_d L'_q - L'^2_{dq}))$ which is constant for a certain operating point and can be identified experimentally. A new error signal $\varepsilon' = \varepsilon + K_c$ can then be used in the PLL to compensate for the cross saturation effect. This method is implemented in this work to realize the closed-loop sensorless control.

B. Operating Limits

The torque map in the current dq -plane (obtained using FEM-based simulations) of the prototype PMSynRel machine is shown in Fig. 2. In this work, 336 operating points were simulated taking approximately 6 hours using JMAG-Studio ver. 10.0 on a dual core 3.3 GHz processor with 8 Gb memory. The trajectory corresponding to $\Delta L' = 0$ is represented as a solid line in Fig. 2. The maximum torque-per-ampere (MTPA) current trajectory is illustrated with circles in the figure.

1) *Mapping the Feasible Region:* The feasible region is defined in [24] as the region, in terms of i_d and i_q , where $\Delta L' > 0$. It is evident that the PMSynRel in consideration cannot be operated sensorless along the entire MTPA trajectory since the effective saliency, due to saturation, becomes too small and even reverses sign for large torques. Experimental evaluation of low-speed sensorless control following the MTPA trajectory is only possible up to around 50 Nm which is only 44% of the rated torque and it is in line with the results depicted in Fig. 2.

From (8), it can be seen that the mutual differential inductance L'_{dq} not only introduces a phase shift in the error signal ε but it also modulates its amplitude. Hence, mapping the feasible region using solely ε is not straight forward since

Fig. 2. Constant torque loci (FEM-based simulation). The solid line represents the $\Delta L' = 0$ condition, the dashed line represents the $L'_{dq} = 0$ condition and the circles (o) represent the MTPA trajectory.

the amplitude of ε is not always zero along the $\Delta L' = 0$ trajectory. The term K_e , given by

$$K_e = \frac{V_c}{4\omega_c} \frac{\sqrt{\Delta L'^2 + 4L'^2_{dq}}}{L'_d L'_q - L'^2_{dq}} \text{sign}(\Delta L') \quad (10)$$

is introduced in [25] to map the feasible region without the knowledge of the differential inductances. Note that the *absolute value* of K_e equals the amplitude of the position error signal ε in (8) and that the sign of K_e equals the sign of $\Delta L'$. By computing L'_q , L'_d and L'_{dq} using FEM-based simulations, K_e can be obtained for different operating points. It is shown in [25] that K_e can be extracted from the measured error signal ε since the amplitude of ε decreases along the q -axis direction as $\Delta L'$ is decreasing. For a certain value of i_q , the magnitude of ε reaches its minimum value (when $\Delta L' = 0$) and then starts to increase again (when $\Delta L'$ changes sign). The FEM-based simulation results, based on (10), and the corresponding experimental results are shown in Fig. 3. As seen, a reasonably good agreement between the experimental results and the FEM-based simulation is obtained.

Remark: The FEM-based simulations were carried out for a fixed rotor position corresponding to $\theta = 0^\circ$. From the results shown in Fig. 3, it is evident that K_e can be obtained with reasonable accuracy. However, limitations when adopting this simplified modeling approach will be highlighted in Section IV-B.

2) *Impact of Noise and Harmonics:* For operating points where the differential saliency becomes very low, the noise-to-signal ratio in ε can render the sensorless control unstable [26]. In [26], the *sensorless safety operation area* (SSOA) is introduced to quantify this effect. Additionally, the work in [18] highlights that spatial harmonics in the error signal can also have a substantial impact on the performance when operating sensorless at low speeds.

III. STABLE LOW-SPEED SENSORLESS CONTROL WITH REDUCED COPPER LOSSES

From the discussion above, it is evident that to obtain accurate position estimates in the low-speed range, the magnitude of K_e cannot be too small in any operating point. At the same time, the operating points, in terms of i_d and i_q , should be selected so that the resistive losses, which dominates the total machine losses at low speeds, are as small as possible. In order to obtain both of these targets, an optimization procedure, using results from a set of FEM-based simulations, is presented below.

The proposed optimization procedure is illustrated in Fig. 4 as a flowchart where the torque reference T_e^{ref} is input. A number of FEM-based simulations are first carried out in which the flux linkages ψ_d ψ_q are computed and stored for each operating point (set of i_d and i_q). In Step 1 of the optimization procedure, all operating points fulfilling the given torque reference are obtained. In Step 2, K_e is computed for all selected operating points (using (10)). To guarantee a sufficient signal-to-noise ratio, the operating points fulfilling

$K_e > K_{e,\min}$ are selected in Step 3. In the final step (Step 4), the resulting operating point is obtained by selecting the combination of i_d and i_q that minimizes the copper losses while, at the same time, fulfills all of the above conditions.

Fig. 4. Flowchart of the proposed optimization procedure.

Fig. 3. Contour surfaces of K_e together with the MTPA trajectory represented as black circles: (a) FEM-based simulation; (b) Experimental result (the black dots represent the measured operating points) [25].

IV. EXPERIMENTAL EVALUATION

In this section, experimental results are presented verifying the validity of the proposed optimization procedure for selecting suitable operating points when operating sensorless at low speeds.

A. Experimental Setup

The PMSynRel machine described in Section II is kept rotating at constant speed corresponding to an electrical frequency of 10 Hz. The converter is operating using pulse width modulation with a switching frequency of 5 kHz. The bandwidth of the current controller is selected to 80 Hz. Since the parameters of the machine is changing with each operating point, the parameters of the current controller are selected following the method outlined in [27]. The actual rotor position θ is measured using a resolver mounted on the rotor shaft.

With access to θ , the position estimation error $\tilde{\theta}$ is varied from 0° to 180° during 1 s. Now,

$$\mathbf{v}_h = V_c \cos \omega_c t \begin{bmatrix} \cos \tilde{\theta} & -\sin \tilde{\theta} \end{bmatrix}^T \quad (11)$$

is added on top of the fundamental excitation ($V_c = 115$ V, $f_c = \omega_c / (2\pi) = 500$ Hz). The currents in dq -reference frame are obtained and $\hat{i}_{cq}$ is then computed as

$$\hat{i}_{cq} = i_d \sin \tilde{\theta} + i_q \cos \tilde{\theta}. \quad (12)$$

The error signal ε is now obtained according to (7). The fundamental component of ε is then computed and K_e and φ are identified. A sample experimental results is reported in Fig. 5 where the phase shift φ is evident. To capture such relatively small error signals, current transducers with high accuracy are used incorporate with 16-bit analog-to-digital (A/D) converters. To verify the tests, all measurements were repeated and same results were obtained.

Fig. 5. Sample experimental result to obtain K_e and φ from ε (solid line). The dashed line represents the fundamental component of ε .

B. Optimal Operating Points in the Feasible Region

1) *Optimization Procedure:* A FEM-based simulation was first carried out in where the flux linkages ψ_d and ψ_q were computed for different values of i_d and i_q while keeping the rotor position at $\theta = 0^\circ$. The obtained values for ψ_d and ψ_q were then used to compute the differential inductance L'_d , L'_q and L'_{dq} . Finally, the optimization procedure proposed in Section III was applied for $K_{e,\min} = 0.1$ A, $K_{e,\min} = 0.3$ A, and $K_{e,\min} = 0.6$ A. The resulting operating points for different torque levels are reported in Fig. 6. As seen, for small torque levels, the MTPA current trajectory is followed. For larger torques, the trajectory is diverging from the MTPA trajectory in order to maintain K_e above its specified level.

Fig. 6. Optimized operating trajectories for different values of $K_{e,\min}$. The bold and dashed lines refer to $\Delta L' = 0$ and $L'_{dq} = 0$, respectively. The circles (o) refer to the MTPA trajectory. Asterisks (*), square (□) and diamond (◇) refer to $K_{e,\min} = 0.1$ A, $K_{e,\min} = 0.3$ A, and $K_{e,\min} = 0.6$ A, respectively.

2) *Corresponding Experimental Tests:* Experimental tests were now carried out to verify the results above. The results are reported in Fig. 7–8 which show measured and predicted values of K_e and φ for different torque levels along the three trajectories obtained. For the predicted values of K_e and φ , two sets of FEM-based simulations has been used. In the first set, discussed above, the FEM-based simulations were carried

out for a single rotor position only. In the second set, ψ_d and ψ_q are computed for a complete electric revolution (180 angular steps are performed to obtain a reasonable angular resolution) and their average values are used to compute L'_d , L'_q and L'_{dq} . As seen in Fig. 7, a reasonable agreement in K_e between the experiments and the two FEM-based simulations is obtained. The small offset can be due to the strong saturation of the rotor bridges used to fixate the magnets in the rotor [15]; an effect which is difficult to model exactly in a FEM-based simulation without access to BH-curves for the electrical sheets up to very high values of H (often not supplied by the steel sheet manufacturer). Additionally, the effect of the punching or laser cutting of the laminations locally affects the magnetic properties of the sheets, an effect which has not been included in the FEM-based simulation model used in this paper.

In Fig. 8, measured and predicted values of the phase shift φ are shown. Here, however, a substantial deviation between the experimental results and the FEM-based simulation taking only a single rotor position into account is obtained. The phase shift φ is due to cross saturation as outlined in Section II-A. Hence, although the winding of the PMSynRel in consideration is of the conventional distributed type, the impact of spatial harmonics have to be taken into consideration when evaluating the impact of magnetic cross saturation on low-speed sensorless control performance.

Fig. 7. Measured and predicted values of K_e for different torque levels. Circles (o) refer to FEM-based predictions evaluated for a single rotor position only, asterisks (*) refer to FEM-based predictions averaged over a complete electrical evolution and the boxes (□) refer to the experimental results: (a) $K_{e,\min} = 0.1$ A; (b) $K_{e,\min} = 0.3$ A; (c) $K_{e,\min} = 0.6$ A.

3) *Closed-Loop Sensorless Control:* To experimentally demonstrate the effectiveness of the proposed approach, the

Fig. 8. Measured and predicted values of φ for different torque levels. Circles (o) refer to FEM-based predictions evaluated for a single rotor position only, asterisks (*) refer to FEM-based predictions averaged over a complete electrical evolution and the boxes (□) refer to the experimental results: (a) $K_{e,\min} = 0.1$ A; (b) $K_{e,\min} = 0.3$ A; (c) $K_{e,\min} = 0.6$ A.

closed-loop sensorless control algorithm was also implemented. When the machine was operated following the current trajectory corresponding to $K_{e,\min} = 0.1$ A, the control system failed due to the small signal-to-noise ratio when $T_e^{\text{ref}} \geq 60$ Nm. For $K_{e,\min} = 0.3$ A and $K_{e,\min} = 0.6$ A, however, stable sensorless was achieved for the proposed current trajectories. Fig. 9 shows a sample experimental result for $K_{e,\min} = 0.3$ A. As seen, a high frequency torque ripple can be found in Fig. 9. The ripple is oscillating at around 180 Hz, independent of the rotor speed, and it is likely attributed to the mechanical resonance frequency due to the finite stiffness of the torque transducer and the associated mechanical coupling. The achieved maximum torque when operating sensorless is 110 Nm which is more than twice the value obtained when following the MTPA trajectory. Compared to the MTPA trajectory at 110 Nm, the copper losses are increased with around 8% which must be considered as a moderate increase considering that the operating point is selected only in the low speed range. As the speed increases, a transition to the MTPA trajectory can be implemented with ease.

C. Optimal Operating Points in the Unfeasible Region

The *unfeasible region* is defined in [24] as the region, in terms of i_d and i_q , where $\Delta L' < 0$. When initiating operation from zero rotor speed and $i_d = i_q = 0$, the unfeasible region cannot be reached easily due to the vanishing saliency.

Fig. 9. Sample experimental results when operating with $K_{e,\min} = 0.3$ A: (a) Measured torque (solid line) and torque reference (dashed line); (b) Measured position estimation error.

However, at higher speeds, where back-EMF based sensorless algorithms can be utilized, the vanishing saliency does not pose any limitation and the unfeasible region could potentially be exploited when the rotor speed decreases. Another way of entering the unfeasible region can be to initiate sensorless operation from zero speed using sensorless algorithms not relying on signal injection but which can still handle an initial startup procedure, e.g., [28], [29].

1) *Optimization Procedure:* Finding optimized operating points in the unfeasible region can be determined by relaxing Step 3 in the flowchart in Fig. 4 so that $|K_e(i_d, i_q)| \geq K_{e,\min}$ must hold. Resulting operating points for different torque levels are reported in Fig. 10. As can be seen, to minimize the copper losses, some of the operating points are located in the unfeasible region.

2) *Corresponding Experimental Tests:* Measured and predicted values of K_e and φ are shown in Fig. 11 and Fig. 12, respectively. Regarding the FEM-based predictions, only results based on average values for a complete electric revolution are shown. As seen, a good agreement with the FEM-based prediction and the measurements is obtained. In Fig. 12, some operating points, even with high $|K_{e,\min}|$, possess a significant phase shift in the unfeasible region. This phenomenon can be explained by studying (8) and has previously been reported in [30]; the error signal is not only amplitude-modulated but also rotated by the mutual differential inductance L'_{dq} .

3) *Closed-Loop Sensorless Control in the Unfeasible Region:* Fig. 13 (a) shows an experimentally obtained position estimation error when the PMSynRel is operating in closed-loop sensorless control at 80 Nm with $i_d = -23$ A and $i_q = 26$ A (i.e., operation in the unfeasible region with $K_e = -0.3$). The measured error signal for the operating point is shown in Fig. 13 (b). As seen, the magnitude of the spatial harmonics is substantial (compare with Fig. 5)

Fig. 10. Optimized operating trajectories for different values of $K_{e,\min}$ when $|K_e(i_d, i_q)| \geq K_{e,\min}$ must hold. The bold and dashed lines refer to $\Delta L' = 0$ and $L_{dq}' = 0$, respectively. The circles ($\circ$) refer to the MTPA trajectory. Asterisks (*), square ($\square$) and diamond ($\diamond$) refer to $K_{e,\min} = 0.1$ A, $K_{e,\min} = 0.3$ A, and $K_{e,\min} = 0.6$ A, respectively.

Fig. 11. Measured and predicted values of K_e for different torque levels. Asterisks (*) refer to FEM-based predictions and the boxes ($\square$) refer to the experimental results: (a) $|K_{e,\min}| = 0.1$ A; (b) $|K_{e,\min}| = 0.3$ A; (c) $|K_{e,\min}| = 0.6$ A.

which easily can render the system unstable. Hence, it is concluded that the impact of spatial harmonics is substantial when operating in the unfeasible region for the PMSynRel machine in consideration. An interesting topic for further research is to determine which design measures are most suitable in order to enable sensorless control at all possible

Fig. 12. Measured and predicted values of φ for different torque levels. Asterisks (*) refer to FEM-based predictions and the boxes ($\square$) refer to the experimental results: (a) $|K_{e,\min}| = 0.1$ A; (b) $|K_{e,\min}| = 0.3$ A; (c) $|K_{e,\min}| = 0.6$ A.

operating points without limiting the output torque.

Fig. 13. Sample experimental result when operating with $|K_{e,\min}| = 0.3$ A (70 Nm, $i_d = -23$ A, $i_q = 26$ A): (a) Measured position estimation error; (b) Measured error signal ε at the given operating point.

V. CONCLUSION

In this paper, an optimization procedure, based on results from finite-element (FEM) based simulations, was proposed in order to compute current reference trajectories in which

copper losses are kept at minimum while the capability for low-speed sensorless control is still maintained. The results from the FEM-based simulations were in good agreement with corresponding experimental results. For the experimental prototype in consideration, the torque limit when operating sensorless at low-speed was increased substantially from below 45% when operating along the MTPA current reference trajectory to around 95% of its rated value with only slightly increased copper losses. Additionally, the impact of position dependent harmonics on the magnetic cross saturation affecting the steady-state position estimation error were found to be substantial. This highlights that this spatial variation should be taken into consideration for accurate prediction of performance during sensorless operation even if the winding of the machine is of the conventional distributed type. An interesting topic for further research is to extend the work in [14]–[18] in order to develop more general guidelines for how low-speed sensorless control can be enabled without affecting other machine performance characteristics negatively.

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